Generative AI for Brane Configurations and Gauge Theories

Rak-Kyeong Seong

Quantum100 x AI Workshop - Universität Münster
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rakkyeongseong.github.io
Mathematical Physics and AI Lab

National Research Foundation of Korea

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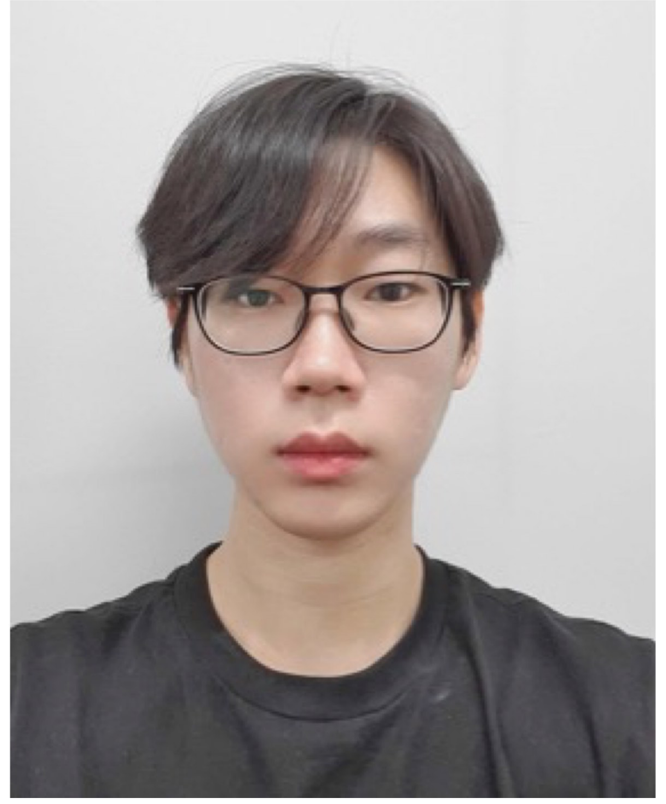


Eugene Choi

Master/PhD Combined (Year 3)

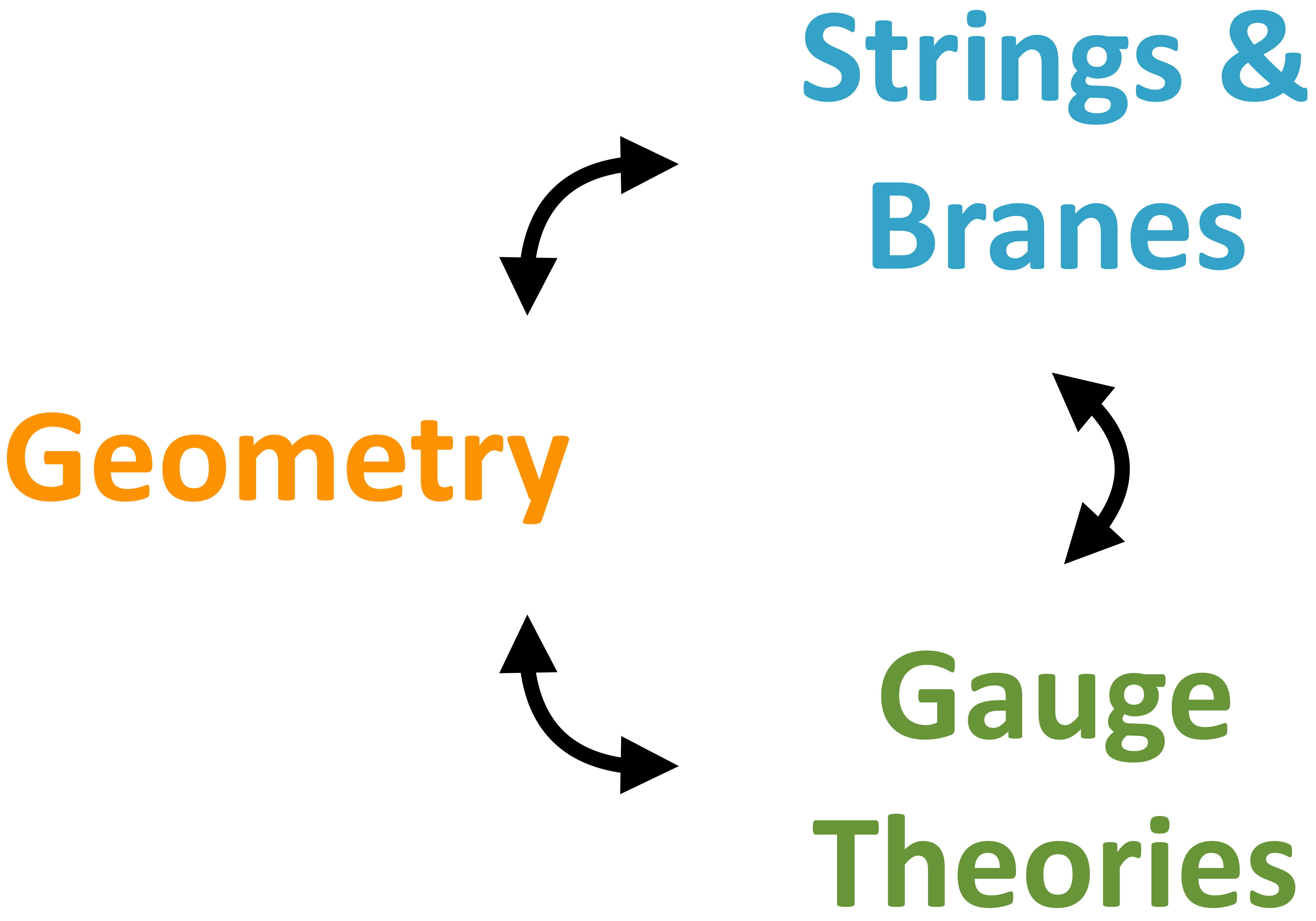
Juno Kwon

Master/PhD Combined (Year 1)

Geometry

Strings & Branes

Gauge Theories


Strings &
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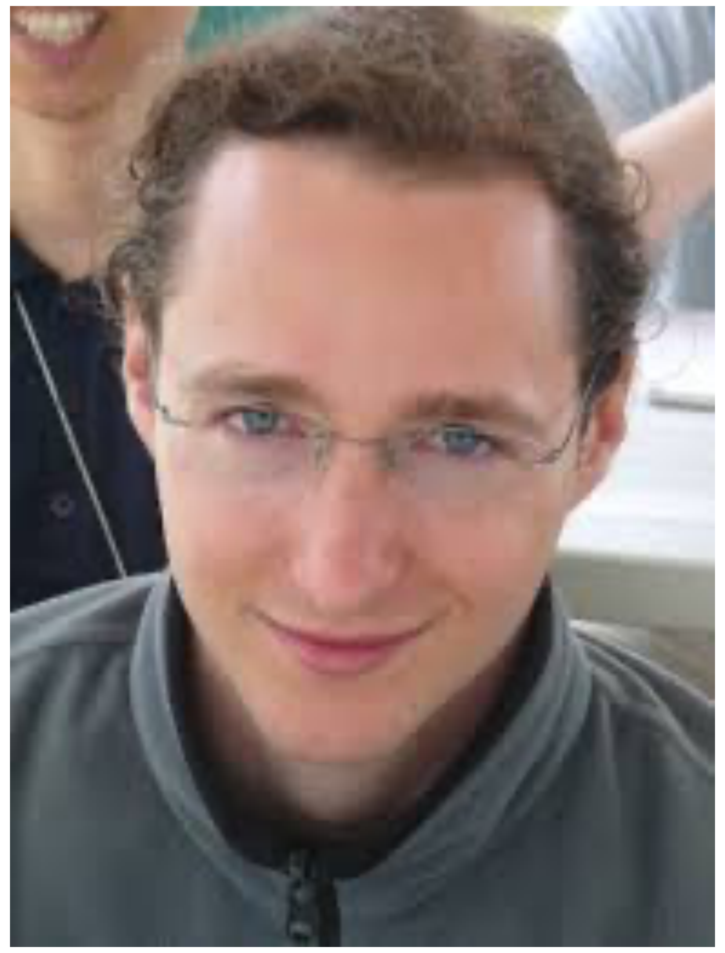
2017



Daniel Krefl

University of Geneva

PHYSICAL REVIEW D **96**, 066014 (2017)

Machine learning of Calabi-Yau volumes

Daniel Krefl[1] and Rak-Kyeong Seong[2]

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We employ machine learning techniques to investigate the volume minimum of Sasaki-Einstein base manifolds of noncompact toric Calabi-Yau three-folds. We find that the minimum volume can be approximated via a second-order multiple linear regression on standard topological quantities obtained from the corresponding toric diagram. The approximation improves further after invoking a convolutional neural network with the full toric diagram of the Calabi-Yau three-folds as the input. We are thereby able to circumvent any minimization procedure that was previously necessary and find an explicit mapping between the minimum volume and the topological quantities of the toric diagram. Under the AdS/CFT correspondence, the minimum volumes of Sasaki-Einstein manifolds correspond to central charges of a class of $4d$ $\mathcal{N} = 1$ superconformal field theories. We therefore find empirical evidence for a function that gives values of central charges without the usual extremization procedure.

DOI: 10.1103/PhysRevD.96.066014

I. INTRODUCTION

In recent years, machine learning has become a cornerstone for many fields of science, and it has been adopted more and more as a valuable toolbox. Machine learning has attracted much interest due to significant theoretical progress and due to increased availability of large amounts of data, computing power (GPUs) and easy to use software implementations of standard machine learning techniques.

Despite these developments, applications of machine learning techniques to mathematical physics have been limited to our knowledge. One of the reasons for this is that machine learning aims to empirically approximate the underlying probability density function of a given data set. Making use of machine learning to identify hidden structures in data sets, which teach us about new phenomena in string theory and mathematics, has not been systematically considered before.

This work aims to change the status quo and to provide

worldvolume of a stack of D3-branes probing toric Calabi-Yau three-folds, characterized by convex lattice polygons known as toric diagrams [1–4]. These theories are expected to flow at low energies to a superconformal fixed point.

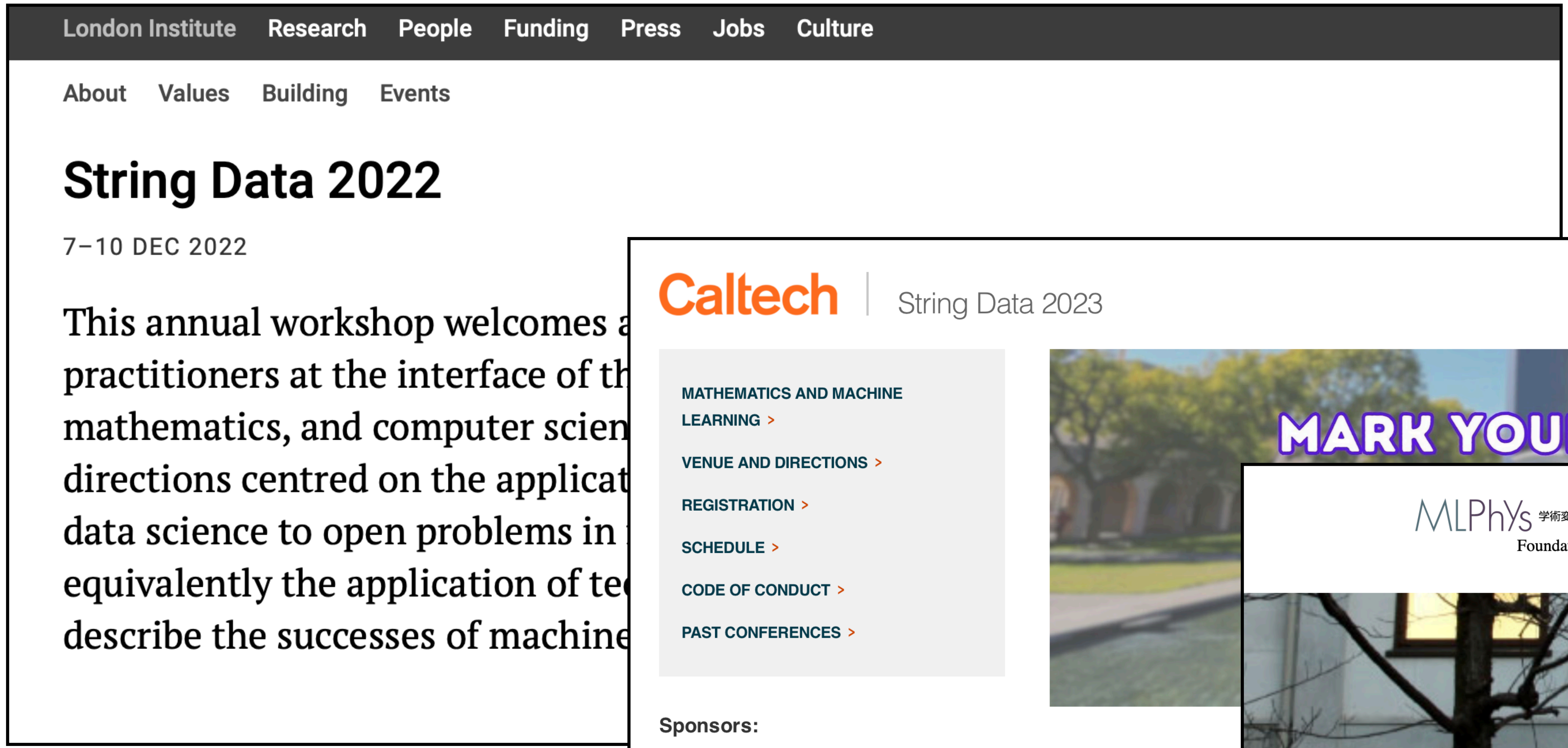
From a machine learning perspective, this work studies the minimum volumes of Sasaki-Einstein five-manifolds. These are the base manifolds of the probed toric Calabi-Yau three-folds. The minimum volume is of particular interest because under the AdS/CFT correspondence, it is expected to be related to the maximized a-function that gives the central charge of the $4d$ $\mathcal{N} = 1$ superconformal field theory [5–9].

Using a large data set of toric Calabi-Yau three-folds, our aim is to train a machine learning model in such a way that it approximates a functional relationship between topological quantities of the toric Calabi-Yau three-fold and the minimum volume of the Sasaki-Einstein base manifold. Such a functional relation would be of great use because it

Krefl, Seong

Phys. Rev. D **96**, 066014 (2017)

[arXiv:1706.03346]



String Data 2022
University of Cambridge (UK)

String Data 2023
Caltech (USA)

String Data 2024
Kyoto University (Japan)

QUESTION

Can machine learning tell us about **critical phenomena** in physics (and mathematics)?

Critical Phenomena in Physics

- Phase Transitions

  Critical phase transitions are one of the most fascinating physical phenomena in nature.

water (liquid)

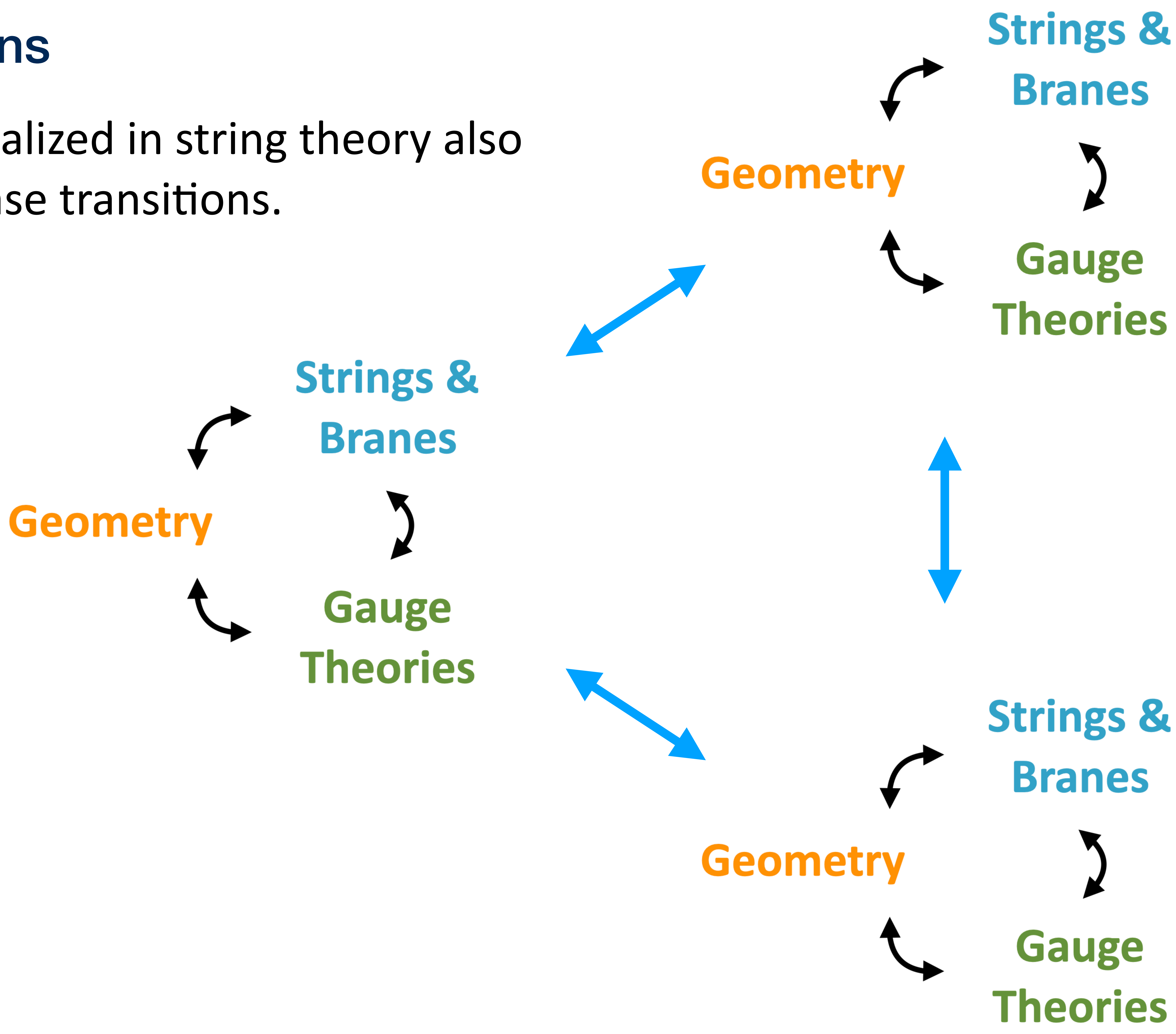
ice (solid)

vapour (gas)

Critical Phenomena in Physics

- Phase Transitions

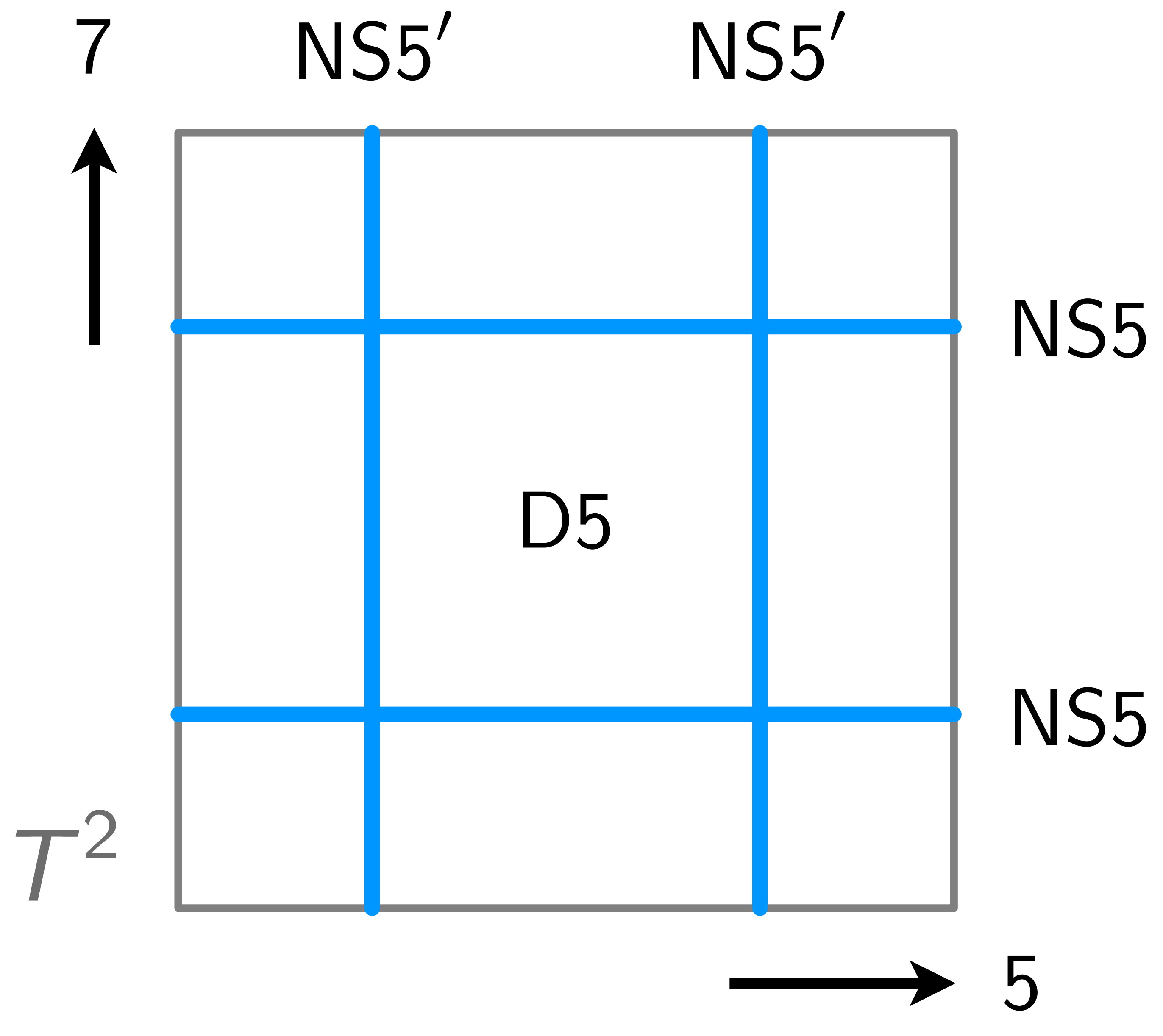
  Gauge theories realized in string theory also exhibit critical phase transitions.



Brane Engineering of Gauge Theories

Brane Boxes and Brane Tilings

Brane boxes consisting of D5-branes and NS5-branes realize **4d N=1 supersymmetric gauge theories**.



5,7-directions in 10d

brane configuration in 10d string theory

	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	·	×	·	×	·	·
NS5	×	×	×	×	×	×	·	·	·	·
NS5′	×	×	×	×	·	·	×	×	·	·

Brane Engineering of Gauge Theories

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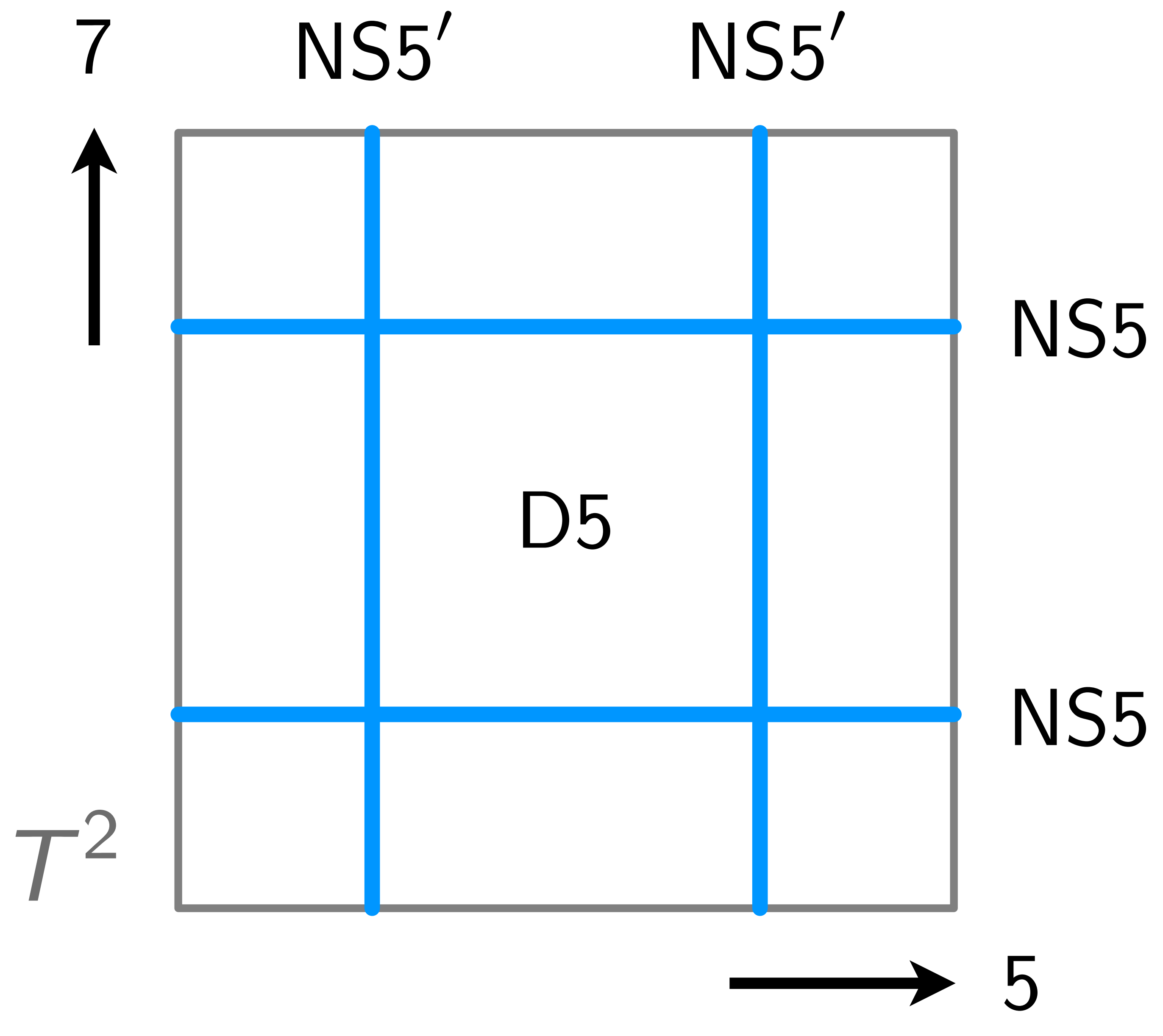

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NS5′	×	×	×	×	·	·	×	×	·	·

4d $\mathcal{N}=1$ gauge theory

Brane Engineering of Gauge Theories

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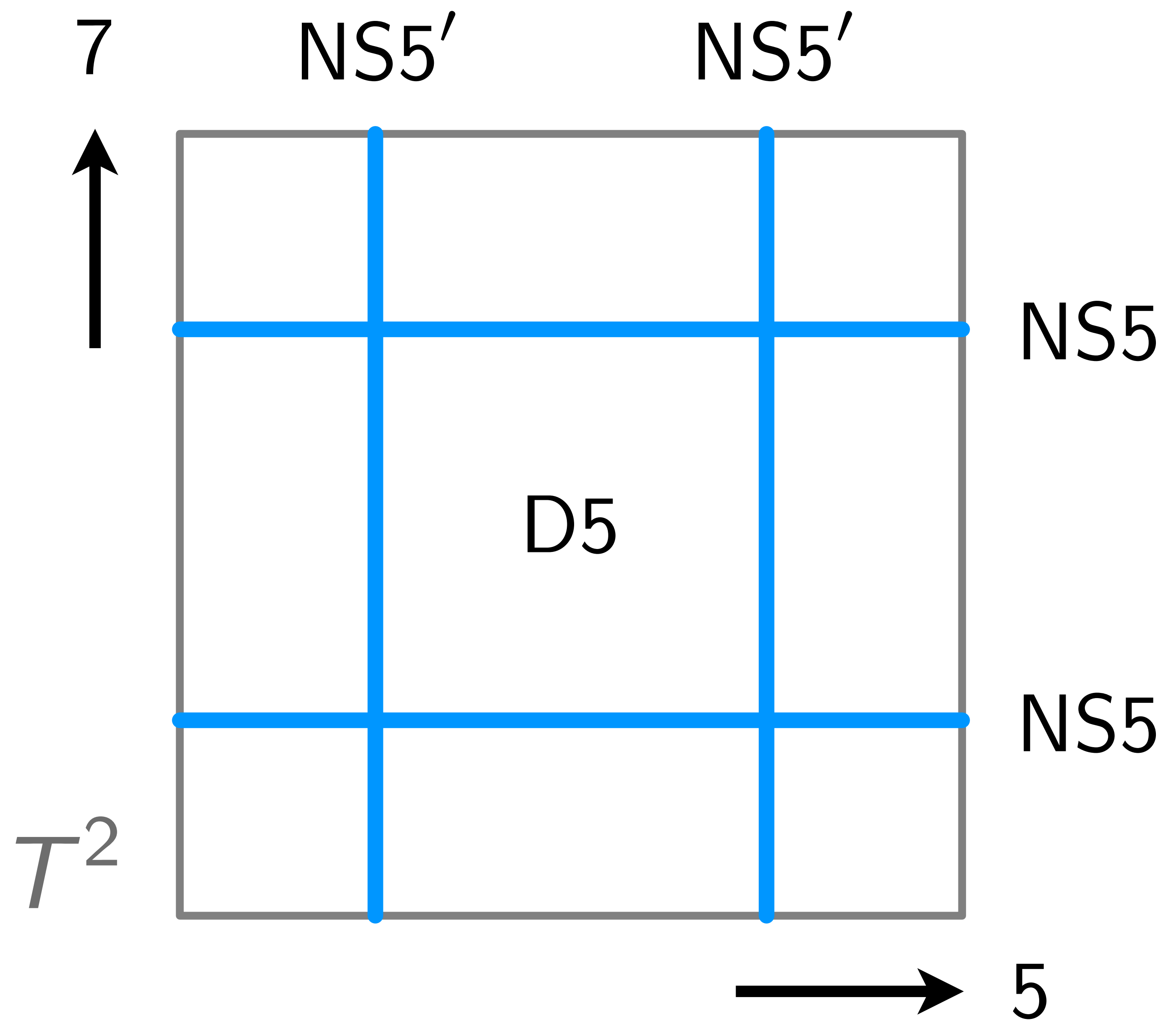

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NS5′	×	×	×	×	·	·	×	×	·	·

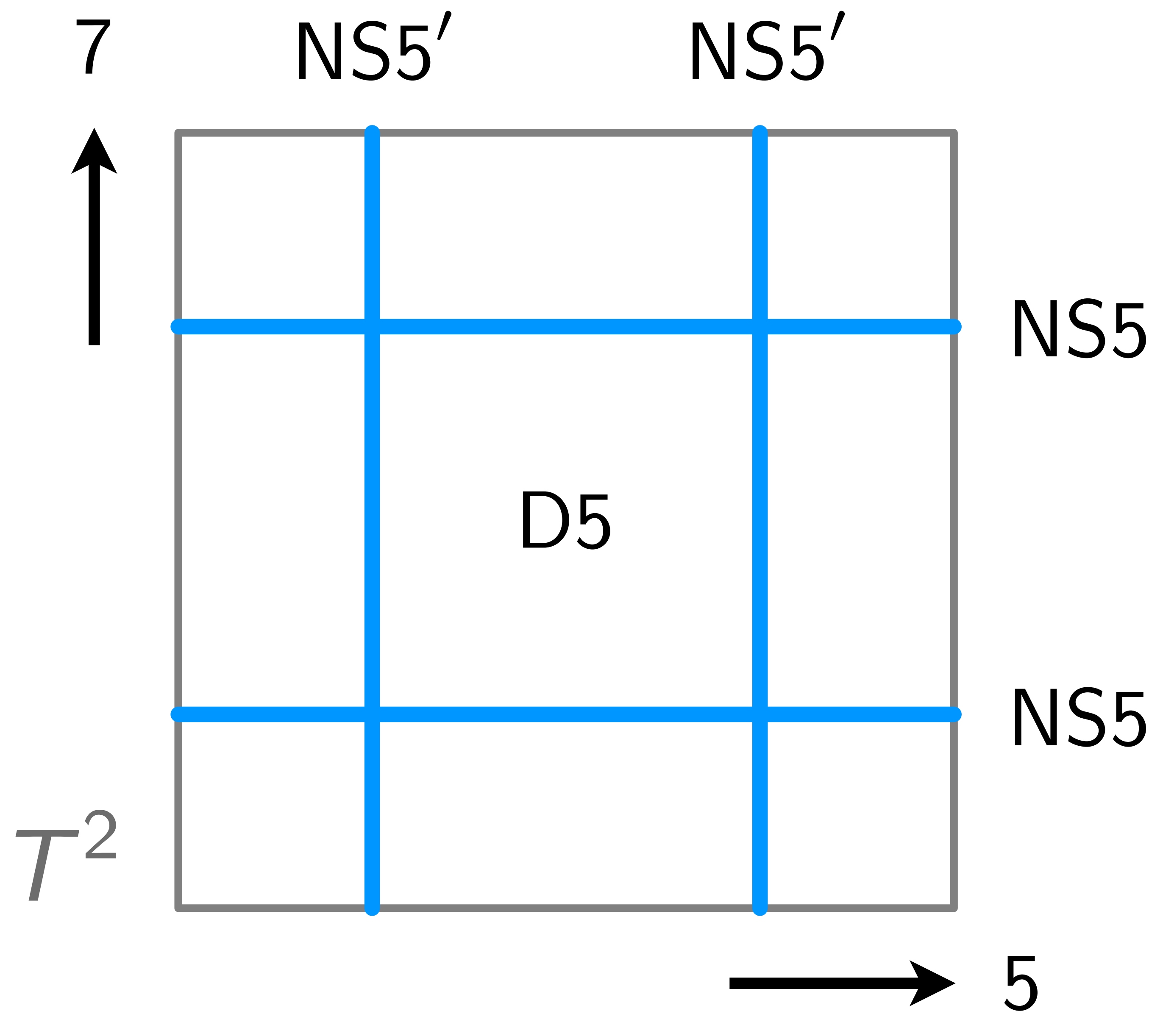
2-torus T^2

$(x^5, x^7) \in T^2$

Brane Engineering of Gauge Theories

Brane Boxes and Brane Tilings

We can combine the NS5-branes to a single NS5-brane wrapping a **holomorphic curve**.



