AI-assisted Mathematical Discovery

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Where are we with AI for Mathematical Discovery?

Bottom-Up as a formal logical system

Top-Down as a creative/intuitive art

Meta-Mathematics as a language

Reviews:

YHH: Machine-Learning Mathematical Structures, 2101.06317 *IJMSDS 2021*

M. Douglas: Machine learning as a tool in theoretical science, *Nature Rev. Phys, 2022*

S. Gukov, J. Halverson, F. Ruehle: Rigor with machine learning *Nature Rev. Phys, 2024*

YHH: A Triumvirate of AI Driven Theoretical Discovery, 2405.19973 *Nature Rev. Phys, 2024*

M. Doulgas, KH. Lee: Mathematical Data Science *2025*

I. Bottom-Up

Proof Co-Pilots

Mathlib

- Hilbert, Russel-Whitehead: Formalism ("We must know, we will know")
- `https://leanprover-community.github.io/mathlib-overview.html` [Launched 2013]
- Buzzard, Commelin, Massot: *Formalising perfectoid spaces* [2019]
- Gowers-Green-Manner-Tao: Freiman-Rusza-Marton Conj [2023]
- Buzzard: ICM 2022: XenaProject (2013 - 2023)

Lean Community

- Lean F(ocused) R(esearch) O(rganization): 2023 - towards auto-formalization & the future of formalization
- *Process-Driven Autoformalization in Lean 4*: Lu et al. [2024]
- Buzzard's Formalizing Fermat EPSRC Grant [2024 - 7]
- Buzzard, YHH, Mehta, & Hill: London Lean Monthly: Imperial/UCL/LIMS [2025-]
- 2025 - Deepmind AlphaProof & AlphaEvolve (as opposed to AlphaGeo & AlphaGeo2): incorporates Lean
- 2024 - PhysLean: Tooby-Smith

II. Top-Down

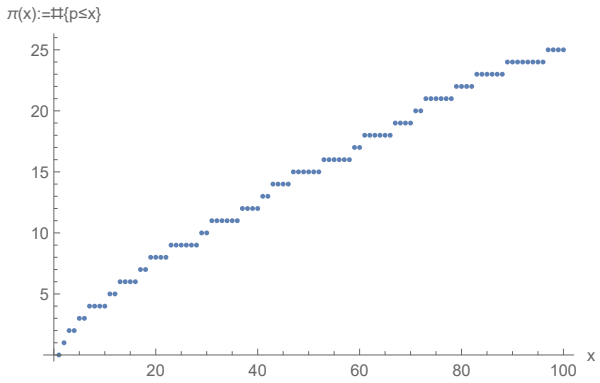
Experimental Mathematics

AI-guided intuition

- (somewhat) Brouwer: Intuitionism (mathematics is a creation of the mind; no more law of excluded middle $A \vee \neg A$)
- "Mathematics is a branch of physics where the experiments are cheap" - V. Arnol'd
- Theoretical physics: not bottom-up
- Conjecture Formulation from mathematical data
- M. Douglas: Platonic Data

