Heavy quark dynamics in the pre-equilibrium phase



QCD Challenges from pp to AA collisions 2024



Pre-equilibrium dynamics



Berges, Heller, Mazeliauskas, Venugopalan [2005.12299]

High-energy collisions

Stitching together effective theories



Schlichting [Initial Stages (2016)]

High-energy collisions

Pre-equilibrium stages



*Gelis [Int.J.Mod.Phys.A28(2013)]

High-energy collisions

Pre-equilibrium stages



*Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney [Phys.Rev.C99(2019)]





Color Glass Condensate

An EFT for high energy QCD



Small-x limit of QCD \leftrightarrow evolution

*Gelis, lancu, Jalilian-Marian, Venugopalan [Ann.Rev.Nucl.Part.Sci.60(2010)] Artwork by T. Ullrich



Color Glass Condensate

An EFT for high energy QCD







Classical Yang-Mills equations





CGC fields





Glasma fields

Numerical field after the collision



- Known CGC fields at $\tau < 0$

- Boundary condition at $\tau=0$
- Unknown Glasma fields at $\tau>0$

Milne coordinates (τ, η) $\tau = \sqrt{2x^+x^-}, \ \eta = \ln(x^+/x^-)/2$

Boost-invariant approximation fields = $indep(\eta)$

 $\begin{array}{l} \mbox{Numerical solution of Yang-Mills} \\ \mbox{equations} \Rightarrow \mbox{Glasma}^{\star} \end{array}$





Saturation scale, flux tubes, anisotropy



- Relevant scale saturation momentum Q_s
- Initial longitudinal color flux tubes
- \blacktriangleright Fields dilute after $\tau\simeq Q_s^{-1}$
- Fields arranged in correlation domains of $\delta x_T \simeq Q_s^{-1}$
- ► Longitudinal ≠ transverse pressures ⇒ anisotropy

Gelis [Rept.Prog.Phys.84(2021)]



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Lappi [Phys.Lett.B643(2006)]

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Epelbaum, Gelis [Phys.Rev.Lett.111(2013)]

Next stages of pre-equilibrium

Effective kinetic theory



Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney [Phys.Rev.C99(2019)]

Bottom-up thermalization

Equilibration at weak coupling



Schlichting, Teaney [Ann.Rev.Nucl.Part.Sci.69(2019)]

Effective kinetic theory

 \grave{A} la AMY^{\dagger} and KZ^{\star}

Trajectories for different initial conditions*



[†]Arnold, Moore, Yaffe [JHEP01(2003)] *Kurkela, Zhu [Phys.Rev.Lett.115(2015)] Boltzmann equation



- Soft scale $m_D \ll$ hard scale Q_s
- Overoccupied $f \sim 1/\alpha_s$ at $Q_s \tau \sim 1$
- Boost-invariance $p_z \ll p_T$





Classical fields, soft particles, energy loss



- Stage o Overoccupied gluon fields Anisotropy ξ , coupling $\lambda = 4\pi N_c \alpha_s$
- Stage *

Maximum anisotropy, hard modes

Stage •

Minimum occupancy, bath of soft modes

Stage v

Almost isotropic, hard modes radiated Thermalization $\tau_{\rm BMSS}\sim \alpha_s^{-13/5}Q_s^{-1}$



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Initial stage of pre-equilibrium Heavy quarks in glasma



Ipp, Müller, Schuh [Phys.Lett.B810(2020)]

Nucleus A

Initial stage of pre-equilibrium

Heavy quarks in glasma

Approaches

- Numerical trajectories
- Correlator method

Quantities

- Momentum broadening $\langle \delta p^2 \rangle$
- Fransport coefficient κ
- ▶ Observables $R_{AA}^{\frown P u}$, $\mathcal{C}(\Delta \phi)$ Nucleus B

Figure from A. Ipp, D. Müller, D. Schuh [2009.14206]

Particles in Yang-Mills fields



Wong's equations of motion

► Approach: numerical trajectories of classical particles in glasma fields

 $\begin{array}{l} \mbox{Wong's equations} \leftrightarrow \mbox{classical equations of motion for particles } (x^\mu,p^\mu,Q) \\ \mbox{evolving in a Yang-Mills background field } A^\mu \end{array}$