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combine into a single NS5-brane

Brane Engineering of Gauge Theories

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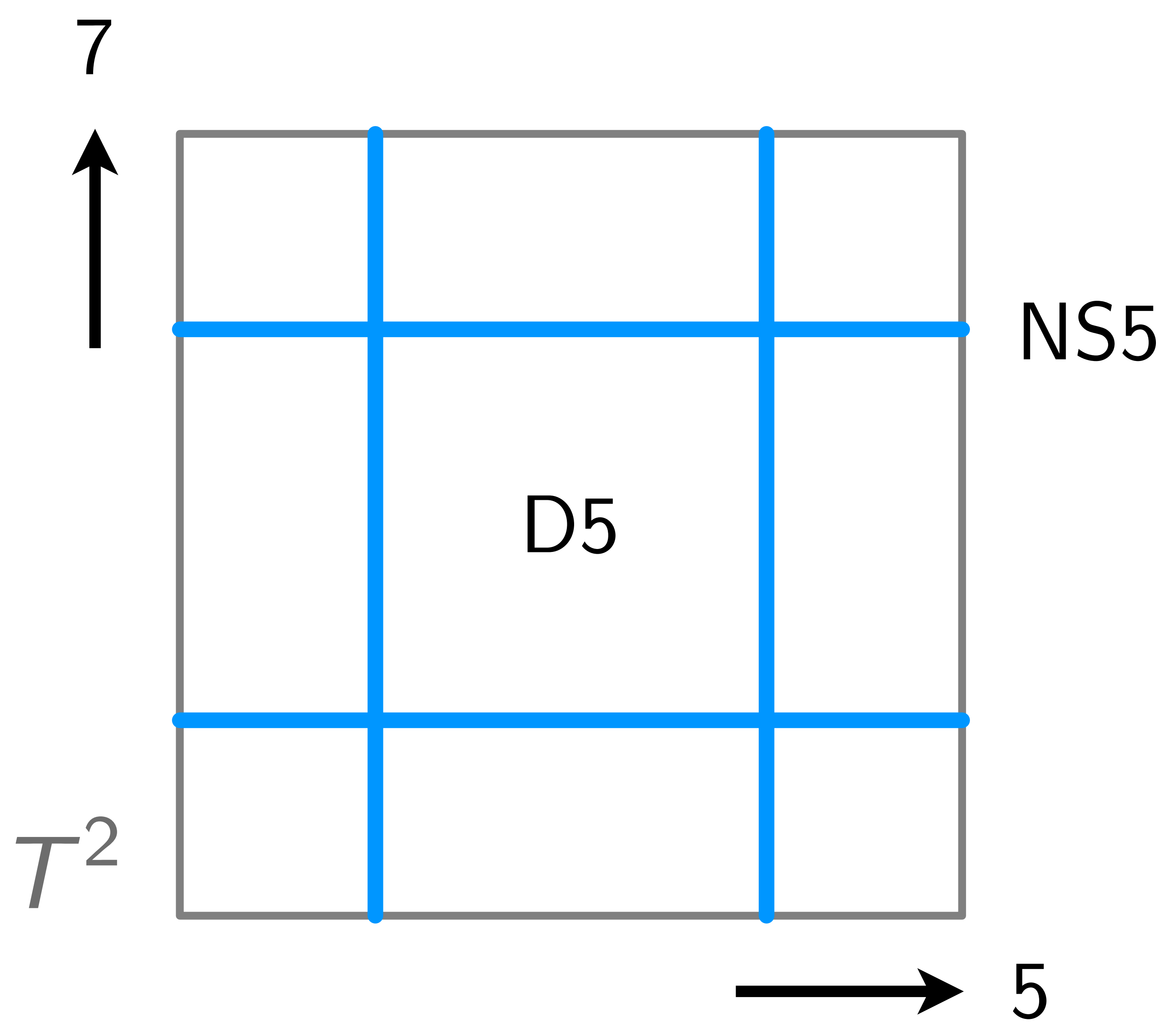


5,7-directions in 10d

brane configuration in 10d string theory

	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	·	×	·	×	·	·
NS5	×	×	×	×	$-\Sigma(x,y)-$				·	·

$$x \equiv x^4 + ix^5$$
$$y \equiv x^6 + ix^7$$

$x, y \in \mathbb{C}^*$

holomorphic curve $\Sigma(x,y)$: $P(x,y) = 0$

(characteristic polynomial $P(x,y)$)

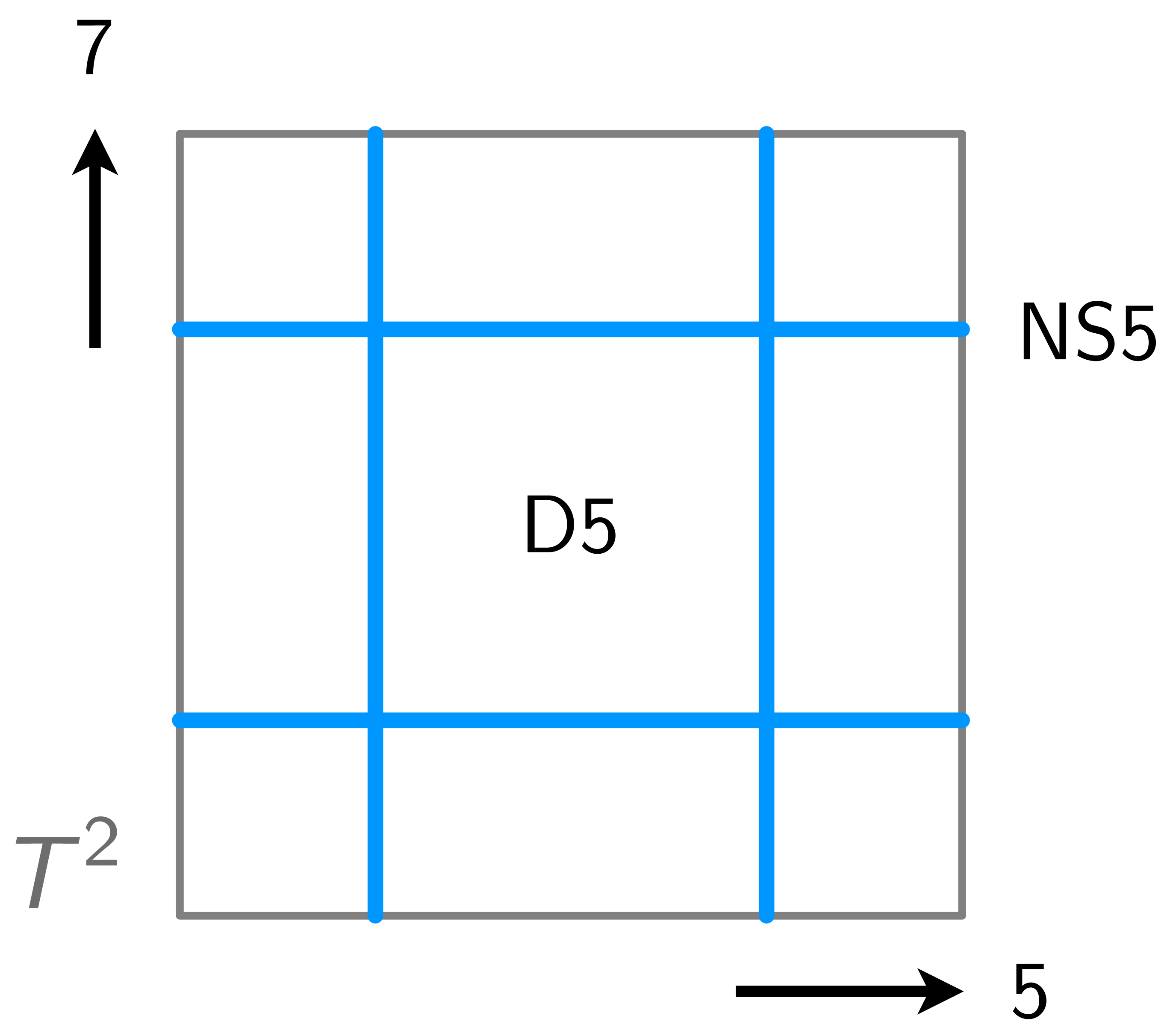
mirror curve of a toric Calabi-Yau 3-fold

Brane Engineering of Gauge Theories

- Brane Boxes and Brane Tilings

Different Calabi-Yau mirror curves give **different brane configurations** realizing different 4d supersymmetric gauge theories.

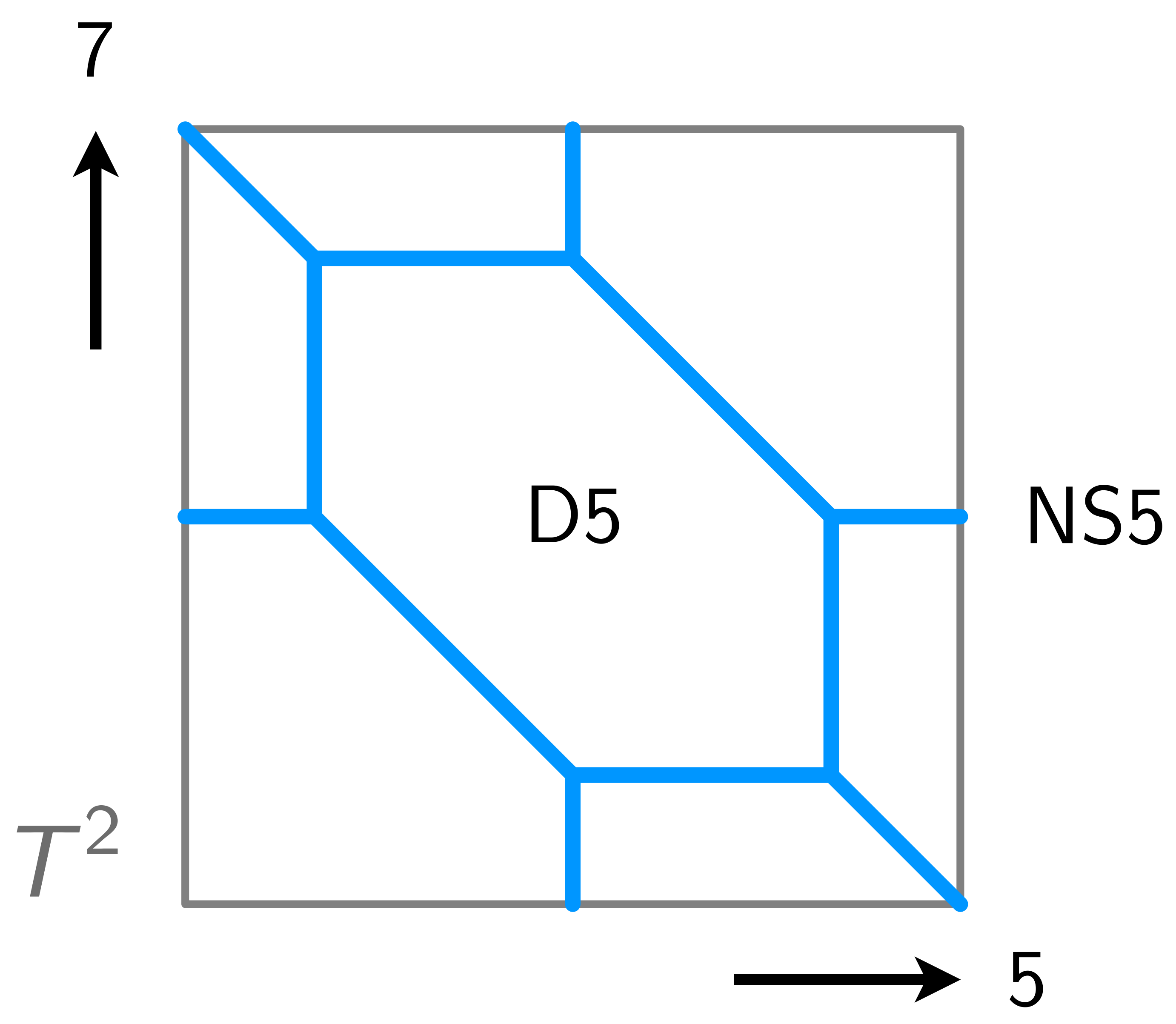
Example
Hirzebruch
F_0



$$\Sigma(x,y)\ :\ P(x,y)=0$$

$$P(x,y)=x+\frac{1}{x}+y+\frac{1}{y}+1$$

Example
del Pezzo
dP_0



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Brane Engineering of Gauge Theories

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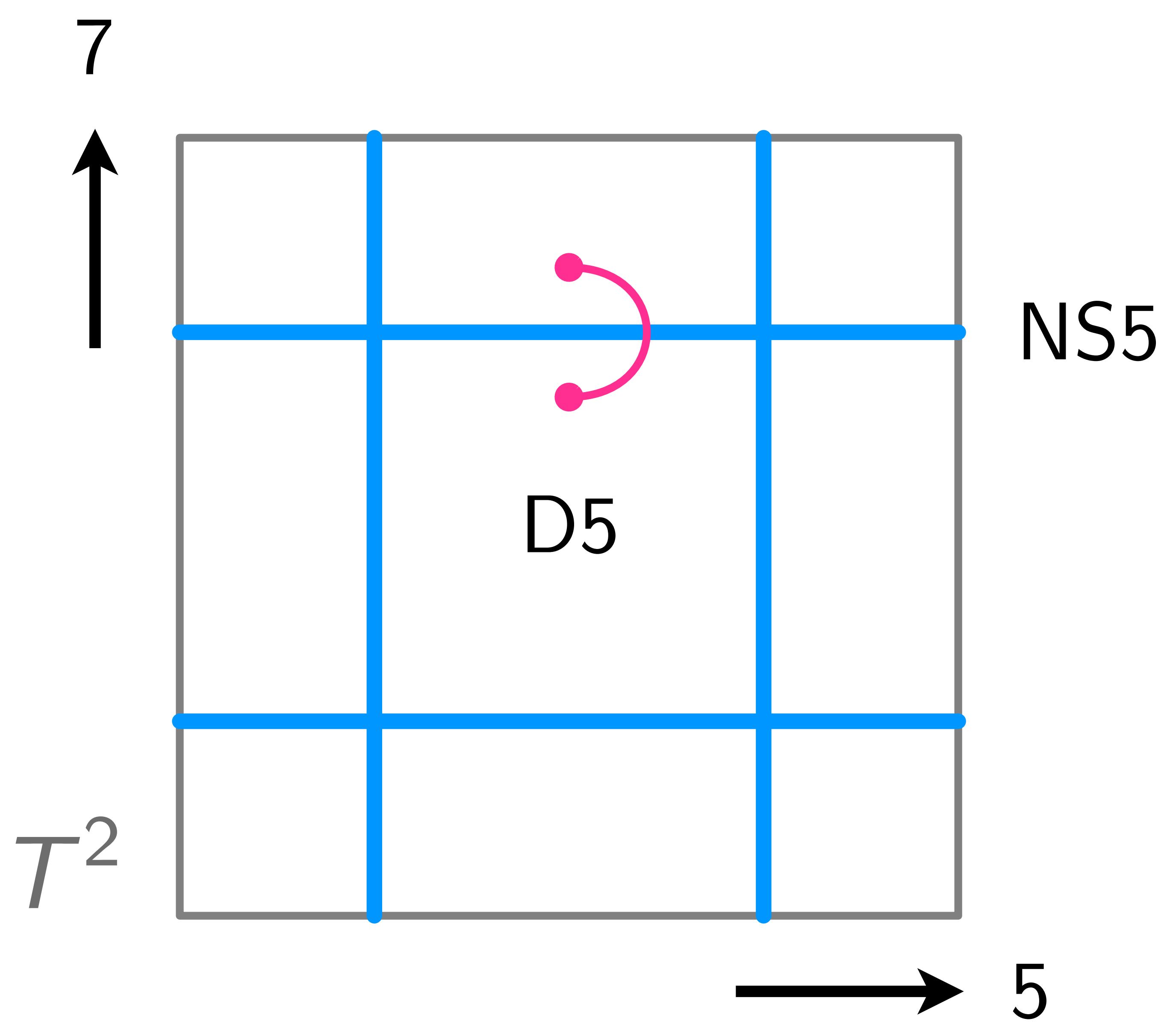

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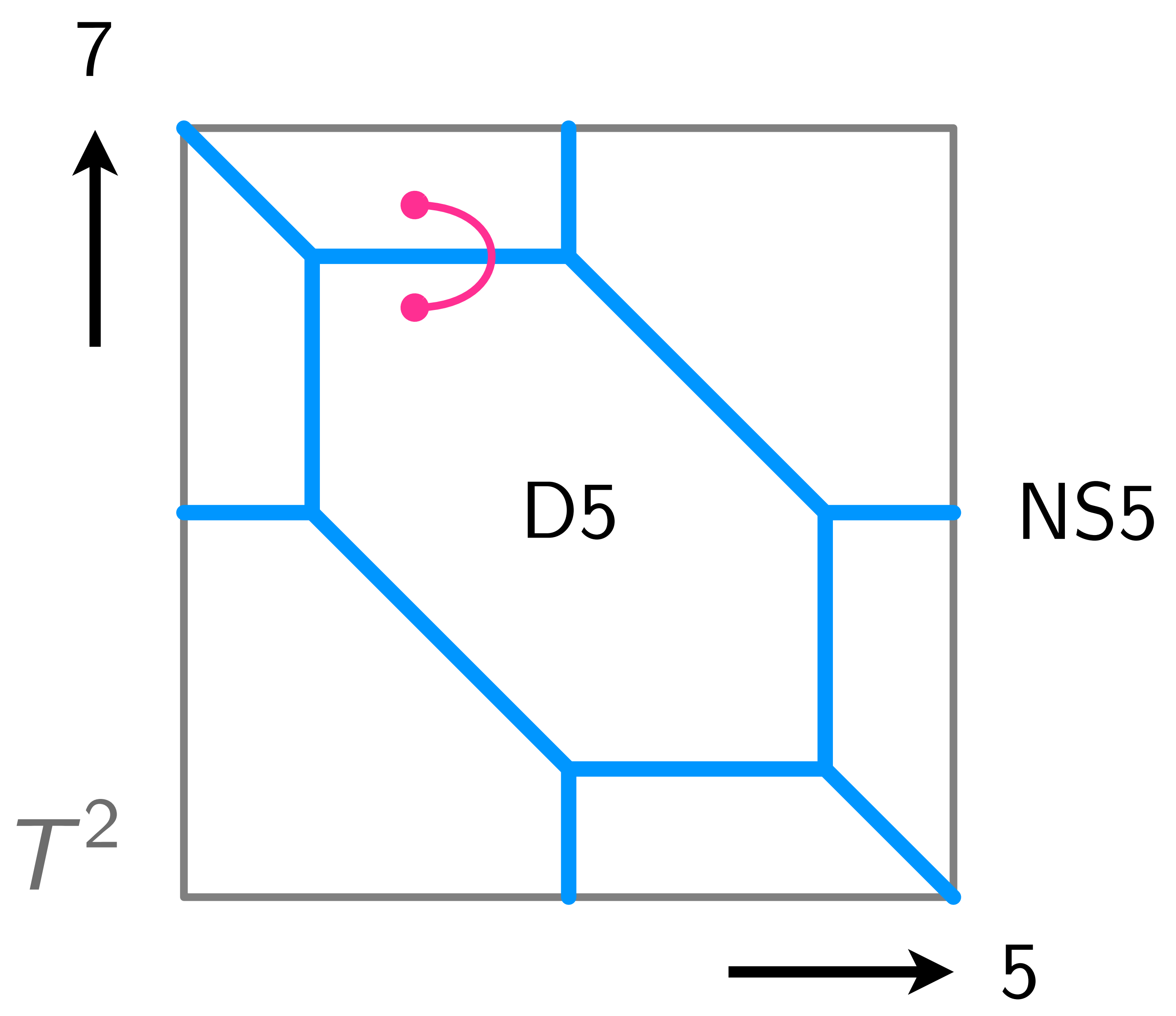

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Brane Engineering of Gauge Theories

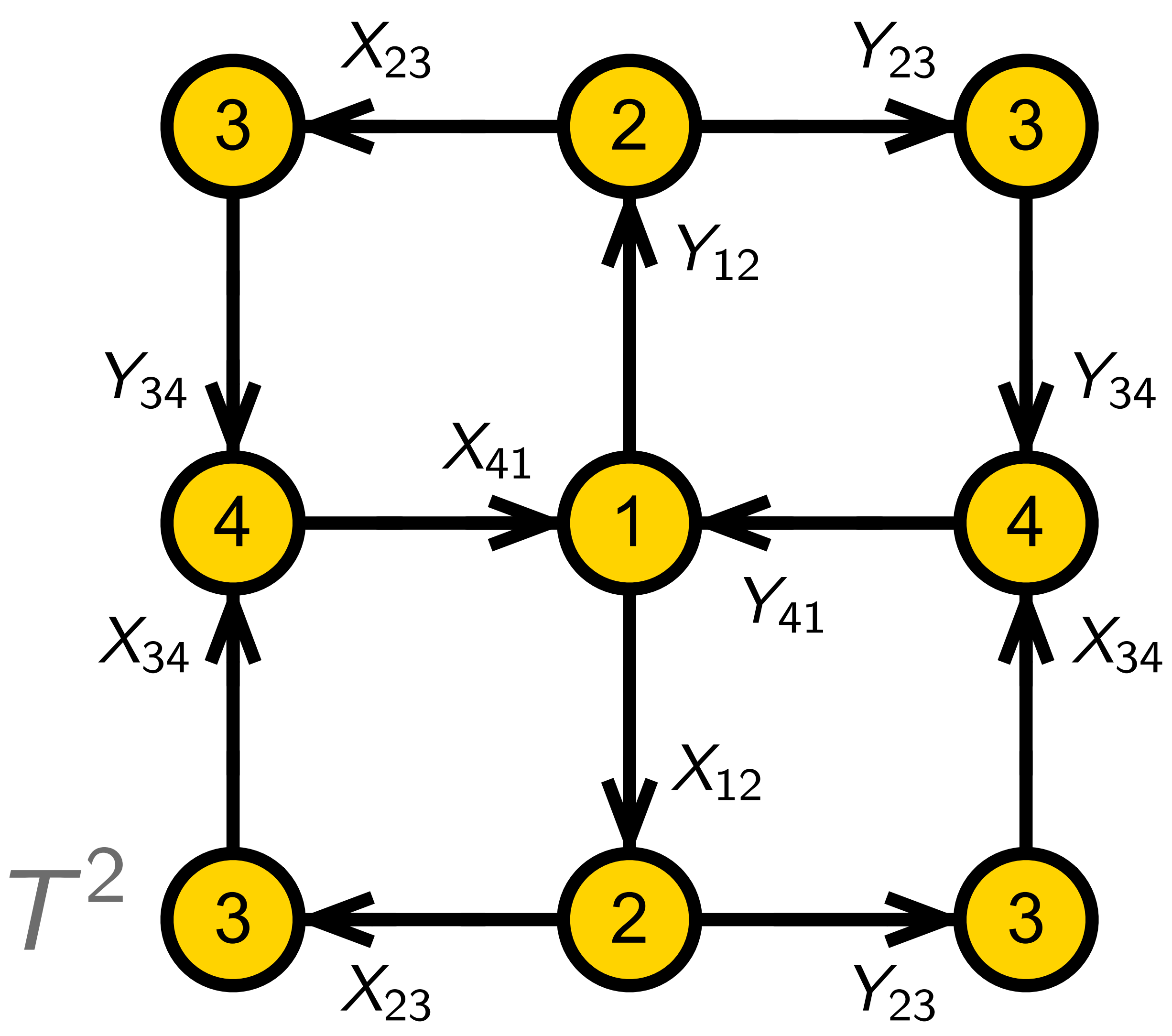
- **4d N=1 Gauge Theories**

The corresponding 4d supersymmetric gauge theories are given by **periodic quivers** wrapping the 2-torus along the 5,7-directions.

Example

Hirzebruch

F_0



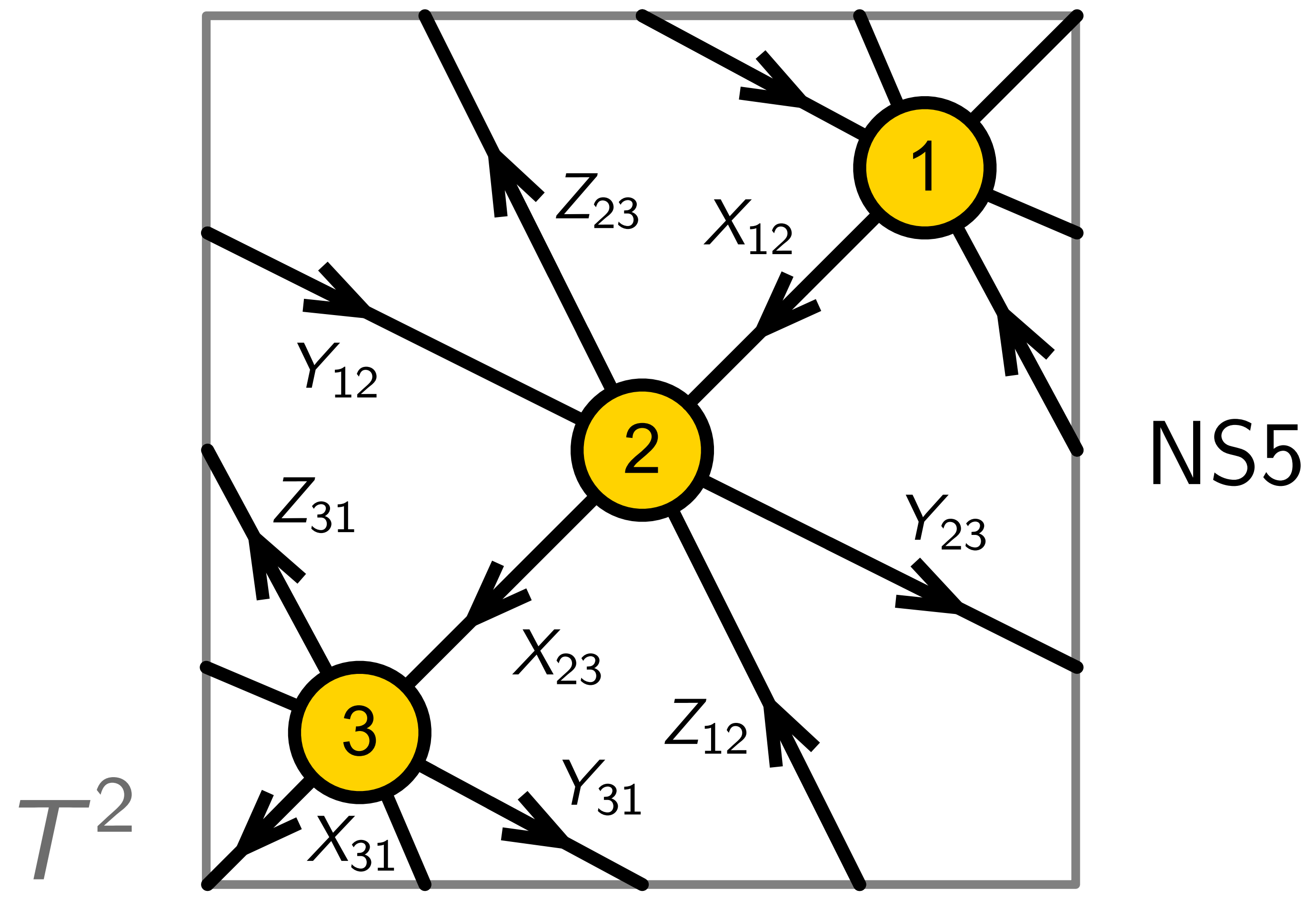
$$\Sigma(x,y) \;:\; P(x,y) = 0$$

$$P(x,y) = x + \frac{1}{x} + y + \frac{1}{y} + 1$$

Example

del Pezzo

dP_0



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$$P(x,y) = x + y + \frac{1}{xy} + 1$$

Brane Engineering of Gauge Theories

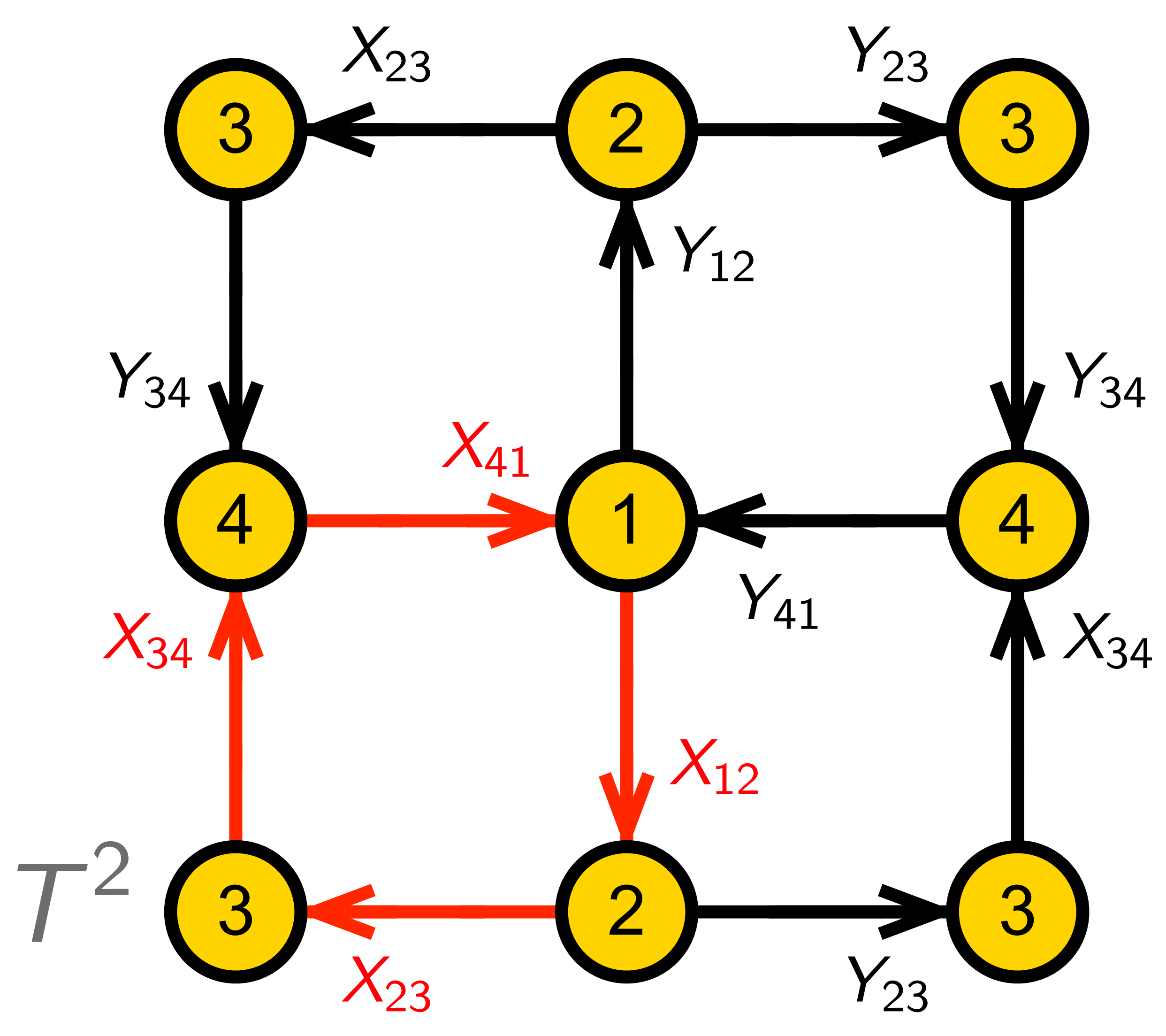
- 4d N=1 Gauge Theories

  We can identify the **4d supersymmetric gauge theory** from the periodic quiver on the 2-torus.

Example

Hirzebruch

F_0



positive W-term
(clockwise)

abelian theory (N=1)

chiral matter fields

	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$
X_{12}, Y_{12}	+1	-1	0	0
X_{23}, Y_{23}	0	+1	-1	0
X_{34}, Y_{34}	0	0	+1	-1
X_{41}, Y_{41}	0	0	+1	-1

superpotential

$$W = X_{12}X_{23}X_{34}X_{41} + Y_{12}Y_{23}Y_{34}Y_{41}$$
$$-X_{12}Y_{23}X_{34}Y_{41} - Y_{12}X_{23}Y_{34}X_{41}$$

Brane Engineering of Gauge Theories

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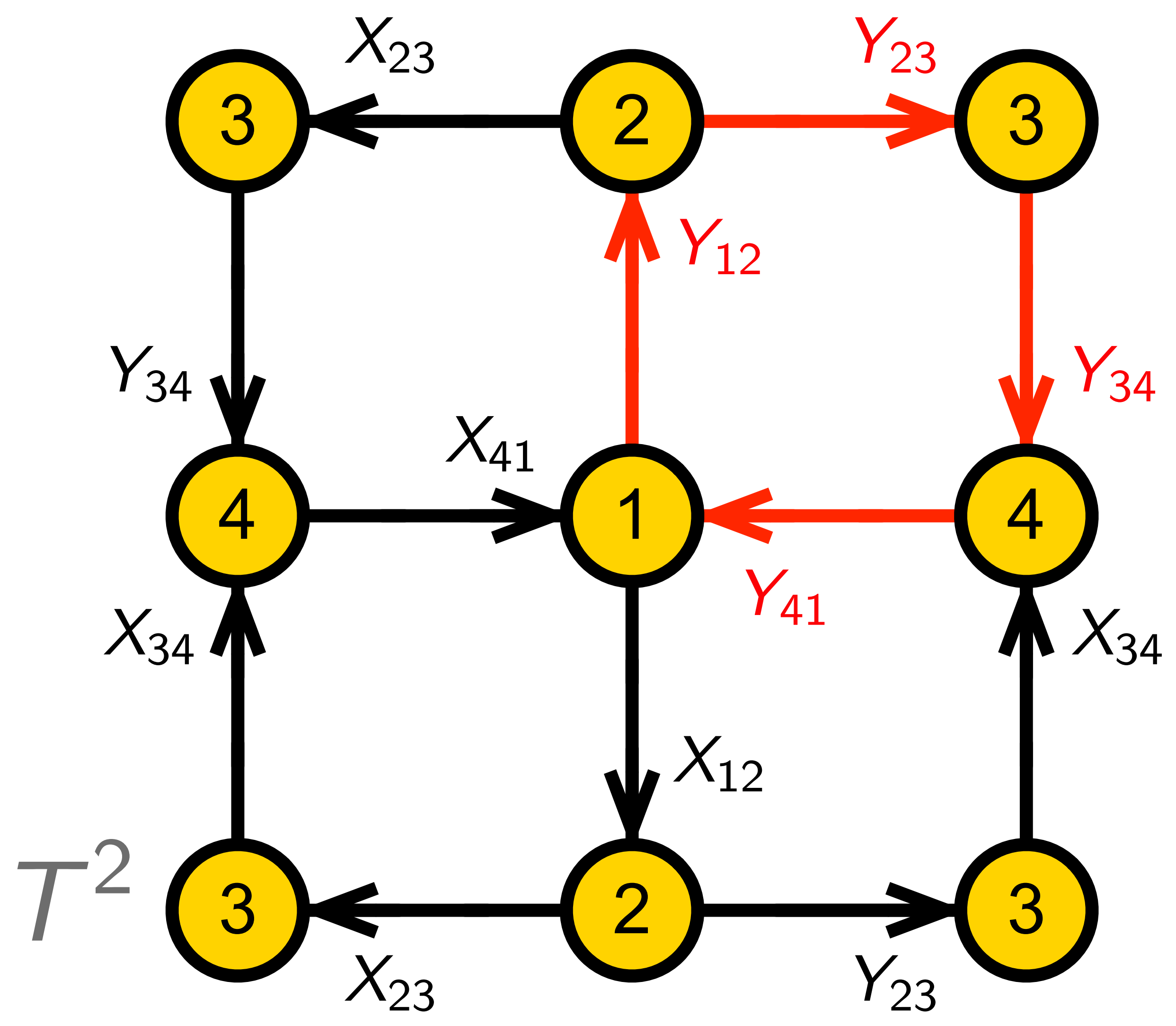

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Brane Engineering of Gauge Theories