intuition, experience, experimentation

- C19th: Gauss



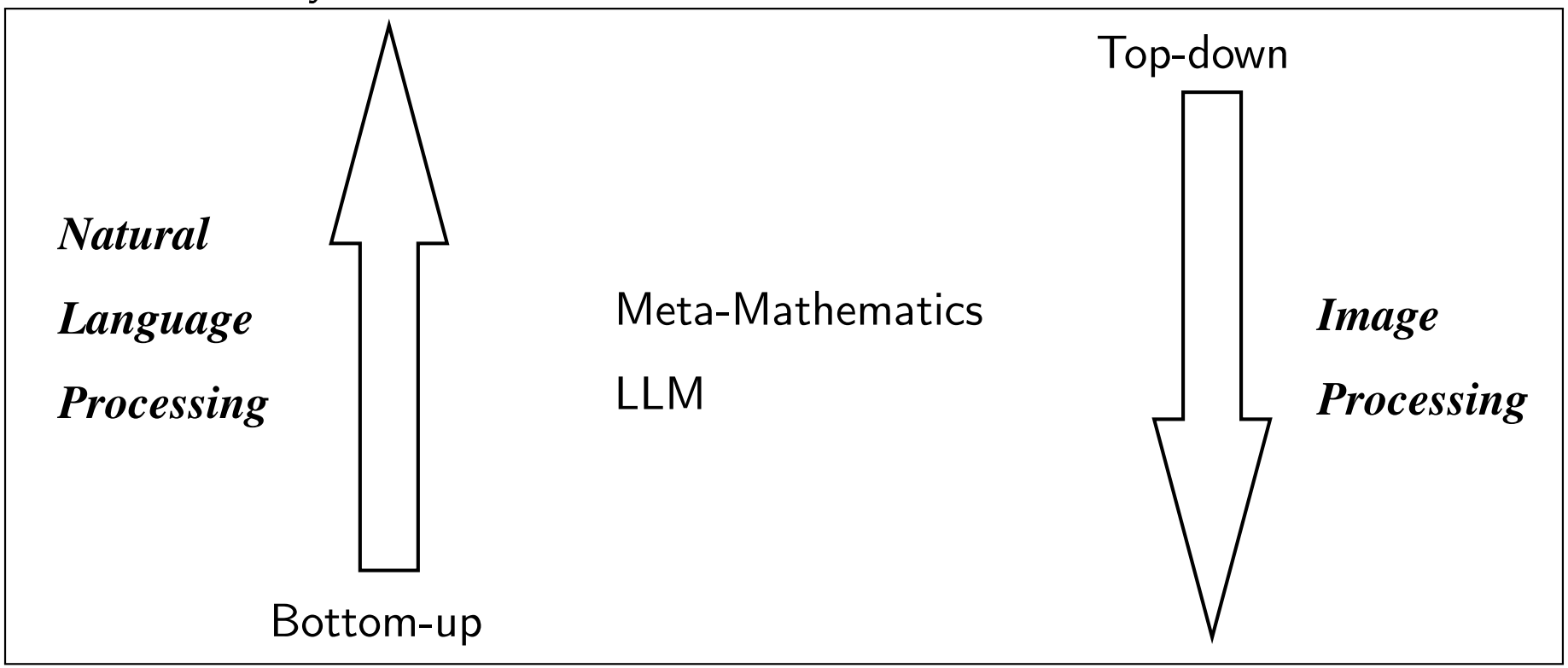
(w/o computer and before complex analysis [50 years before Hadamard-de la Vallée-Poussin's proof]): PNT $\pi(x) \sim x/\log(x)$

- C20th: BSD computer experiment of Birch & Swinnerton-Dyer [1960's] on plots of rank r & N_p on elliptic curves
- 2 / 6 remaining Millenium Prize Problems (RH, BSD) came from experimentation

AI-guided theoretical/mathematical Discovery

YHH: A Triumvirate of AI Driven Theoretical Discovery, 2405.19973

Nature Rev. Phys, 2024:

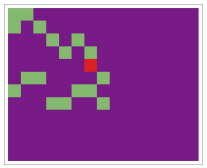


Algebraic Geometry as Image Processing

- A typical calculation:

$$X = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \boxed{\text{What Bourbaki teaches us}} \longrightarrow h^{2,1}(X) = 22$$

- 2017: (YHH, Deep learning the landscape) think of it as an image processing problem



$\longrightarrow$ What Machine-Learning teaches us $\longrightarrow 22$

2017

YHH (1706.02714); Krefl-Seong (1706.03346); Ruehle (1706.07024); Carifio-Halverson-Krioukov-Nelson (1707.00655)

Sophia: Hanson Robotics, HongKong

- First Transformer
- AlphaZero
- Sophia becomes a "human" citizen (in Saudi Arabia)
- Beginning of `String_Data` Annual conference series

Progress in String Theory: Start Dates of Annual Series

1986- "Strings" Conference

2002- "StringPheno" Conference

2006 - 2010 String Vacuum Project (NSF)

2008 - ISGT Integrability in String/Gauge

2011- "String-Math" Conference (2020 - , M-theory & Maths Workshop)

2012- "Amplitudes"

2014- String/Theoretical Physics Session in SIAM Conference

2017- "String-Data" Conference

(2025: Cambridge/InfoSys/LIMS (YHH, Krippendorf, Mishra)

AI4Maths Annual Series

2017 - String-Data (w/ Krippendorf, Mishra, Halverson, Ruehle, Grimm, Hashimoto, Gukov, et al)

2020 - DANGER (w/ Kaspryzyk, Hirst, Heyes, Oliver, Lee): 2020 -

2021- Math-AI @ NeurIPS

2024 - ICMS, Harvard (Douglas et al)

2024 - AI for Math Workshop @ ICML

The Birch Test: "AI + N" @ "AI"

