

# Hadronization of light and heavy flavor across collision systems

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Università degli Studi di Catania**

**INFN-LNS**

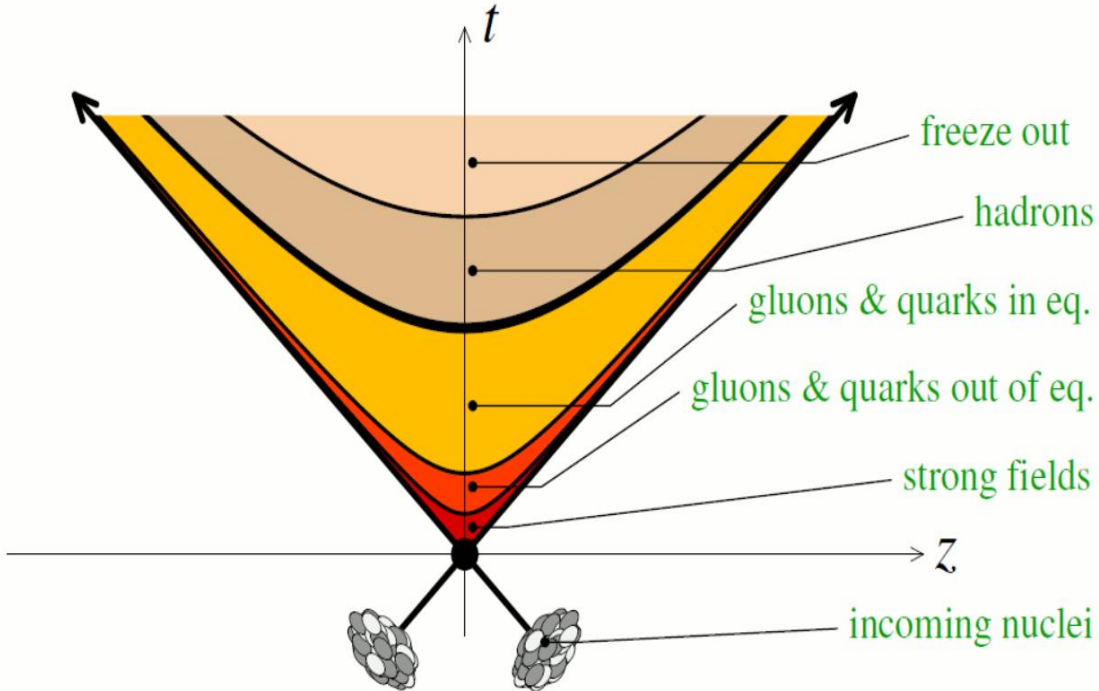


**Thanks to:**

**V. Minissale, M.L. Sambaturo, S. K. Das, Y. Sun, V. Greco**



# Hadronisation in heavy ion collisions



- **Hadronisation:**  
*the mechanism by which quarks and gluons produced in hard partonic scattering processes form the hadrons*
- **No first-principle description of hadron formation**
  - *Non-perturbative problem*
  - *Necessary to resort to models*
- **Hadronisation of the QGP medium at the pseudo-critical temperature**  
*Transition from a deconfined medium composed of quarks, antiquarks and gluons to color-neutral hadronic matter*
- **Hadronization (impossible to neglect)**

## Hadronization is relevant:

- *how hadron are produced? Is it a universal process in  $e^+e^-$ ,  $e^-p$ ,  $pp$ ,  $pA$  and  $AA$ ?*
- *source of systematic uncertainty in final observable  $R_{AA}$  and  $v_2, v_3, \dots$*   
→ *systematic uncertainty in extracting transport coefficients.*
- *Crucial role in the interpretation of results of Polarization for light hadrons, open heavy flavors, and quarkonia.*
- *Impact in predictions for multi-charm production  $PbPb$  vs  $KrKr$  vs  $ArAr$ . Relevant for ALICE3*

# Hadronization: fragmentation and coalescence

## Proton to pion ratio Enhancement:

In vacuum from fragmentation functions

the ratio is small  $\frac{D_{q \rightarrow p}(z)}{D_{q \rightarrow \pi}(z)} < 0.25$

## Elliptic flow splitting:

For  $p_T > 2$  GeV Both hydro and fragmentation predicts similar  $v_2$  for pions and protons

## Another hadronization mechanism is by coalescence:

Formalism originally developed for light-nuclei production from coalescence of nucleons on a freeze-out hypersurface.

Extended to describe meson and baryon formation in AA collision from the quarks of QGP through  $2 \rightarrow 1$  and  $3 \rightarrow 1$  processes

V. Greco, C.M. Ko, P. Levai PRL 90, 202302 (2003).

V. Greco, C.M. Ko, P. Levai PRC 68, 034904 (2003).

R.J. Fries, B. Muller, C. Nonaka, S.A. Bass PRL 90, 202303 (2003).

