Thoughts on Hadronization

(… of Heavy Quarks in QGP)

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"QCD Challenges from pp to AA Collisions"
Münster (Germany), Sept. 02-06, 2024

Outline

1.) Introduction

2.) Conceptual Issues

-- Relation to QCD Matter

3.) Theoretical Considerations

-- Conservation Laws + Constraints

4.) Practical Implementations (HICs)

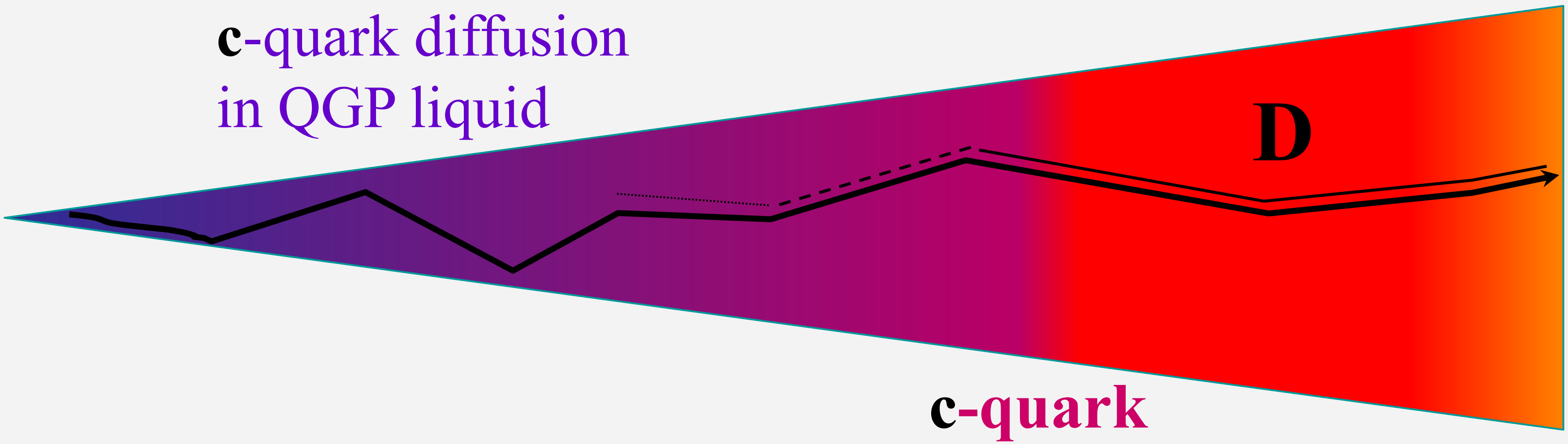
-- Space-Momentum Correlations

-- Off-Equilibrium Quark Distributions

5.) Summary

2.) Heavy Flavor in QCD Matter

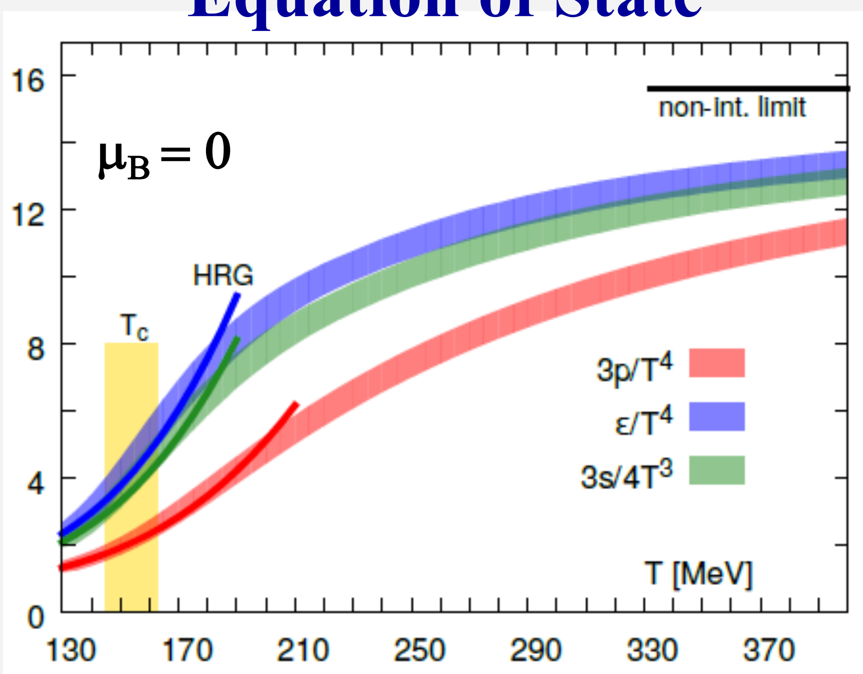
c-quark diffusion
in QGP liquid



D-meson
diffusion
in hadron
liquid

c-quark
hadronization

Equation of State



- Change in dofs above **T~160MeV**
- No "discontinuities" in interaction
- Strongly coupled quantum many-body system

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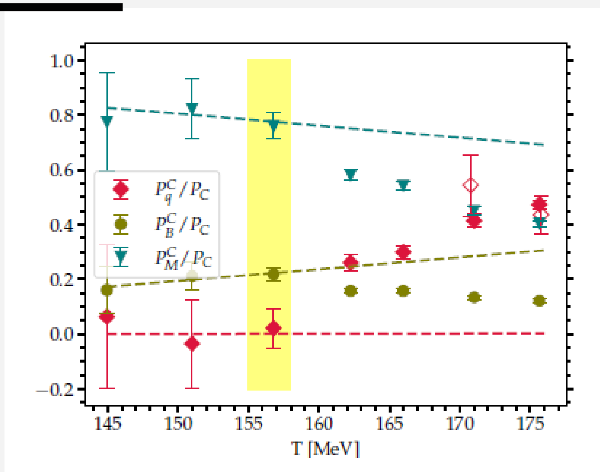
3.) Theoretical Considerations



- **Color neutralization near equilibrium:**
  - spectral functions of the system transit from partonic to hadronic across $\mathbf{T_{pc}}$
  - strong coupling ⇒ broad spectral functions
  - embed heavy quarks in a realistic medium with realistic transition rates
- **Consistency with equilibrium limits**