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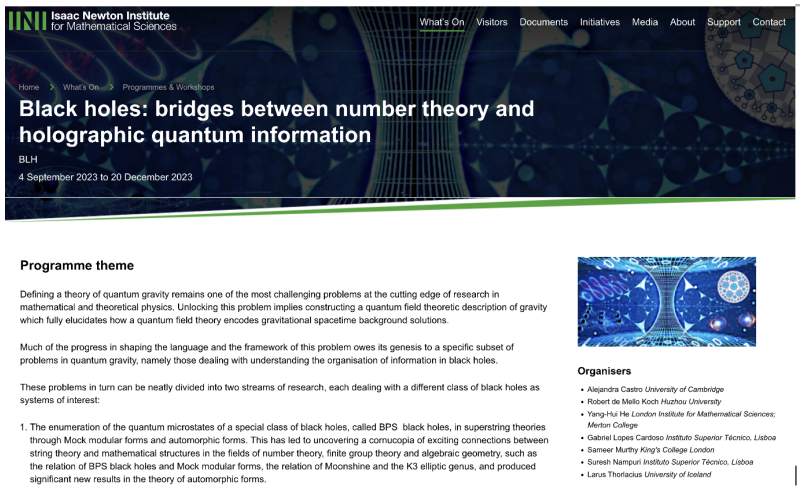

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Black holes: bridges between number theory and holographic quantum information

BLH

4 September 2023 to 20 December 2023

Programme theme

Defining a theory of quantum gravity remains one of the most challenging problems at the cutting edge of research in mathematical and theoretical physics. Unlocking this problem implies constructing a quantum field theoretic description of gravity which fully elucidates how a quantum field theory encodes gravitational spacetime background solutions.

Much of the progress in shaping the language and the framework of this problem owes its genesis to a specific subset of problems in quantum gravity, namely those dealing with understanding the organisation of information in black holes.

These problems in turn can be neatly divided into two streams of research, each dealing with a different class of black holes as systems of interest:

1. The enumeration of the quantum microstates of a special class of black holes, called BPS black holes, in superstring theories through Mock modular forms and automorphic forms. This has led to uncovering a cornucopia of exciting connections between string theory and mathematical structures in the fields of number theory, finite group theory and algebraic geometry, such as the relation of BPS black holes and Mock modular forms, the relation of Moonshine and the K3 elliptic genus, and produced significant new results in the theory of automorphic forms.

Organisers

- Alejandra Castro *University of Cambridge*
- Robert de Mello Koch *Huzhou University*
- Yang-Hui He *London Institute for Mathematical Sciences; Merton College*
- Gabriel Lopes Cardoso *Instituto Superior Técnico, Lisboa*
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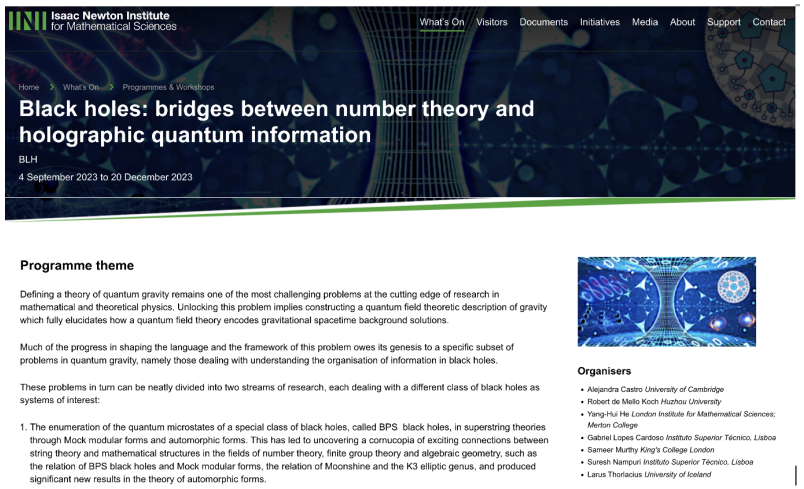