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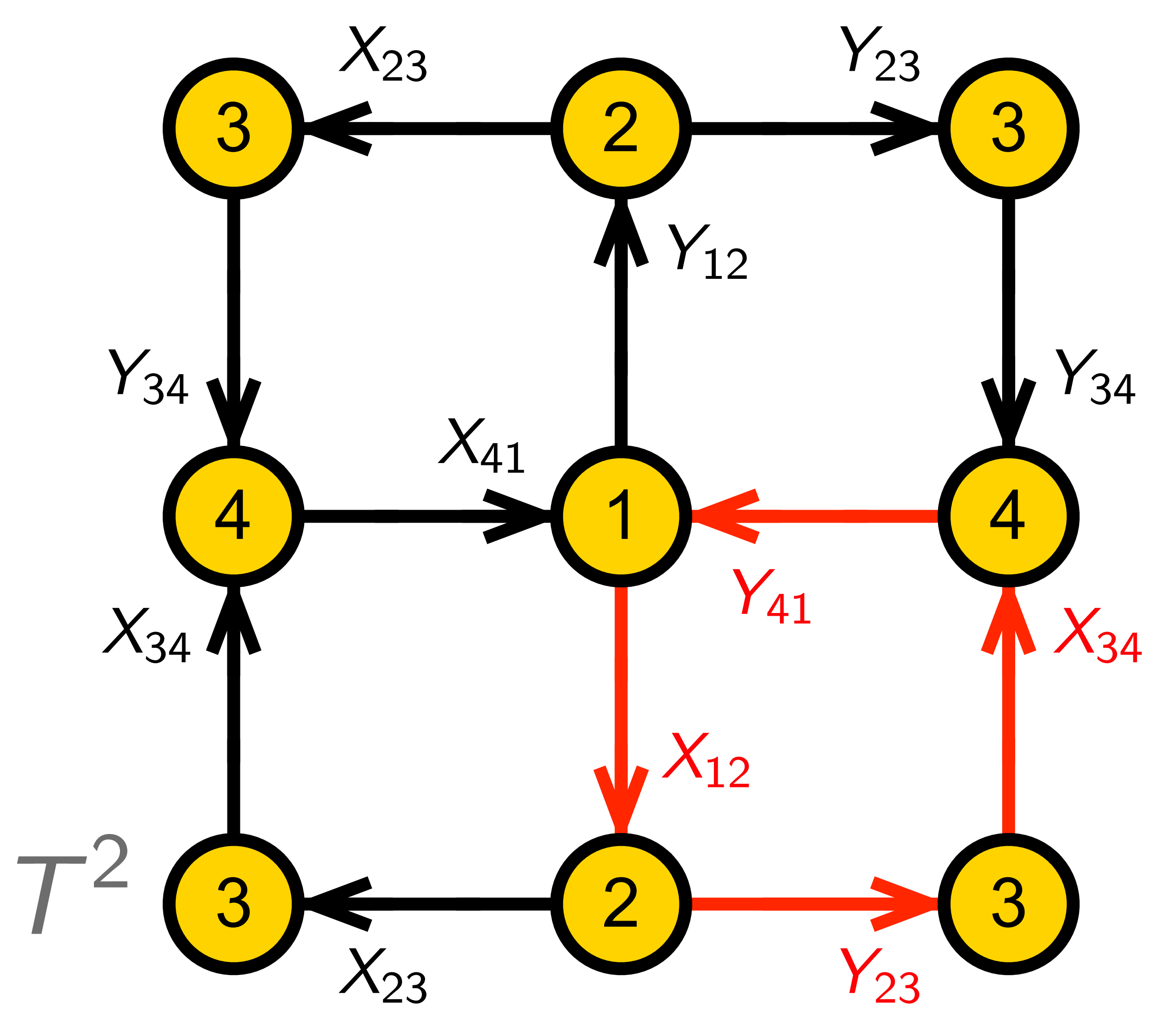

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Brane Engineering of Gauge Theories

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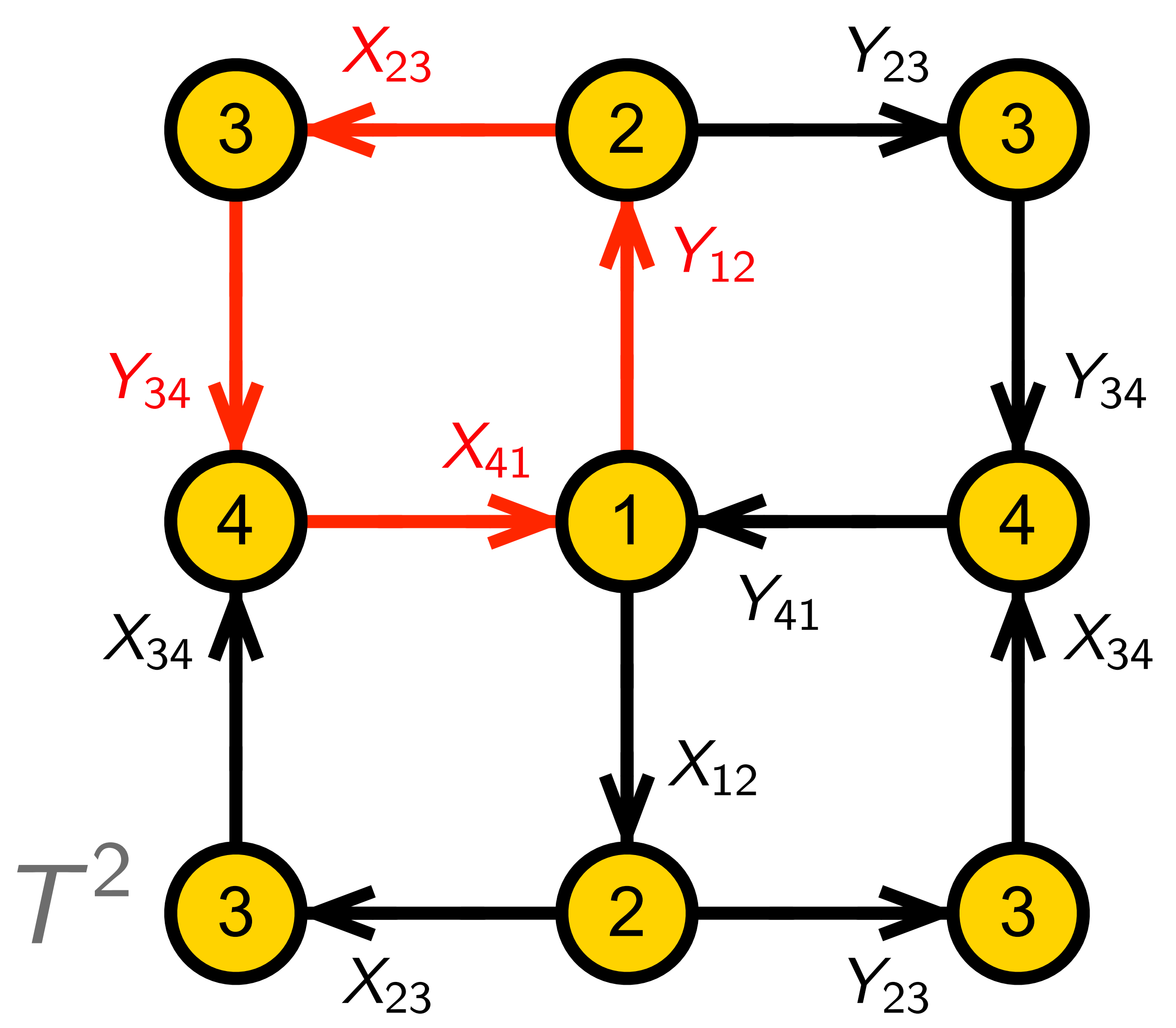

negative W-term
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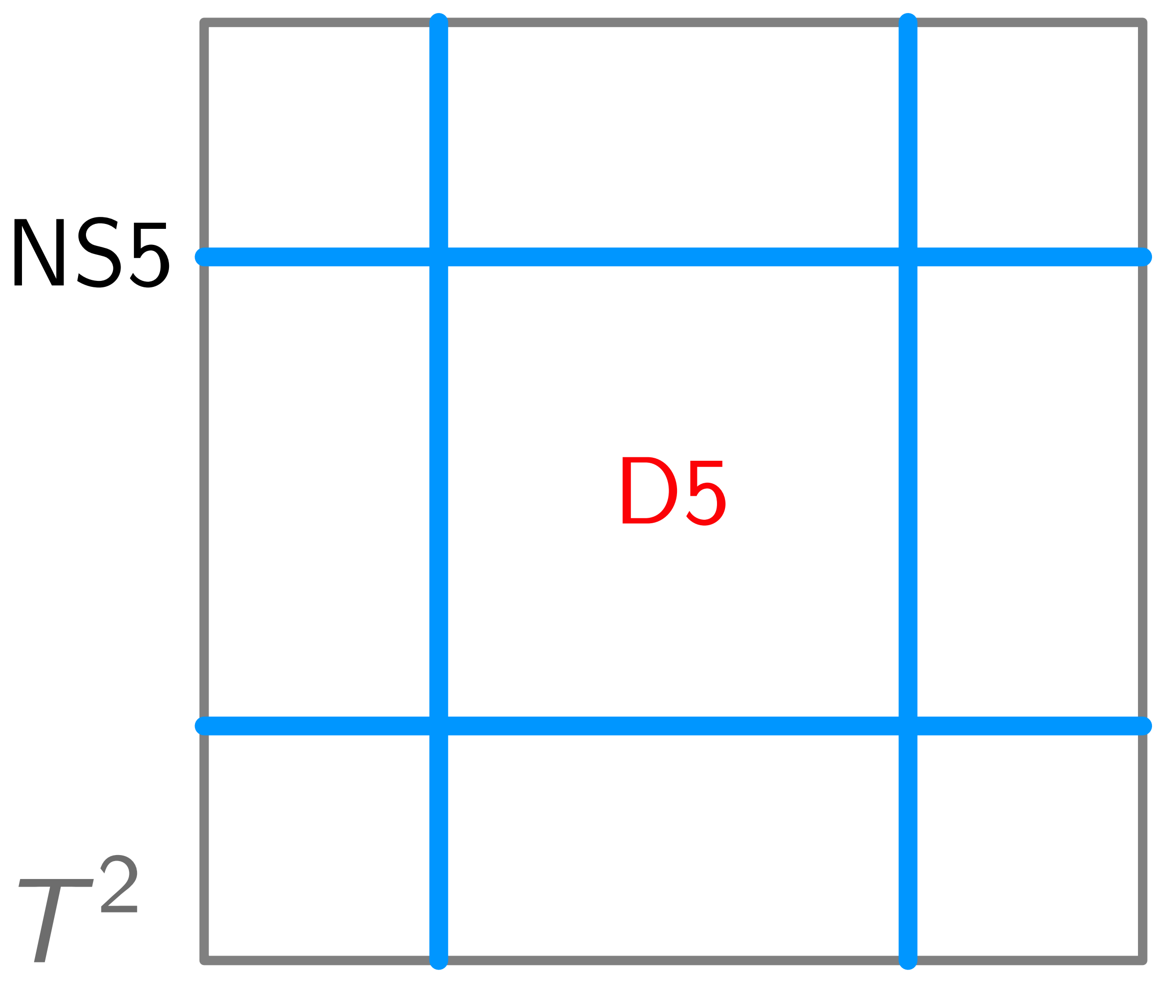
Brane Engineering of Gauge Theories

[Franco, Hanany, Kennaway, Vegh, Wecht 2005]

- **Brane Tilings (Dimers)**

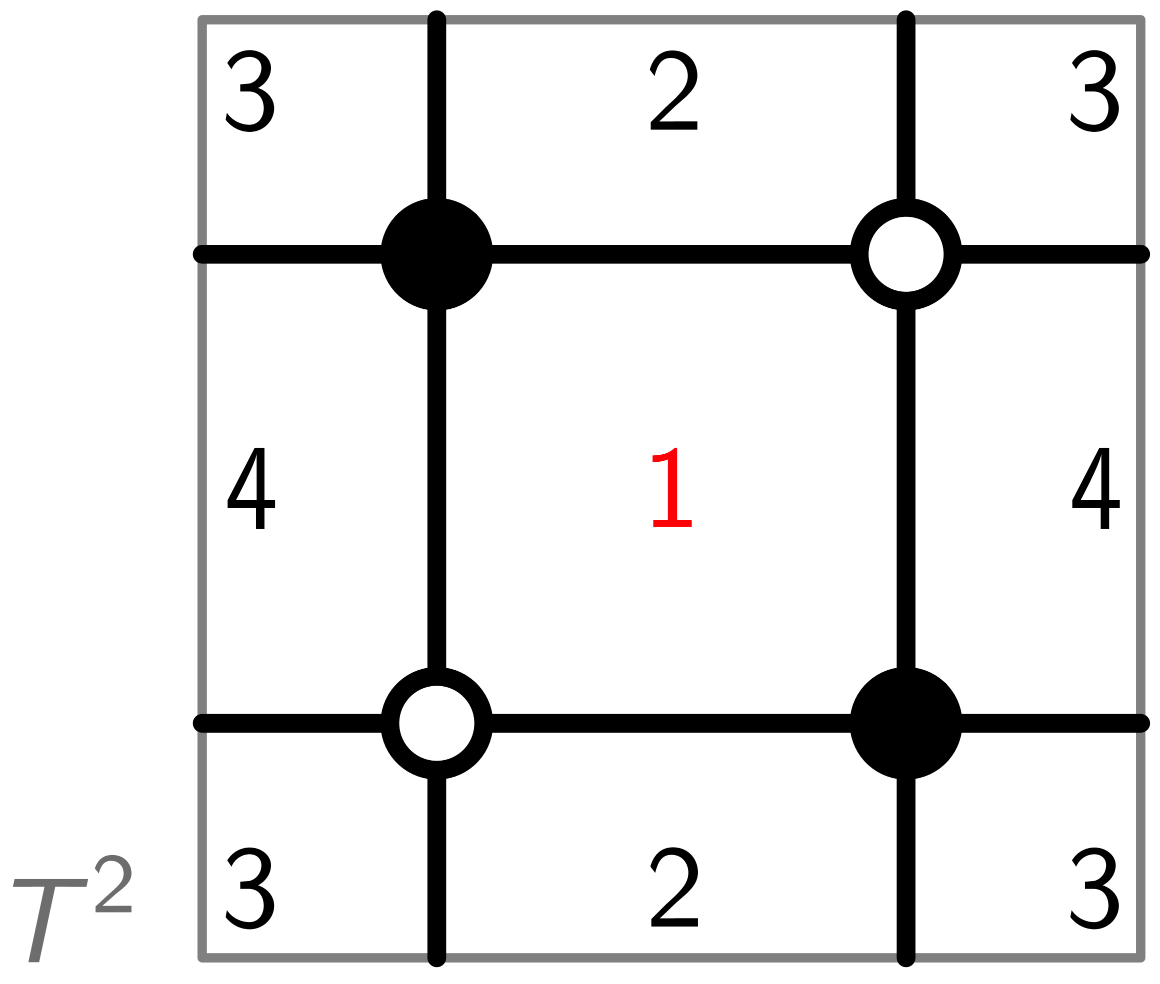
We can encode the 4d gauge theory with a bipartite graph on the 2-torus known as a **dimer** model.

brane configuration



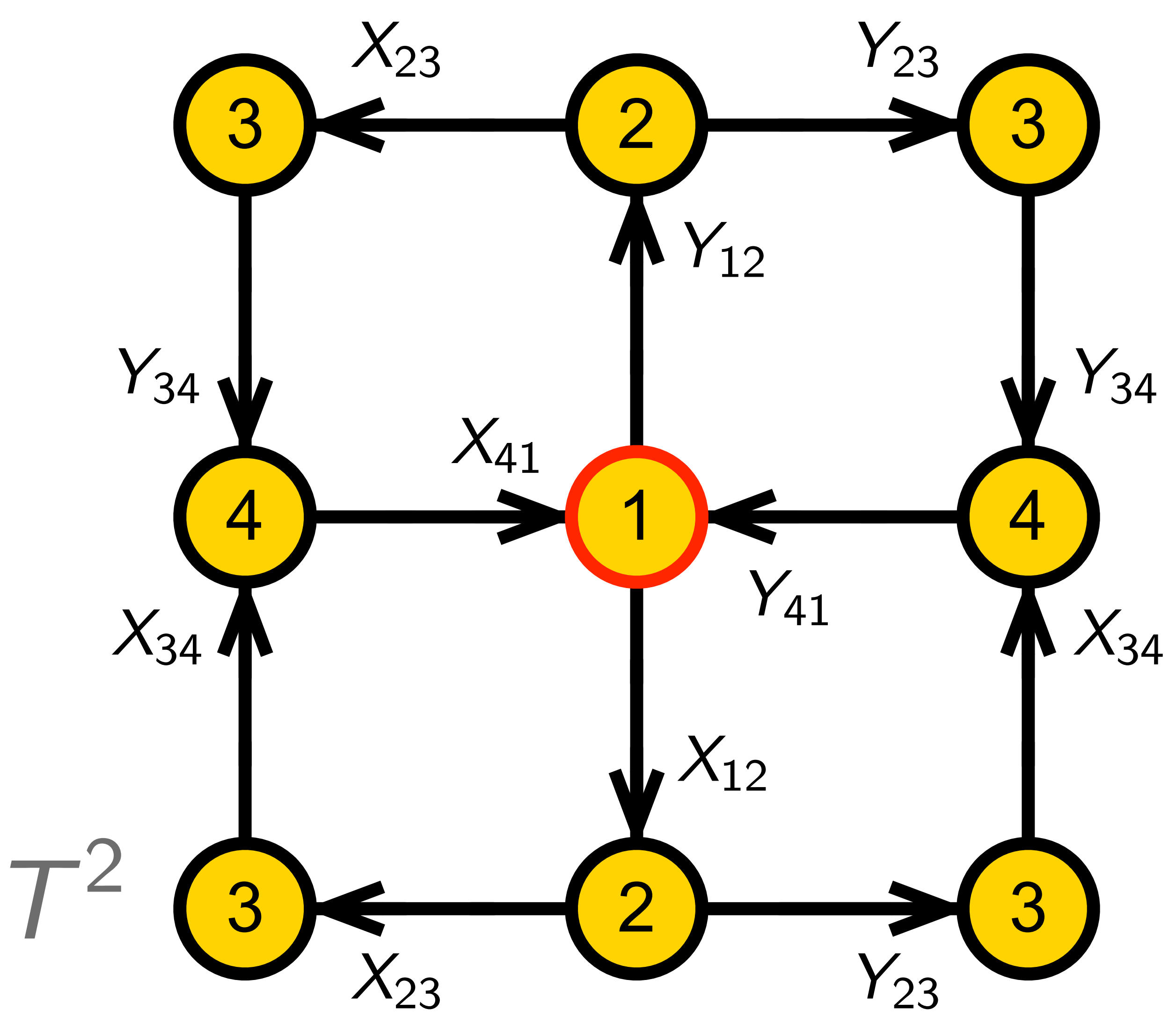
D5-brane

brane tiling (dimer)



face

quiver



gauge group

Brane Engineering of Gauge Theories

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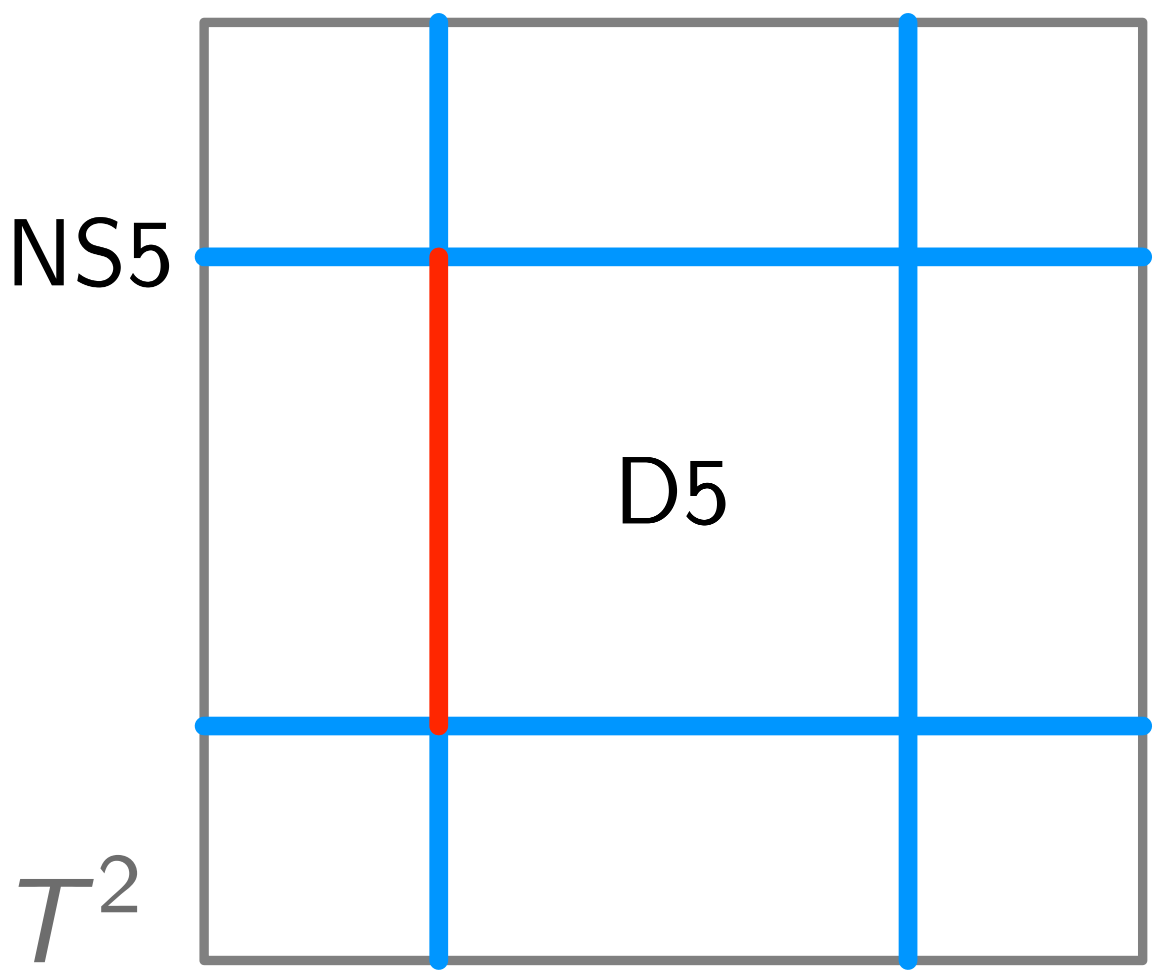
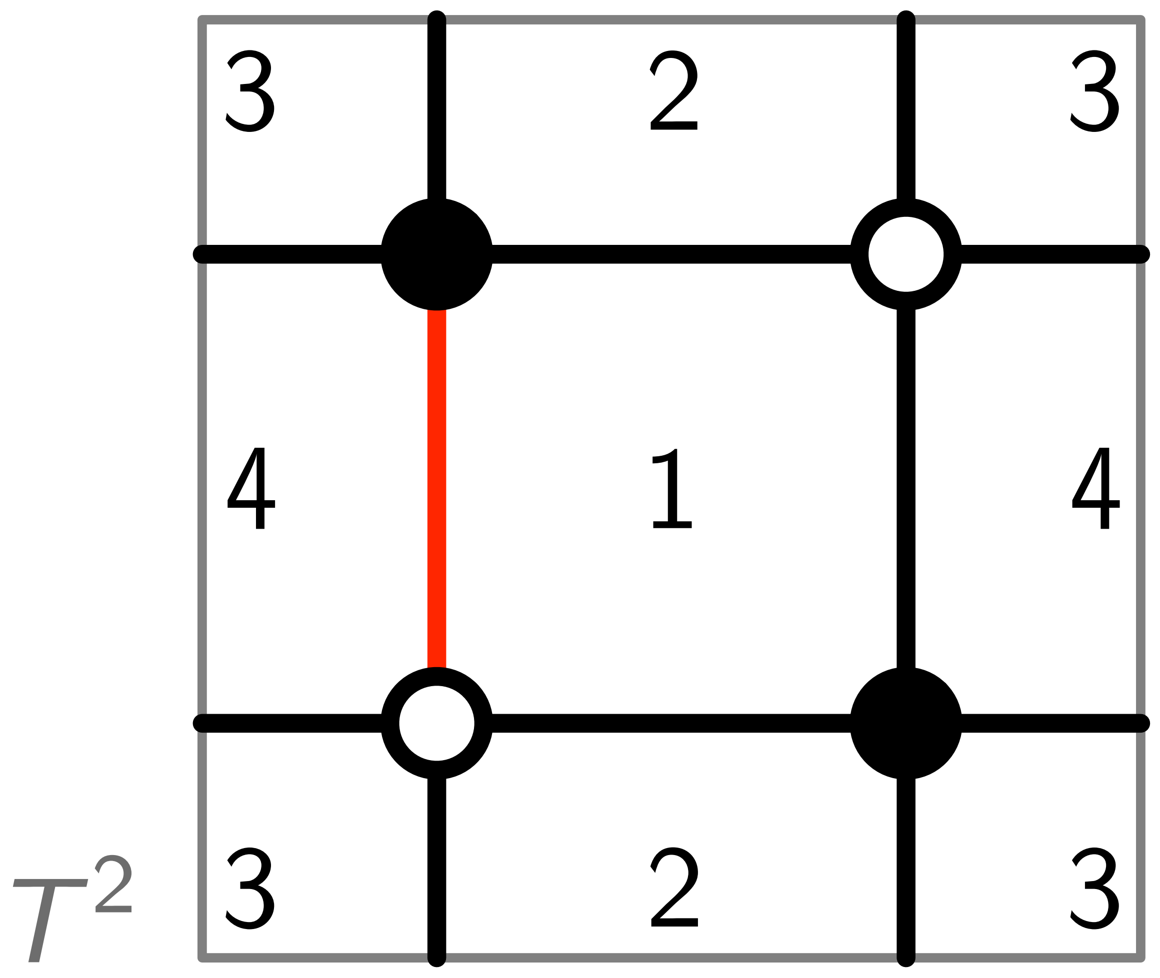


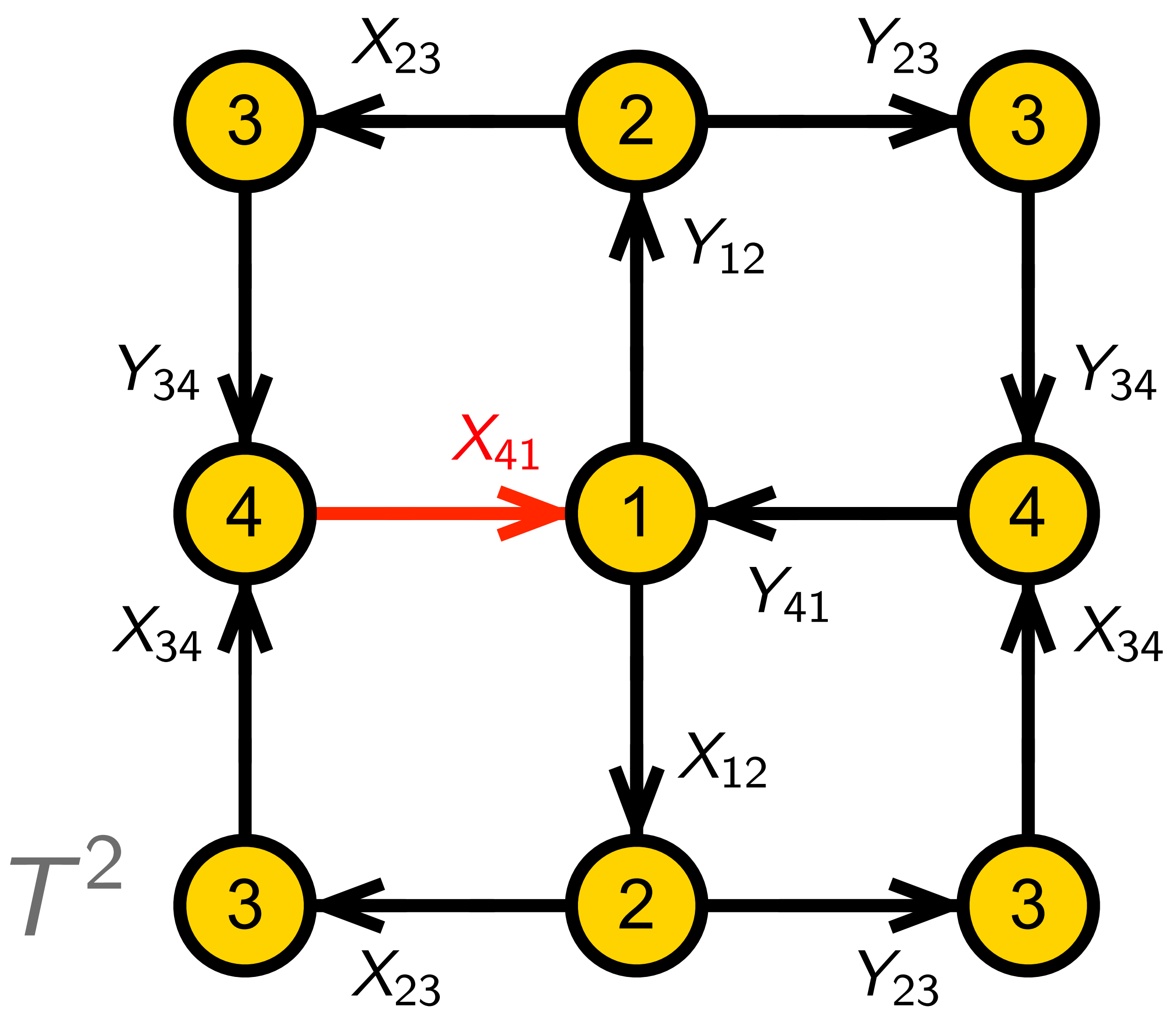
open string
between two D5-branes

brane tiling (dimer)



edge

quiver



bifundamental chiral field

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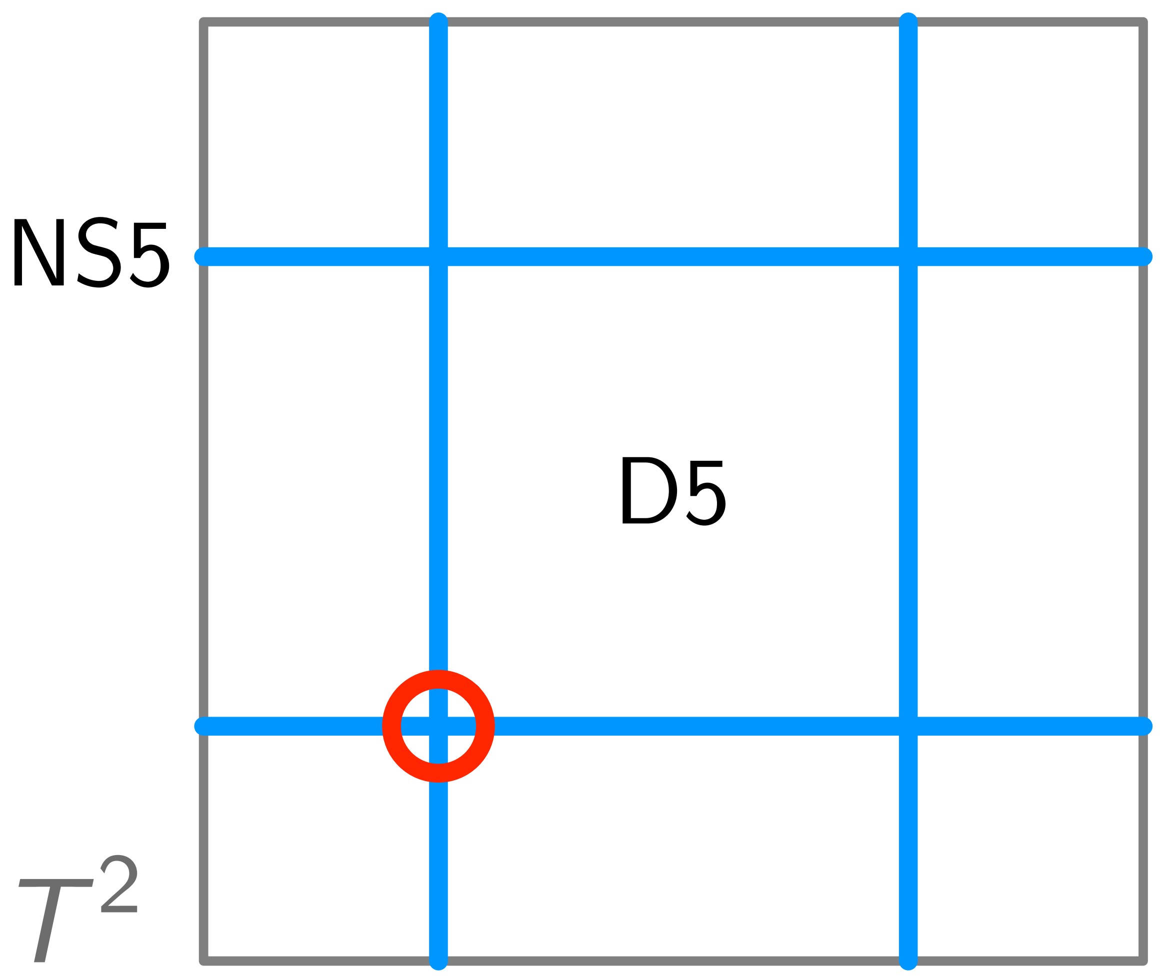
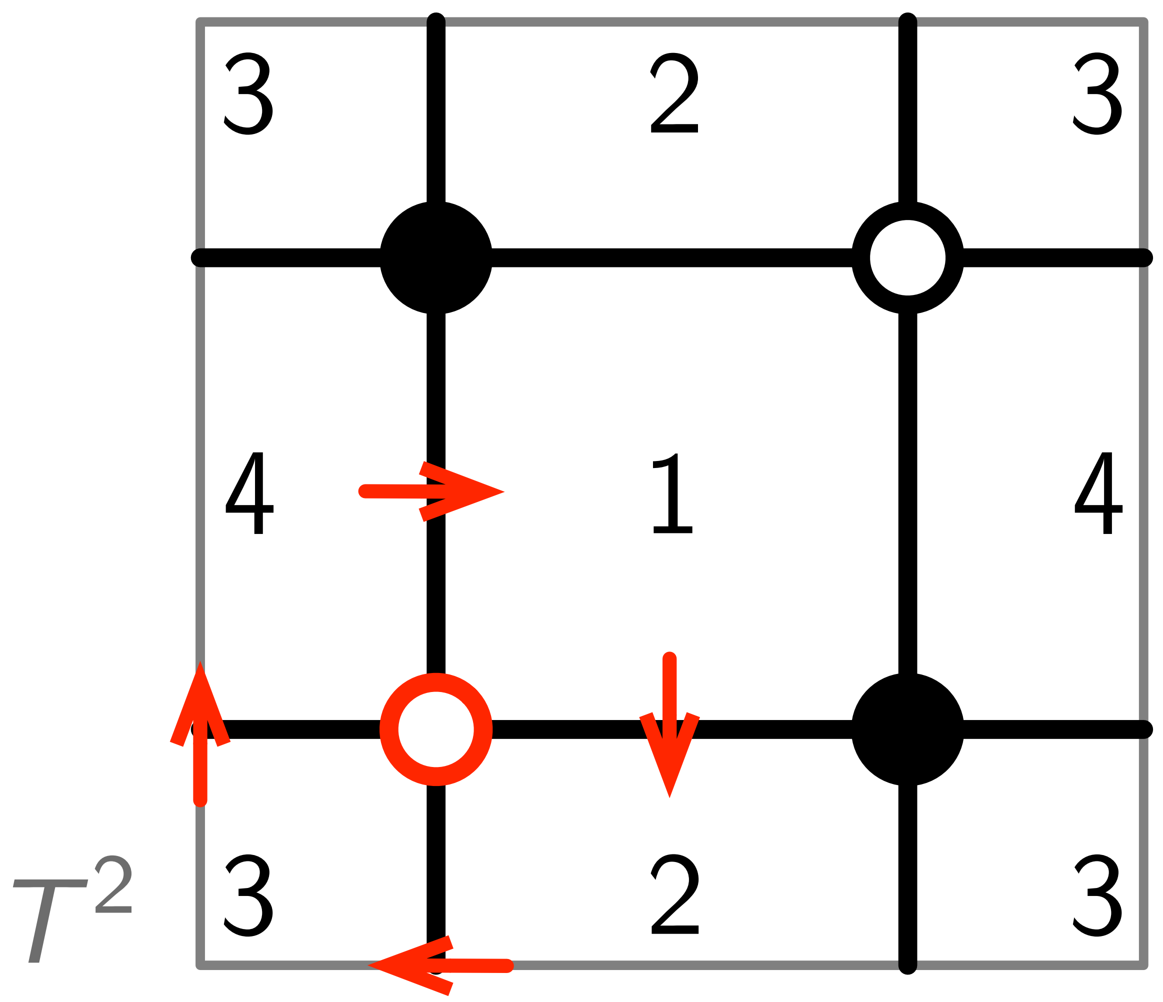