Particles in Yang-Mills fields

Vizualizing the trajectories*



*Avramescu, Băran, Greco, Ipp, Müller, Ruggieri [Phys.Rev.D107(2023)]

Particles in Yang-Mills fields

Quantities: averaged over particle trajectories and glasma events

- Momentum broadening $\langle \delta p_i^2 \rangle(\tau) = \langle p_i^2(\tau) p_i^2(\tau_{\text{form}}) \rangle$ Transport coefficient* $\kappa_i = \frac{d}{d\tau} \langle \delta p_i^2(\tau) \rangle$ theoretical

• Transverse momentum spectra $\frac{dN}{dnr}(\tau)$ using FONLL input $\frac{dN}{dnr}(\tau_{\text{form}})$

• Nuclear modification factor
$$R_{AA} = \frac{\mathrm{d}N^{AA}/\mathrm{d}p_T}{A^2\mathrm{d}N^{pp}/\mathrm{d}p_T}$$

observables[†]

*More on transport coefficients in QGP from Maria-Lucia and Salvatore [†]More on HF observables from Mattia

• Azimuthal correlation $C(\Delta \phi) = \frac{1}{N_{\text{partice}}} \frac{\mathrm{d}N}{\mathrm{d}\Delta\phi}$ for $Q\overline{Q}$ pairs



Numerical trajectories results







R_{pA} for D-mesons

- First study of HQs in glasma
- SU(2) glasma, static box
- Proton Q_s from hot spot model
- ► FONLL input + fragmentation





Hybrid R_{AA} and v_2

- SU(2) glasma, static box
- Compared with Fokker-Planck
- ▶ Including glasma increases v₂

Sun, Coci, Das, Plumari, Ruggieri, Greco [Phys.Lett.B798(2019)]




Momentum variance σ_p

- SU(2) glasma, static box
- Compared with Langevin
- Glasma correlation domains

Liu, Das, Greco, Ruggieri [Phys.Rev.D103(2021)]





Momentum variance σ_p

- SU(2) glasma, longitudinal expansion
- Compared with Langevin
- Glasma correlation domains

Khowal, Das, Oliva, Ruggieri [Eur.Phys.J.Plus137(2022)]





Momentum broadening $\langle \delta p^2 \rangle$ Transport coefficient κ

- SU(3) glasma, longitudinal expansion
- Colored-particle-in-cell solver
- Compared with correlator method

Avramescu, Băran, Greco, Ipp, Müller, Ruggieri [Phys.Rev.D107(2023)]





R_{AA} with nPDF effects

- SU(3) glasma, longitudinal expansion
- Colored-particle-in-cell solver
- ► FONLL + EPPS16 input calculation

Avramescu, Greco, Lappi, Mäntysaari, Müller [in preparation]





Azimuthal decorrelation $\mathcal{C}(\Delta \phi)$

- First study of $Q\overline{Q}$ correlations in glasma
- SU(3) glasma, longitudinal expansion
- Colored-particle-in-cell solver
- Extraction of decorrelation widths $\sigma_{\Delta\phi}$

Avramescu, Greco, Lappi, Mäntysaari, Müller [in preparation]

Particles in Yang-Mills fields



Correlator method

► Approach: infer particle dynamics from background field correlators

$$\langle \delta p_i^2(\tau) \rangle = g^2 \int_{\tau_0}^{\tau} \mathrm{d}\tau' \int_{\tau_0}^{\tau} \mathrm{d}\tau'' \left\langle \mathrm{Tr} \left[\widetilde{\mathcal{F}}_i(\tau') \widetilde{\mathcal{F}}_i(\tau'') \right] \right\rangle$$

$$\text{gauge invariant force correlator}$$

 $\text{Lorentz force } \mathcal{F}_i = F_{i\mu} \frac{p^{\mu}}{p^{\tau}} \xrightarrow[]{\text{gauge invariant}} \text{ parallel transport on lattice } \widetilde{\mathcal{F}}_i$