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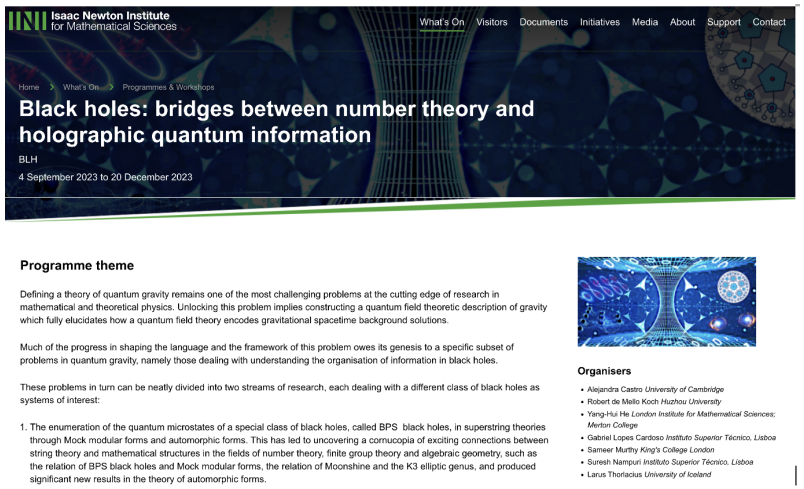
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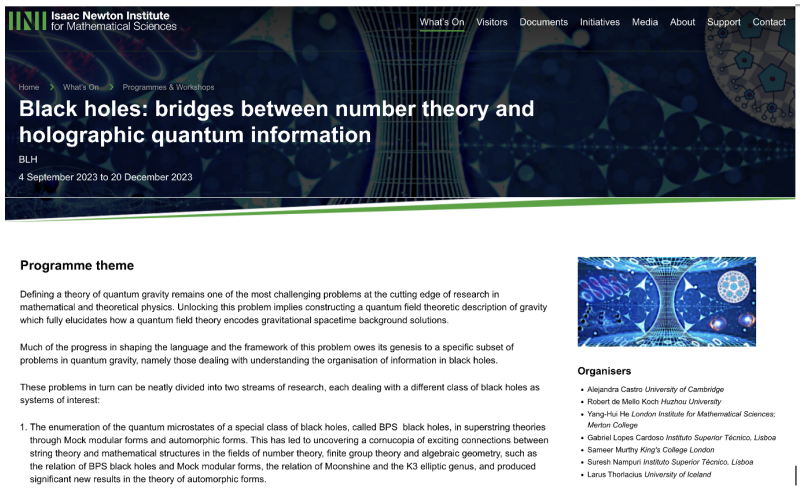