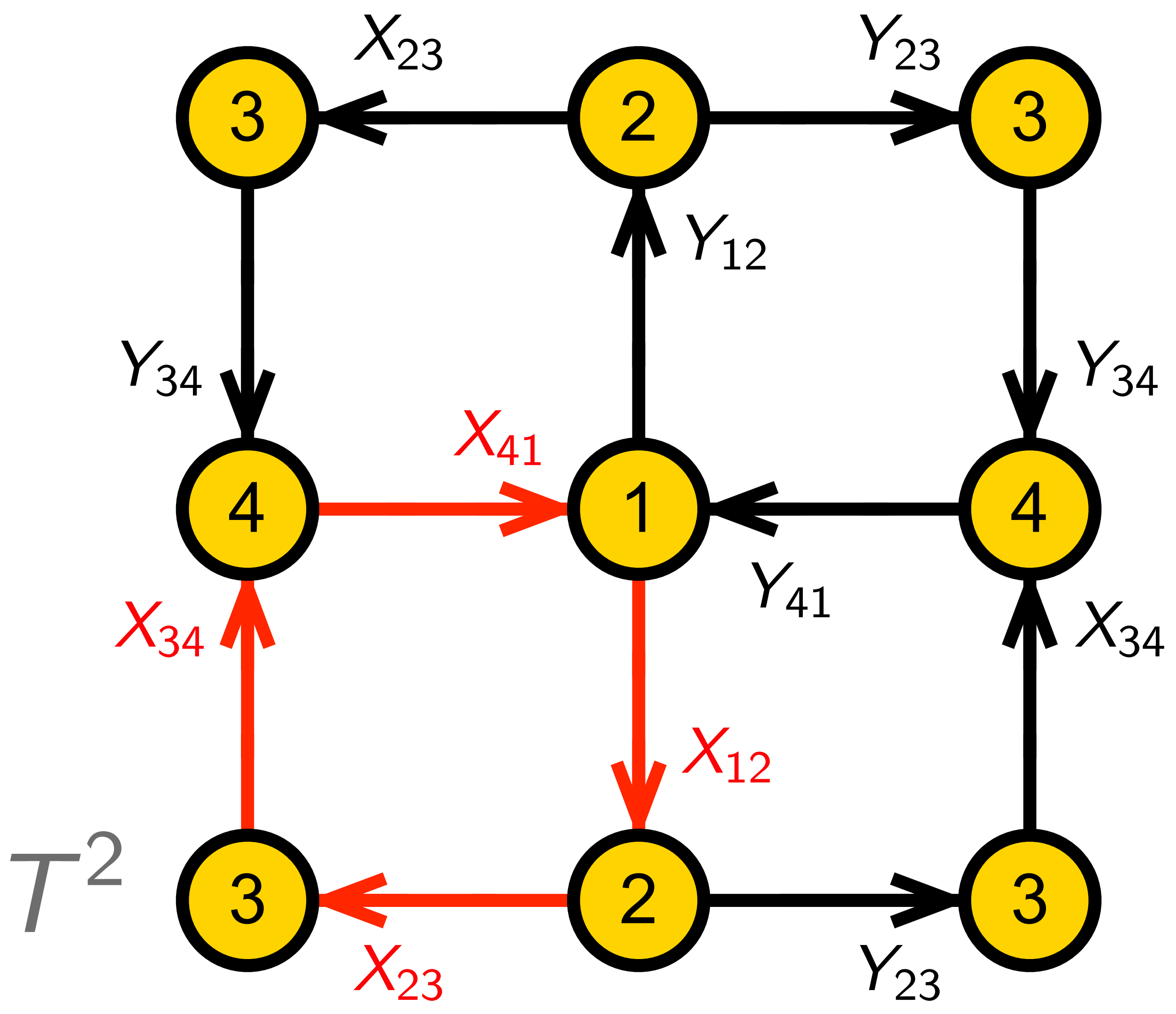
multiple open strings meet

brane tiling (dimer)



white node (clockwise)

quiver



positive W-term

Brane Engineering of Gauge Theories

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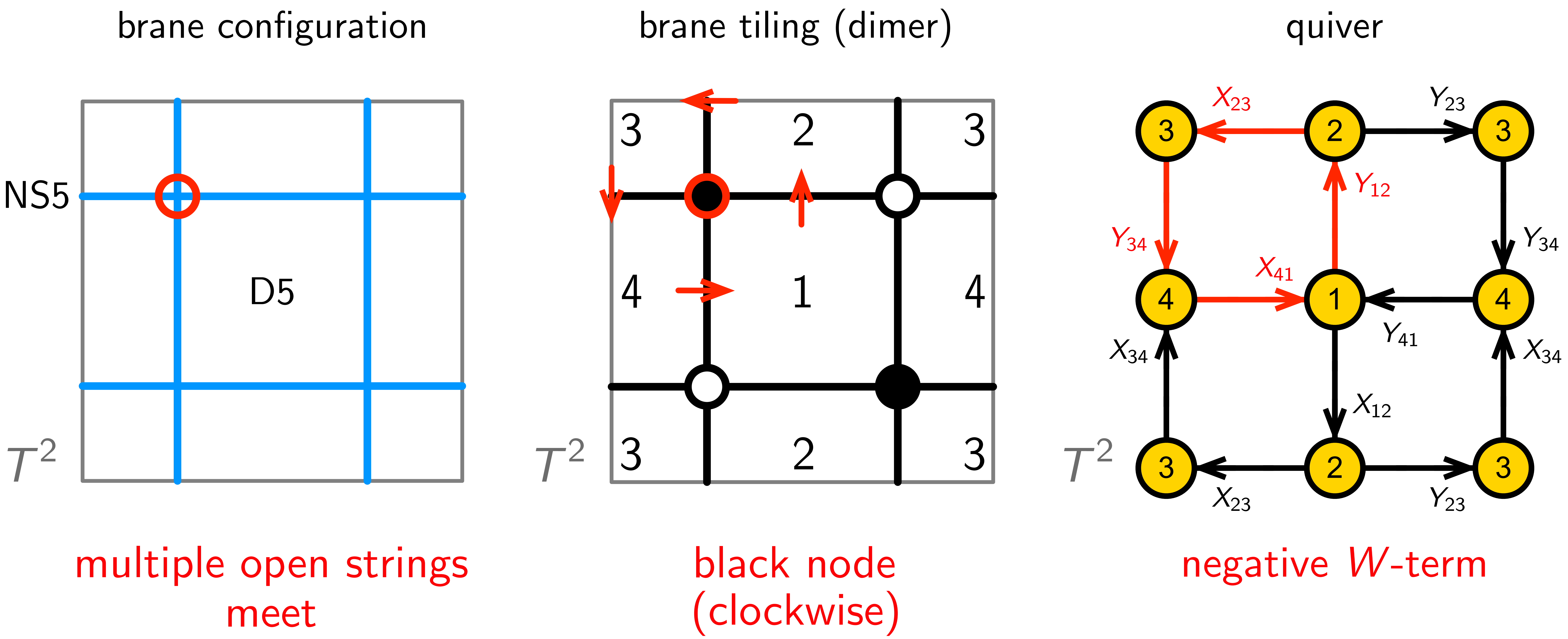


Brane Engineering of Gauge Theories

[Franco, Hanany, Kennaway, Vegh, Wecht 2005]

- **Brane Tilings (Dimers)**

  We can encode the 4d gauge theory with a bipartite graph on the 2-torus known as a **dimer** model.

$$\mathcal{L} = \frac{1}{16\pi} \int d^2\theta \; \tau_f \mathrm{Tr}(W_f^{\alpha} W_{f\alpha}) + \boxed{\int d^4\theta \; {\Phi_e}^{\dagger} e^{V_f} \Phi_e e^{-V_f}} + \boxed{\int d^2\theta \; W} + \text{h.c.}$$

chiral field term

superpotential term

Lagrangian of the 4d N=1 Theory

Brane Engineering of Gauge Theories

- Toric Varieties and Toric Diagrams

Newton polynomial $$P(x,y) = \sum_{(n_x,n_y)\in\Delta} c_{(n_x,n_y)}\, x^{n_x} y^{n_y} \qquad x,y \in \mathbb{C}^*$$

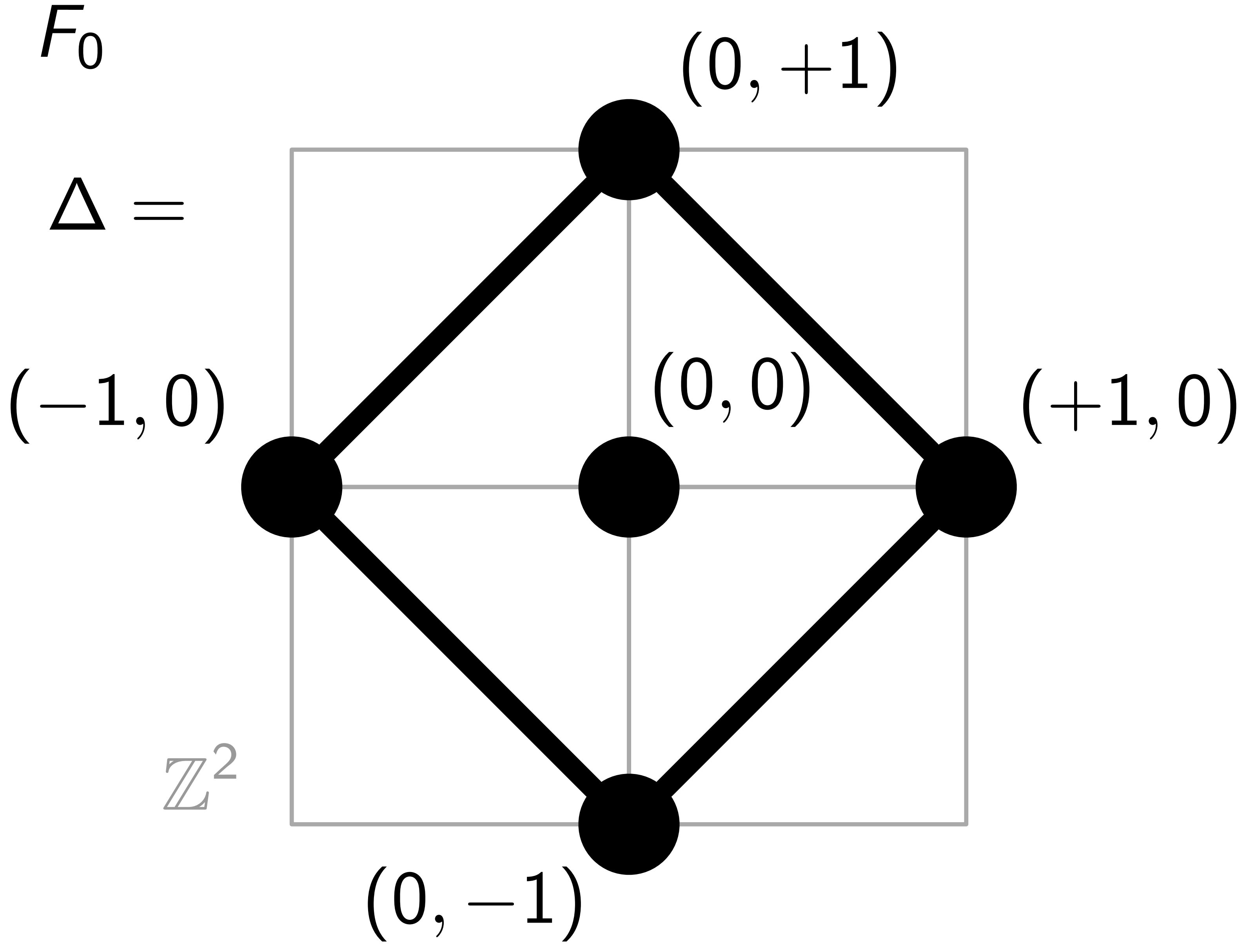
complex structure moduli

Example

Hirzebruch

F_0

$\Delta =$



$$P(x,y) = x^{+1}y^0 + x^{-1}y^0 + x^0y^{+1} + x^0y^{-1} + x^0y^0$$

$$= x + \frac{1}{x} + c_1\left(y + \frac{1}{y}\right) + c_2$$

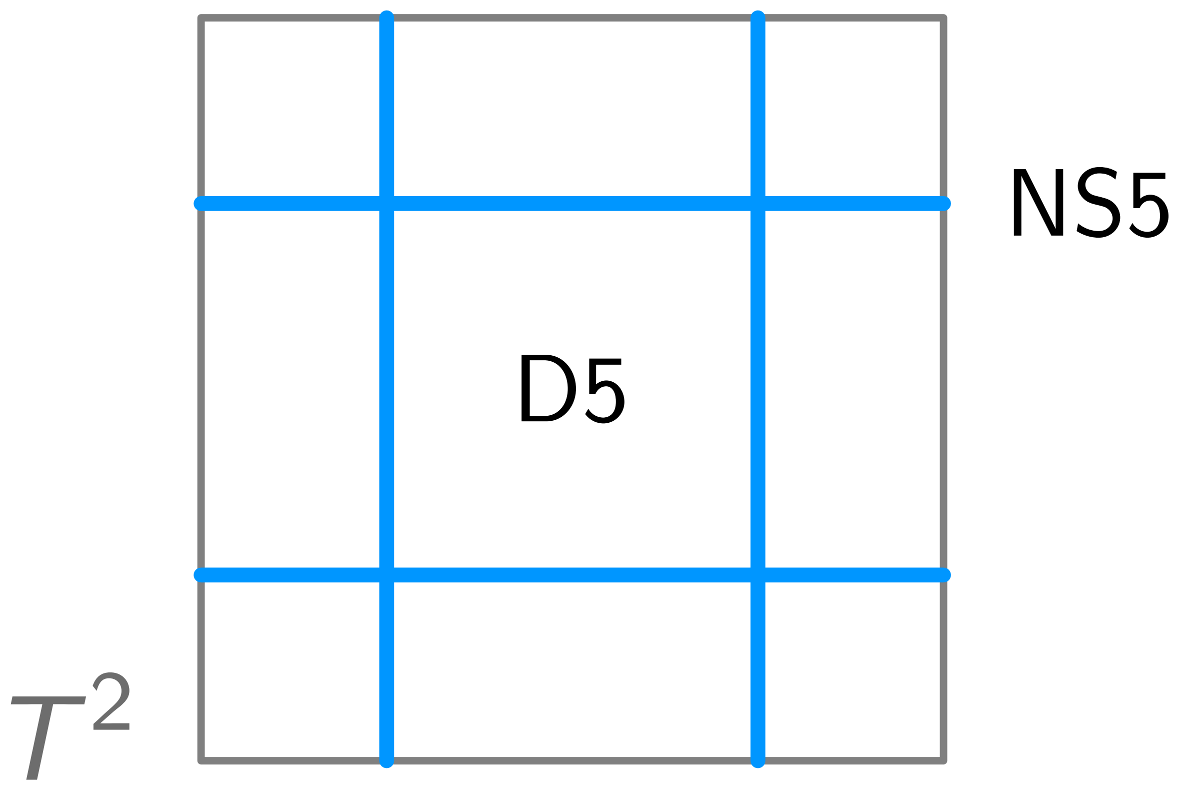
independent complex structure moduli

mirror curve $\Sigma \; : \; P(x,y) = 0$

Brane Engineering of Gauge Theories

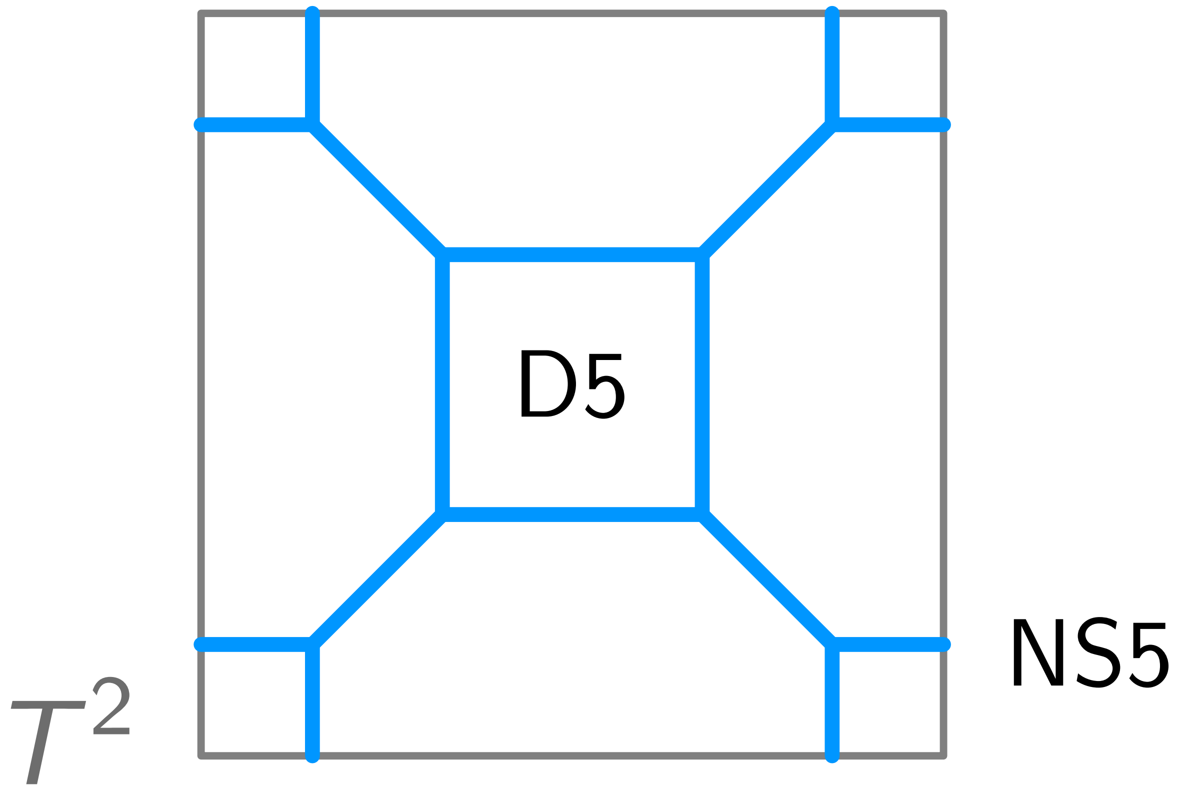
- Brane Tilings and Calabi-Yau Mirror Symmetry

Changing the **complex structure moduli** changes the NS5- and D5-brane configuration.



$$P(x,y) = x + \frac{1}{x} + c_1\left(y + \frac{1}{y}\right) + c_2 \qquad x, y \in \mathbb{C}^*$$

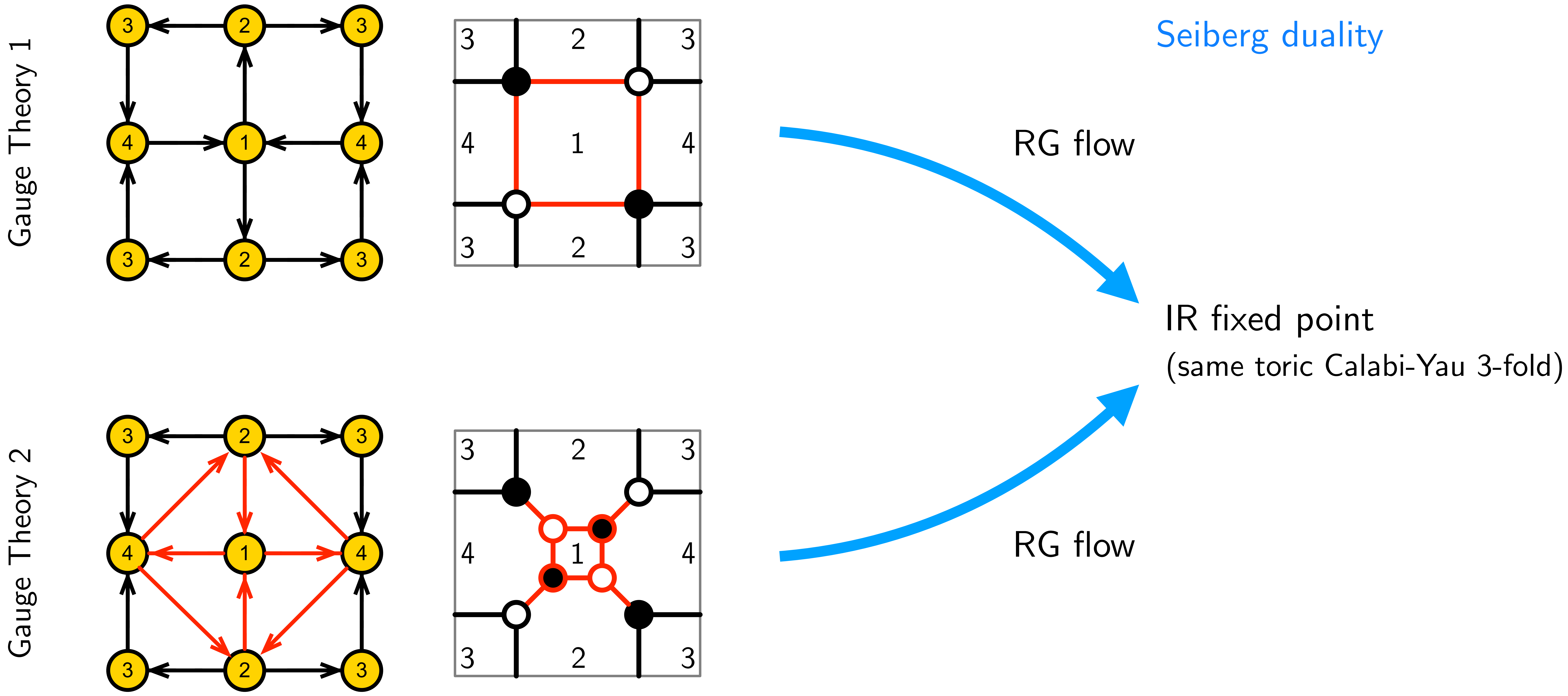
complex structure moduli



$$P(x,y) = x + \frac{1}{x} + c_1'\left(y + \frac{1}{y}\right) + c_2' \qquad x, y \in \mathbb{C}^*$$

Brane Engineering of Gauge Theories

- 4d N=1 Gauge Theory Duality (Seiberg Duality)



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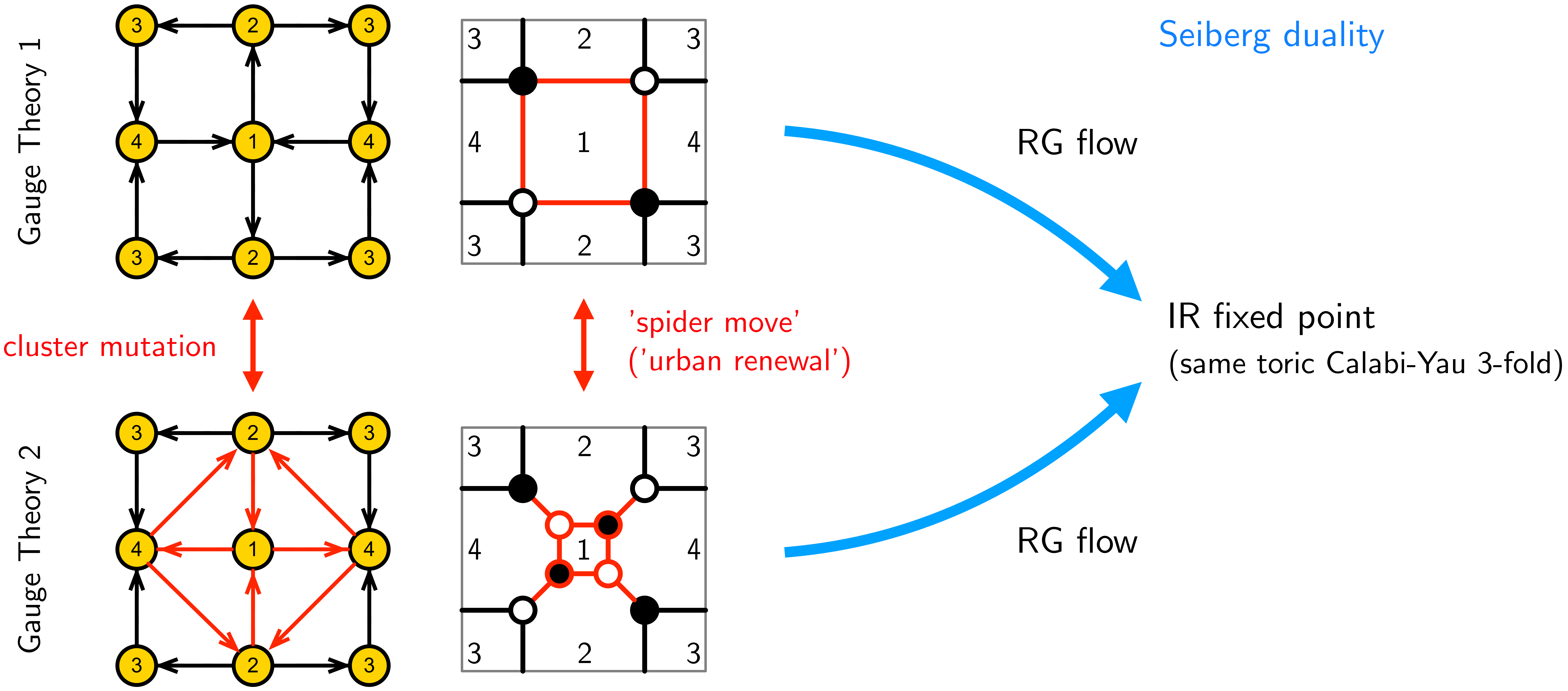

Brane Engineering of Gauge Theories

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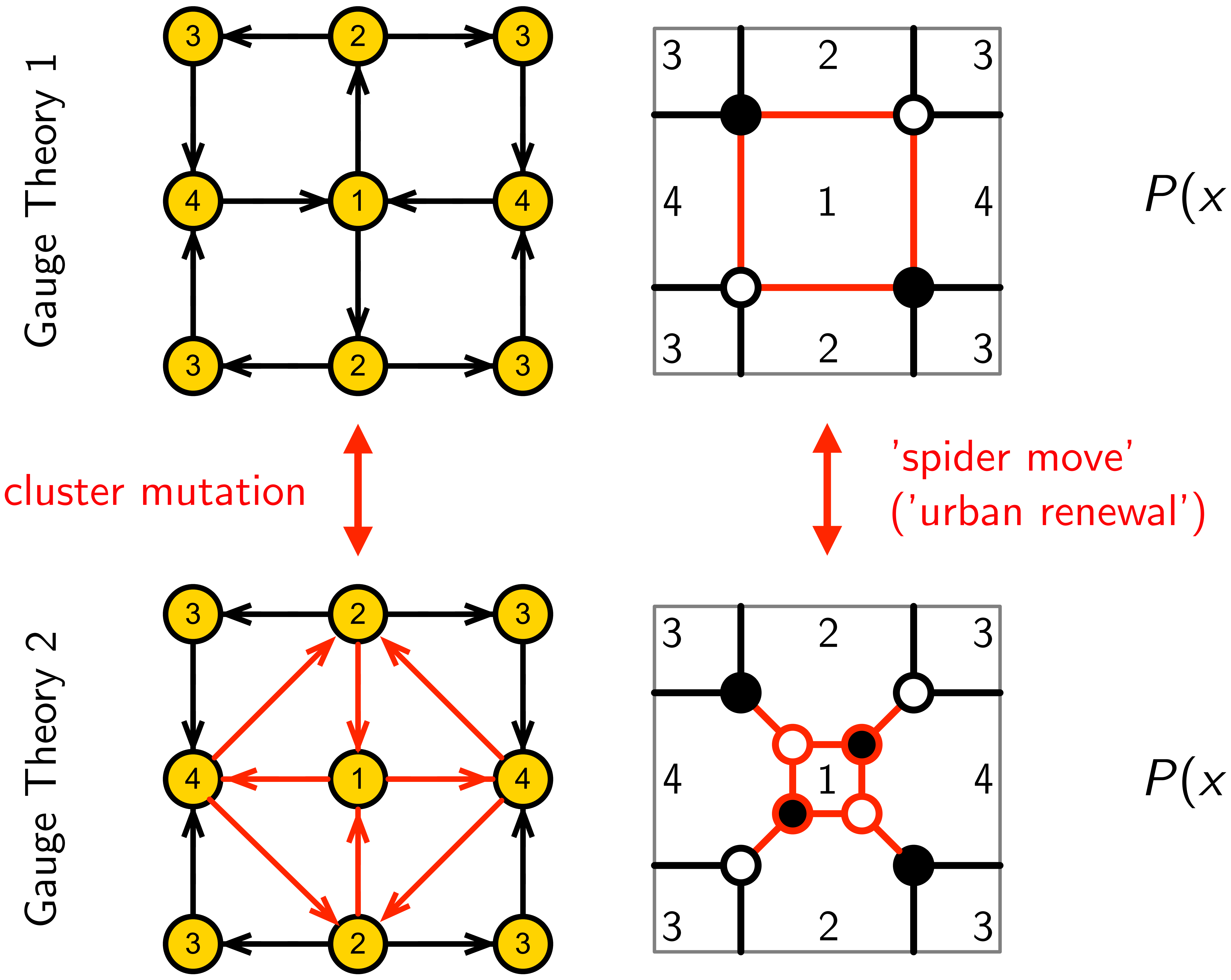


$$P(x,y) = x + \frac{1}{x} + c_1\left(y + \frac{1}{y}\right) + c_2$$

complex structure moduli

$$P(x,y) = x + \frac{1}{x} + c_1'\left(y + \frac{1}{y}\right) + c_2'$$

[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

QUESTION

Given a mirror curve with a choice of complex structure moduli,
can we visualize the **shape of the curve** and
the corresponding **brane configuration**?

Tropical Geometry

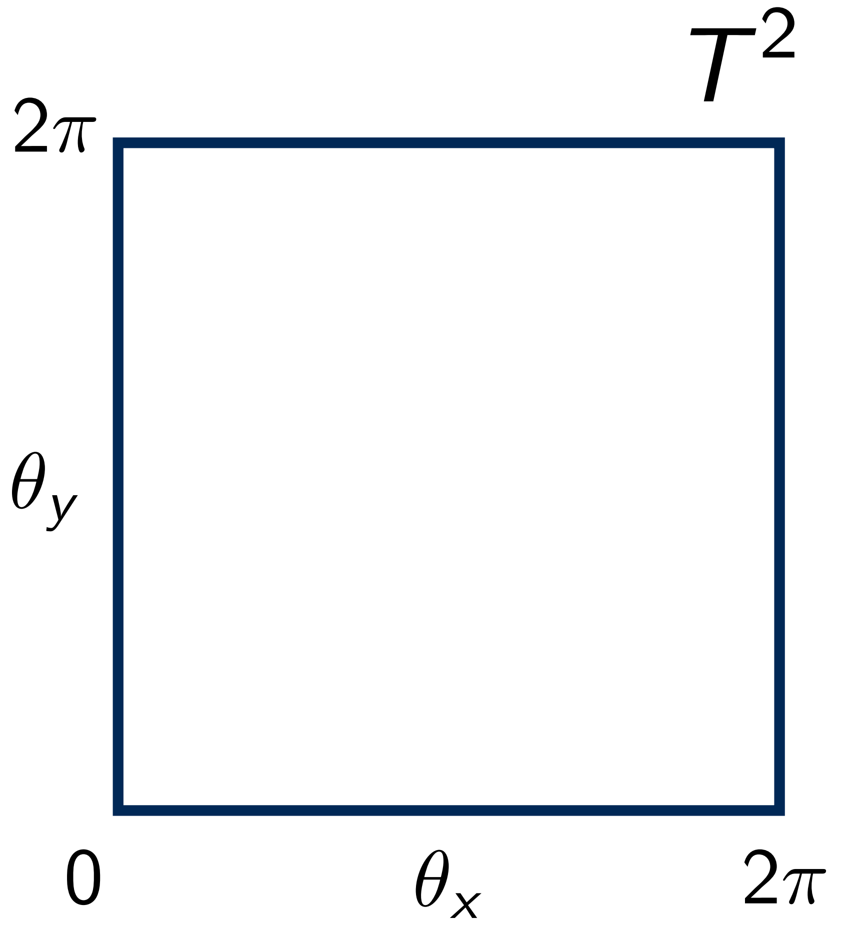
[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

- **Coamoeba Projection**

We can visualize the mirror curve under a **projection onto the 2-torus** known as the coamoeba projection.

mirror curve $\Sigma \ : \ P(x,y) = 0 \qquad x, y \in \mathbb{C}^*$

Coamoeba $\big(x = r_x e^{i\theta_x}, y = r_y e^{i\theta_y}\big) \ \mapsto \ (\theta_x, \theta_y) \in T^2$



mapped solutions: $N_\theta = 0$

UNIST

Tropical Geometry

[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

- **Coamoeba Projection**

We can visualize the mirror curve under a **projection onto the 2-torus** known as the coamoeba projection.