  - kinetic: thermal partons → thermal hadron
  - chemistry: excited states (feeddown)
- **4-momentum conservation**
  - Not trivial for **c + q → D** bound state (spectral functions?!)
- **Entropy** conservation …

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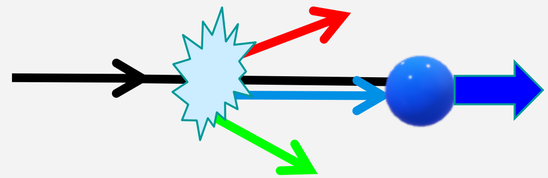
4.1 Hadronization of Heavy Quarks

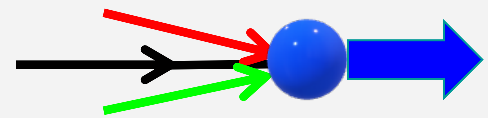
- **Fragmentation**



$c \rightarrow$ D, D*, D_s, Λ_c, …

- determined by **empirical fragmentation functions** $D_{c \rightarrow H_c}(z)$
- in principle universal ("vacuum": e^+e^- collisions or high p_T)

- **Coalescence / Recombination**



c + q (s) $\rightarrow$ D (D_s), D*,… ; c + q + q (s) $\rightarrow$ Λ_c (Ξ_c), …

depends on environment (phase space of surrounding anti-/quarks)
- instantaneous coalescence (spatial wave functions), global
- resonance recombination (momentum space), local
- string recombination, local

4.2 Heavy-Quark Recombination

- **Instantaneous Coalescence Models (ICMs)** **[Hwa '80, Likhoded et al '83, … Greco et al + Fries et al '03,…]**

$$f_h(\mathbf{p}'_h) = \int \Big[\prod_i d\mathbf{p}_i f_i(\mathbf{p}_i)\Big] W(\{\mathbf{p}_i\})\delta(\mathbf{p}'_h - \sum_i \mathbf{p}_i)$$