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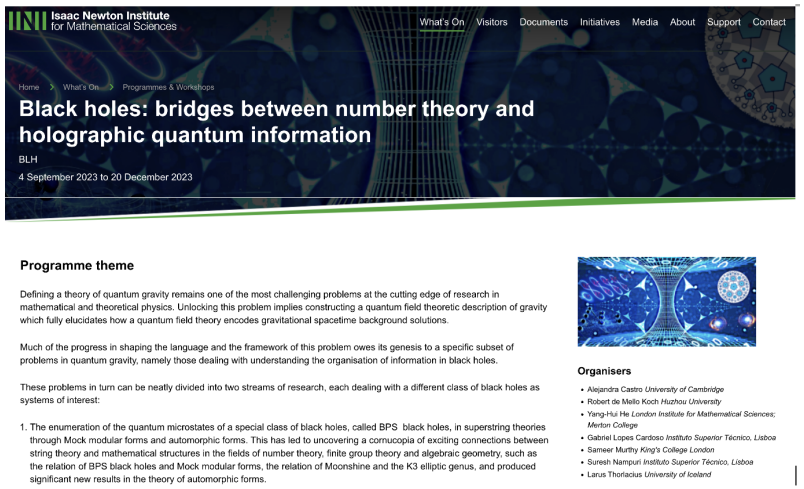
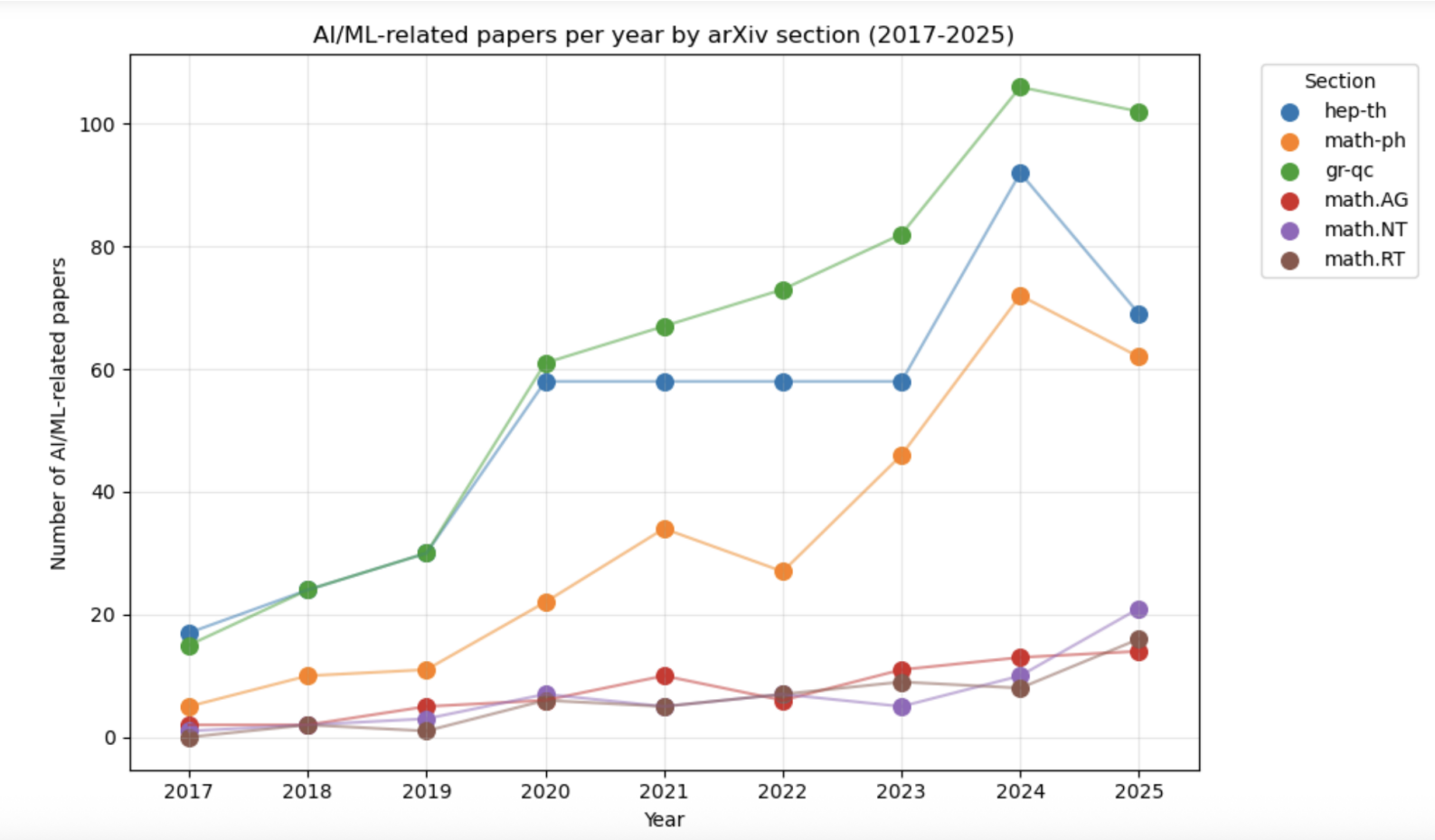


- (Automaticity) be generated by AI
- (Interpretability) concrete enough to be a conjecture
- (Non-Triviality) for the community to work on it
- make Birch happy

A Growing Field



Three Closest Examples

- 2022 ML Knots

  new knot invariants relations from saliency [Davies et al, DeepMind] (fails N)

- 2022 Murmuration Conjectures

  [YHH-Lee-Oliver-Podznyakov, 2022, YHH-Lee-Oliver-Podznyakov-Sutherland, tbc] A patten in L-functions reflecting Chebyshev's bias in the primes (fails A)

- 2025 Discovering Singularities

  [Wang et al, DeepMind (fails A)

Platonic Data

- ~ 1 Terabyte free online
- Finite Groups: GAP
- Algebraic Geometry: GrDB , CY Databases
- Number Theory: lmfDB
- Knot Theory: SnaPY
- ...

III. Meta-Mathematics

LLMs @ Math

Rapid Progress

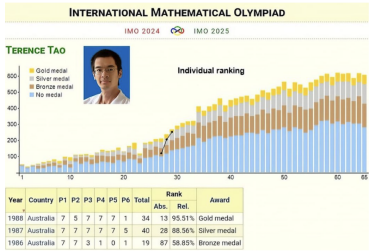
2022 ChatGPT passes the Turing Test

2023 DeepMind's FunSearch, Meta-AI's LLama

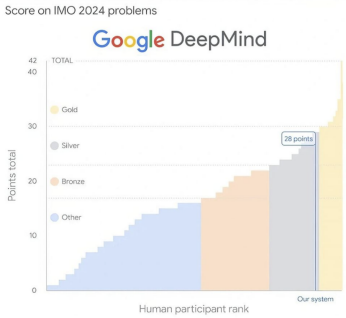
2024 DeepMind's AlphaGeometry, AlphaGeometry2,