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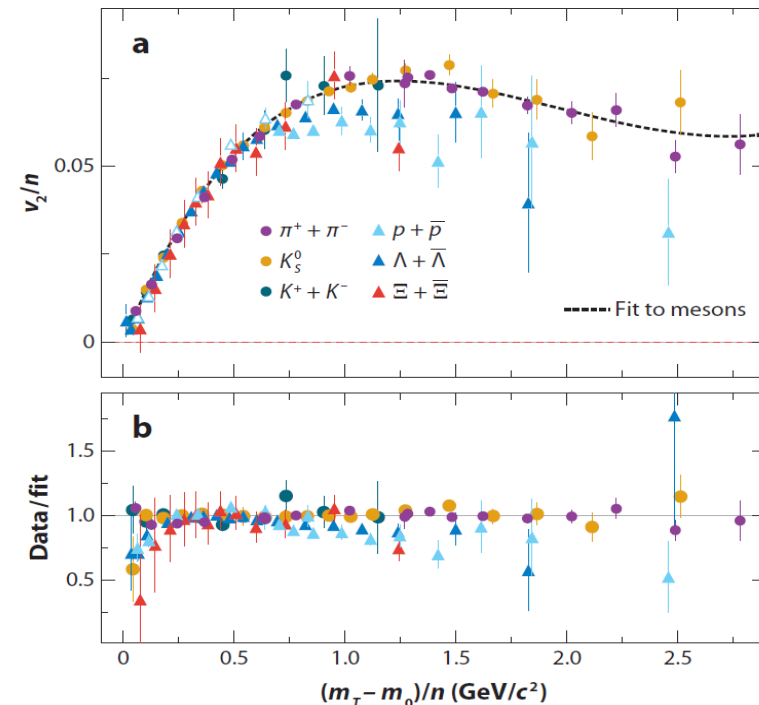
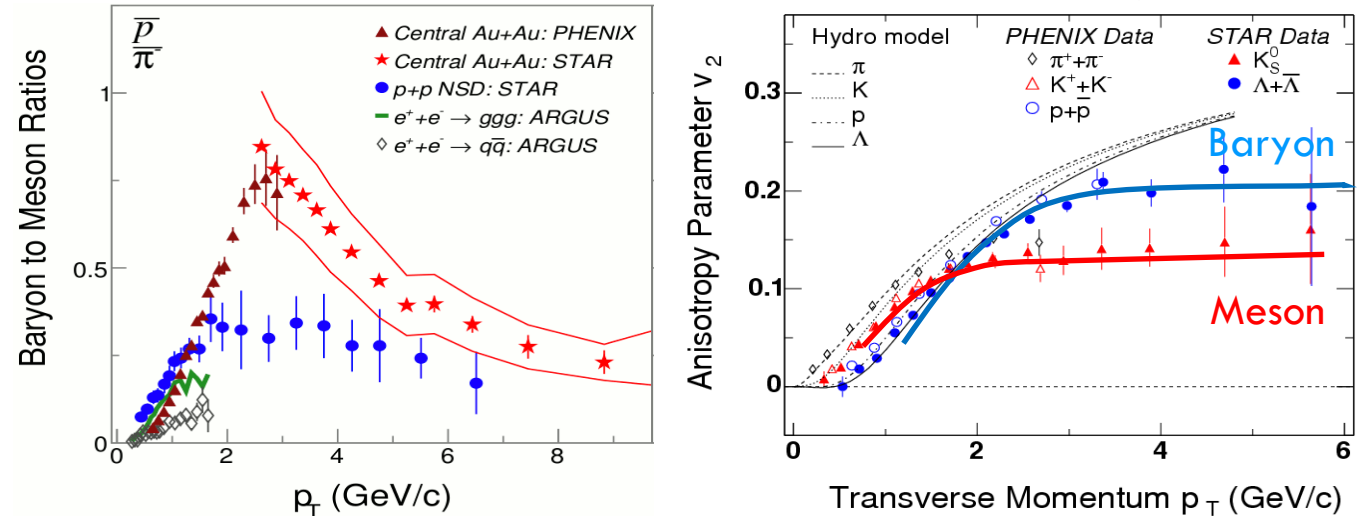
## Resonance Recombination model

