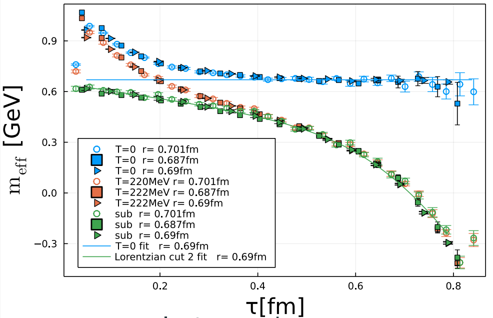
Lattice-QCD determination of transport coefficients for heavy quarks and quarkonia

Viljami Leino
Helmholtz Institute Mainz, JGU Mainz

new results based on collaboration with:
Nora Brambilla, Saumen Datta, Marc Janer, Julian Mayer-Steudte, Péter Petreczky, Antonio Vairo
In various combinations

QCD Challenges from pp to AA collisions,
Münster,
03.09.2024

Motivation



- Lattice QCD allows non-perturbative study of QCD at finite temperature
- However is limited to Euclidean operators X
- To connect to real world: invert Laplacian convolution equation

$$G_X(\tau, T) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_X(\omega, T) K(\omega, \tau, T)$$

- Relevant operators: Diffusion, finite temperature static potential, $\hat{q}$, quarkonium correlators,...
- Here: focus on diffusion; other operators see P. Gossiaux talk yesterday
- Nuclear modification factor R_{AA} and elliptic flow v_2 described by spatial diffusion coefficient D_x see other talks in this track
- Lattice limited to zero momenta

Figure: HOTQCD Phys.Rev.D 109 (2024)

Heavy Quark diffusion: Langevin perspective

- Heavy quark energy changes only little when colliding with medium

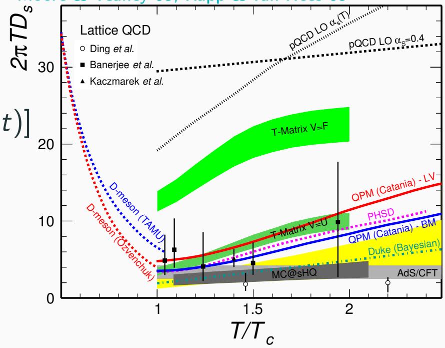
$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium

$\rightarrow$ Brownian motion; Langevin dynamics can be used

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT}p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

Svetitsky 88; Mustafa et.al.97;
Moore & Teaney 05; Rapp & van Hees 05



Associated Fokker-Planck equation

$$\frac{\partial f_Q(p,t)}{\partial t} = -\frac{\partial}{\partial p_i}[p_i\eta_{\rm D}(p)f_Q(p,t)] + \frac{\partial^2}{\partial p_i\partial_j}[\kappa_{ij}(p)f_Q(p,t)]$$

- Single coefficient κ gives access to multiple interesting quantities:

$D_{\rm s} = 2T^2/\kappa$	$\eta_{\rm D} = \kappa/(2MT)$	$\tau_{\rm Q} = \eta_{\rm D}^{-1}$
Spatial diffusion	Drag coefficient	Relaxation time

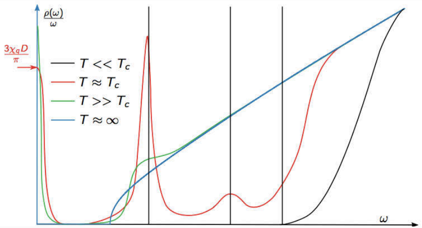
Figure: X. Dong CIPANP (2018)

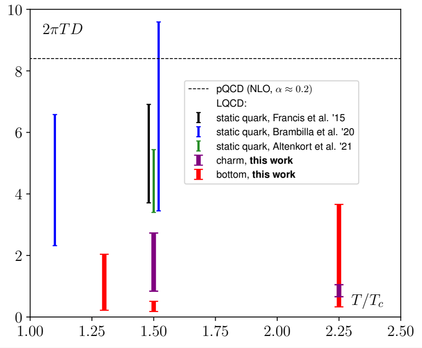
Heavy Quark Current-Current correlators

- For Heavy quark vector current $\hat{\mathcal{J}}$ define force $\mathcal{F} = M d\hat{\mathcal{J}}/\mathrm{dt}$

$$\kappa^{(M)}(\omega) \equiv \frac{1}{3\chi} \int_{-\infty}^{\infty} \mathrm{dt}\, \mathrm{e}^{\mathrm{i}\omega\mathrm{t}} \int_{\mathrm{x}} \left\langle \frac{1}{2} \left\{ \mathcal{F}^{\mathrm{i}}(\mathrm{t},\mathrm{x}), \mathcal{F}^{\mathrm{i}}(0,0) \right\} \right\rangle$$