Example

Hirzebruch
F_0

$$\Sigma \ : \ P(x,y) = x + \frac{1}{x} + c_1 \left(y + \frac{1}{y} \right) + c_2 = 0 \qquad x, y \in \mathbb{C}^*$$

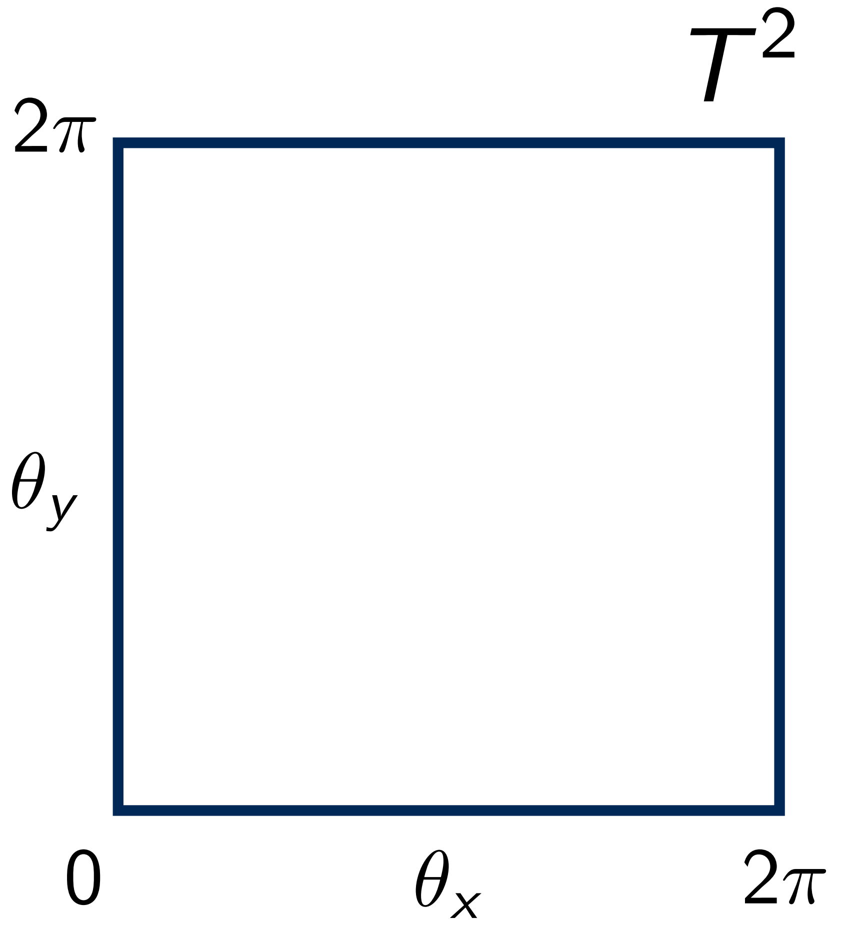
Coamoeba

$$(x = r_x e^{i\theta_x}, y = r_y e^{i\theta_y}) \ \mapsto \ (\theta_x, \theta_y) \in T^2$$

choice:

$c_1 = -3 + 6i$

$c_2 = -3 - 6i$



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Coamoeba

$$(x = r_x e^{i\theta_x}, y = r_y e^{i\theta_y}) \;\mapsto\; (\theta_x, \theta_y) \in T^2$$

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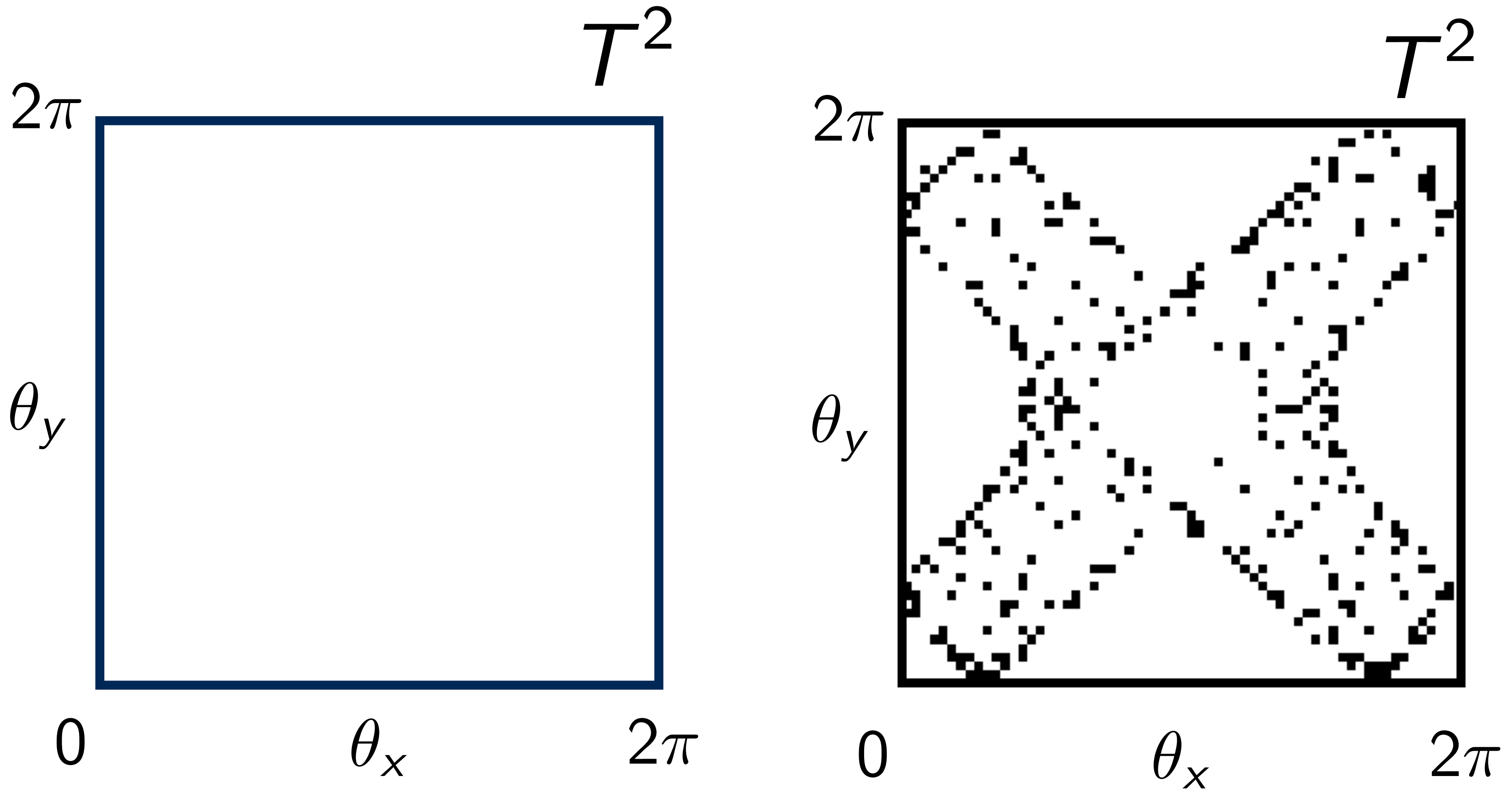

mapped solutions: $N_\theta = 0$ $N_\theta = 100$

Tropical Geometry

[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

- **Coamoeba Projection**

We can visualize the mirror curve under a **projection onto the 2-torus** known as the coamoeba projection.

Example

Hirzebruch

F_0

$$\Sigma \ : \ P(x,y) = x + \frac{1}{x} + c_1 \left(y + \frac{1}{y} \right) + c_2 = 0 \qquad x, y \in \mathbb{C}^*$$

Coamoeba

$$(x = r_x e^{i\theta_x}, y = r_y e^{i\theta_y}) \ \mapsto \ (\theta_x, \theta_y) \in T^2$$

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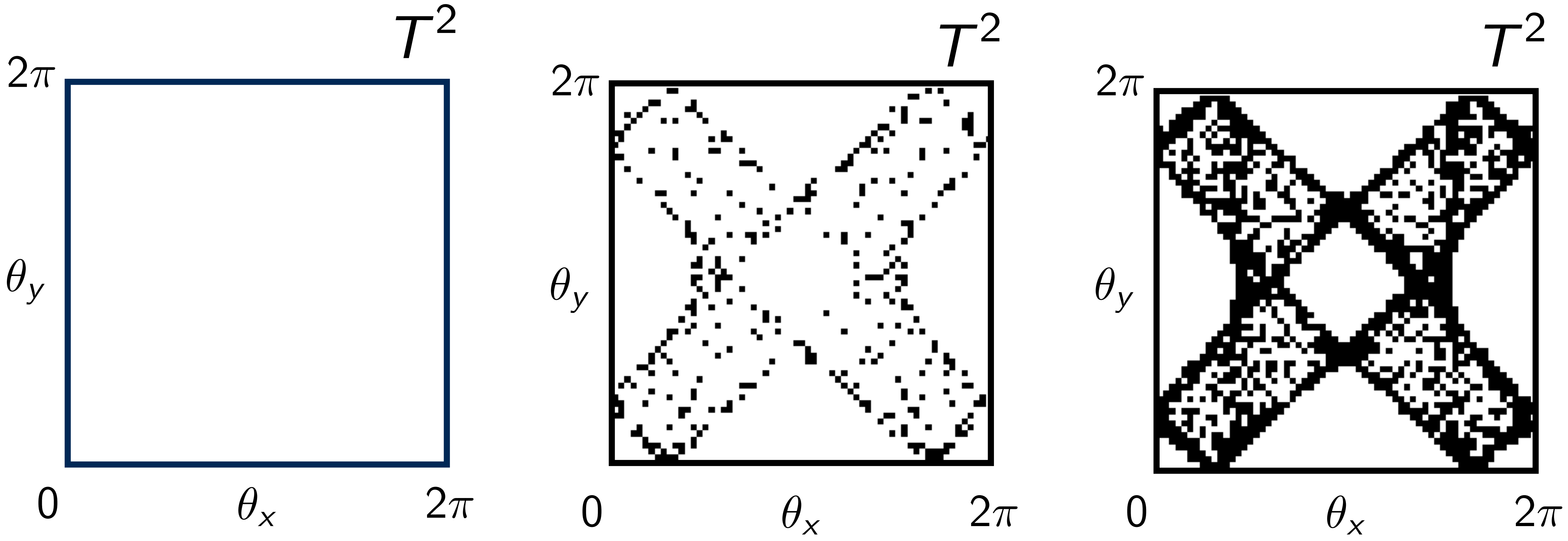

mapped solutions: $N_\theta = 0$ $N_\theta = 100$ $N_\theta = 1000$

Tropical Geometry

[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

- **Coamoeba Projection**

We can visualize the mirror curve under a **projection onto the 2-torus** known as the coamoeba projection.

Example

Hirzebruch

F_0

$$\Sigma \; : \; P(x,y) = x + \frac{1}{x} + c_1 \left(y + \frac{1}{y} \right) + c_2 = 0 \qquad x, y \in \mathbb{C}^*$$

Coamoeba

$$(x = r_x e^{i\theta_x}, y = r_y e^{i\theta_y}) \; \mapsto \; (\theta_x, \theta_y) \in T^2$$

choice:

$c_1 = -3 + 6i$

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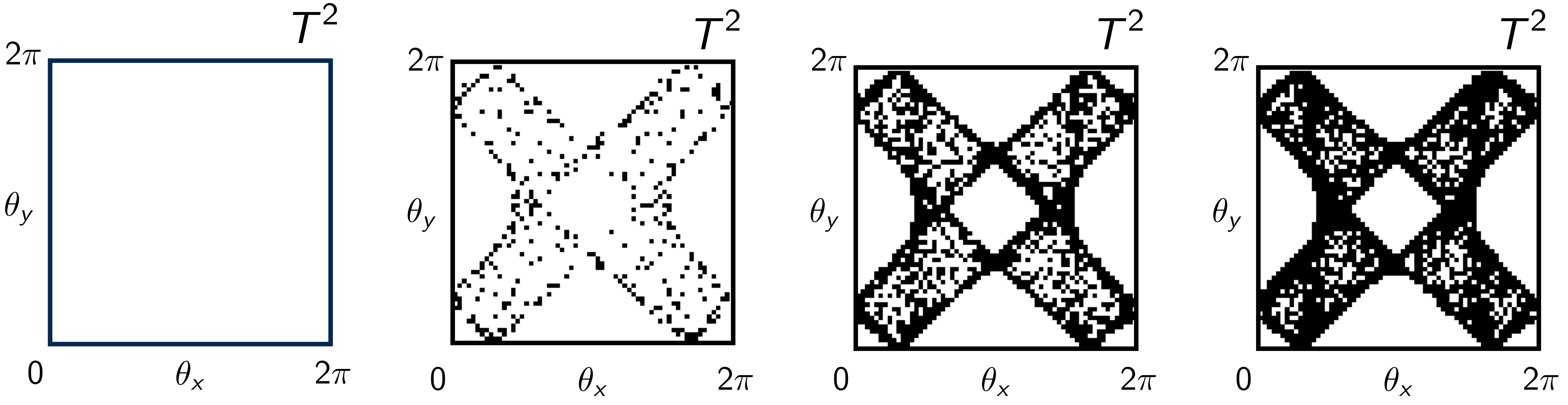


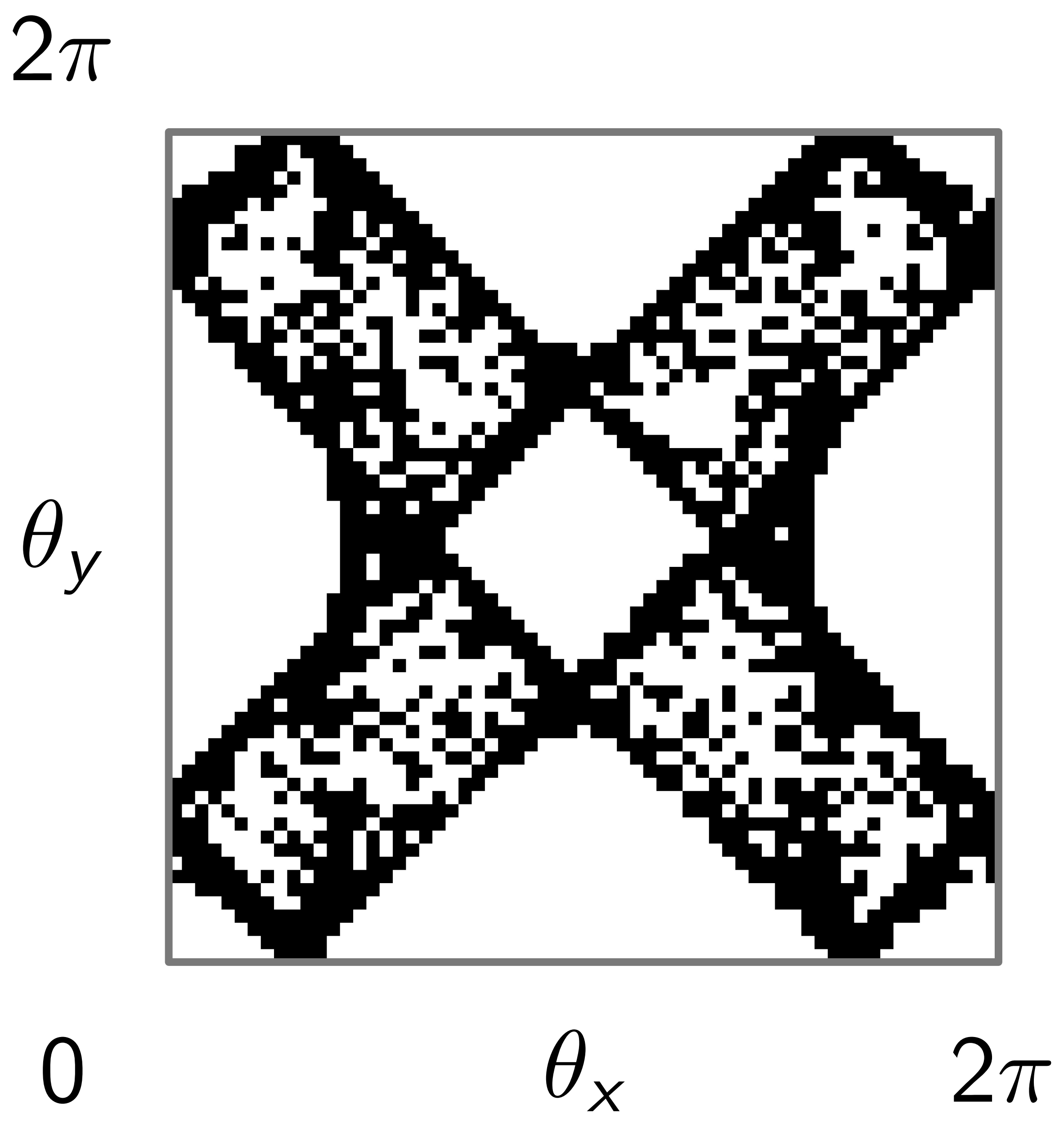
mapped solutions: $N_\theta = 0$ $\quad N_\theta = 100$ $\quad N_\theta = 1000$ $\quad N_\theta = 2000$

Tropical Geometry

[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

Example Hirzebruch F_0

$$P(x,y) = x + \frac{1}{x} + c_1\left(y + \frac{1}{y}\right) + c_2 \qquad c_1, c_2 \in \mathbb{C}^*$$

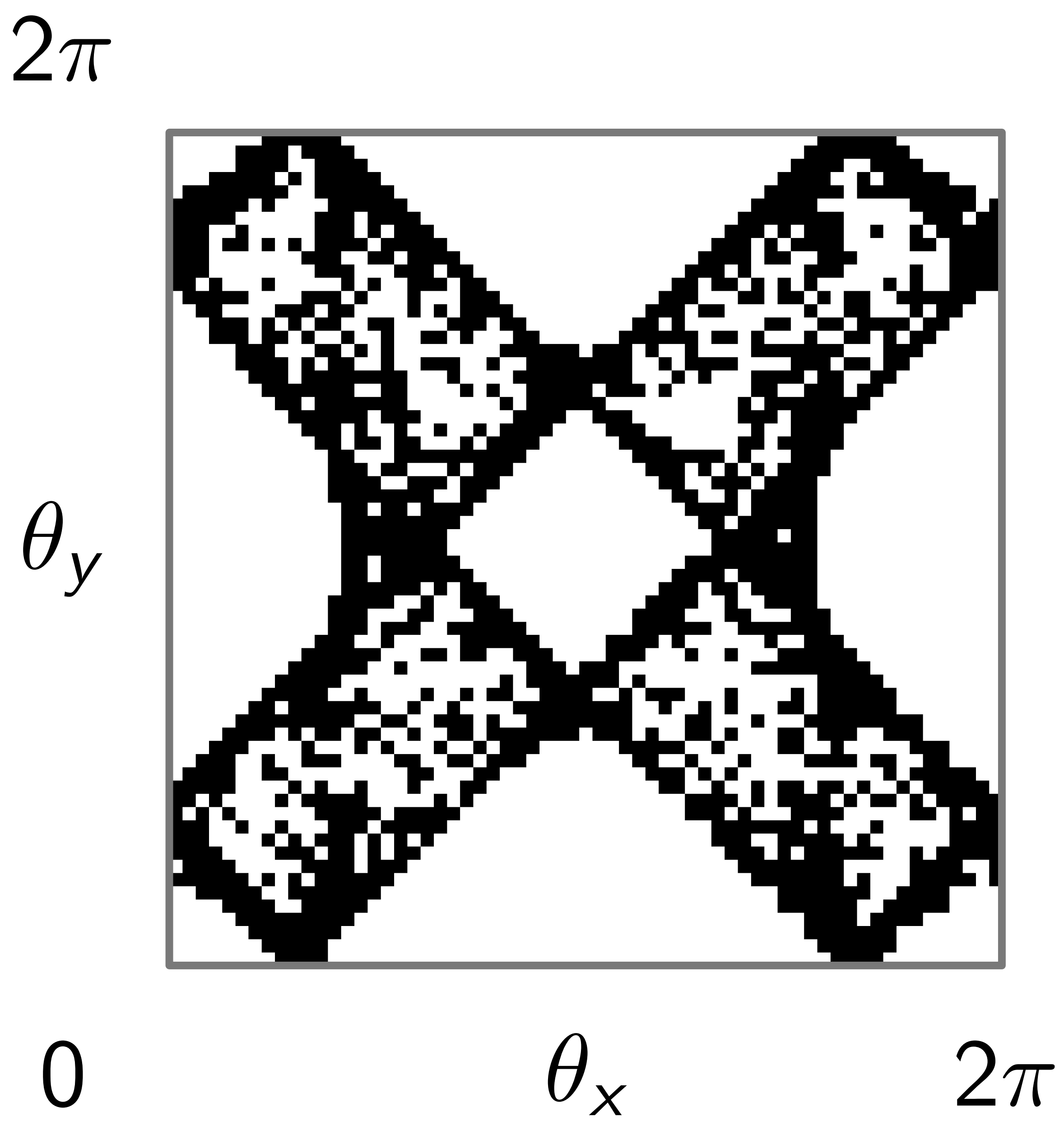


$c_1 = -3 - 6i\ ,\ c_2 = -3 + 6i$

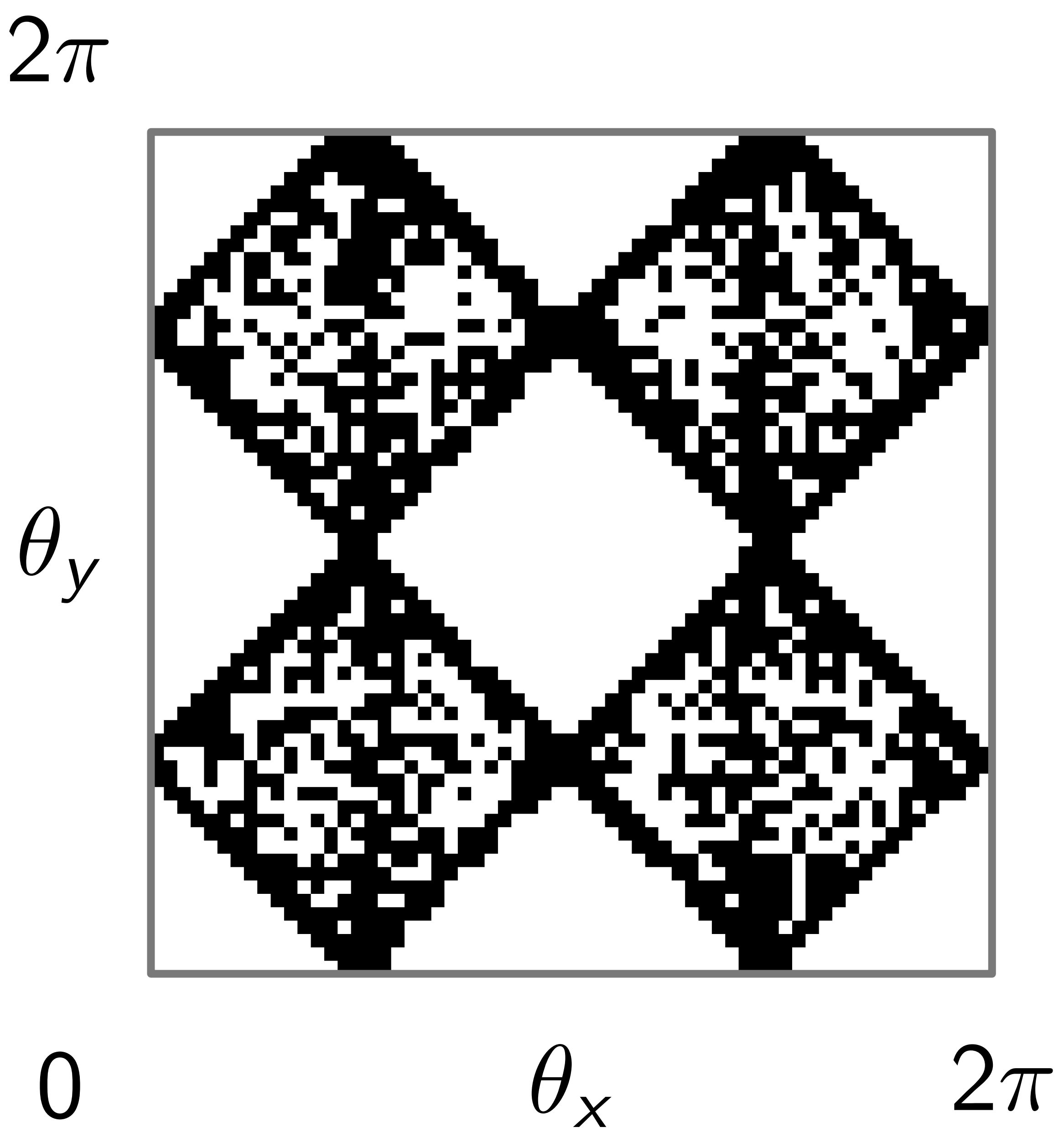
Tropical Geometry

[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

Example Hirzebruch F_0 $P(x,y) = x + \frac{1}{x} + c_1\left(y + \frac{1}{y}\right) + c_2$ $c_1, c_2 \in \mathbb{C}^*$



$c_1 = -3 - 6i\ ,\ c_2 = -3 + 6i$



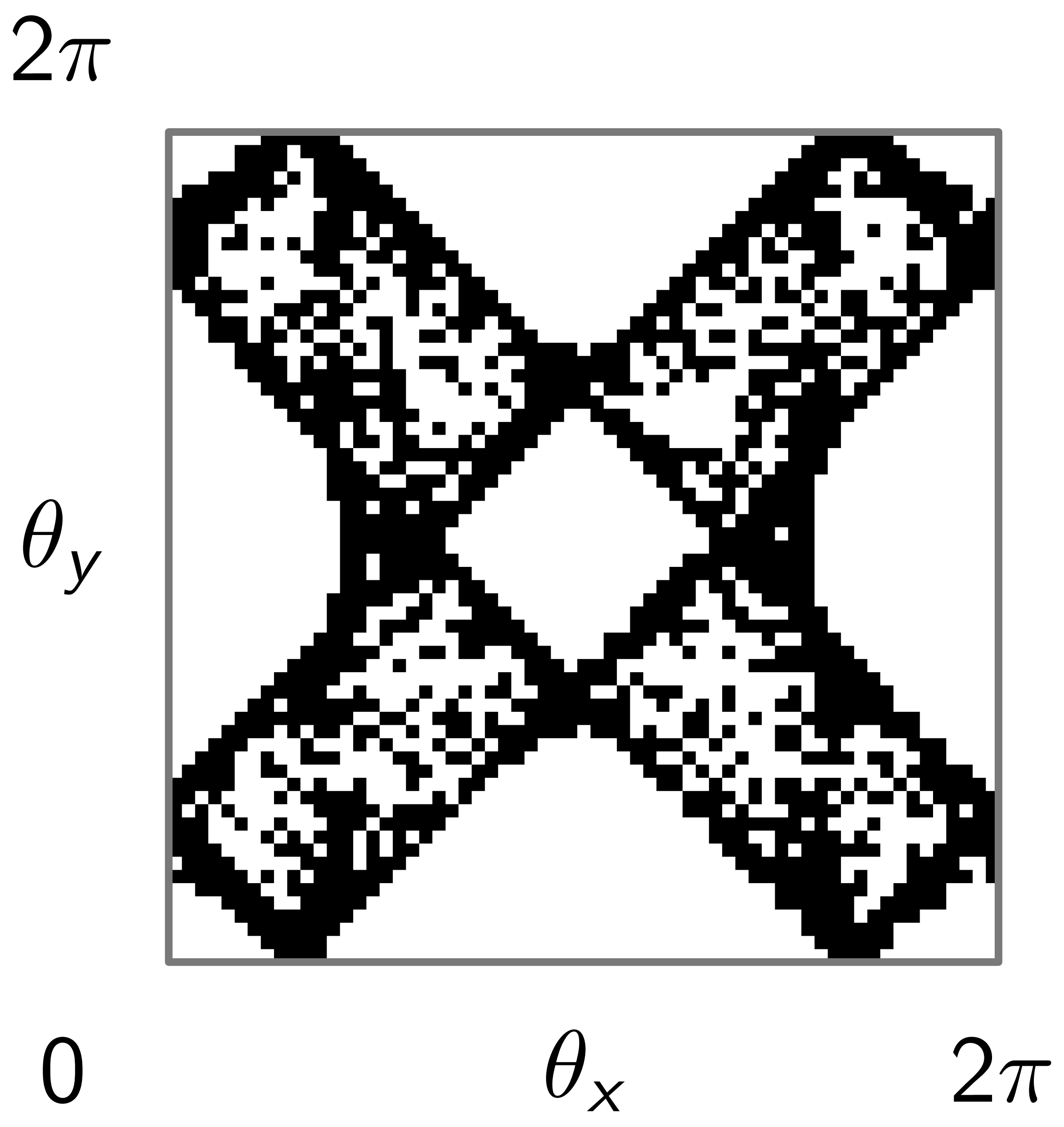
$c_1 = -9i\ ,\ c_2 = -3 - 3i$

Tropical Geometry

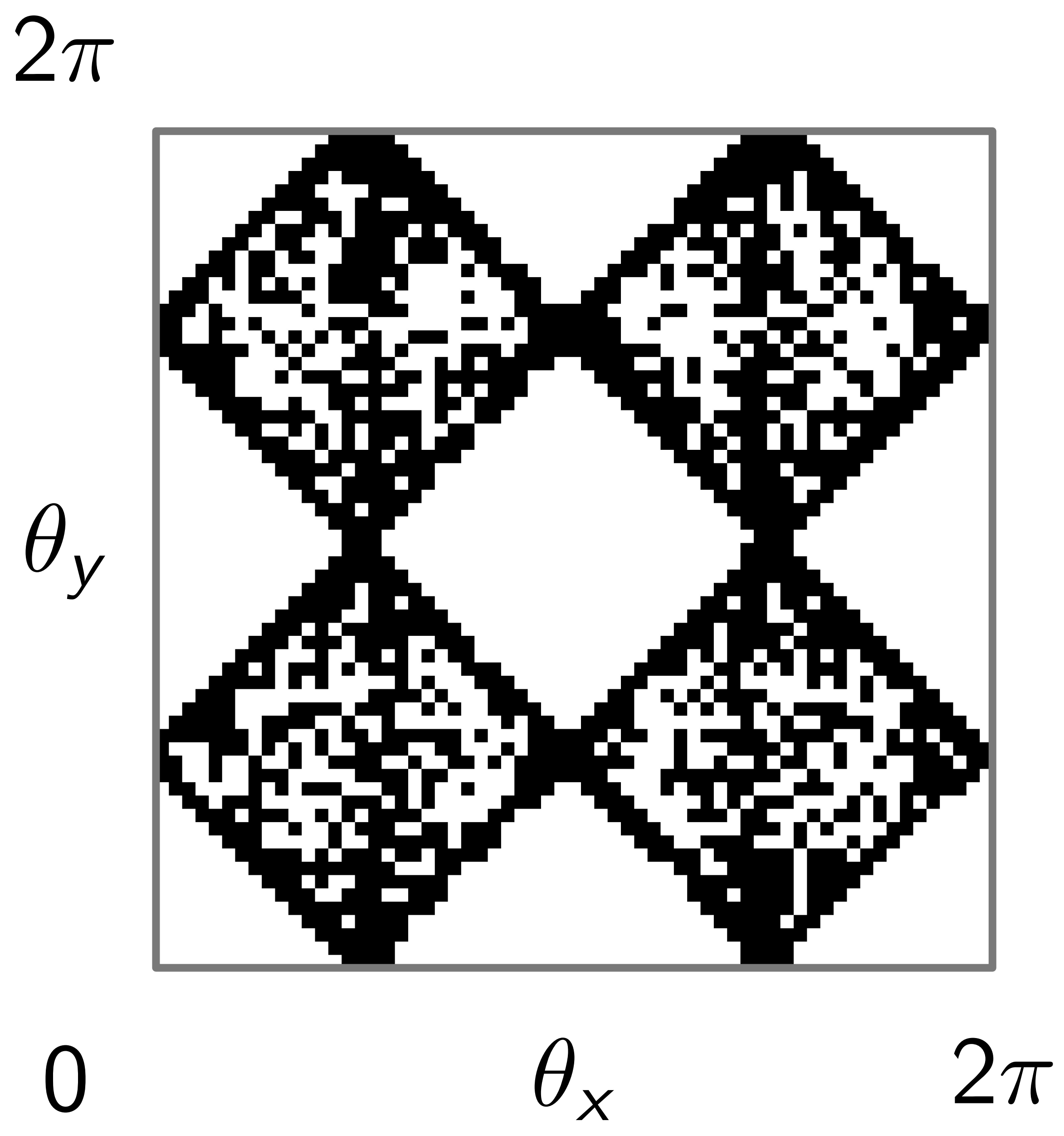
[Feng, He, Kennaway, Vafa 2005]

[Seong 2023]

Example Hirzebruch F_0 $P(x,y) = x + \frac{1}{x} + c_1\left(y + \frac{1}{y}\right) + c_2$ $c_1, c_2 \in \mathbb{C}^*$



$c_1 = -3-6i\ ,\ c_2 = -3+6i$



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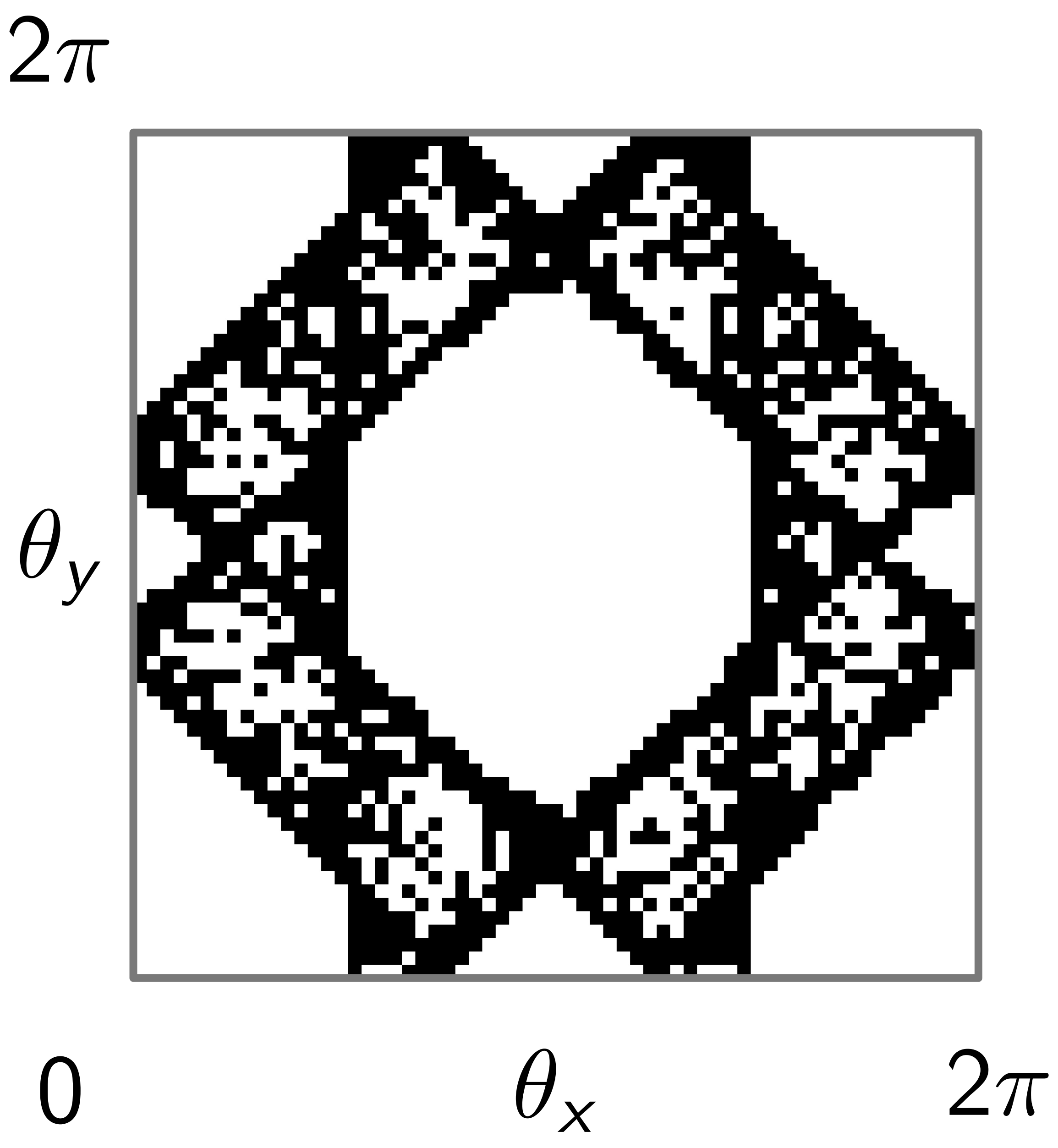
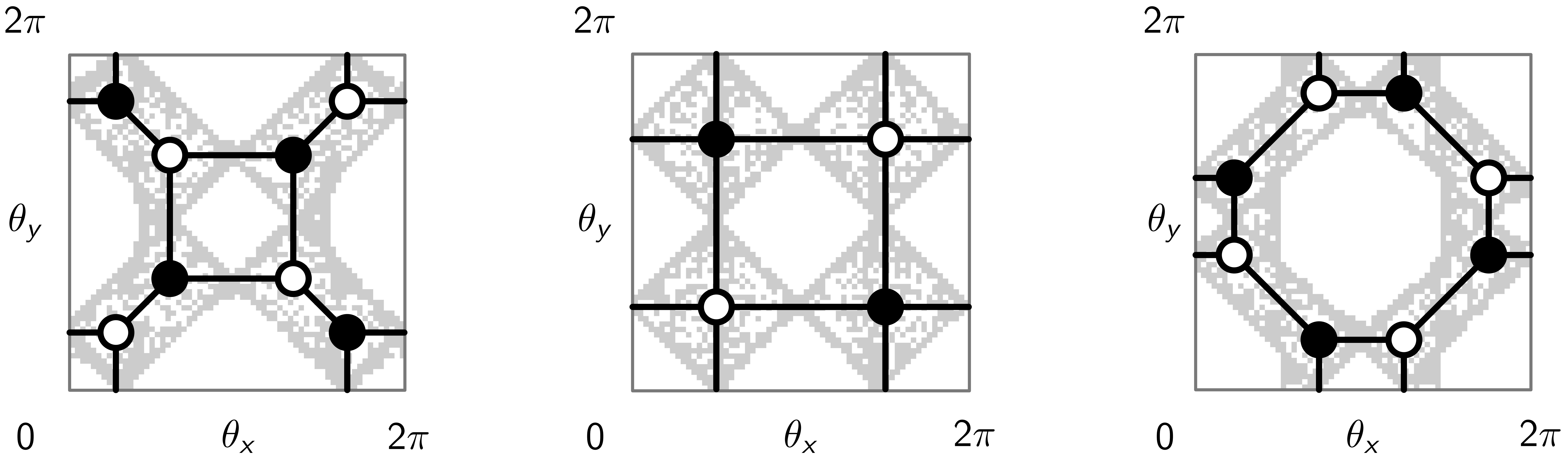