Reformulation of coalescence approaches based on a transport

L. Ravagli and R. Rapp, Phys. Lett. B 655, 126 (2007).

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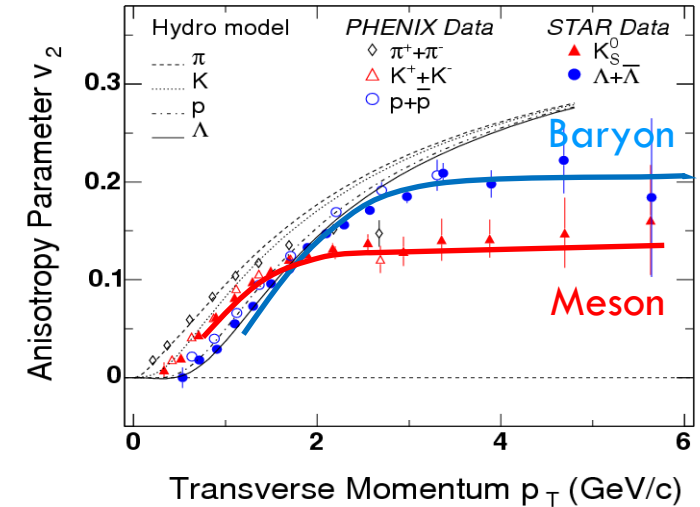
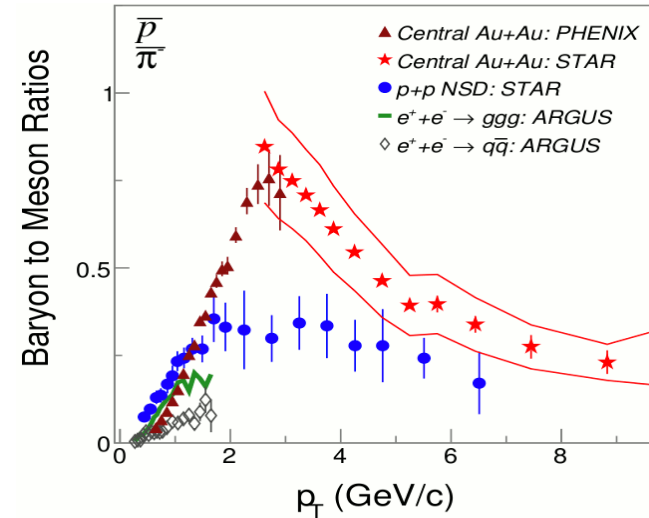
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## HF hadronization have stimulated new developments:

- PYHTIA beyond Leading Color (LC)  $\rightarrow$  Color Reconnection (CR) in pp
- SHM applied to pp
- Coalescence+Fragmentation approach applied to pp
- Local color recombination: POWLANG in AA and in pp
- Inclusion of HF Coalescence+ Fragmentation in EPOS (pp & AA)

# Hadronization schemes

- **Independent fragmentation**

$$q \rightarrow \pi, K, p, \Lambda \dots$$

$$c \rightarrow D, D_s, \Lambda_c, \dots$$

- **String fragmentation (PYTHIA)**

T. Sjostrand et al., JHEP 05 (2006), 026

C. Bierlich et al., SciPost Phys. Codeb. 2022 (2022), 8

- **In medium hadronization with Cluster decay**

A. Beraudo et al., EPJC82(2022) [AA]

A. Beraudo et al., PRD109(2024) [pp]

- **Coalescence/recombination**

S. Plumari, V. Minissale et al, Eur. Phys. J. **C78** no. 4, (2018) 348

S. Cao et al. , Phys. Lett. B 807 (2020) 135561

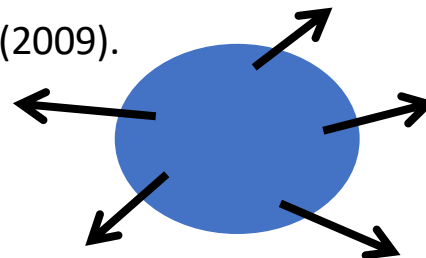
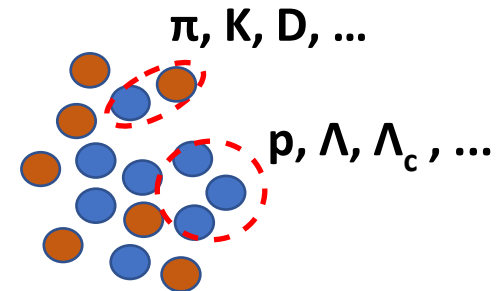
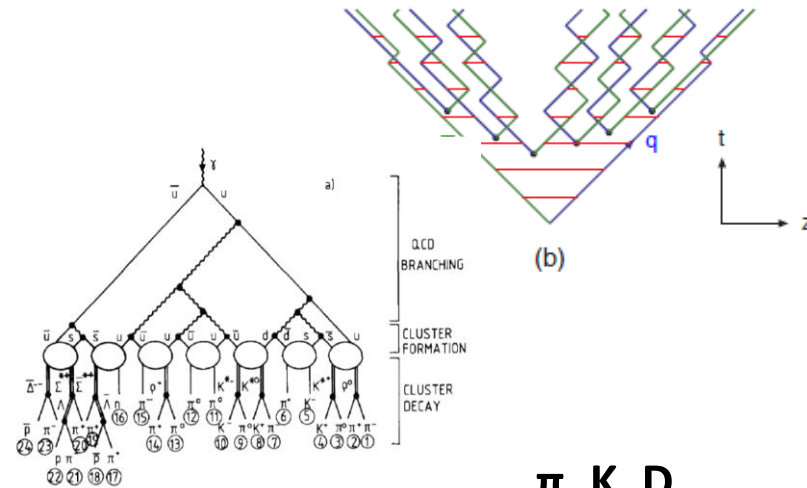
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- **Statistical hadronization model (SHM)**