- Very narrow transport peak at zero
- Competitive results for κ only recently
- Allows to measure mass shifts and thermal widths





Figures: up: courtesy to H-T. Shu, down: Ding *et.al.*PRD104 2021

HQET picture

- Expand the force in $1/M$

$$\mathcal{F}^i = \phi^\dagger \left[-gE^i + \frac{[D^i, D^2 + c_b g\sigma \cdot B]}{2M} + \cdots \right] \phi$$

- Note also Lorentz force

$$F(t) = \dot{p} = q\,(E + v \times B))\,(t)$$

- Switch to Euclidean space correlation function:

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re\,Tr}\,[U(\beta,0)]\rangle},$$

$$G_{\mathrm{B}}(\tau) = \sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\,[U(1/T,\tau)B_i(\tau,0)U(\tau,0)B_i(0,0)]\rangle}{3\,\langle \mathrm{Re\,Tr}\,U(1/T,0)\rangle}$$

$$\kappa_{\mathrm{E,B}} = \lim_{\omega\to 0} \frac{2T}{\omega}\rho(\omega) \qquad G_{\mathrm{E,B}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}}$$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)
A. Bouttefeux and M. Laine JHEP 12 (2020) 150, M. Laine JHEP 06 (2021) 139

Heavy Quarkonium Diffusion

- Quarkonium in medium can be described by Limbland equation by using pNRQCD and open quantum systems
- Three possible interactions Brambilla *et.al.*TUM-EFT 191/24
  - Singlet $\rightarrow$ Octet : dissociation
  - Octet $\rightarrow$ Singlet : recombination
  - Octet $\rightarrow$ Octet
- Each process described by two parameters κ_{xx} and γ_{xx}
- κ_{so} is related to the thermal width and describes heavy quarkonium diffusion
- γ_{so} is related to the mass shift $\gamma = -\frac{1}{3N_c}\int_0^\infty \frac{d\omega}{2\pi}\frac{\rho(\omega)}{\omega}$
- Width and Shift can be measured on the lattice for un-quenched estimate
- $\kappa_{so} \simeq \kappa_{HQ}$ at two-loop level
- γ for normal heavy quarks expected to be zero
- Also related: Diffusion of an adjoint static quark

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018), Brambilla *et.al.*PRD100 (2019), Eller *et.al.*PRD99 (2019), Scheihing-Hitschfeld & Yao PRD108 (2023), V.L. Lattice2023

Heavy Quarkonium Diffusion on the lattice

- Euclidean correlators similar to HQ-case, but with adjoint Wilson line
- κ_{so} and κ_{os} given by

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\langle \mathrm{Re}\,\mathrm{Tr}\;[gE_i(\tau,0)\Phi(\tau,0)gE_i(0,0)]\rangle \;,$$

- Separating κ_{so} and κ_{os} on lattice still work in progress
- κ_{oo} very similar to HQ-diffusion

$$G_{\mathrm{EE}}^{8.\mathrm{symm}} \cdot L_8 \equiv \frac{1}{3}\sum_{i=1}^{3}\langle \Phi^{\mathrm{A}}_{xa}(N_T,t) d_{abc} E^{i,c}(t) G^{\mathrm{A}}_{bz}(t,0) d_{zxg} E^{i,g}(0)\rangle$$

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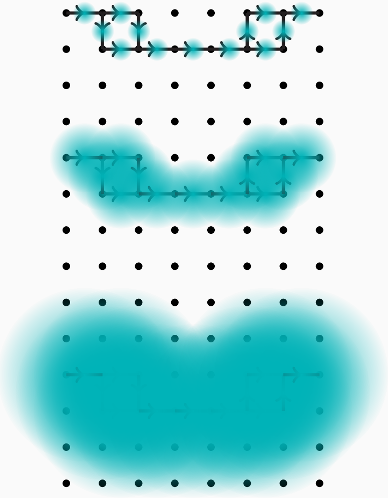
- Adjoint lines need renormalization on the lattice Gubta *et.al.*PRD77 2008