$$W_s = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 \mathbf{k}^2}$$

Wigner function, σ ~ radius parameter for each hadron h

- energy not conserved → challenge for chemical + thermal equilibrium

- **Resonance Recombination Model (RRM)** **[Ravagli et al '07, He et al '12]**

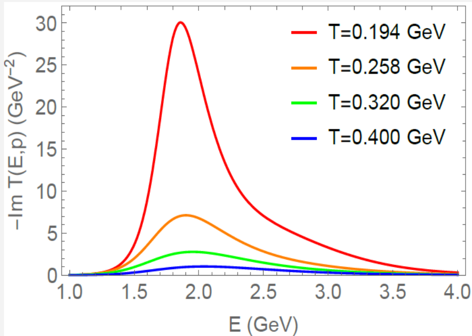
- derived from Boltzmann equation

$$f_M(\vec{x},\vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x},\vec{p}_1) f_{\bar{q}}(\vec{x},\vec{p}_2)\ \sigma_M(s) v_{\mathrm{rel}}(\vec{p}_1,\vec{p}_2)\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2)$$

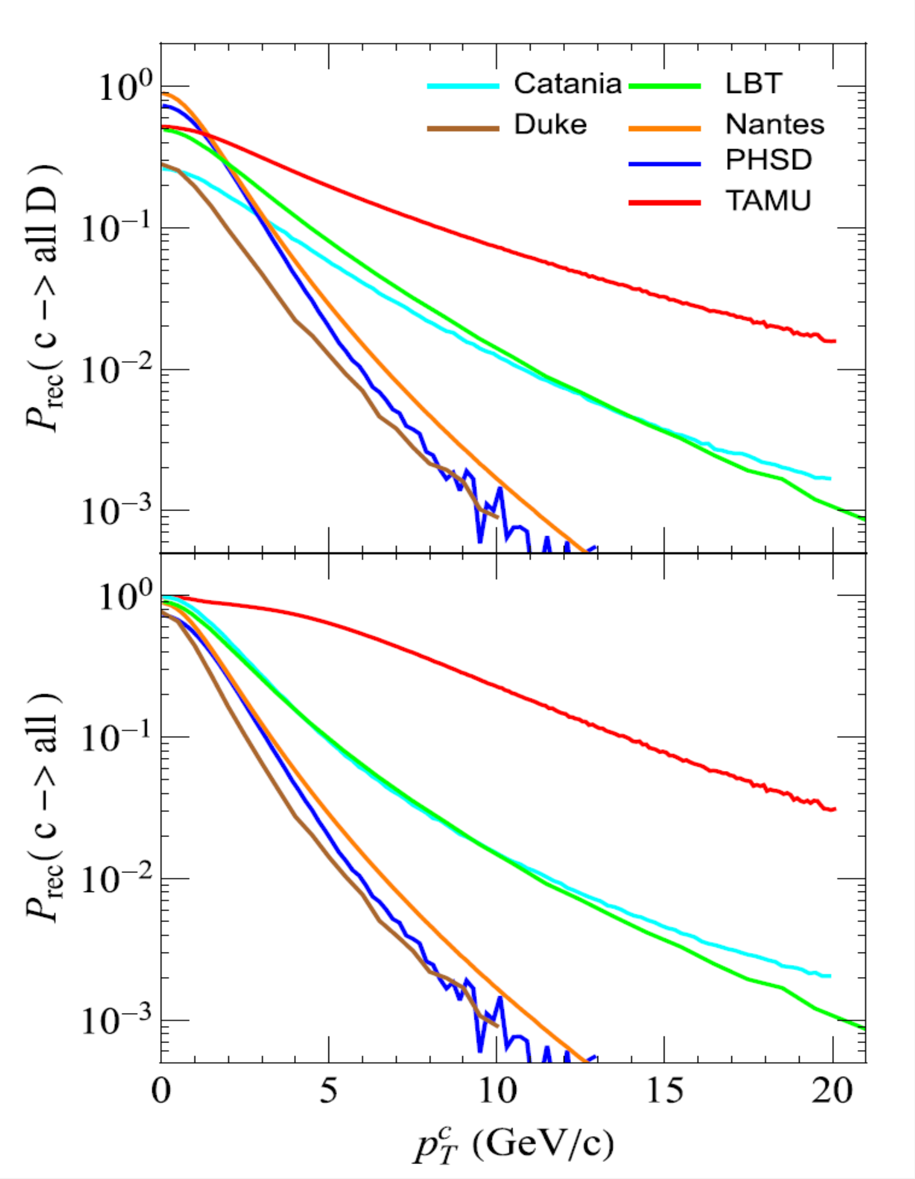
$\sigma_M(s)\ v_{rel} \sim |T_{Qj}|^2$: resonant heavy-light scatt. amplitude