A. Andronic et al, JHEP 07 (2021) 035

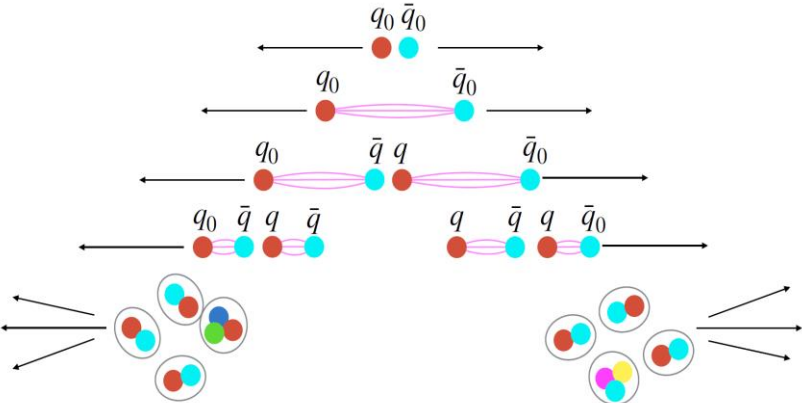


For recent reviews of HF hadronization see:  
 J. Altmann, arXiv:2405.19137 [hep-ph]  
 J. Zhao, et al., PRC **109**, no.5, 054912 (2024)

# PYTHIA

T. Sjostrand et al., JHEP 05 (2006), 026  
 C. Bierlich et al., SciPost Phys. Codeb. 2022 (2022), 8

PYTHIA model hadronization from e+e- up to pp collisions recent extension (Angantyr) to study AA collisions (C. Bierlich et al., JHEP 10, 134 (2018))

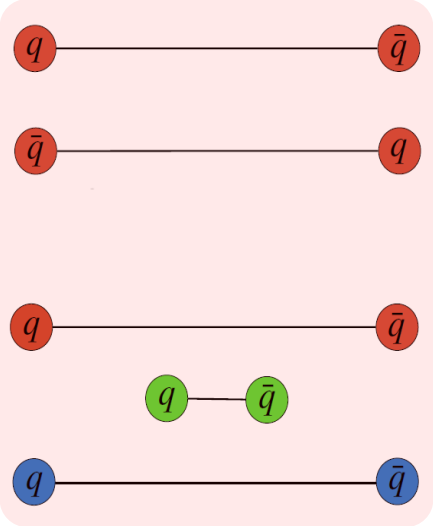


The string breaking is modelled via Schwinger mechanism for QCD  $\sim e^{-\left(\frac{\pi m_{\perp q}^2}{k}\right)}$   $k$  is the string tension

high- $p_T$  excitations and heavy quarks are produced by perturbative mechanisms (partonic scattering processes and parton showers) and not by string breaking

Leading Color ( $N_c \rightarrow \infty$ ): Prob. of Local Color neutralization  $\rightarrow 0$  only "dipole" string configurations are allowed