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018), Brambilla *et.al.*PRD100 (2019),
Eller *et.al.*PRD99 (2019), Scheihing-Hitschfeld & Yao PRD108 (2023), V.L. Lattice2023

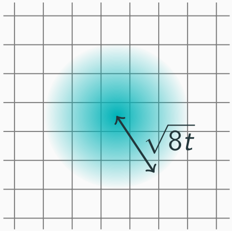
Gradient Flow



$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu} \,,$$

$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \,.$$

$$B_{0,\mu} = A_\mu \quad \leftarrow \text{ the original gauge field}$$



- Field strength tensor components discretized (clover, 2plaq, corner)
- On lattice there is a self-energy contribution that has to be renormalized
- Gradient flow automatically renormalizes $\sqrt{8\tau_f} > a$
- Avoid oversmearing/overlap $\sqrt{8\tau_f} < \tau/2$
- For chromomagnetic fields there is also a finite anomalous dimension and renormalization is required

See e.g. M. Lüscher JHEP 08 (2010)

General procedure

- Normalize the data with perturbative LO result (also tree-level improve)

$$G^{\mathrm{norm}}_{\mathrm{E,B}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- On Lattice E has non-physical self-energy contribution

  Removed by the flow. Previous studies used the 1-loop result:

  $Z_{\mathrm{E}} = 1 + g_0^2 \times 0.137718569\ldots + \mathcal{O}(g_0^4)$ (Christensen and Laine PLB02 (2016))

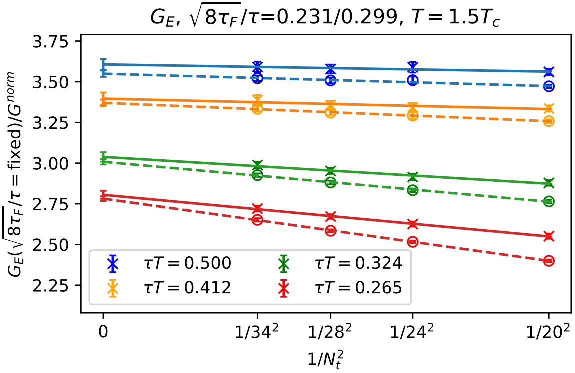
- κ_{B} more complicated, requires renormalization

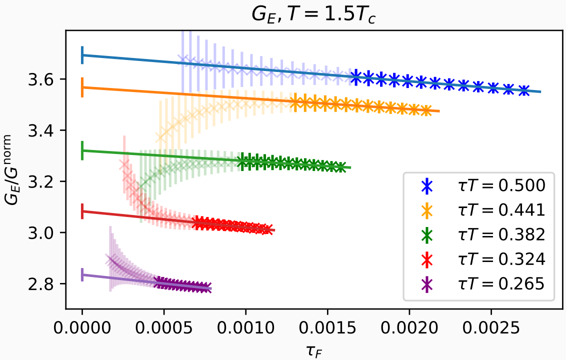
$$G_{\mathrm{B}}^{\mathrm{flow,UV}}(\tau,\tau_{\mathrm{F}}) = (1 + \gamma_0 g^2 \ln(\mu\sqrt{8\tau_{\mathrm{F}}}))^2 Z_{\mathrm{flow}}\, G_{\mathrm{B}}^{\overline{\mathrm{MS}},\mathrm{UV}}(\tau,\mu) + h_0 \cdot (\tau_{\mathrm{F}}/\tau)\,,$$

- Still need additional normalization, normalize to 1 at small τT
- Set scale such that NLO UV contribution vanishes
- Then invert the spectral function