$c_1 = 9+9i\ ,\ c_2 = 9-6i$

Tropical Geometry

[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

Example Hirzebruch F_0 $$P(x,y) = x + \frac{1}{x} + c_1\left(y + \frac{1}{y}\right) + c_2 \qquad c_1, c_2 \in \mathbb{C}^*$$



$c_1 = -3-6i\ ,\ c_2 = -3+6i$ $c_1 = -9i\ ,\ c_2 = -3-3i$ $c_1 = 9+9i\ ,\ c_2 = 9-6i$

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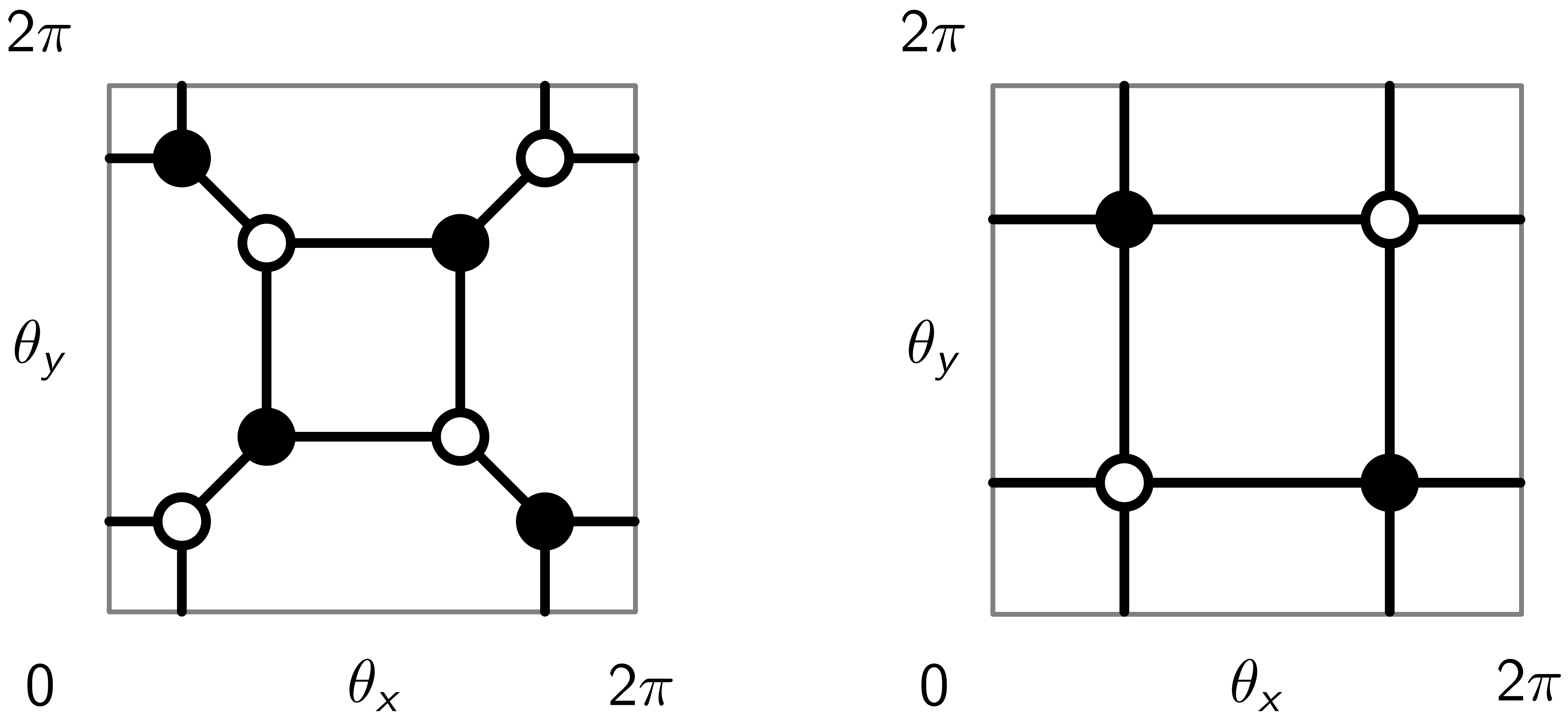
Tropical Geometry

[Feng, He, Kennaway, Vafa 2005]
[Seong 2023]

Example Hirzebruch F_0 $$P(x,y) = x + \frac{1}{x} + c_1\left(y + \frac{1}{y}\right) + c_2 \qquad c_1, c_2 \in \mathbb{C}^*$$

change in the choice of complex structure moduli generates local mutations of the dimer

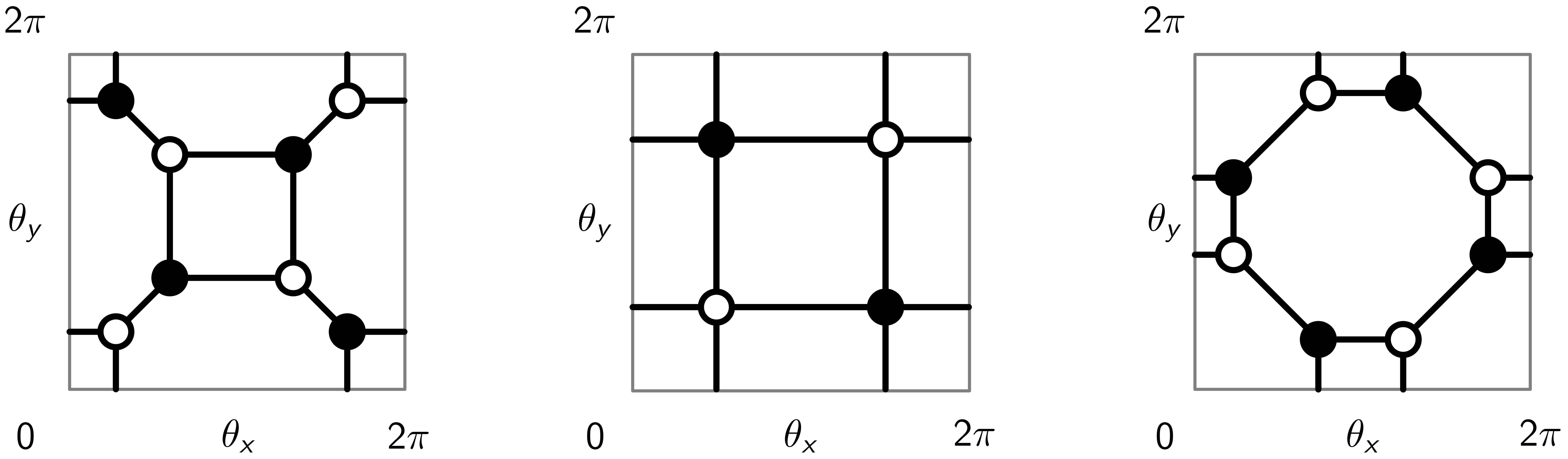


$c_1 = -3 - 6i\ ,\ c_2 = -3 + 6i$ $c_1 = -9i\ ,\ c_2 = -3 - 3i$ $c_1 = 9 + 9i\ ,\ c_2 = 9 - 6i$

Tropical Geometry

[Seong 2023]

different choices of complex structure moduli correspond to Seiberg dual 4d N=1 theories



$c_1 = -3 - 6i \ , \ c_2 = -3 + 6i$ $\qquad$ $c_1 = -9i \ , \ c_2 = -3 - 3i$ $\qquad$ $c_1 = 9 + 9i \ , \ c_2 = 9 - 6i$

Seiberg dual $\qquad$ **Seiberg dual**

Tropical Geometry

[Seong 2023]

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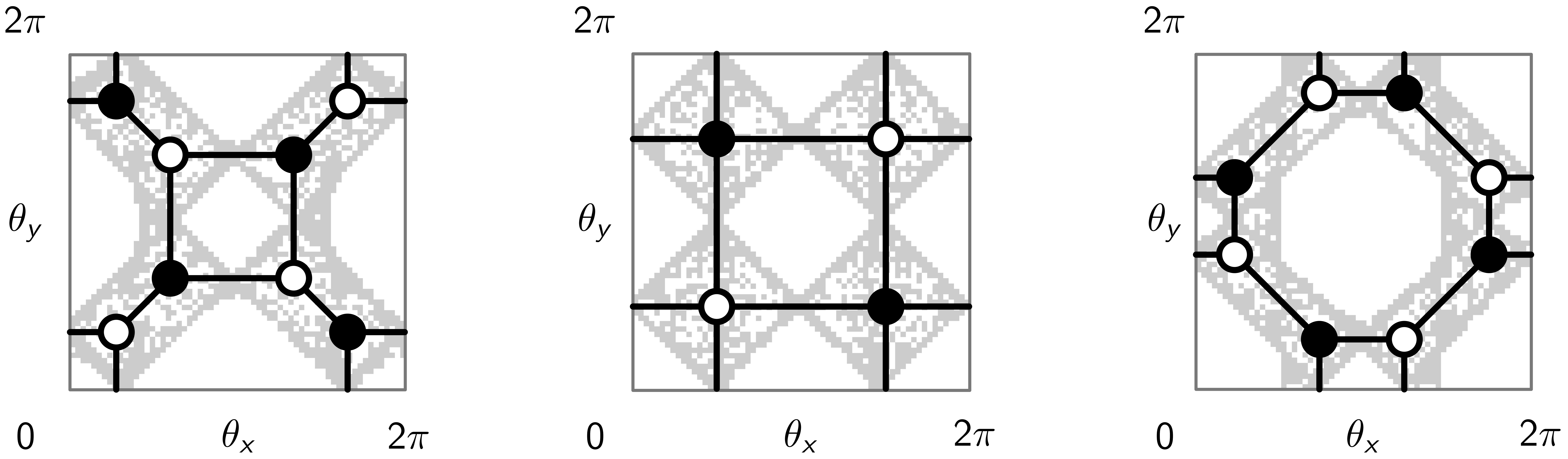

$c_1 = -3 - 6i\ ,\ c_2 = -3 + 6i$ $\qquad$ $c_1 = -9i\ ,\ c_2 = -3 - 3i$ $\qquad$ $c_1 = 9 + 9i\ ,\ c_2 = 9 - 6i$

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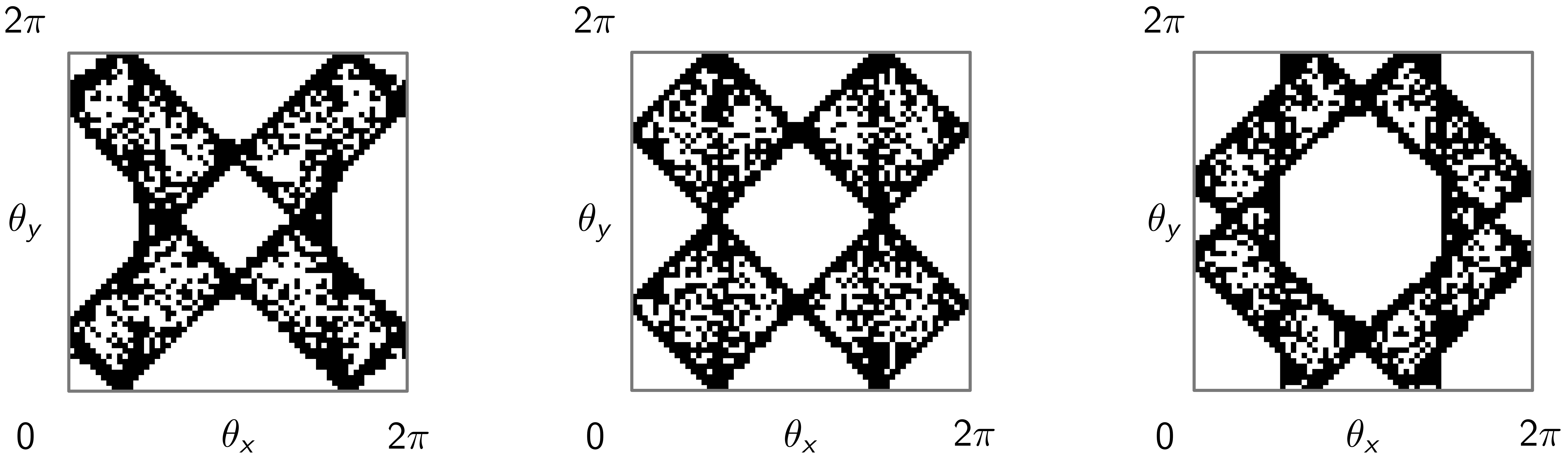

$c_1 = -3 - 6i \; , \; c_2 = -3 + 6i$ $\quad$ $c_1 = -9i \; , \; c_2 = -3 - 3i$ $\quad$ $c_1 = 9 + 9i \; , \; c_2 = 9 - 6i$

Seiberg dual $\quad$ **Seiberg dual**

[Seong 2023]

QUESTION

Can an unsupervised ML model learn about the following?

1. Phase Transitions and Seiberg Duality

2. Phase Spaces of SUSY Gauge Theories

2023

[Seong 2023]

PHYSICAL REVIEW D **108,** 106009 (2023)

Unsupervised machine learning techniques for exploring tropical coamoeba, brane tilings and Seiberg duality

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(Received 13 September 2023; accepted 13 October 2023; published 8 November 2023)

We introduce unsupervised machine learning techniques in order to identify toric phases of $4d$ $\mathcal{N}=1$ supersymmetric gauge theories corresponding to the same toric Calabi-Yau 3-fold. These $4d$ $\mathcal{N}=1$ supersymmetric gauge theories are world volume theories of a D3-brane probing a toric Calabi-Yau 3-fold and are realized in terms of a type IIB brane configuration known as a brane tiling. It corresponds to the skeleton graph of the coamoeba projection of the mirror curve associated to the toric Calabi-Yau 3-fold. When we vary the complex structure moduli of the mirror Calabi-Yau 3-fold, the coamoeba and the corresponding brane tilings change their shape, giving rise to different toric phases related by Seiberg duality. We illustrate that by employing techniques such as principal component analysis and t-distributed stochastic neighbor embedding, we can project the space of coamoeba labeled by complex structure moduli down to a lower-dimensional phase space with phase boundaries corresponding to Seiberg duality. In this work, we illustrate this technique by obtaining a 2-dimensional phase diagram for brane tilings corresponding to the cone over the zeroth Hirzebruch surface F_0.

DOI: 10.1103/PhysRevD.108.106009

I. INTRODUCTION

The world volume theories of a D3-brane probing a toric Calabi-Yau 3-fold [1,2] form a very rich class of $4d$ $\mathcal{N}=1$ supersymmetric gauge theories [3–8]. These supersymmetric gauge theories are realized by a type IIB brane configuration that takes the form of a bipartite periodic graph on a 2-torus. Such bipartite graphs have been extensively studied in mathematics as dimers [9,10], and

where $P(x,y)$ is the Newton polynomial in $x, y \in \mathbb{C}^*$ of the toric diagram Δ corresponding to the toric Calabi-Yau 3-fold. The Newton polynomial is defined as

$$P(x,y) = \sum_{(n_x,n_y)\in\Delta} c_{(n_x,n_y)} x^{n_x} y^{n_y}, \qquad (1.2)$$

where $(n_x,n_y) \in \mathbb{Z}^2$ are the coordinates of the vertices

Seong
Phys. Rev. D **108**, 106009 (2023)
[arXiv:2309.05702]

UNIST

Unsupervised Machine Learning

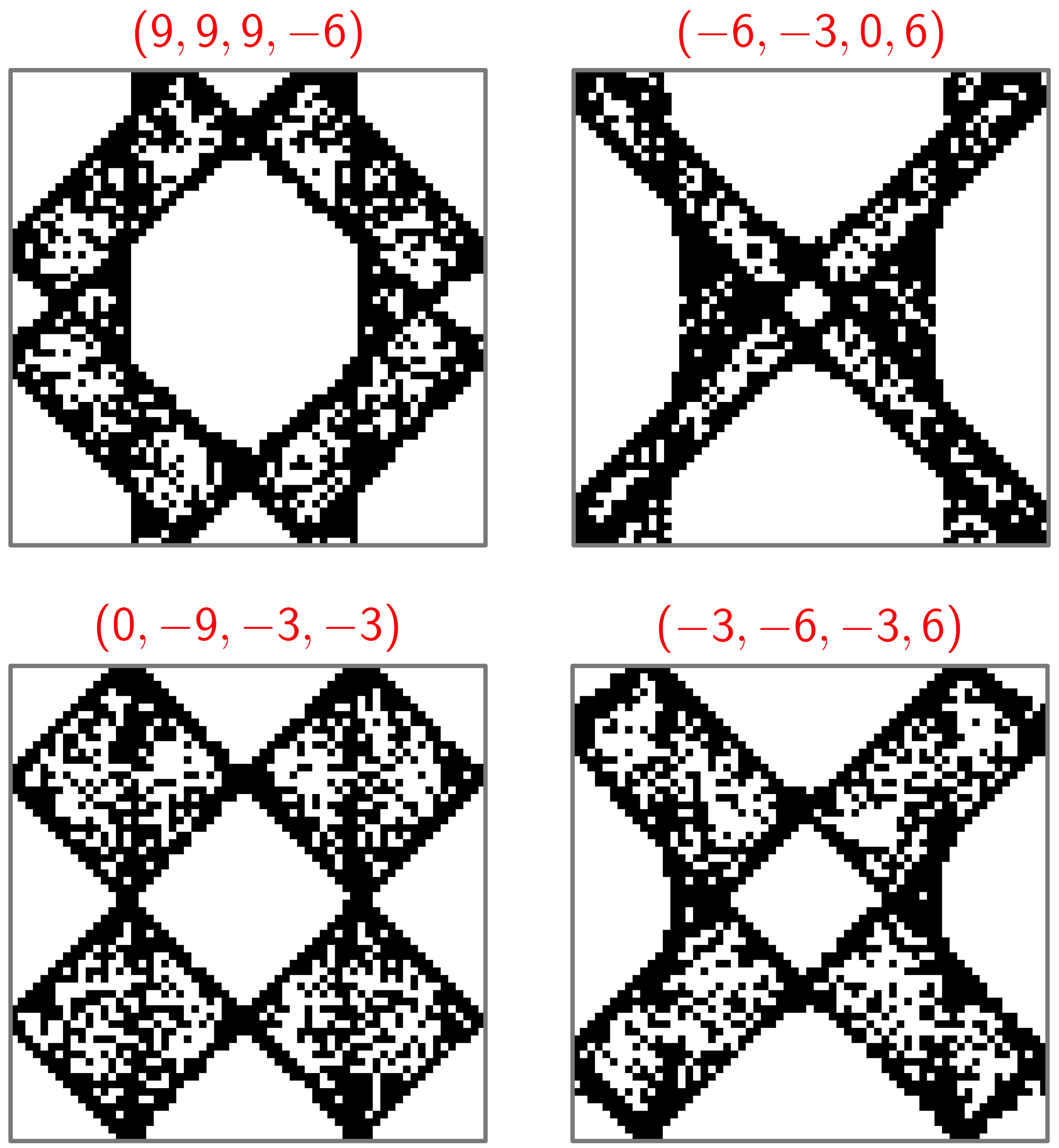
[Seong 2023]

- **Principal Component Analysis (CPA)**

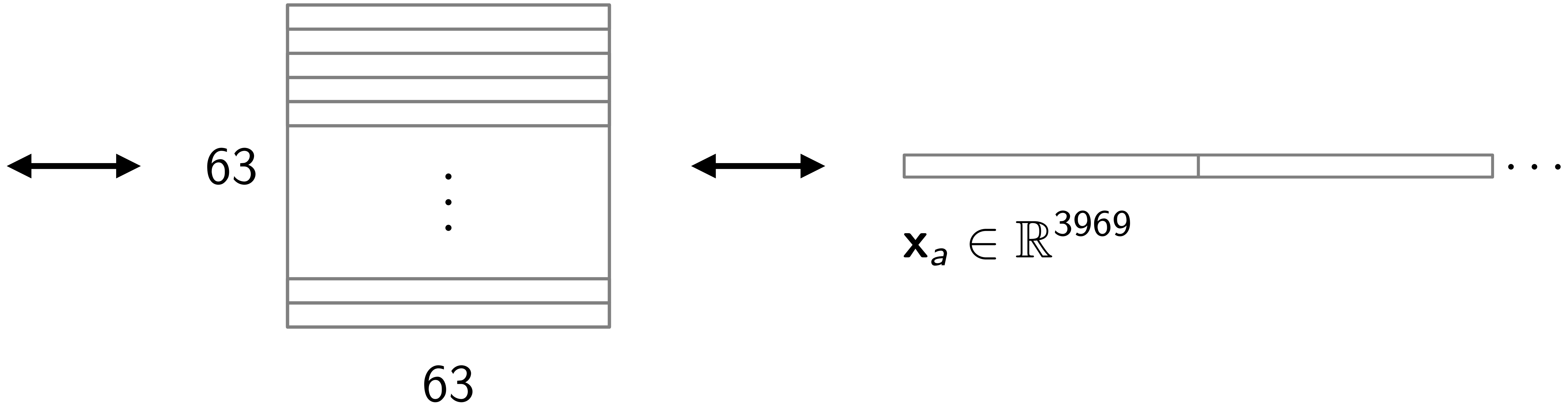
$$\Sigma \ : \ P(x,y) = x + \frac{1}{x} + (c_{11} + i c_{12})\left(y + \frac{1}{y}\right) + (c_{21} + i c_{22}) = 0$$

sample domain: $c_{ij} \in \{-9,-6,-3,0,3,6,9\}$ $\quad N = 2304$

The coamoeba projections of the mirror curve for **different complex structure moduli** become a set of 3969-dimensional vectors.



flatten the coamoeba projection as a vector in $\mathbb{R}^{3969}$



Unsupervised Machine Learning

[Seong 2023]

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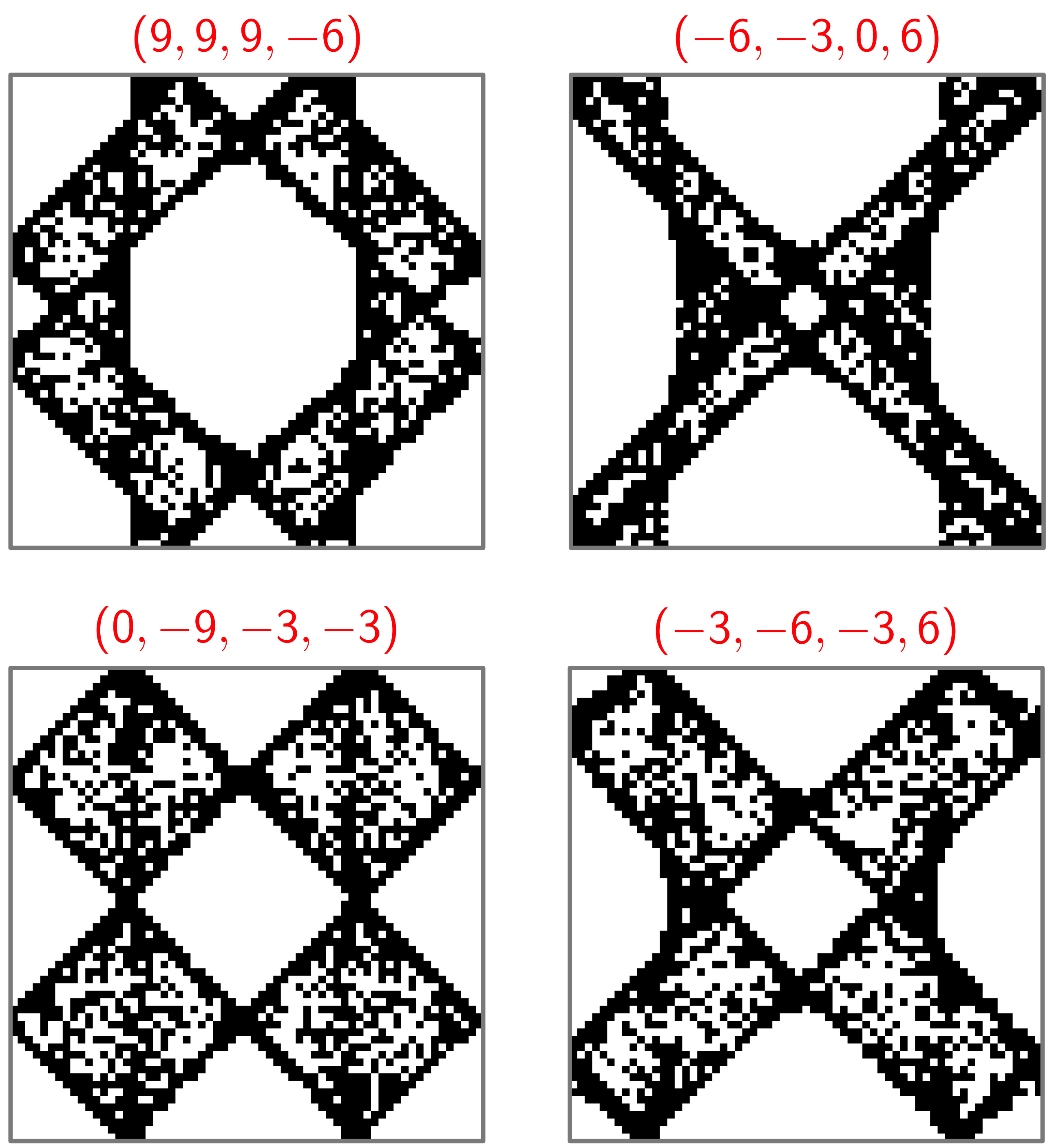

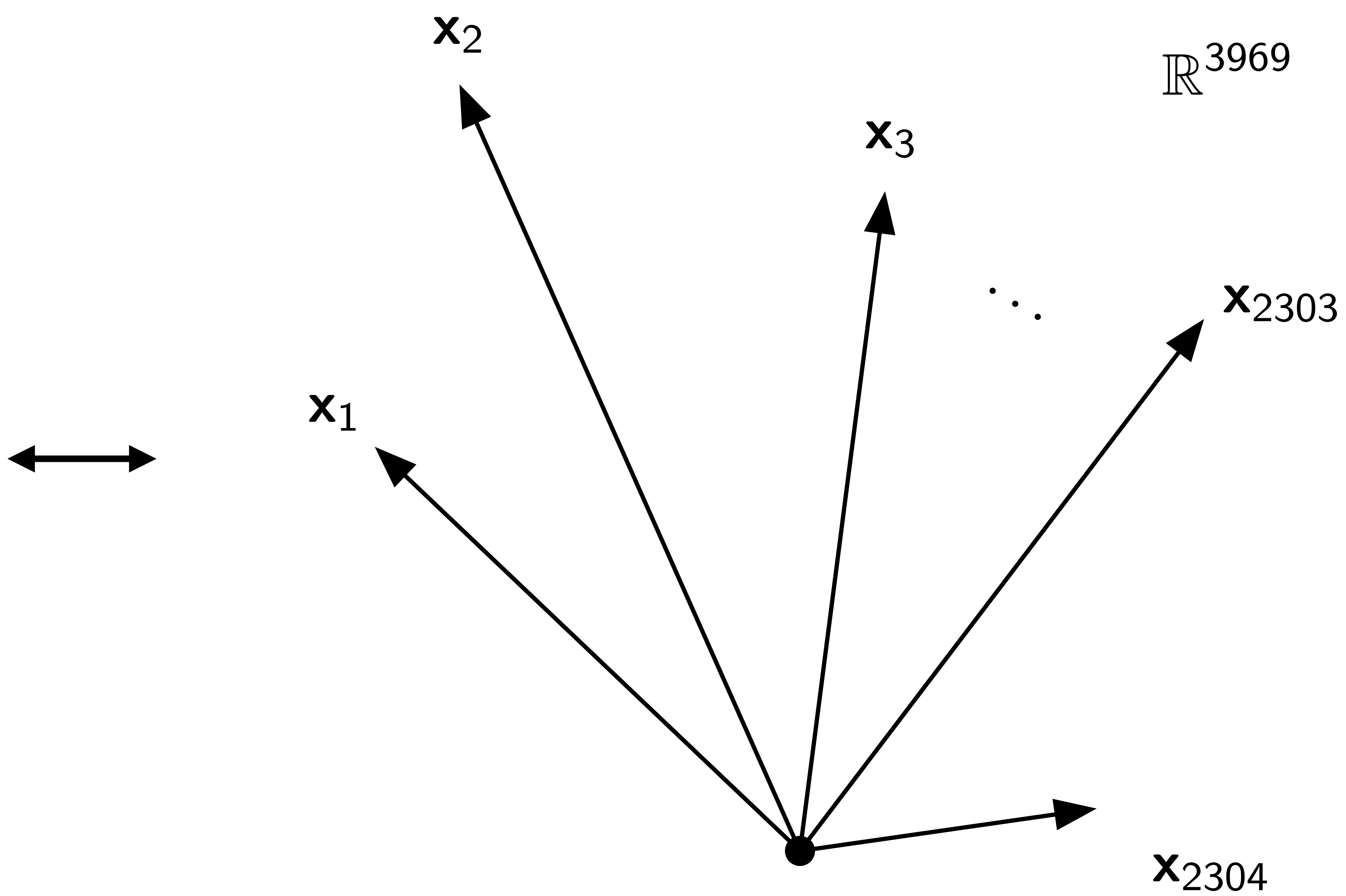

Unsupervised Machine Learning

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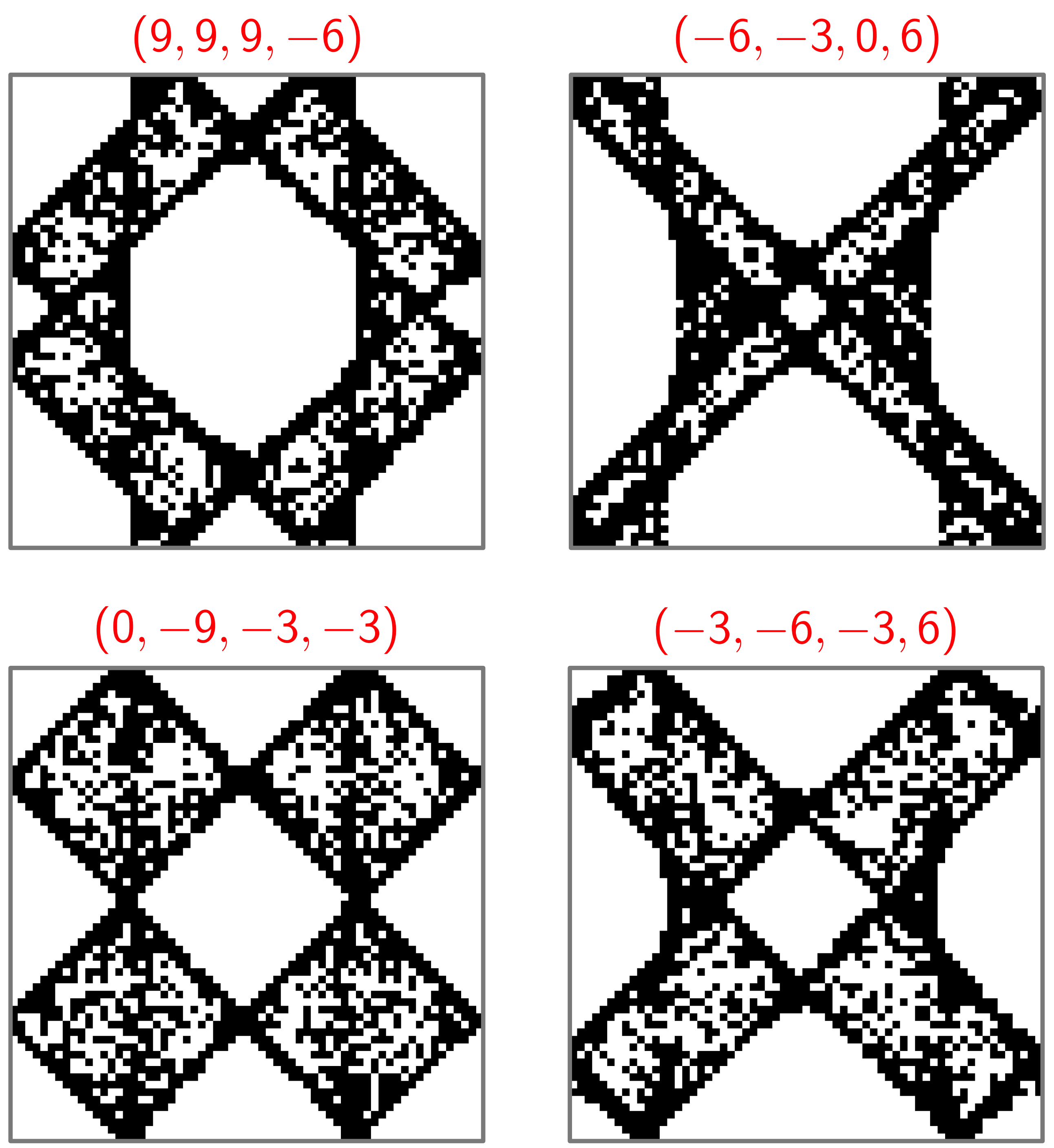
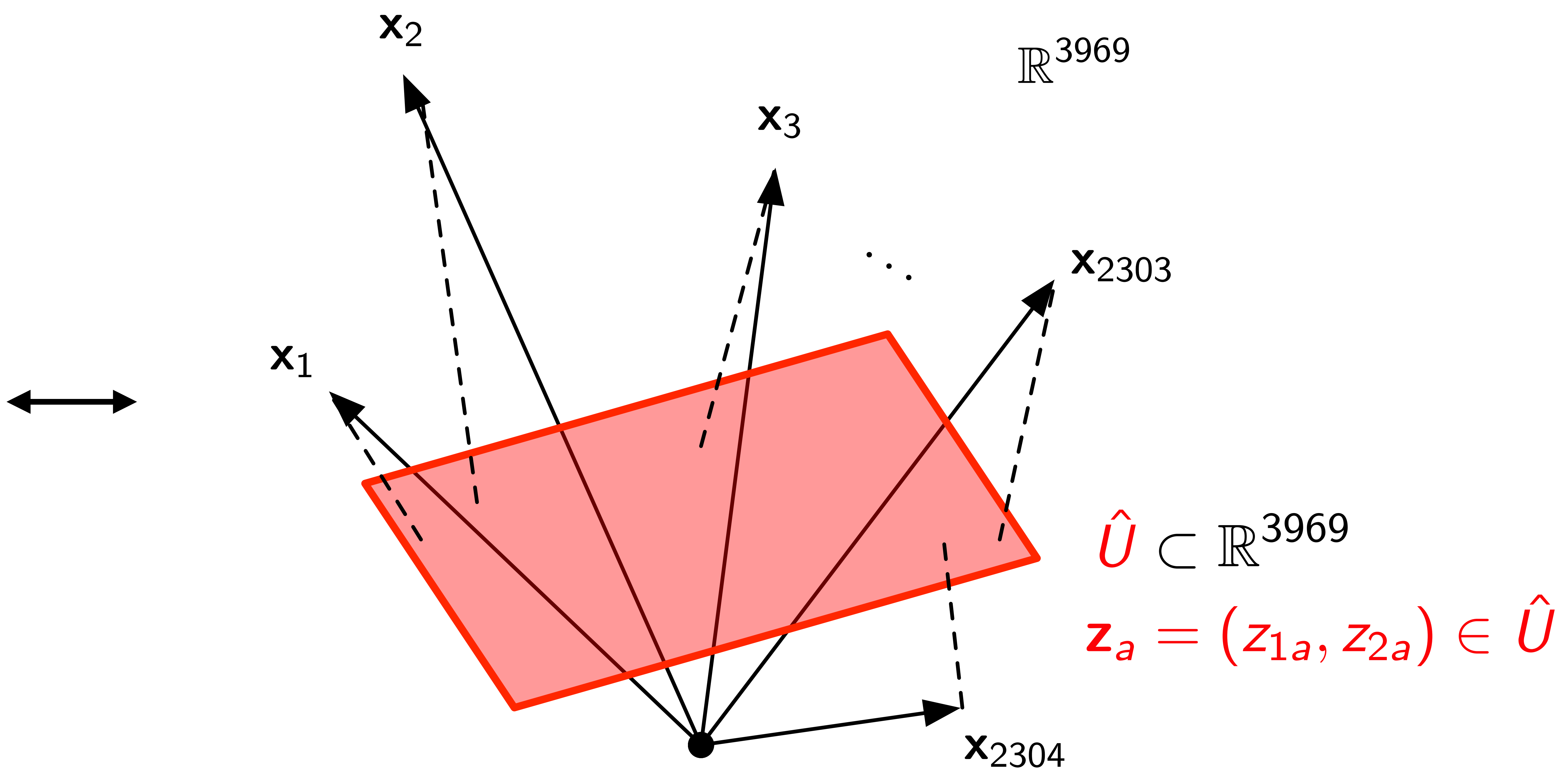
sample domain: $c_{ij} \in \{-9,-6,-3,0,3,6,9\}$ $\quad N = 2304$

Using PCA, we want to find an **optimal 2-dimensional subspace** of the original 3969-dimensional vector space representation of the coamoeba.