In Leading Color HF baryon only by [di-quark+HF] with HF as string end point

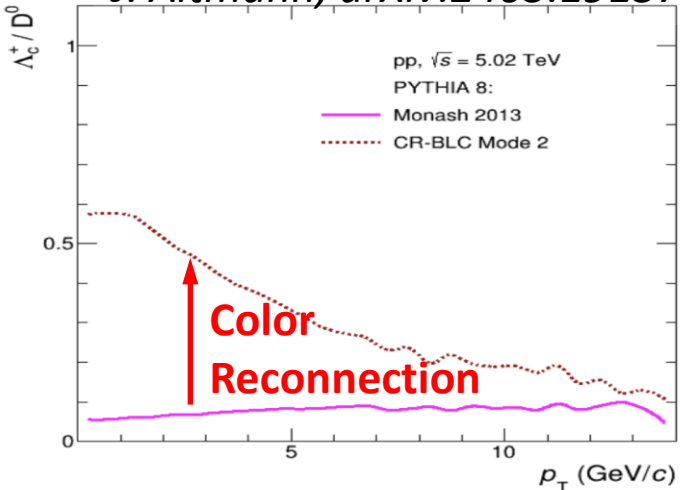


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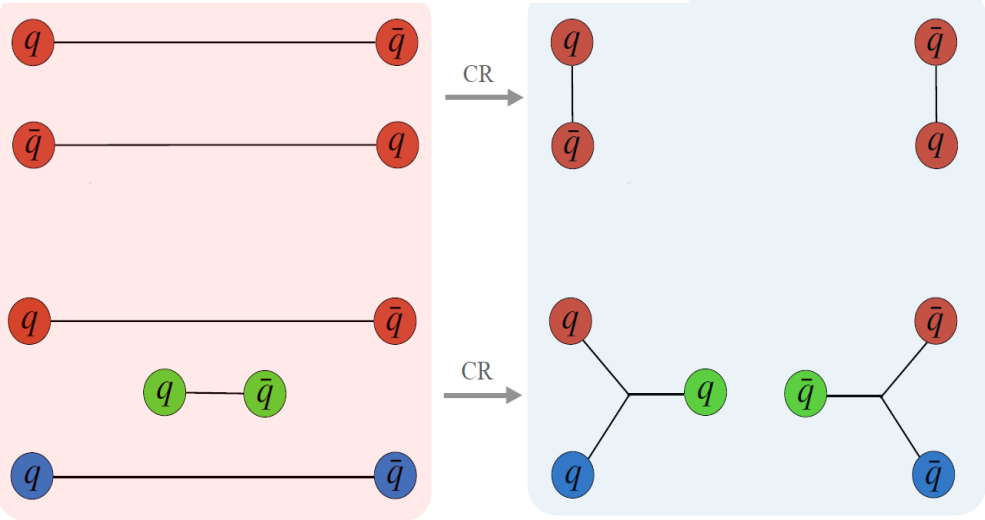


The string breaking is modelled via Schwinger mechanism for QCD  $\sim e^{-\left(\frac{\pi m_{1q}^2}{k}\right)}$  *k is the string tension*

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When string color reconnection is switched-on in pp according to SU(3) counting:

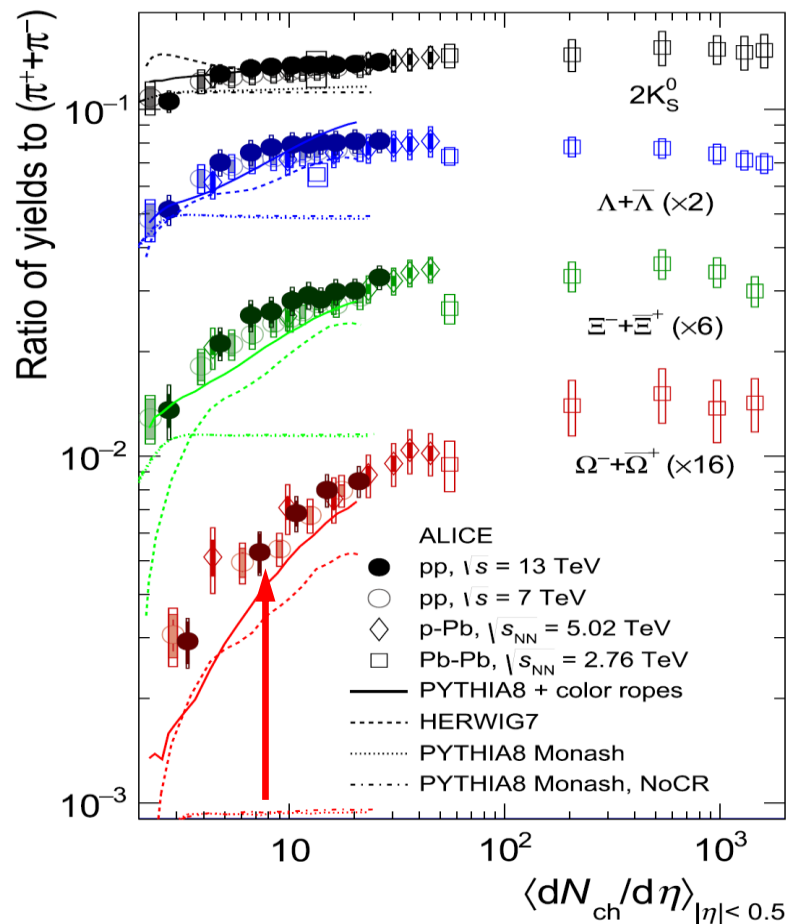
J. R. Christiansen and P. Z. Skands JHEP 08 (2015) 003

- Very large baryon  $\Lambda_c, \Sigma_c$  enhancement
  - not that relevant for D, similar to coalescence+fragmentation
- Local reconnection  $\rightarrow$  string energy minimization  $\rightarrow$  smaller invariant mass and breaking of long y correlation**



PYTHIA model hadronization from e+e- up to pp collisions recent extension

(Angantyr) to study AA collisions (C. Bierlich et al., JHEP 10, 134 (2018))



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Local reconnection  $\rightarrow$  string energy minimization  $\rightarrow$  smaller invariant mass and breaking of long  $y$  correlation

Rope hadronization has been proposed to describe strangeness enhancement  $\rightarrow$  increase string tension in densely packed environments  $\rightarrow$  to higher strange-hadron production.