2024-5 AlphaProof & AlphaEvolve (incoporates Lean)

July 2025: AI $\simeq$ 12 year old Terry Tao



Year	Country	P1	P2	P3	P4	P5	P6	Total	Rank Abs.	Rank Rel.	Award
1988	Australia	7	5	7	7	7	1	34	13	95.51%	Gold medal
1987	Australia	7	7	7	7	7	5	40	28	88.56%	Silver medal
1986	Australia	7	7	3	1	0	1	19	87	58.85%	Bronze medal



Graph showing performance of our AI system relative to human competitors at IMO 2024. We earned 28 out of 42 total points, achieving the same level as a silver medalist in the competition.

Terence Tao, 28 points - Silver Medal; AlphaGeo2, 28 points - Silver Medal (2024), 35 points - Gold (2025)

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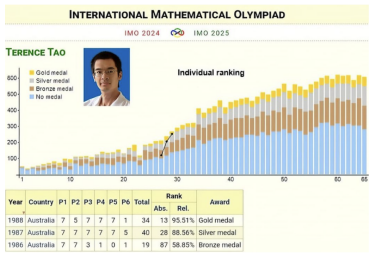

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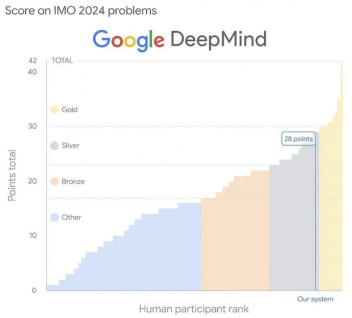

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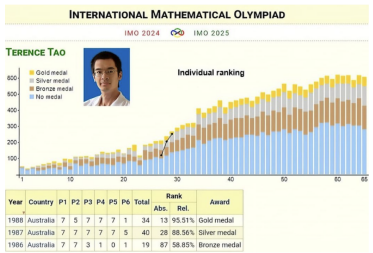

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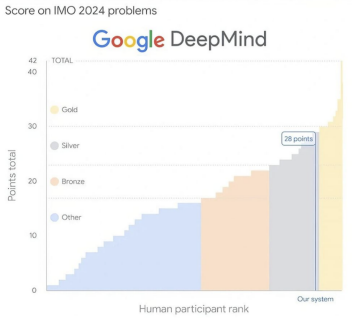

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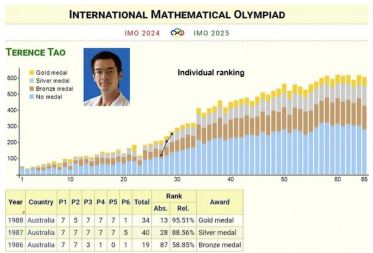

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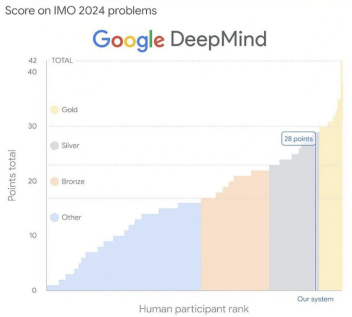

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- and May 2025, 30 of us were flown in to Berkeley HQ for Tier 4

Scientific American Financial Times

Methods of Proof

- STANDARD: Proof by contradiction, induction, syllogism

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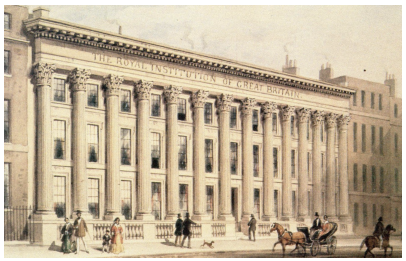
  ...
- LLMs: Proof by vibe, by citation, by authority

The London Institute for Mathematical Sciences

- UK's only independent research institute for maths; modelled after IAS, Princeton
- Founded in 2011 by Dr. Thomas Fink
- Housed in the Faraday Suites of the Royal Institution of Great Britain



- 1 of 4 themes: AI for Maths Discovery

  https://lims.ac.uk/event/ai-assisted-maths-discovery/

- Apply to our Cognia AI/Math JRF, Khodorkovsky Fellowships