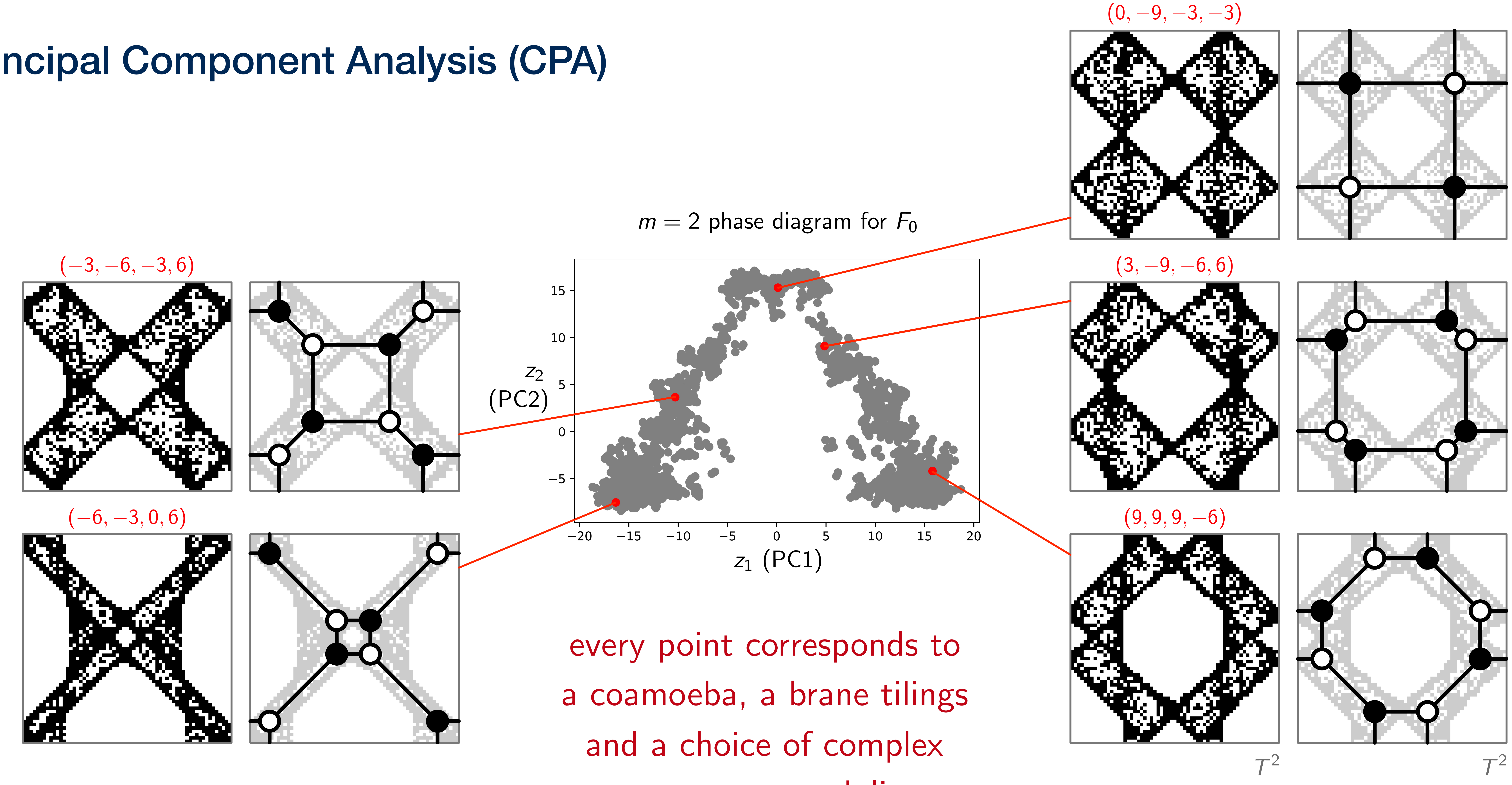




ML-generated Phase Space

[Seong 2023]

- Principal Component Analysis (CPA)



every point corresponds to a coamoeba, a brane tilings and a choice of complex structure moduli

Generative AI

[Seong 2024]

QUESTION

Can a Generative AI model learn the following?

1. Coamoeba

2. Brane Configurations

3. 4d N=1 Gauge Theories

UNIST

2024

[Seong 2024]

PHYSICAL REVIEW D **111,** 086013 (2025)

Generative AI for brane configurations and coamoeba

Rak-Kyeong Seong*
Department of Mathematical Sciences, and Department of Physics,
Ulsan National Institute of Science and Technology, 50 UNIST-gil, Ulsan 44919, South Korea

(Received 26 November 2024; accepted 19 March 2025; published 10 April 2025)

We introduce a generative AI model to obtain type IIB brane configurations that realize toric phases of a family of 4D $\mathcal{N}=1$ supersymmetric gauge theories. These 4D $\mathcal{N}=1$ quiver gauge theories are world volume theories of a D3-brane probing a toric Calabi-Yau 3-fold. The type IIB brane configurations are given by the coamoeba projection of the mirror curve associated with the toric Calabi-Yau 3-fold. The shape of the mirror curve and its coamoeba projection, as well as the corresponding type IIB brane configuration and the toric phase of the 4d $\mathcal{N}=1$ theory, all depend on the complex structure moduli parametrizing the mirror curve. We train a generative AI model, a conditional variational autoencoder (CVAE), that takes a choice of complex structure moduli as input and generates the corresponding coamoeba. This enables us not only to obtain a high-resolution representation of the entire phase space for a family of 4D $\mathcal{N}=1$ theories corresponding to the same toric Calabi-Yau 3-fold, but also to continuously track the movements of the mirror curve and the branes wrapping the curve in the corresponding type IIB brane configurations during phase transitions associated with Seiberg duality.

DOI: 10.1103/PhysRevD.111.086013

I. INTRODUCTION

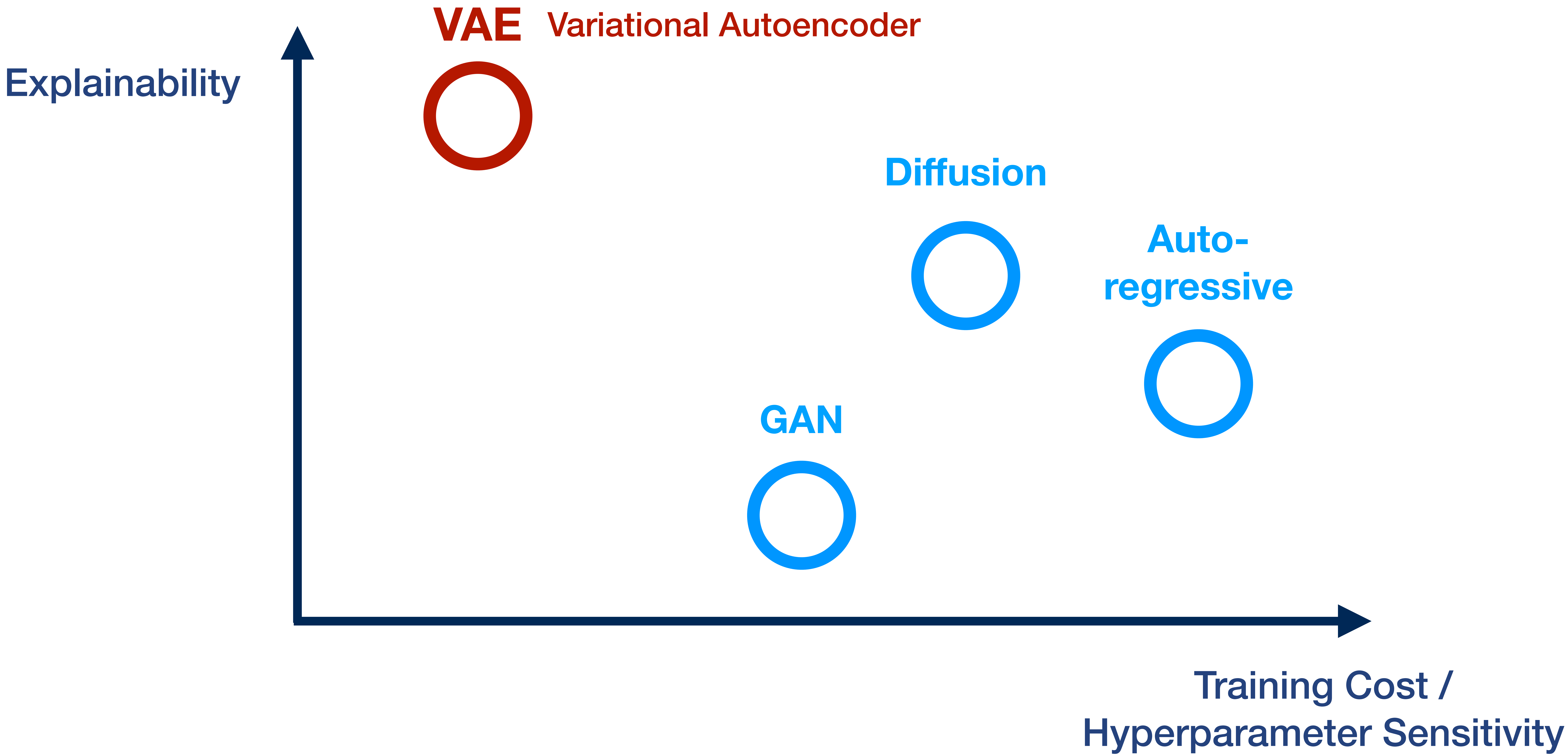
Since the pioneering works in [1–5], machine learning has pushed the boundaries of what is computationally possible in quantum field theory and string theory [6–21] as well as in geometry and topology [22–36]. In this work, we advance this development by introducing a generative AI model in order to study a family of type IIB brane configurations that realize 4D $\mathcal{N}=1$ supersymmetric gauge theories corresponding to a toric Calabi-Yau 3-fold. These 4D $\mathcal{N}=1$ theories are world volume theories of a D3-brane

quintessential role in the type IIB brane configuration realizing the family of 4D $\mathcal{N}=1$ theories that we are studying in this work. Under T-duality, the probe D3-brane at the Calabi-Yau 3-fold singularity becomes a D5-brane suspended between a NS5-brane that wraps the holomorphic curve Σ in the type IIB brane configuration. Changes to the complex structure moduli in Σ lead to changes in the configuration of D5- and NS5-branes giving rise to transitions between distinct type IIB brane configurations. These transitions can be interpreted as *Seiberg duality* [49] between corresponding *toric phases* of the 4D $\mathcal{N}=1$

Seong
Phys. Rev. D **111**, 086013 (2025)
[arXiv:2411.16033]

Generative AI

[Seong 2024]



Generative AI

[Seong 2024]

- Conditional Variational Autoencoder (CVAE)

$$p(\mathbf{x}|\mathbf{c}) = \int_L p(\mathbf{x}|\mathbf{z},\mathbf{c})\; p(\mathbf{z}|\mathbf{c})\; d\mathbf{z}$$

marginal likelihood: $p(\mathbf{x}|\mathbf{c})$

$\mathbf{x}$: coamoeba (brane configuration)

$\mathbf{c}$: complex structure moduli

L: latent space

$p(\mathbf{x}|\mathbf{z},\mathbf{c})$: likelihood

$p(\mathbf{z}|\mathbf{c})$: prior

coamoeba for a choice of complex structure moduli are modeled in terms of **probability distributions**

Generative AI

[Seong 2024]

- Conditional Variational Autoencoder (CVAE)

marginal likelihood

$$p(\mathbf{x}|\mathbf{c}) = \int_L p(\mathbf{x}|\mathbf{z}, \mathbf{c})\ p(\mathbf{z}|\mathbf{c})\ d\mathbf{z}$$

- $\mathbf{x}$: coamoeba (brane configuration)
- $\mathbf{c}$: complex structure moduli
- L: latent space
- $p(\mathbf{x}|\mathbf{z}, \mathbf{c})$: likelihood
- $p(\mathbf{z}|\mathbf{c})$: prior

coamoeba for a choice of complex structure moduli are modeled in terms of **probability distributions**

↓

evidence lower bound (ELBO)

estimated likelihood $p(\mathbf{x}|\mathbf{z}, \mathbf{c}) \sim \mathrm{Bernoulli}_{\theta_d}(x_{ij} = 1)$

estimated prior $p(\mathbf{z}|\mathbf{c}) \sim \mathcal{N}(\mathbf{z}; 0, 1)$

$$\log p(\mathbf{x}|\mathbf{c}) \geq E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}\left[\log p(\mathbf{x}|\mathbf{z}, \mathbf{c})\right] + E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}\left[\log p(\mathbf{z}|\mathbf{c})\right] - E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}\left[\log p(\mathbf{z}|\mathbf{x}, \mathbf{c})\right]$$

estimated posterior $p(\mathbf{z}|\mathbf{x}, \mathbf{c}) \sim \mathcal{N}(\mathbf{z}; \mu(\theta_e), \sigma^2(\theta_e))$

Generative AI

[Seong 2024]

- Conditional Variational Autoencoder (CVAE)

evidence lower bound (ELBO)

estimated likelihood $p(\mathbf{x}|\mathbf{z},\mathbf{c}) \sim \text{Bernoulli}_{\theta_d}(x_{ij}=1)$

estimated prior $p(\mathbf{z}|\mathbf{c}) \sim \mathcal{N}(\mathbf{z};0,1)$

$$\log p(\mathbf{x}|\mathbf{c}) \geq E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}[\log p(\mathbf{x}|\mathbf{z},\mathbf{c})] + E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}[\log p(\mathbf{z}|\mathbf{c})] - E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}[\log p(\mathbf{z}|\mathbf{x},\mathbf{c})]$$

estimated posterior $q_{\theta_e}(\mathbf{z}|\mathbf{x},\mathbf{c}) \sim \mathcal{N}(\mathbf{z};\mu(\theta_e),\sigma^2(\theta_e))$

Generative AI

[Seong 2024]

- Conditional Variational Autoencoder (CVAE)

evidence lower bound (ELBO)

estimated likelihood $p(\mathbf{x}|\mathbf{z},\mathbf{c}) \sim \text{Bernoulli}_{\theta_d}(x_{ij}=1)$

estimated prior $p(\mathbf{z}|\mathbf{c}) \sim \mathcal{N}(\mathbf{z};0,1)$

$$\log p(\mathbf{x}|\mathbf{c}) \geq E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}[\log p(\mathbf{x}|\mathbf{z},\mathbf{c})] + E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}[\log p(\mathbf{z}|\mathbf{c})] - E_{p(\mathbf{z}|\mathbf{x},\mathbf{c})}[\log p(\mathbf{z}|\mathbf{x},\mathbf{c})]$$

estimated posterior $q_{\theta_e}(\mathbf{z}|\mathbf{x},\mathbf{c}) \sim \mathcal{N}(\mathbf{z};\mu(\theta_e),\sigma^2(\theta_e))$

Reconstruction Loss

↓

DECODER

θ_d

Kullback-Leibler (KL) Divergence

↓

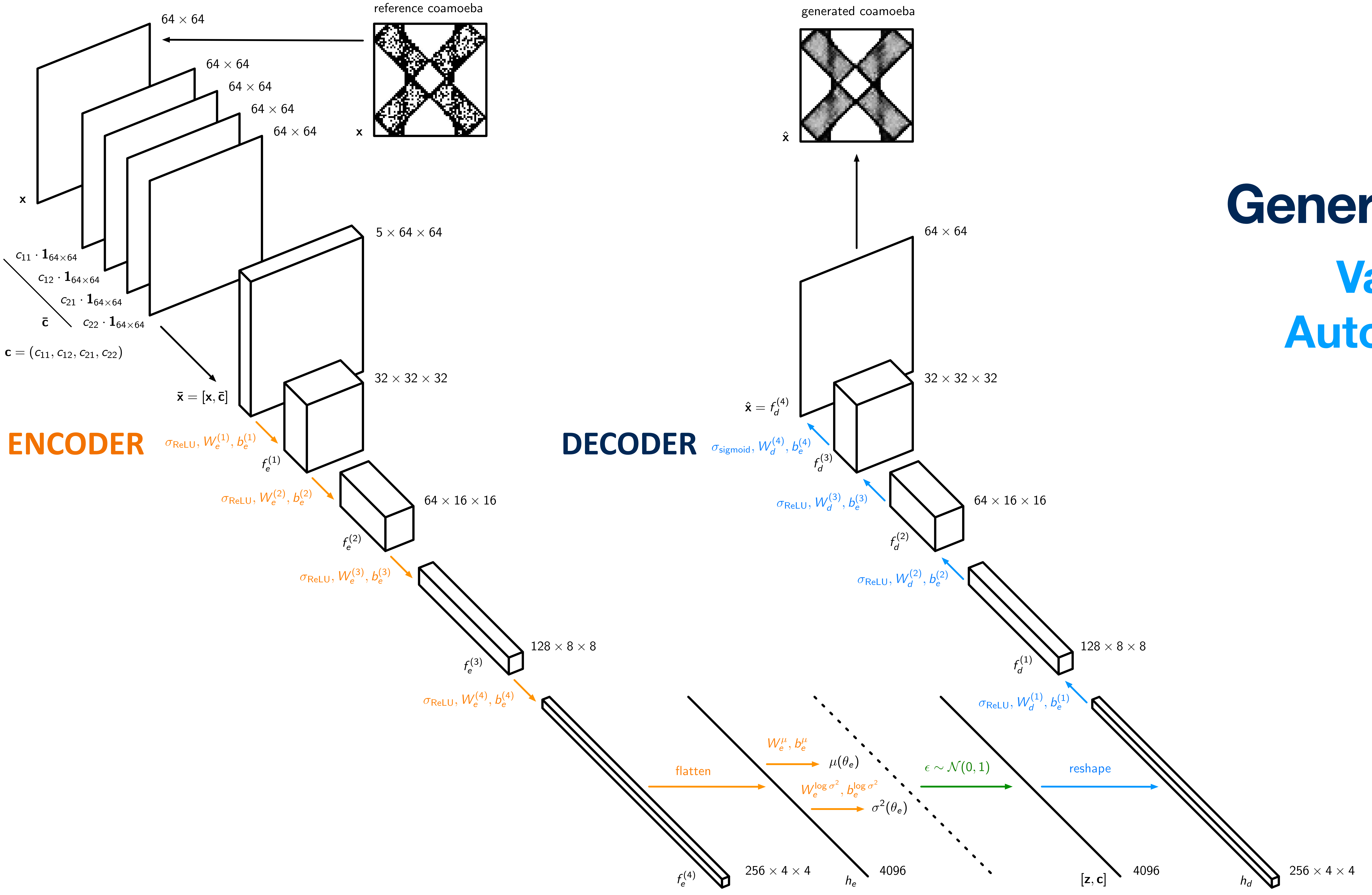
ENCODER

θ_e

[Seong 2024]

Generative AI

Variational Autoencoder

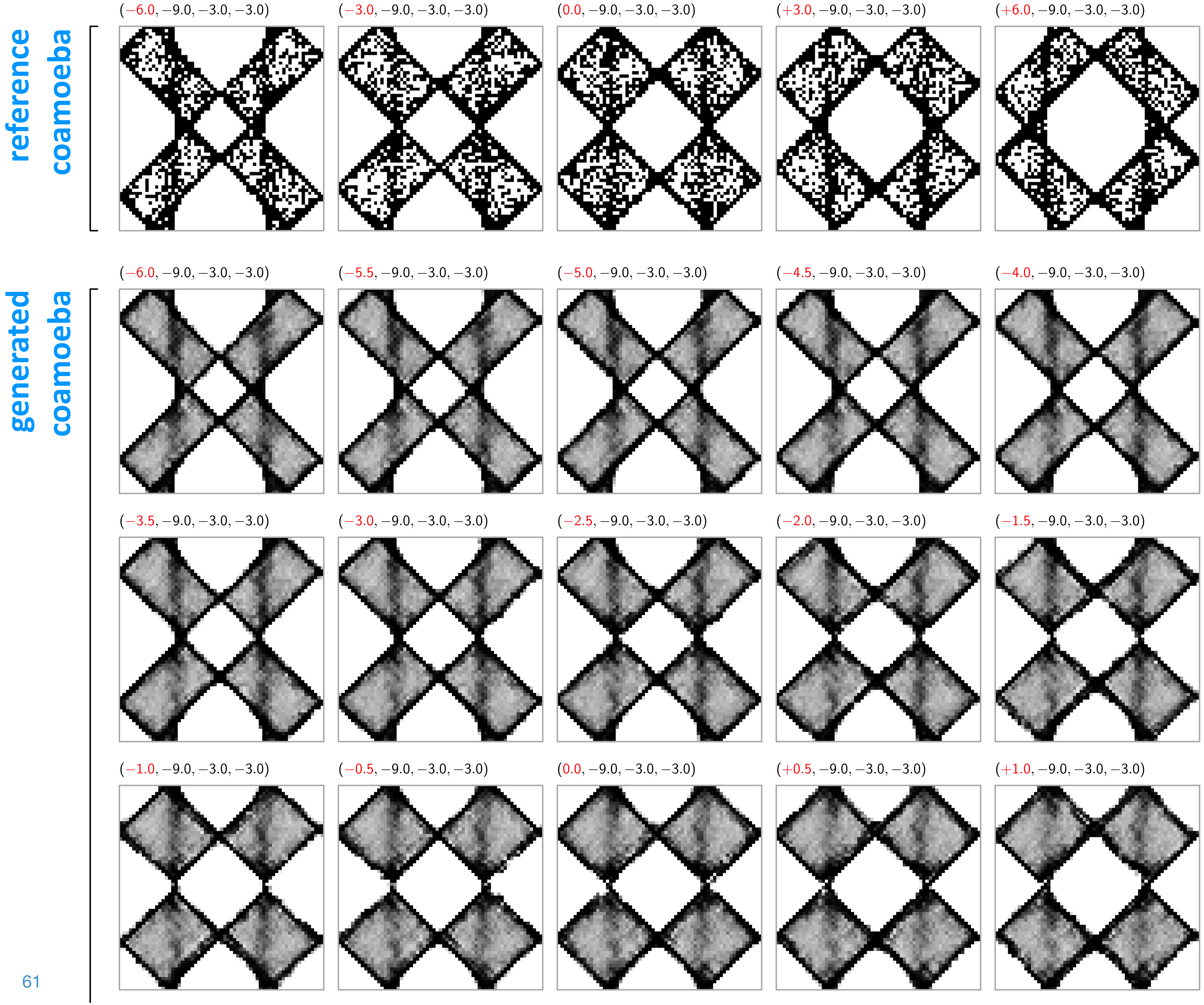


[Seong 2024]

Generative AI

Coamoeba

Brane Configurations



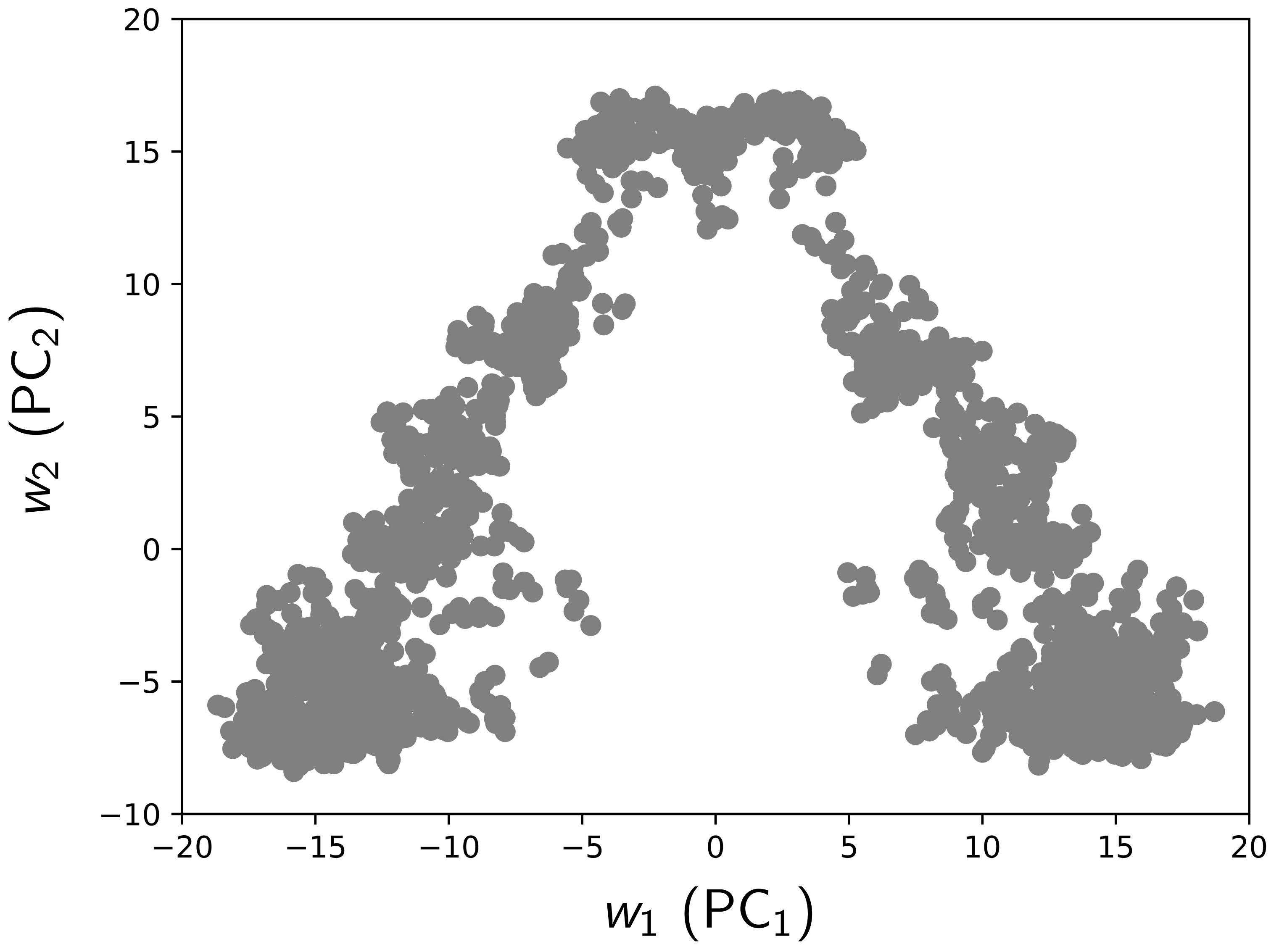
Generative AI

Phase Spaces

Quiver Gauge Theories

reference coamoeba

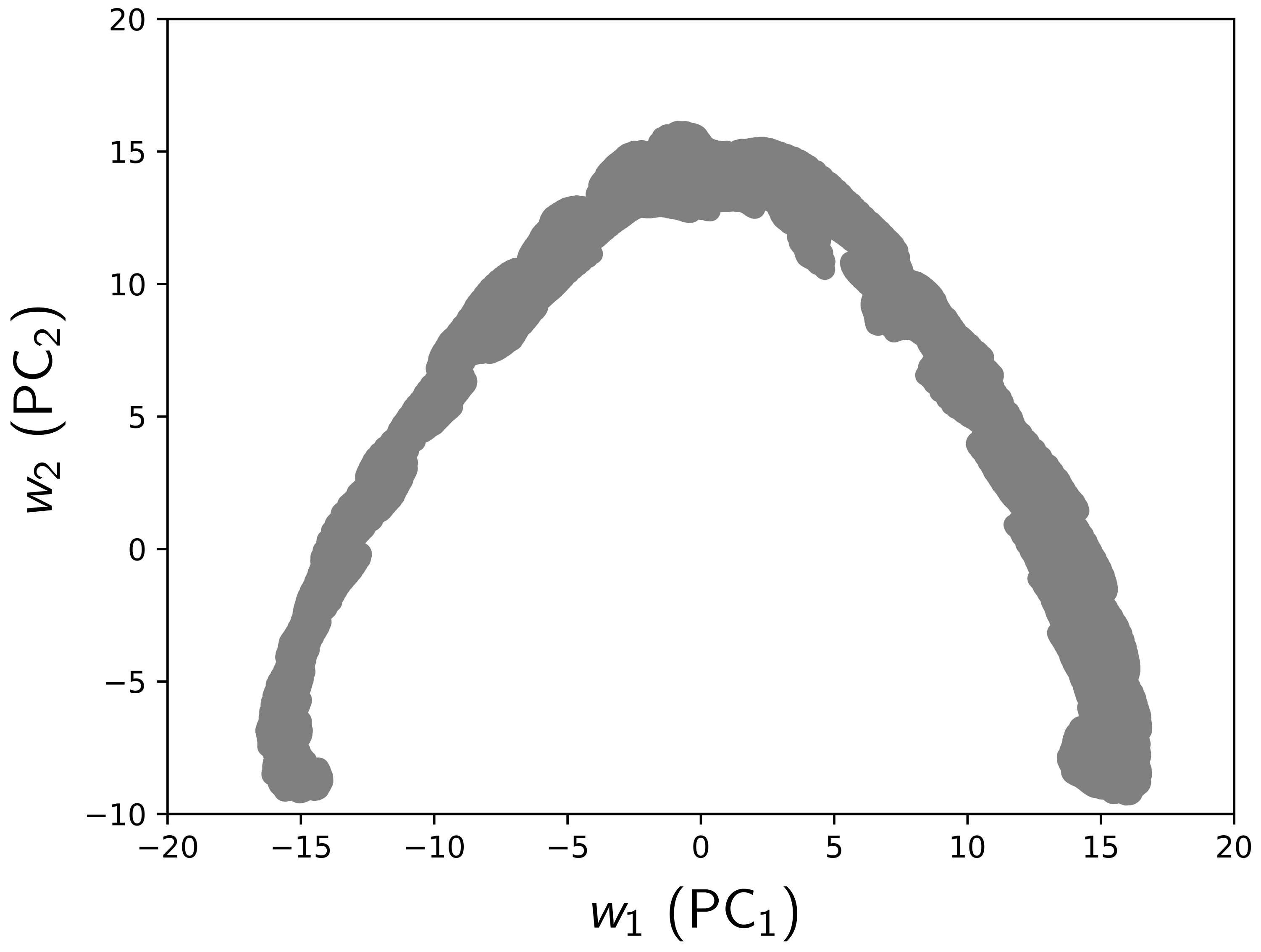
$m_{PCA} = 2$ phase diagram for reference coamoeba for F_0 (N=2304)



[Seong 2023]

generated coamoeba

$m_{PCA} = 2$ phase diagram for generated coamoeba for F_0 (N=389376)

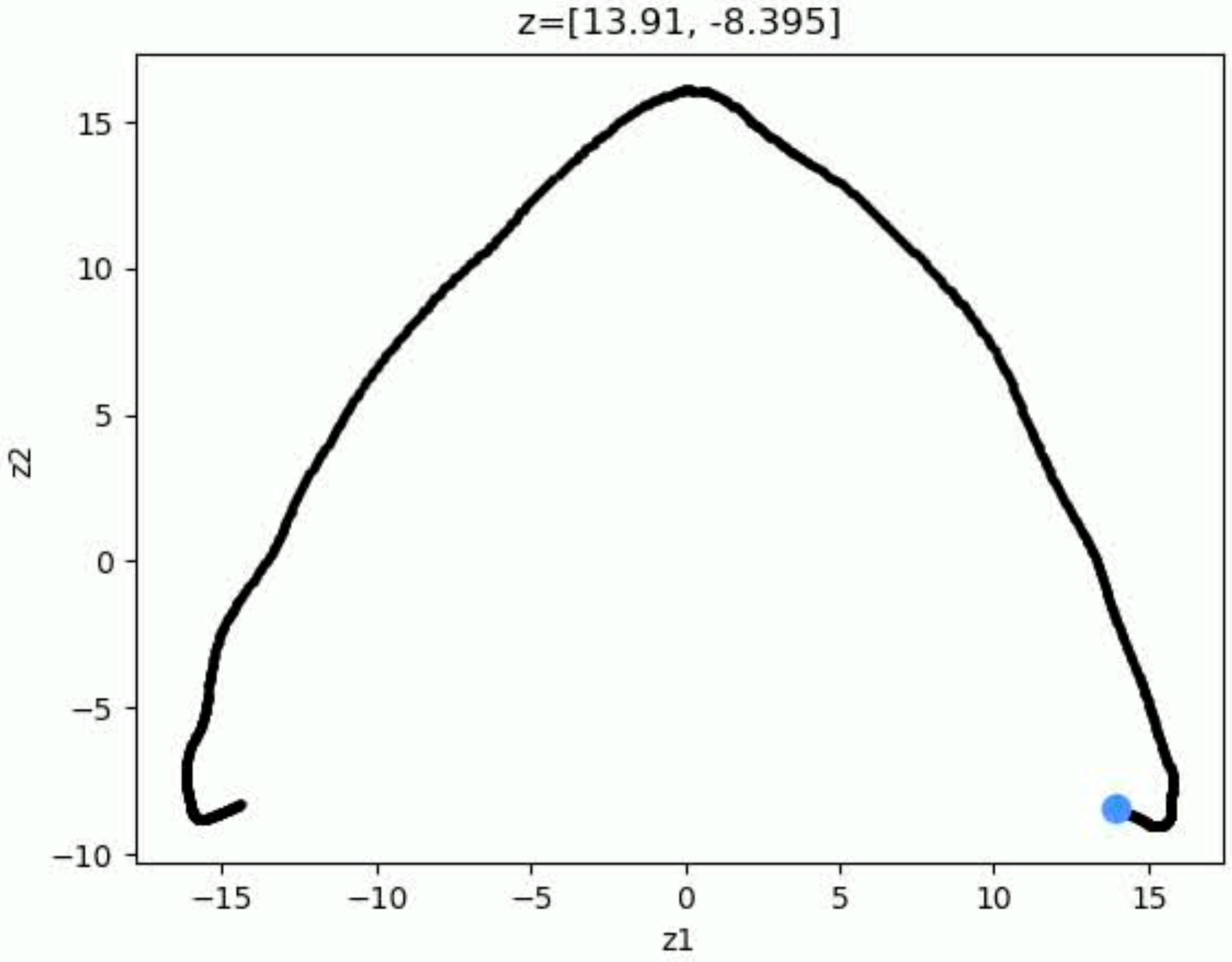


[Seong 2024]