Perturbative results: A.M. Eller 2021, L. Altenkort *et.al.*223

Order of limits matters



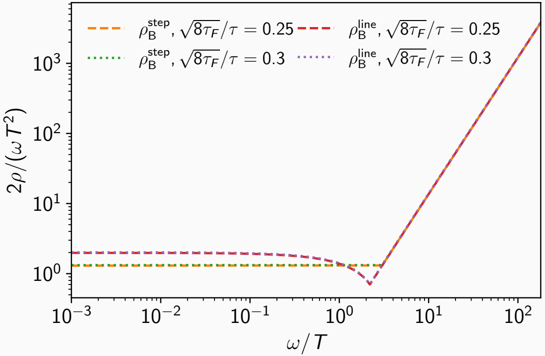


Take limits in correct order:

1. Continuum limit to zero lattice spacing
2. Zero flow time limit
3. invert spectral function

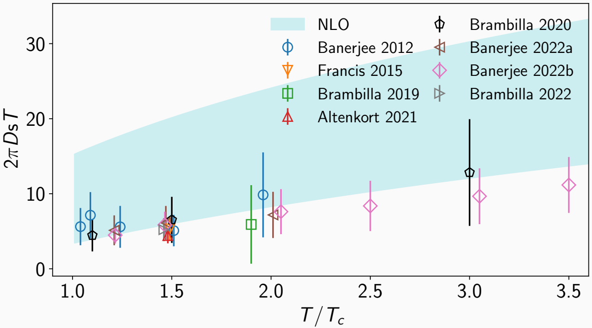
Inversion of the spectral function

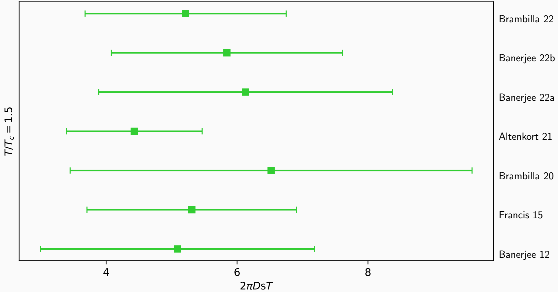
- $\rho(\omega)$ known at IR and UV
- Connect IR and UV with i) step ii) line
- Simple step function $\rho^{\text{step}}_{\text{E,B}}(\omega, T) = \rho^{\text{IR}}_{\text{E,B}}(\omega, T)\,\theta(\Lambda - \omega) + \rho^{\text{UV,T=0}}_{\text{E,B}}(\omega, T)\,\theta(\omega - \Lambda)$
- Physically motivated $\rho(\omega) = \sqrt{\rho^2_{\text{IR}} + c\rho^2_{\text{UV}}}$ Francis *et.al.*2015
- Use more complicated shape (line, polynomial)



- In case of κ_B, the spectral function depends on flow time due to renormalization of the anomalous dimension
- Either compare directly to data, Baggus-Gilbert, MEM...

κ_E results (pure gauge)

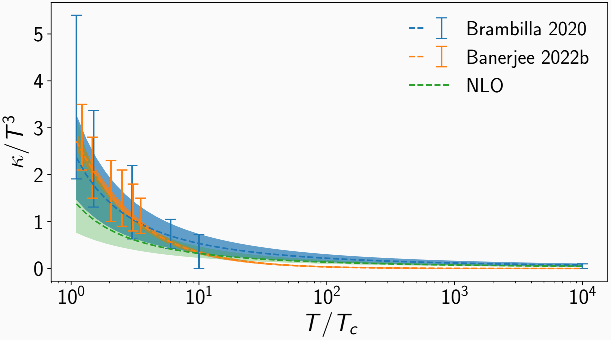




- Results of different groups agree very well
- Error dominated by the systematics from the $\rho(\omega)$ inversion
- Matches well with NLO perturbation theory

κ_{E} temperature dependence (pure gauge)

- Can fit temperature dependence



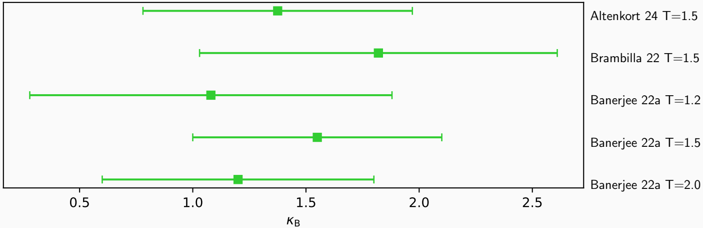
$$\kappa = \frac{g^4 C_{\mathrm{F}} N_{\mathrm{c}}}{18\pi}\left[\ln\frac{2T}{m_{\mathrm{E}}} + \xi + C\frac{m_{\mathrm{E}}}{T}\right]T^3 \quad \text{Brambilla 2020}$$

$$2\pi T D_{\mathrm{s}} = \alpha + \gamma\left(\frac{T}{T_{\mathrm{c}}} - 1\right) \quad \text{Banerjee 2022b}$$