→ directly connect to T-matrix interactions in QGP near T_c

→ encodes equilibrium limits



4.3 Heavy-Quark Recombination Probabilities



- Large model spread
- Non-collinear recombination + excited resonances reach to high $\mathbf{p_T}$

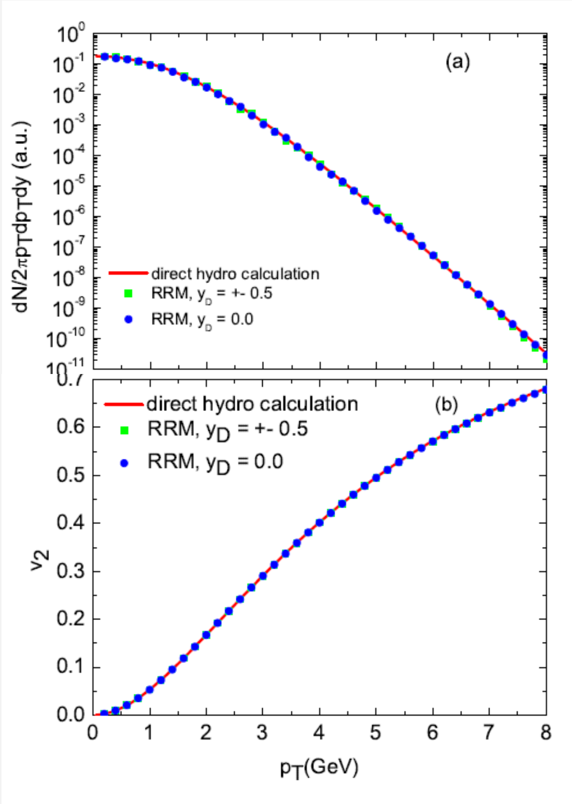
[Zhao et al '24]

4.4 Hydrodynamics + Space-Momentum Correlations

- **Meson Distribution on Hydro Hypersurface**

$$\frac{dN}{p_T dp_T d\phi dy} = \int_\Sigma \frac{p_\mu d\sigma^\mu(\tau, x, y)}{(2\pi)^3} f_M(\tau, x, y; \mathrm{p})$$

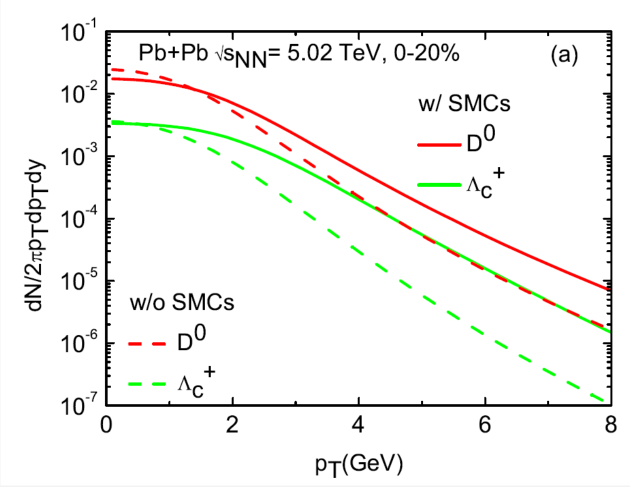
→ thermalized anti-/quarks ⇒ thermal mesons!
(including $\mathbf{v_2}$, ...)