C. Bierlich, EPJ Web Conf. 171 (2018), 14003



# Coalescence approach in phase space

Statistical factor colour-spin-isospin

$$\frac{dN_{Hadron}}{d^2p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta\left(p_T - \sum_i p_{iT}\right)$$

Parton Distribution function

Hadron Wigner function

Wigner function <-> Wave function

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \varphi_M\left(\mathbf{r} + \frac{\mathbf{r}'}{2}\right) \varphi_M^*\left(\mathbf{r} - \frac{\mathbf{r}'}{2}\right)$$

$\varphi_M(\mathbf{r})$  meson wave function

Assuming gaussian wave function

$$f_M(x_1, x_2; p_1, p_2) = A_W \exp\left(-\frac{x_{r1}^2}{\sigma_r^2} - p_{r1}^2 \sigma_r^2\right)$$

For baryon  $N_q=3$

$$f_H(\dots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

Note: only  $\sigma_r$  coming from  $\varphi_M(\mathbf{r})$  or  $\sigma_r^* \sigma_p = 1$  valid for harmonic oscillator with  $V(r)$   $\sigma_r^* \sigma_p > 1$

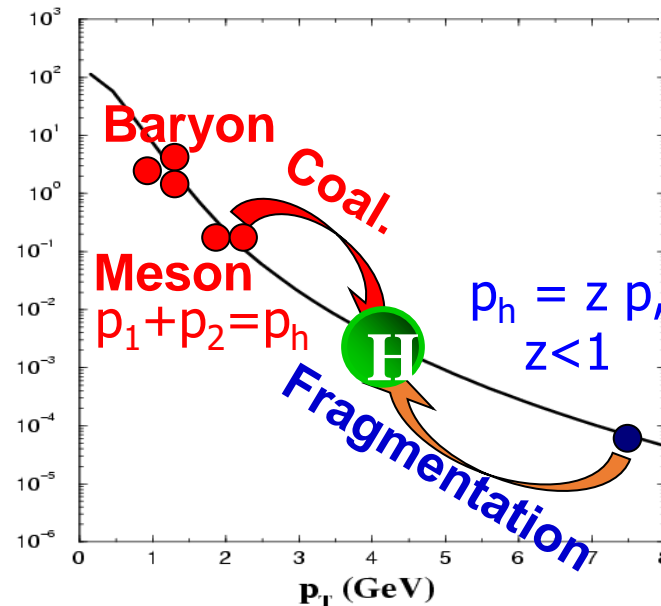
Wigner function width fixed by root-mean-square charge radius from quark model

$$\langle r^2 \rangle_{ch} = \frac{3}{2} \frac{m_2^2 Q_1 + m_1^2 Q_2}{(m_1 + m_2)^2} \sigma_{r1}^2 + \frac{3}{2} \frac{m_3^2(Q_1 + Q_2) + (m_1 + m_2)^2 Q_3}{(m_1 + m_2 + m_3)^2} \sigma_{r2}^2 \quad (8)$$

C.-W. Hwang, EPJ C23, 585 (2002);  
C. Albertus et al., NPA 740, 333 (2004)

$\sigma_{ri} = 1/\sqrt{\mu_i \omega}$  Harmonic oscillator relation

$$\mu_1 = \frac{m_1 m_2}{m_1 + m_2}, \quad \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}$$



# Coalescence approach in phase space for HQ

Statistical factor colour-spin-isospin

Parton Distribution function

Hadron Wigner function

$$\frac{dN_{Hadron}}{d^2p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta\left(p_T - \sum_i p_{iT}\right)$$

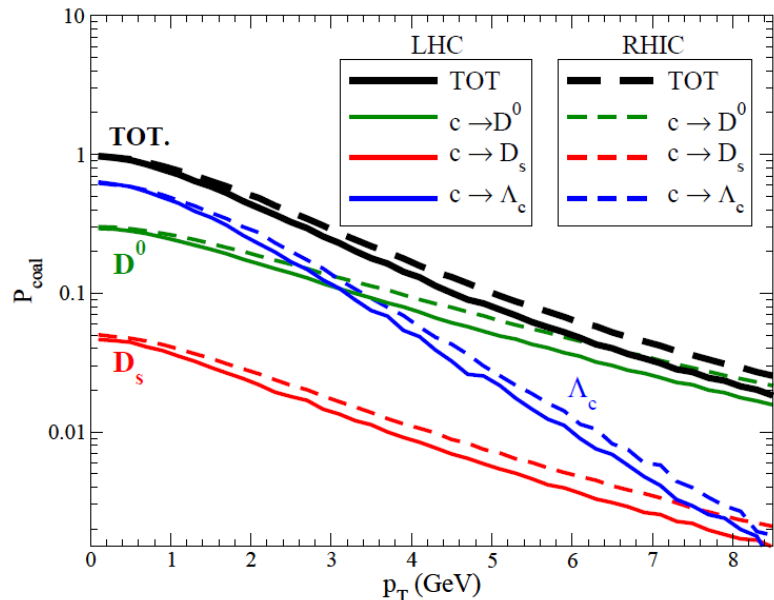
$$f_H(\dots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