- Good agreement between the different approaches, and NLO:

$$\kappa_{\mathrm{E}} = \frac{g^4 T^3}{27\pi}\left[2N_{\mathrm{c}}\left(\ln\frac{2T}{m_{\mathrm{D}}} + \xi\right) + N_f\left(\ln\frac{4T}{m_D} + \xi\right) + \frac{N_c m_D}{T}C\right]$$

κ_B Results (pure gauge)

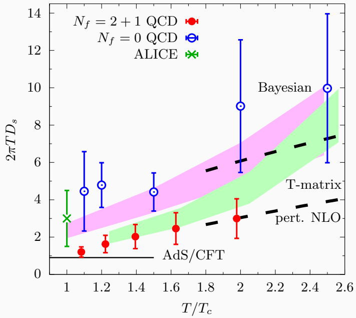


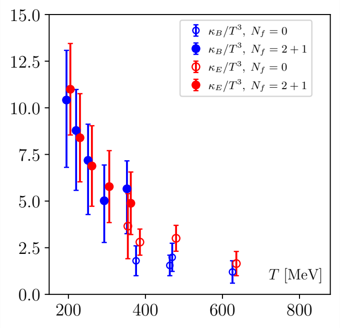
- Good agreement between existing results, close to κ_E
- Minor temperature dependence in the current measured range

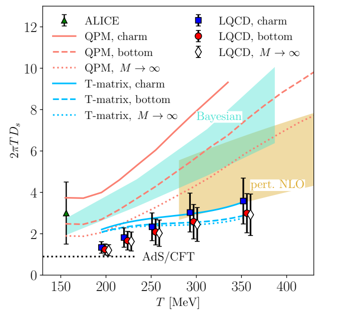
$$\kappa_\mathrm{tot} \simeq \kappa_\mathrm{E} + \frac{2}{3}\langle \mathrm{v}^2 \rangle \kappa_\mathrm{B}$$

- Using $\langle \mathrm{v}^2 \rangle$ from (Petreczky *et.al.* Eur. Phys. J. C62 (2009))
- $\langle \mathrm{v}^2_\mathrm{charm} \rangle \simeq 0.51$ and $\langle \mathrm{v}^2_\mathrm{bottom} \rangle \simeq 0.3$, we get that the mass suppressed effects on the heavy quark momentum diffusion coefficient κ is 34% and 20% for the charm and bottom quarks respectively.

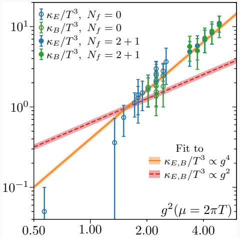
κ_{E} dependence on dynamical fermions





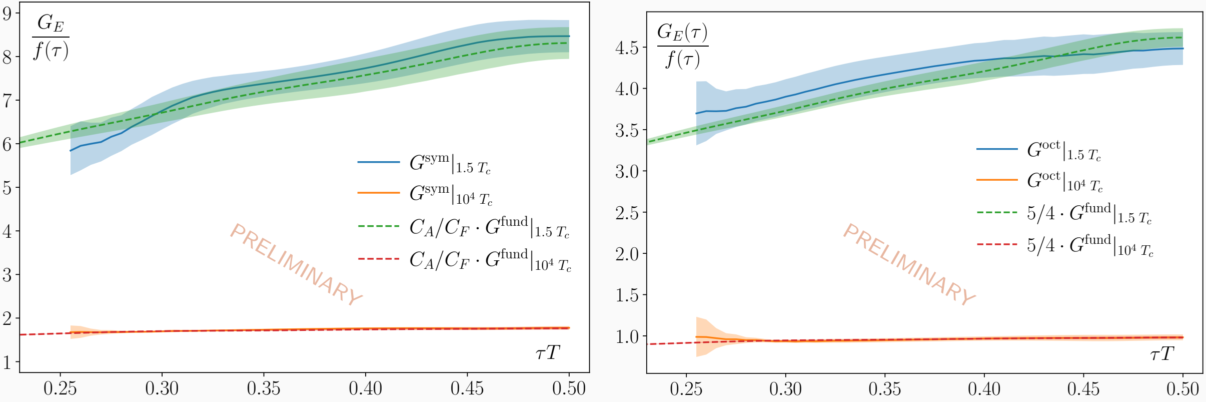


- Most studies have been in pure gauge
- Recent results from HOTQCD
- Main difference to pure gauge is different T_c