Static heavy quarks on lattice*

$$\left. \left\langle \delta p_i^2(\tau) \right\rangle \right|_{\boldsymbol{m} \to \infty} \propto \int_{\tau_0}^{\tau} \mathrm{d}\tau' \int_{\tau_0}^{\tau} \mathrm{d}\tau'' \Big\langle \mathrm{Tr} \big[\boldsymbol{E}_{\boldsymbol{i}}(\tau') \boldsymbol{E}_{\boldsymbol{i}}(\tau'') \big] \Big\rangle$$

*Electric field correlators also used by Vijami and Tom

Field correlators results







Transport coefficient κ Collisional energy loss dE/dx

- Analytical glasma fields in τ expansion
- Glasma $\langle EE \rangle$ and $\langle BB \rangle$ correlators
- Fokker-Planck equation for heavy quarks

Carrington, Czajka, Mrowczynski [Nucl.Phys.A1001(2020)]

Field correlators results







Momentum broadening $\langle \delta p^2 \rangle$ Transport coefficient κ

- Over-occupied classical Yang-Mills
- \blacktriangleright Numerical lattice $\langle EE \rangle$ correlator
- \blacktriangleright Large peak in $\langle \delta p^2 \rangle$
- Oscillations of κ with plasmon frequency

Boguslavski, Kurkela, Lappi, Peuron [JHEP09(2020)]







$\begin{array}{l} \mbox{Momentum broadening } \langle \delta p^2 \rangle \\ \mbox{Transport coefficient } \kappa \end{array}$

- SU(3) glasma, longitudinal expansion
- Numerical lattice $\langle EE \rangle$ correlator
- Comparison with numerical trajectories
- Ordering $\langle \delta p_L^2 \rangle > \langle \delta p_T^2 \rangle$, negative $\kappa_L < 0$

Avramescu, Băran, Greco, Ipp, Müller, Ruggieri [PoSHardProbes2023(2024)]

Field correlators results







Transport coefficient κ

- Glasma-like classical fields
- Numerical lattice $\langle EE \rangle$ correlator
- Non-perturbative gluonic excitations
- Explain the peak in κ

Backfried, Boguslavski, Hotzy [arXiv2408.12646]

Next stages of pre-equilibrium

Heavy quarks during bottom-up



Boguslavski, Kurkela, Lappi, Lindenbauer, Peuron [Phys.Rev.D110(2024)]

Next stages of pre-equilibrium Heavy quarks during bottom-up

Approaches

Effective kinetic theory

Quantities

- Transport coefficient κ
- Drag, diffusion coefficients A_i , B_{ij}

Heavy quarks in EKT

Extracting transport coefficients

- Approach: effective kinetic theory to study the gluon distribution function f(k)
 - Quantities: various transport coefficients κ , A_i , B_{ij} extracted from f(k)



Drag
$$A_i \propto \int d\Gamma_{PS} q_i |\mathcal{M}|^2 f(k) [1 \pm f(k')]$$

Diffusion $B_{ij} \propto \int d\Gamma_{PS} q_i q_j |\mathcal{M}|^2 f(k) [1 \pm f(k')]$

EKT results







Transport coefficient κ

- Energy density ε matched to glasma
- Compare to κ in glasma
- Compare with equilibrium κ_{eq}
- Match for the same m_D , T_\star and ε

Boguslavski, Kurkela, Lappi, Lindenbauer, Peuron [Phys.Rev.D109(2024)]

EKT results







Drag, diffusion A_i , B_{ij}

- Contributions from g, q, g + q
- Angular dependence
- Rescaled coefficients, attractor behavior

Du [Phys.Rev.C109(2024)]

$\mathsf{My} \ \mathsf{questions}$



Theoretical improvements

First stage of bottom-up thermalization

How to connect κ from glasma to EKT? Match using gluon distribution function

Heavy quark energy loss in glasma

Only recently: jet energy loss in glasma from synchrotron radiation*

Experimental observables

Observables sensitive to pre-equilibrium
 What to extract? The most sensitive is the azimuthal correlation

Large initial anisotropy

How to measure anisotropy? Many studies in anisotropic systems †

Compare theory to experiment

▶ How sensitive is data to pre-equilibrium? Simulations of all stages for HQ transport

^{*}Barata, Hauksson, López, Sadofyev [2406.07615]

[†]Hauksson, Jeon [Phys.Rev.C105(2022)]; Barata, Sadofyev, Salgado [Phys.Rev.D105(2022)]

Thank you!