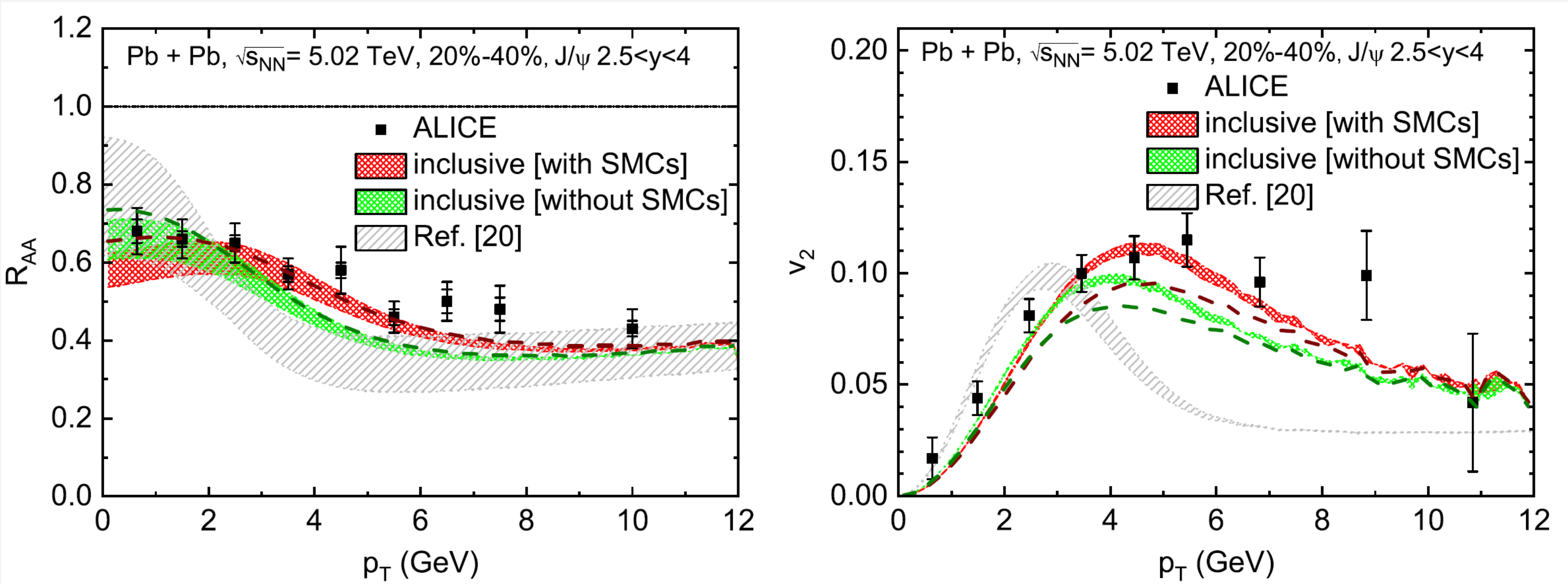
- **Extension to Baryons** [He+RR '19]

$$f_B(\vec{x}, \vec{p}) = \frac{\gamma_B}{\Gamma_B} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2 d^3\vec{p}_3}{(2\pi)^6} \frac{\gamma_{dq}}{\Gamma_{dq}} f_1(\vec{x}, \vec{p}_1) f_2(\vec{x}, \vec{p}_2) \times f_3(\vec{x}, \vec{p}_3) \sigma_{dq}(s_{12}) v_{\mathrm{rel}}^{12} \sigma_B(s) v_{\mathrm{rel}}^{dq3} \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

- **Space-Momentum Correlations**
  - hallmark of hydro evolution
  → fast charm quarks in outer high-flow regions, coalescence contribution out to higher $\mathbf{p_T}$