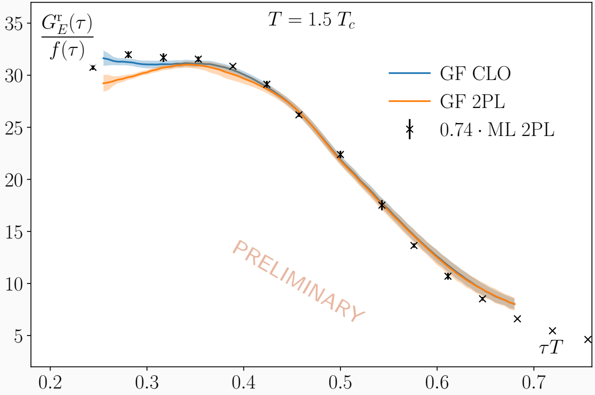
HOTQCD: Phys.Rev.Lett. 130 (2023), Phys.Rev.Lett. 132 (2024), Phys.Rev.D 109 (2024)

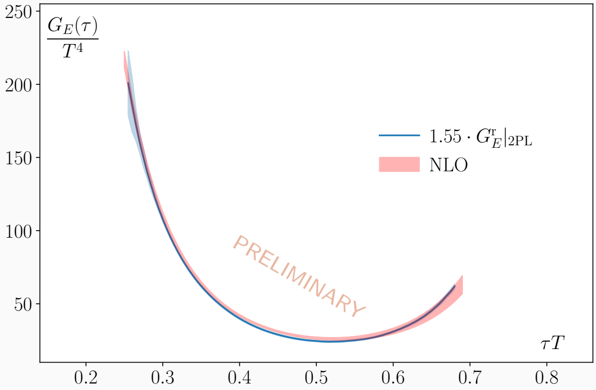
κ_{oo} and adjoint heavy quarks



- Left: adjoint quark, Right: quarkonium
- For symmetric correlators we observe expected (Casimir) scaling nonperturbatively
- These results translate from G_E to κ trivially

κ_{so} and κ_{os}





- Asymmetric correlator on lattice relates to both κ_{so} and κ_{os}
- Spectral reconstruction still pending
- At high temperatures, excellent agreement with the perturbation theory

NLO result: N. Brambilla, P. Panayiotou, S. Säppi, A. Vairo: in preparation

γ

- Only existing determination uses mass shifts for rough estimate
- More recent similar analysis gets $\gamma \sim 0$ (see Tom's talk)
- Euclidean correlators for Quarkonium will allow measurement of γ
- Need to subtract zero temperature contribution
- Promising results on zero temperature measurements
- Combination of zero and finite T still in progress
- stay tuned

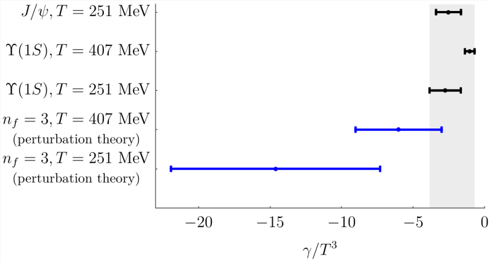


Figure: Brambilla *et.al.*Phys.Rev.D 100 (2019)

Conclusions and Future prospects

- Transport coefficients can be measured on the lattice
- Advancements in recent algorithms such as gradient flow help with calculations
- This talk focused on recent advancements on diffusion coefficients
- Heavy quark diffusion is being well scoped for temperatures and masses
- First heavy quarkonium results very soon
- γ hopefully in future
- Simulations for $1/M$ corrections for quarkonium diffusion ongoing

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Thank you for your attention!