Back-up

Hard probes in Glasma



Classical transport in the very-early stage

Prerequisite: Classical lattice gauge theory $\xrightarrow{\text{solver}}$ Glasma fields

This work: Glasma fields $\stackrel{ ext{background}}{\longleftarrow}$ test particles $\stackrel{ ext{solver}}{\longleftarrow}$ colored particle-in-cel



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CGC as an EFT for high energy QCD



Classical Yang-Mills fields



Two-point function $\langle
ho^a
ho^a
angle \propto Q_s^2$ saturation momentum

Numerical implementation (technicalities)



Boost-invariant Yang-Mills equations for $A_i(\tau, \vec{x}_{\perp}, \mathbf{y})$ and $A_{\eta}(\tau, \vec{x}_{\perp}, \mathbf{y})$



Trace of a plaquette \mapsto gauge invariant Wilson lines on the lattice \leftrightarrow gauge links $U_{x,\mu} = \exp\{igaA_{\mu}(x)\}$ Wilson loops on lattice \leftrightarrow plaquettes $U_{x,\mu\nu} \equiv U_{x,\mu}U_{x+\mu,\nu}U_{x+\mu,\mu}^{\dagger}U_{x,\nu}^{\dagger}$

 $\text{Glasma} \xrightarrow{\text{boost invariance}} \text{transverse gauge links } U_i(\tau, \vec{x}_\perp) \text{, while } A_\eta(\tau, \vec{x}_\perp)$

General features



Relevant scale Q_s Fields dilute after $\delta \tau \simeq Q_s^{-1}$, arrange themselves in correlation domains of $\delta x_T \simeq Q_s^{-1}$



Bjorken expansion

 $au=0.01~{
m [fm/T}$ he fields become dilute after $\delta au\simeq Q_s^{-1} au=0.4~{
m [fm/c]}$





Flux tubes

The fields arrange themselves in correlation domains of $\delta x_T \simeq Q_s^{-1}$



Anisotropy



4

Particles in Yang-Mills fields



Wong's equations of motion

Wong's equations \leftrightarrow classical equations of motion for particles (x^{μ}, p^{μ}, Q) evolving in a Yang-Mills background field A^{μ}



Symplectic numerical solver $\xrightarrow{assurements}$ Wong's equations ervation of Casimir invariants

Heavy quarks in Glasma

Momentum broadening and κ





Jets in Glasma



Momentum broadening and \hat{q}



Large transport coefficients

Plausible in an EKT framework



Large transport coefficients

Plausible in an EKT framework



Two particle correlations

Quantifying the decorrelation



 $\text{Initial } \mathcal{C}(\tau_{\text{form}}) \propto \delta(\Delta \phi - \pi) \delta(\Delta \eta) \xrightarrow{\Delta \tau \text{ in Glasma}} \mathcal{C}(\tau_{\text{form}} + \Delta \tau) \xrightarrow{\text{extract}} \sigma_{\Delta \phi}(\Delta \tau), \sigma_{\Delta \eta}(\Delta \tau)$



Azimuthal decorrelation width



Effect of heavy quark p_T and Glasma Q_s



Azimuthal decorrelation width



Effect of heavy quark $\ensuremath{p_T}$ and Glasma $\ensuremath{Q_s}$



Nuclear modification factor

Sketch of p_T spectra in the Glasma

Heavy quarks $\xrightarrow{\text{FONLL}^*}$ initial p_T distribution $\propto d\sigma^{pp/AA}/dp_T(\sqrt{s}, \text{PDF/nPDF})$



* Fixed Order + Next-to-Leading Logarithms, state-of-the-art resummed heavy quark production
Nuclear modification factor



Extraction of R_{AA} in Glasma



R_{AA} in the Glasma



Temporal evolution and ${\cal Q}_s$ dependence



R_{AA} in the Glasma



Temporal evolution and ${\it Q}_{\it s}$ dependence





Cold nuclear matter effects



R_{AA} in the Glasma with nPDFs



Cold nuclear matter effects