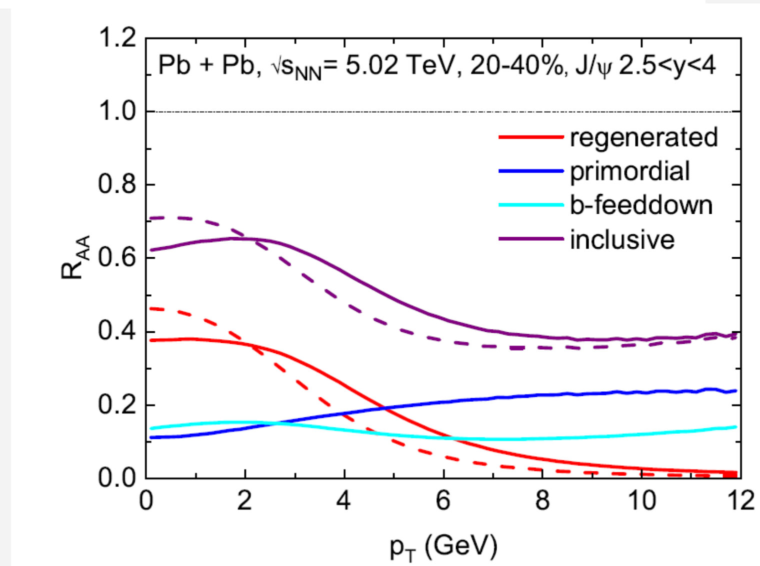
4.5 Off-Equilibrium Quarks + SMCs: J/ψ



- Off-equilibrium **c**-quark spectra + space-momentum correlations

⇒ recombination out to **p_T ~ 8 GeV**

⇒ **much improved description of v_2**

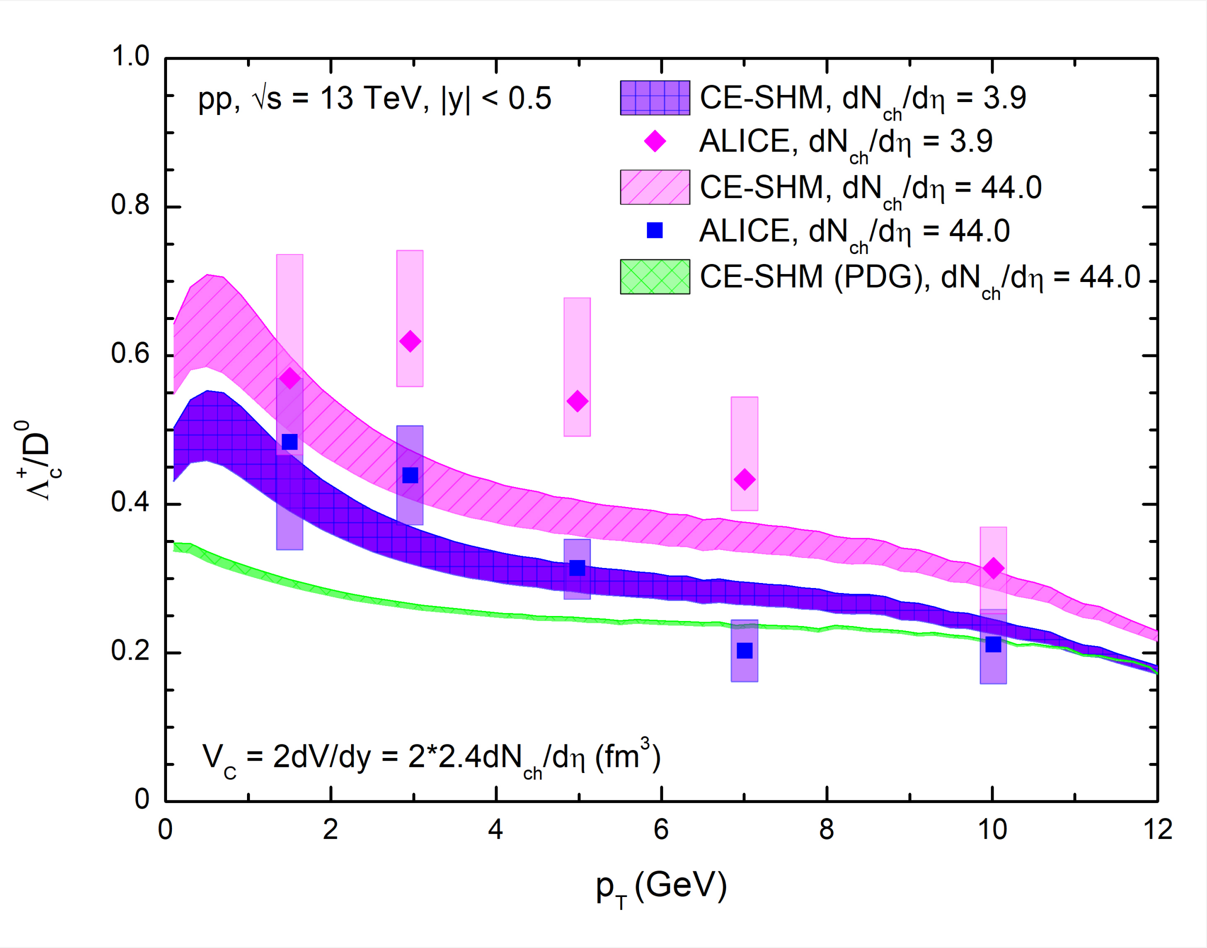


5.) Future Developments

- **Currently:**
  - quarks recombine above threshold with schematic widths in Breit-Wigner
  - no absolute normalization (hypersurface, rather than rate)
- **Future:**
  - implement realistic spectral functions to recombine into bound states
  - utilize production rates to assess absolute yields
  - Ξ_c problem
  - rapidity (multiplicity) dependence
  - model criteria and synergies