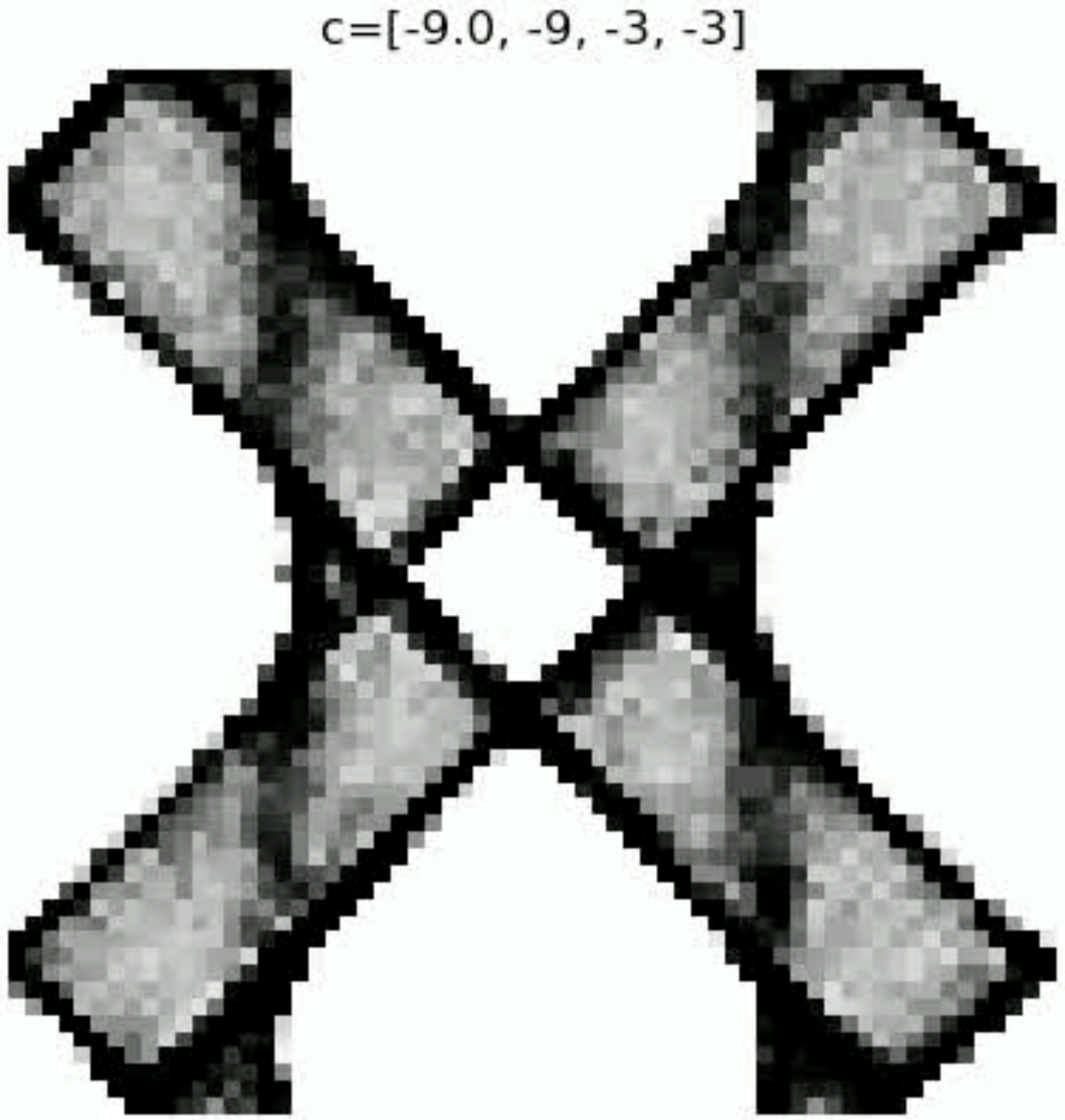
Movements of Branes / Phase Paths

[Seong 2024]

PCA Phase Diagram



Generated Coamoeba

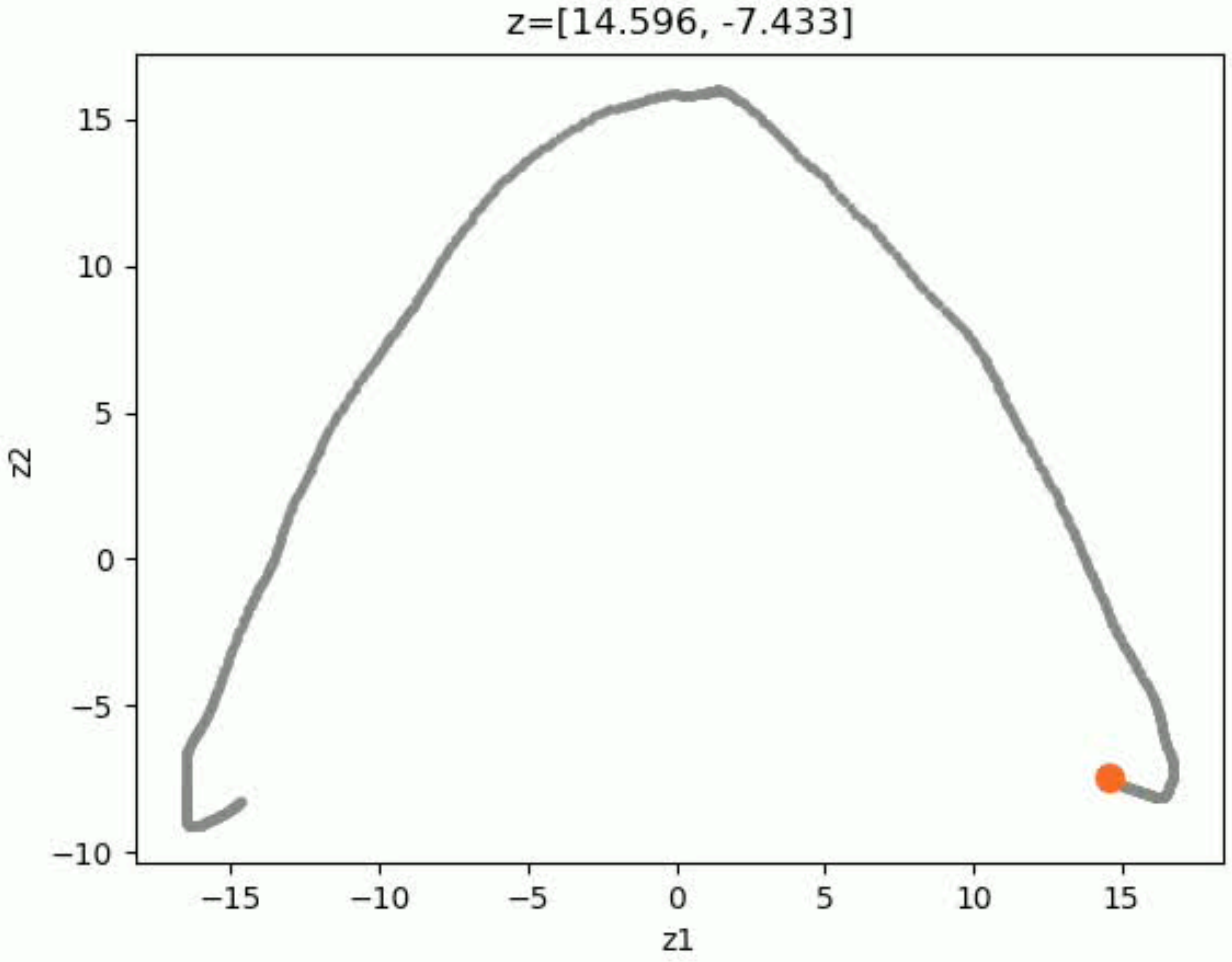


$(c_{11}, -9, -3, -3)$

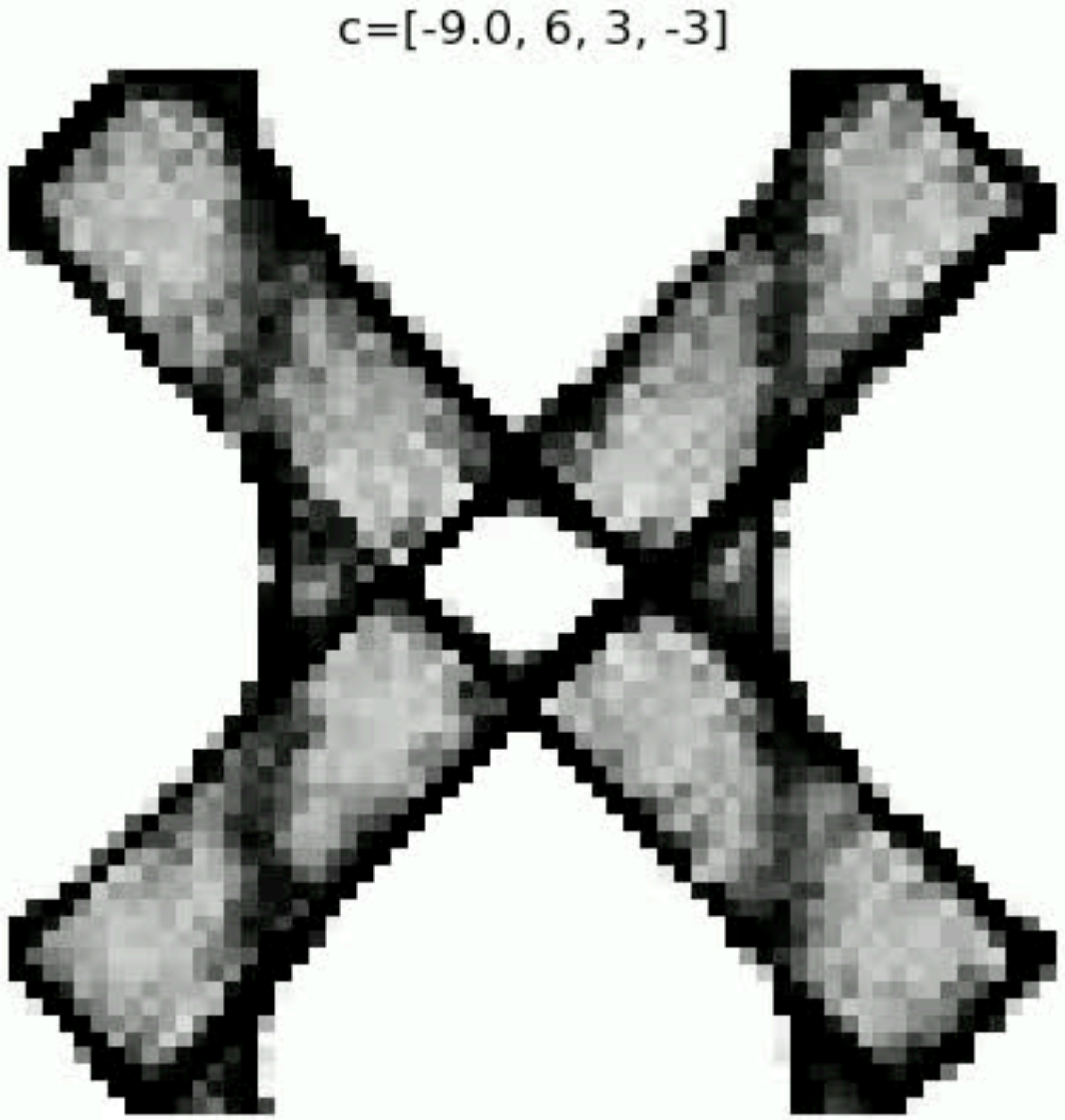
Movements of Branes / Phase Paths

[Seong 2024]

PCA Phase Diagram



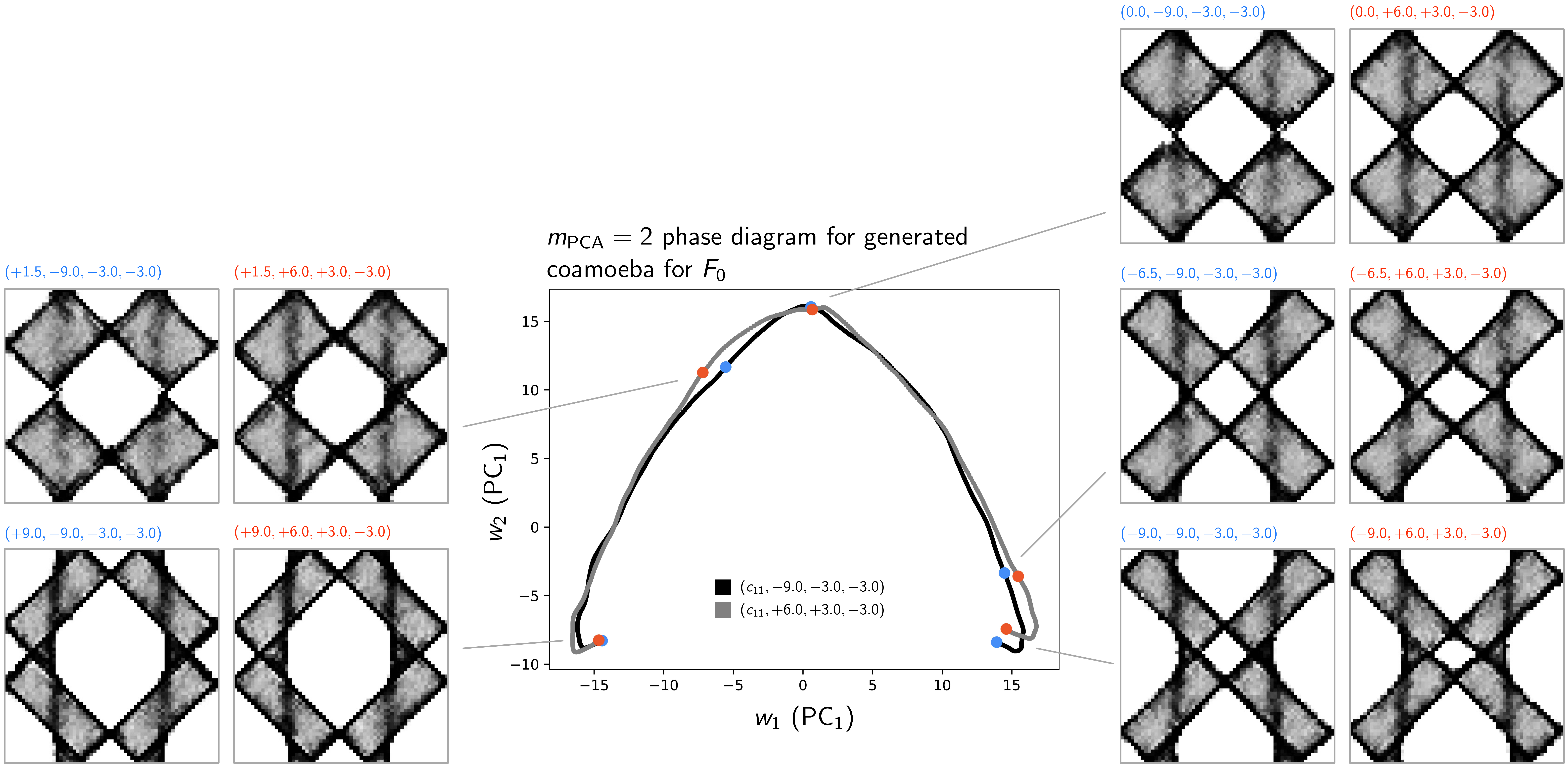
Generated Coamoeba



$(c_{11}, +6, +3, -3)$

Movements of Branes / Phase Paths

[Seong 2024]

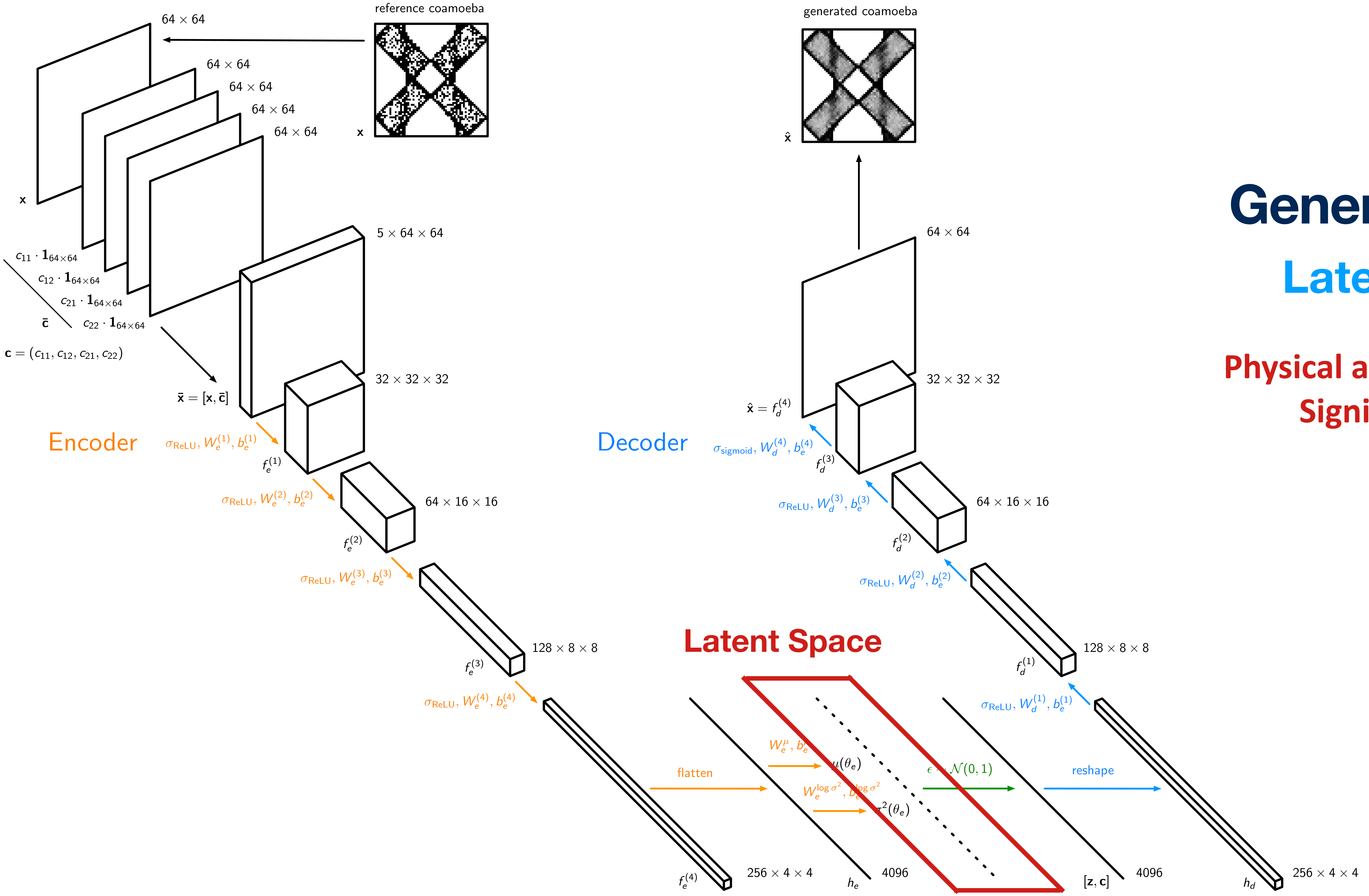


[Seong 2024]

Generative AI

Latent Space

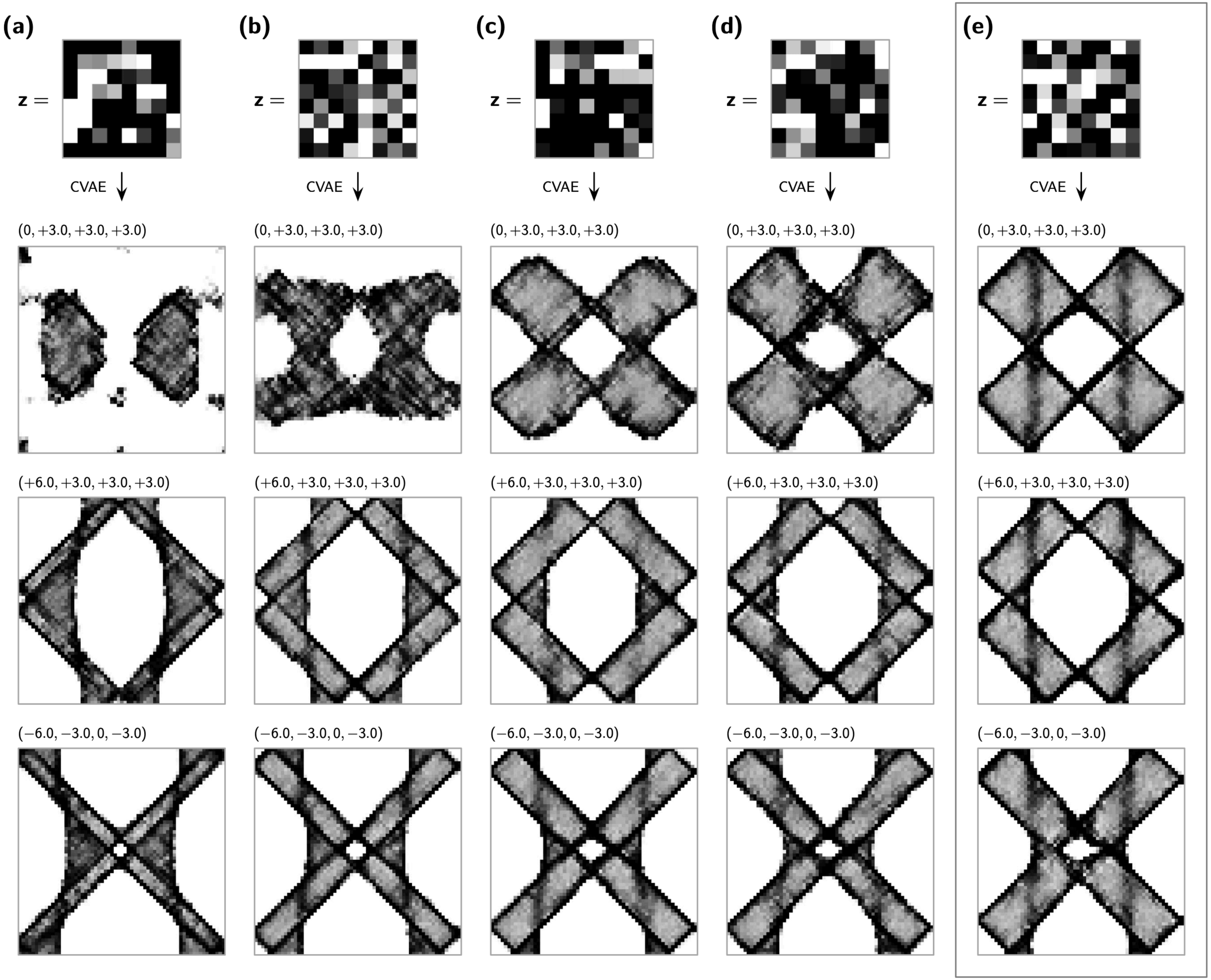
Physical and Geometric Significance of the Latent Space



Generative AI

Latent Space

Physical and Geometric Significance of the Latent Space

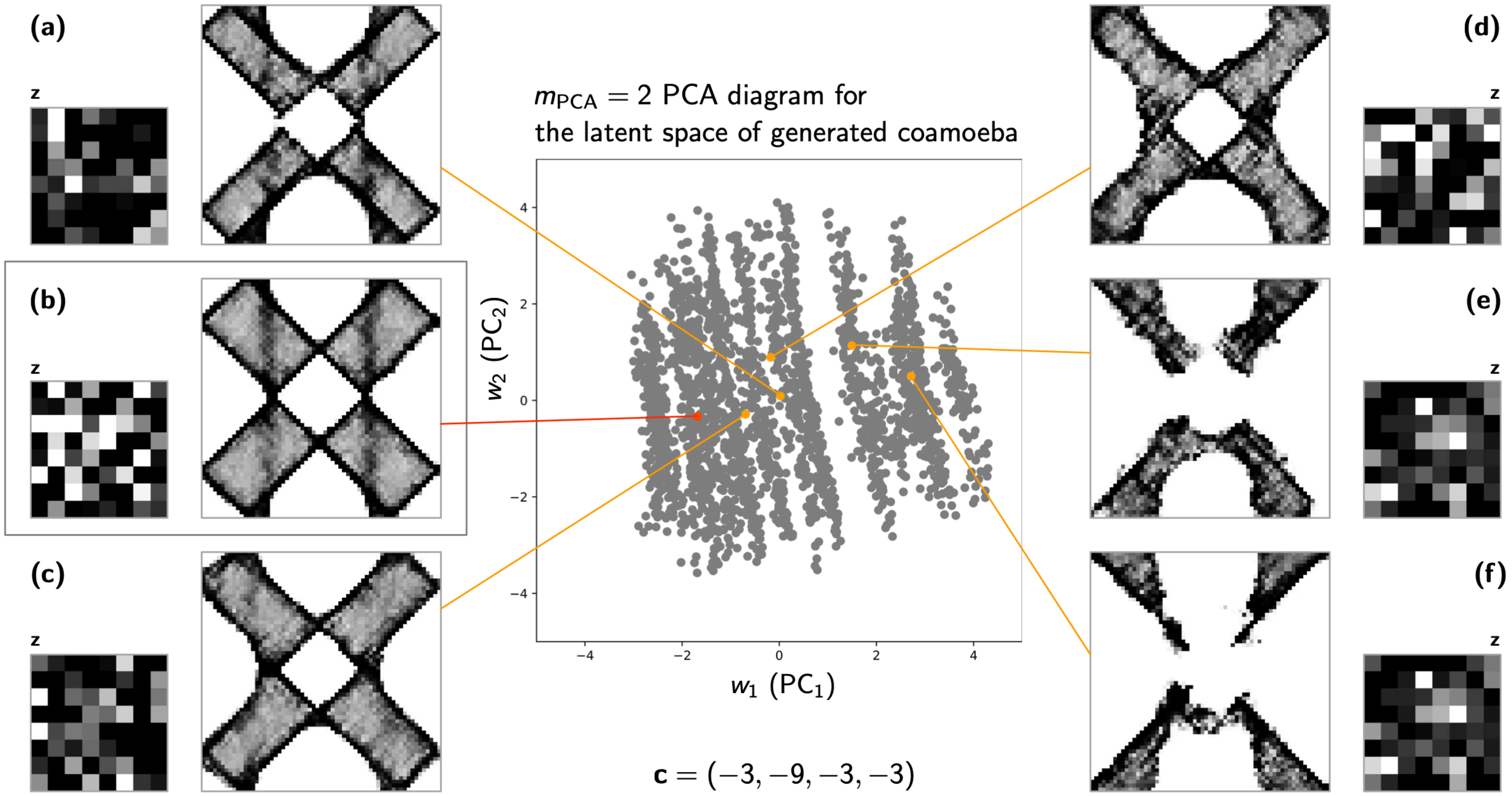


[Seong 2024]

Generative AI

Latent Space

Physical and Geometric Significance of the Latent Space



Beyond Generative AI

[Seong 2025]

Beyond Generative AI

[Seong 2025]

Beyond Generative AI

[Seong 2025]

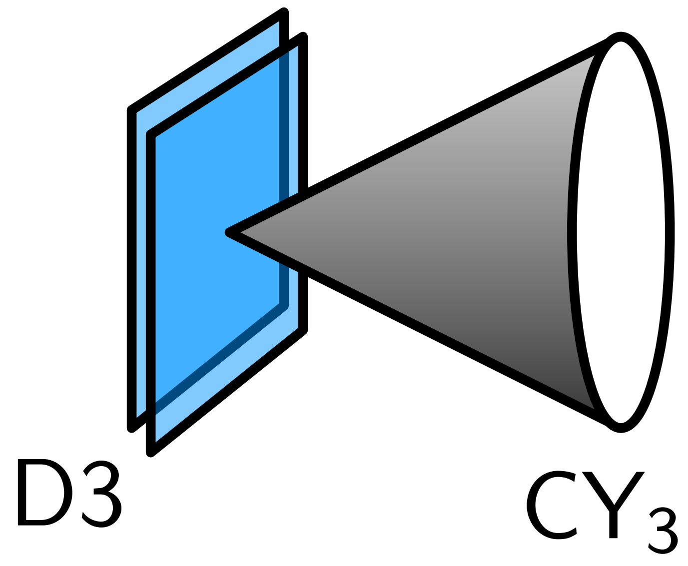
Unification across Dimensions

[Franco, Lee, Seong, Vafa 2017]

Calabi-Yau 3-folds

4d N=1 supersymmetric gauge theory

Brane Tilings (Dimers)

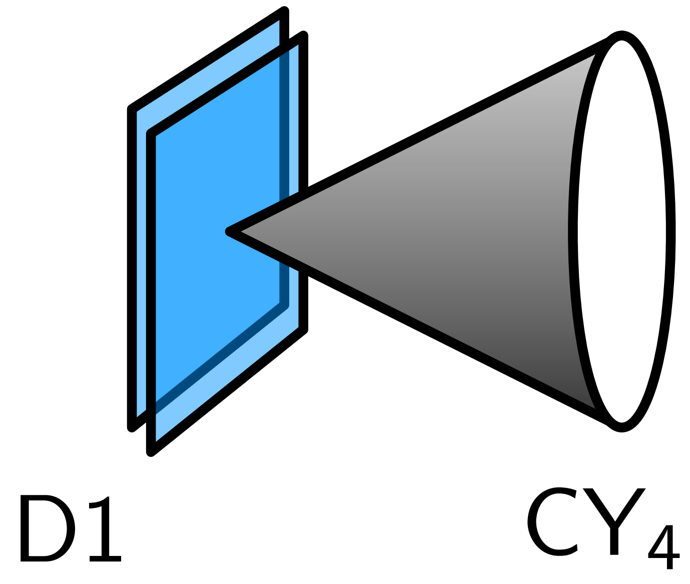


	0	1	2	3	4	5	6	7	8	9
D3	×	×	×	×	·	·	·	·	·	·
CY$_3$	·	·	·	·	×	×	×	×	×	×

Calabi-Yau 4-folds

2d (0,2) supersymmetric gauge theory

Brane Brick Models

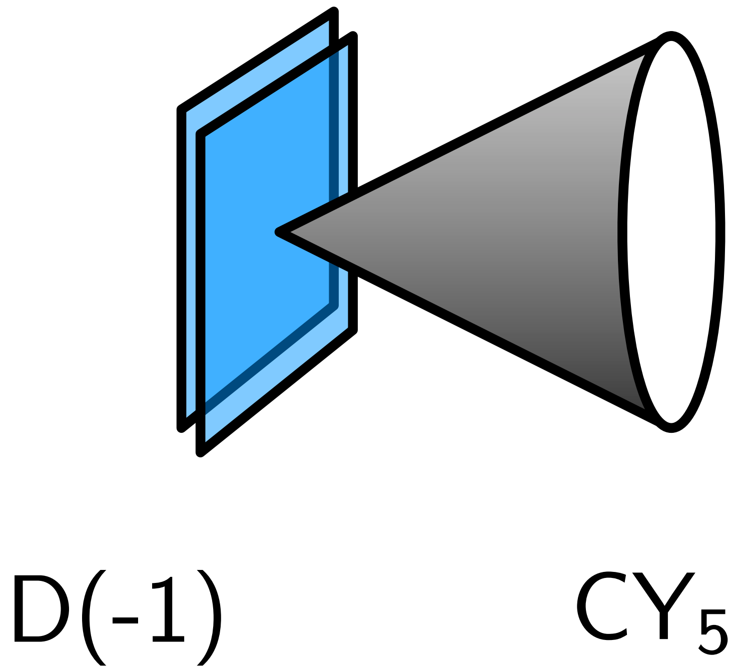


	0	1	2	3	4	5	6	7	8	9
D1	×	×	·	·	·	·	·	·	·	·
CY$_4$	·	·	×	×	×	×	×	×	×	×

Calabi-Yau 5-folds

0d N=1 supersymmetric matrix model

Brane Hyper-Brick Models



	0	1	2	3	4	5	6	7	8	9
D(-1)	·	·	·	·	·	·	·	·	·	·
CY$_5$	×	×	×	×	×	×	×	×	×	×

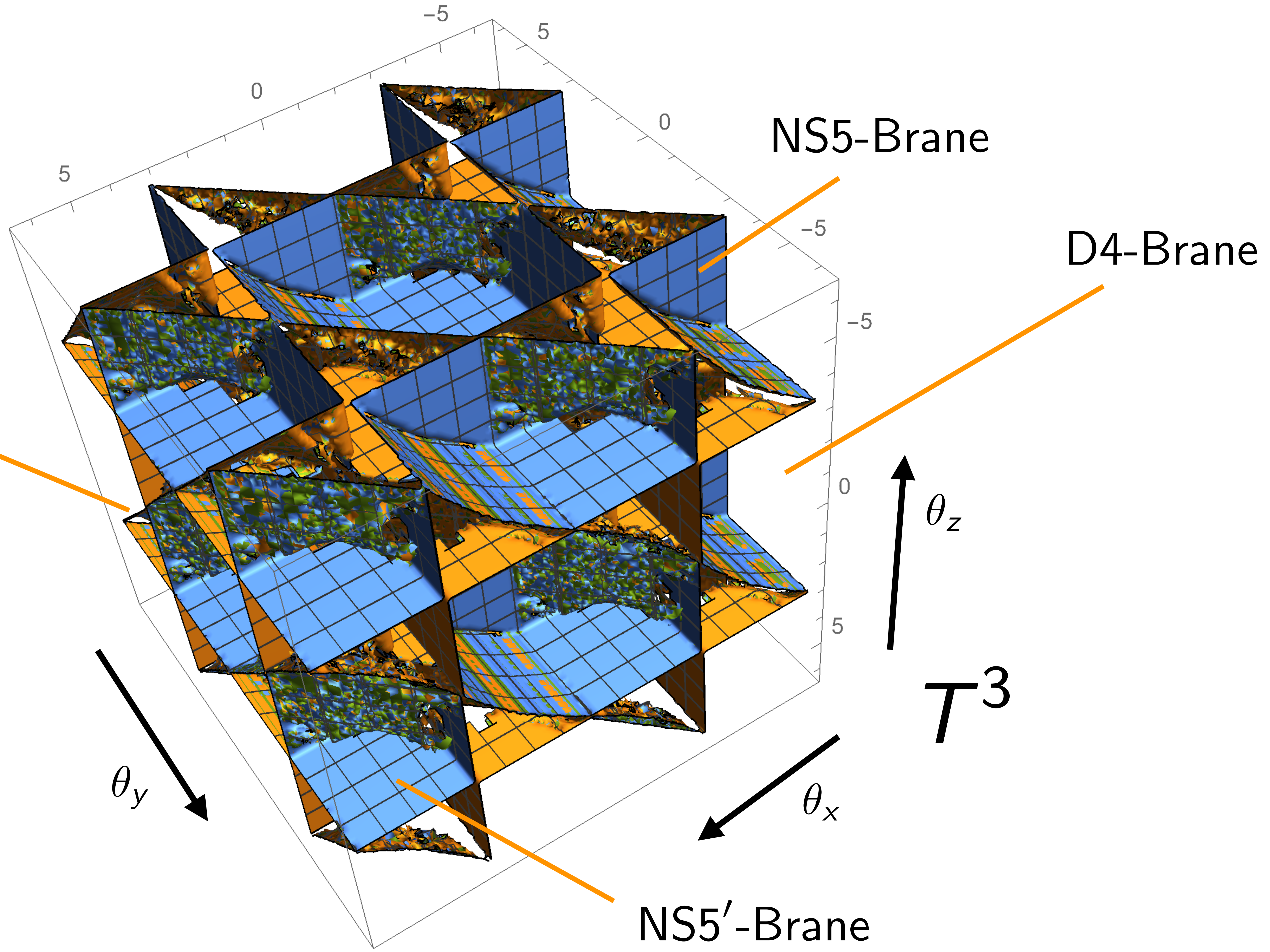
Coamoeba for Calabi-Yau 4-folds

[Franco, Lee, Seong 2016]

- Brane Brick Models

Example

$\mathbb{C}^4$ Model



Thank You

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