  …

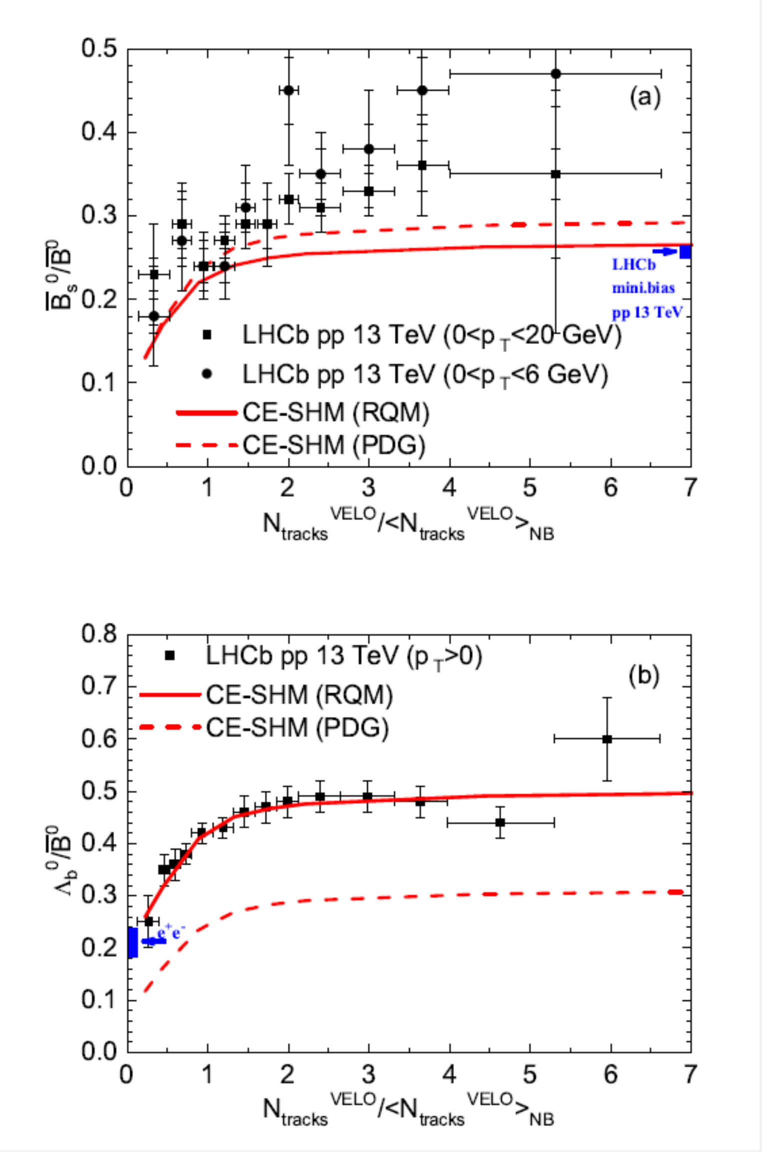
4.6 Multiplicity Dependence of HQ Recombination



- Canonical suppression at low multiplicity (baryon number)

[Chen+He '21]

4.6.2 Multiplicity Dependence: Bottom Sector



- Canonical suppression at low multiplicity

[Dai, Zhao+He '24]

Transport Approaches

- **Boltzmann equation for HQ phase-space distribution** f_Q

$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{\omega_{\boldsymbol{p}}}\frac{\partial}{\partial \boldsymbol{x}} + \boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}\right] f_{\mathrm{Q}}(t,\boldsymbol{x},\boldsymbol{p}) = C[f_{\mathrm{Q}}]$$

- explicit simulation of medium (quasi-) particles in collision term
- semi-classical approximation

- **Fokker-Planck equation**

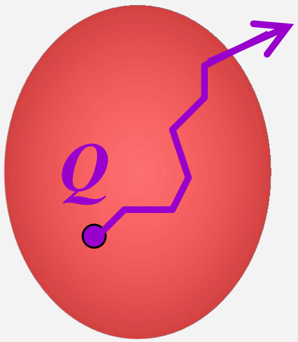
$$\frac{\partial}{\partial t} f_Q(t,\boldsymbol{p}) = \frac{\partial}{\partial p_i}\left\{A_i(\boldsymbol{p}) f_Q(t,\boldsymbol{p}) + \frac{\partial}{\partial p_j}[B_{ij}(\boldsymbol{p}) f_Q(t,\boldsymbol{p})]\right\}$$

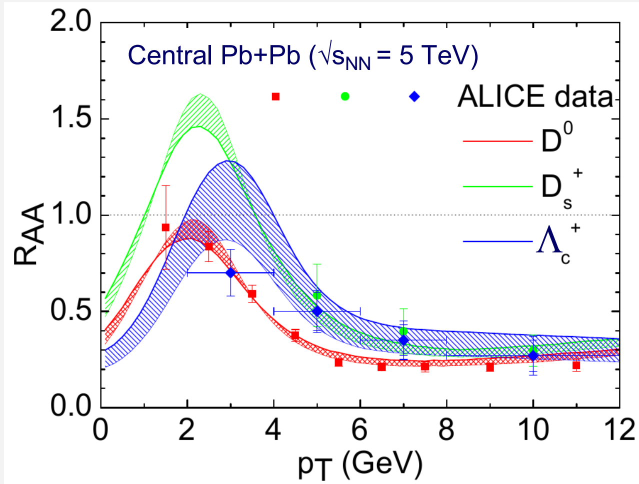
- follows from Boltzmann with $p^2 \sim m_Q T >> q^2 \sim T^2$; ok for $m_Q/T \geq 5$
- **does not require quasi-particle medium**
- well suited for strongly coupled medium where $E_{th} \leq \Gamma_{q,Q} < m_Q$

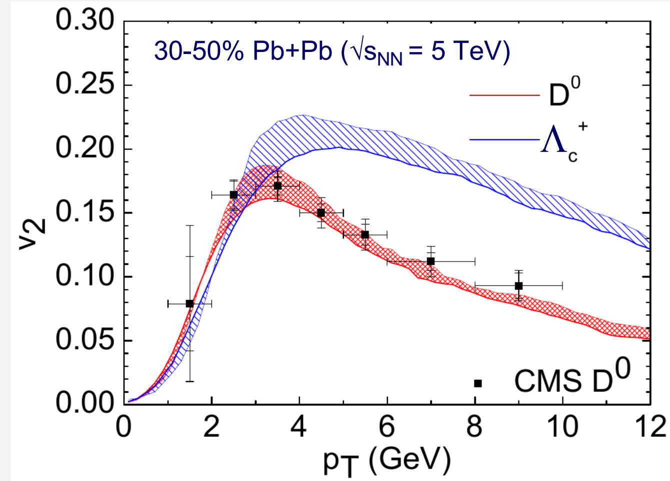
3.) Charmonium p_T Spectra Revisited [He,Wu+RR '22]

- **Main Idea: Use transported c-quark spectra from Langevin simulations**

[He+RR '20]







⇒ **implement into Resonance Recombination Model** [Ravagli et al '07, He et al '12]

(derived from Boltzmann equation)

$$f_M(\vec{x},\vec{p}) = \frac{\gamma_M(p)}{\Gamma_M}\int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x},\vec{p}_1) f_{\bar{q}}(\vec{x},\vec{p}_2)\, \sigma_M(s) v_{\rm rel}(\vec{p}_1,\vec{p}_2)\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2)$$

- $\sigma_M(s)\, v_{rel} \sim |T_{QQ}|^2$: charmonium amplitude
- regeneration yield normalized to rate equation result
- include space-momentum correlation (**SMCs**)

- **As you may know, this series has a special format with emphasis on discussion, triggered by relatively brief presentations (20-25 min.) that rather raise questions than provide answers. In this spirit we propose the following plan for the 3 parallel sessions of our track on both Tue and Wed morning and early afternoon:**

- **We start out with 4-5 talks each day in the morning in the following (informal) order:**
- **Tue: C. Bierlich, A. Ohlson, L. Bianchi, H. van Hees**
- **Wed: D. Bala, J. Wang, R. Rapp, A. Dubla, J. Stachel**
- **roughly corresponding to light flavor on Tue and heavy flavor on Wed (but by no means meant to be exclusive).**
- **We then continue with in-depth discussions for the remainder of the 2nd morning session and in the 1st afternoon session.**

- **We would like to charge each speaker to share their thoughts on the hadronization problem, both for small and large collisions systems, based on their personal expertise and preferences.**