✦ Normalization in  $f_W(\dots)$  fixed by requiring  $P_{coal}(p \rightarrow 0) = 1$  :  
...others modify by hand  $\sigma_r$  to enforce confinement for a charm at rest in the medium

✦ The charm not “coalescing” undergo fragmentation:

$$\frac{dN_{had}}{d^2p_T dy} = \sum \int dz \frac{dN_{fragm}}{d^2p_T dy} \frac{D_{had/c}(z, Q^2)}{z^2}$$

charm number conserved at each  $p_T$ ,  
we have employed  $e^+e^-$  FF now PYTHIA



# Resonance Recombination Model (RRM)

L. Ravagli and R. Rapp, PLB 655, 126 (2007).

L. Ravagli, et al., PRC 79, 064902 (2009).

$$f_M(\vec{x}, \vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x}, \vec{p}_1) f_{\bar{q}}(\vec{x}, \vec{p}_2) \sigma_M(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

The 3-body hadronization process in RRM are conducted in 2 steps

1 :quark-1 and quark-2 recombine into a diquark,

$$q_1(p_1) + q_2(p_2) \rightarrow dq(p_{12})$$

2: the diquark recombines with quark-3 into a baryon

$$dq_1(p_{12}) + q_3(p_3) \rightarrow B$$

$$f_B(\vec{x}, \vec{p}) = \frac{\gamma_B}{\Gamma_B} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2 d^3\vec{p}_3}{(2\pi)^6} \frac{\gamma_{dq}}{\Gamma_{dq}} f_1(\vec{x}, \vec{p}_1) f_2(\vec{x}, \vec{p}_2) \times f_3(\vec{x}, \vec{p}_3) \sigma_{dq}(s_{12}) v_{\text{rel}}^{12} \sigma_B(s) v_{\text{rel}}^{dq3} \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

Recomb. according **not to a w.f. but to a Breit-Wigner** cross section (still a closeness in phase space constrained by  $\Gamma_{M-B} \sim 100-300$  MeV):

→ Assumed a set of additional RQMc-baryons [as in SHM]

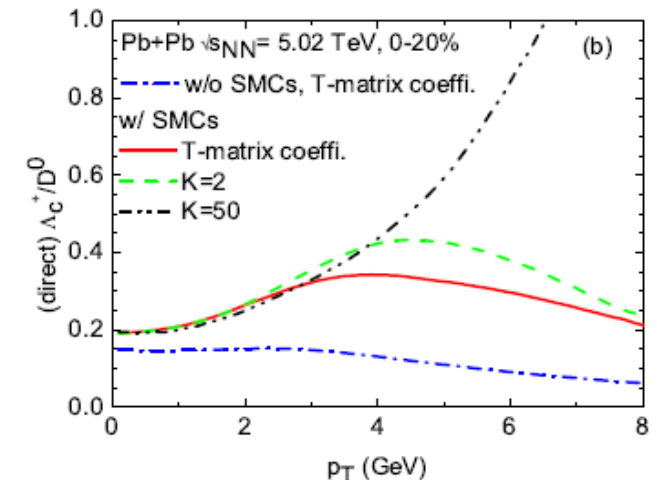
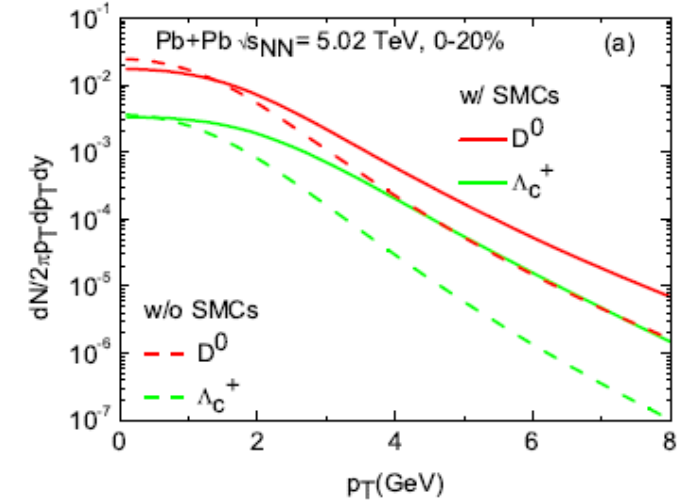
-Increased set of baryons for the  $\Lambda_c$  production:

PDG:  $5\Lambda_c, 3\Sigma_c, 8\Xi_c, 2\Omega_c$

RQM:  $18\Lambda_c, 42\Sigma_c, 62\Xi_c, 34\Omega_c$

→ Similar effects to coalescence on  $R_{AA}$  and  $v_2$  of D &  $\Lambda_c$

Local phase-space recombination with strong: Space-Momentum-Correlation



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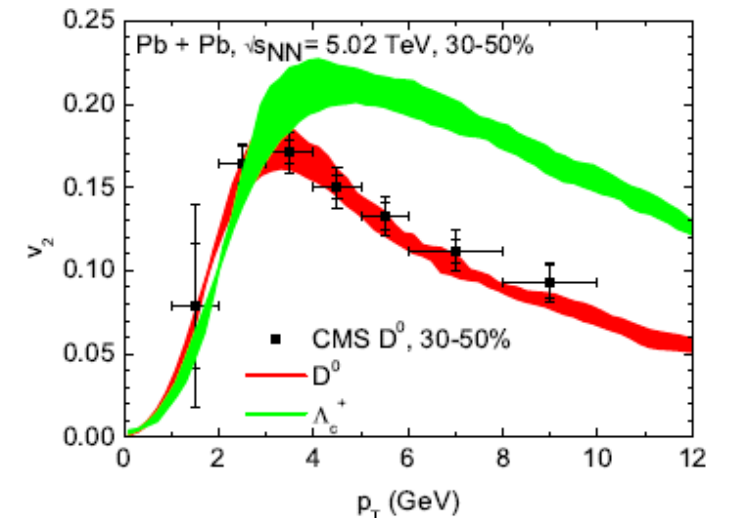
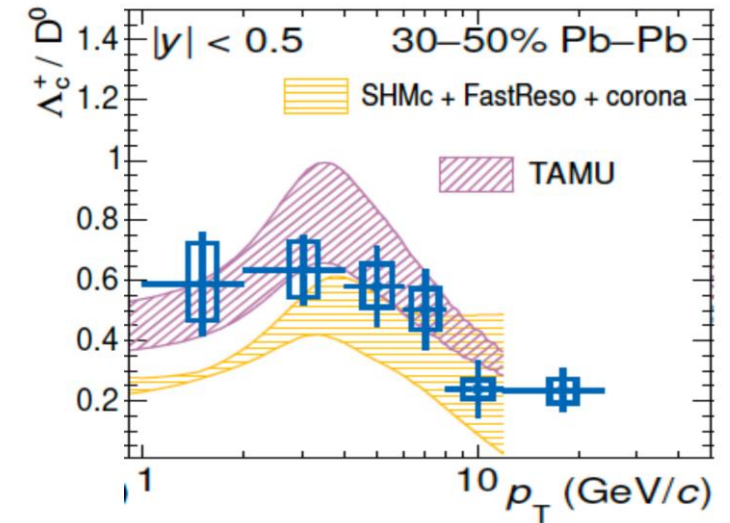
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# POWLANG Local Color Neutralization

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HQ hadronization in the presence of a reservoir of lighter thermal particles:

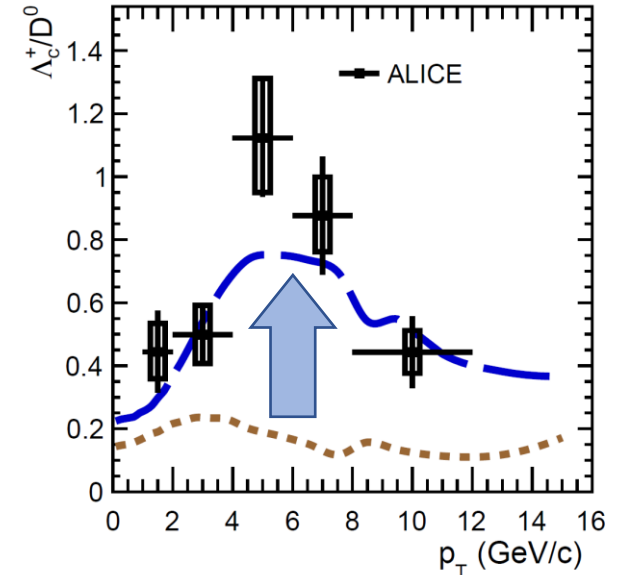
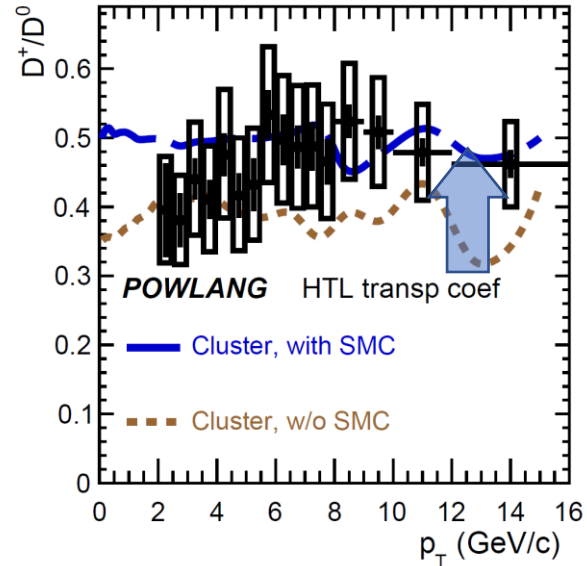
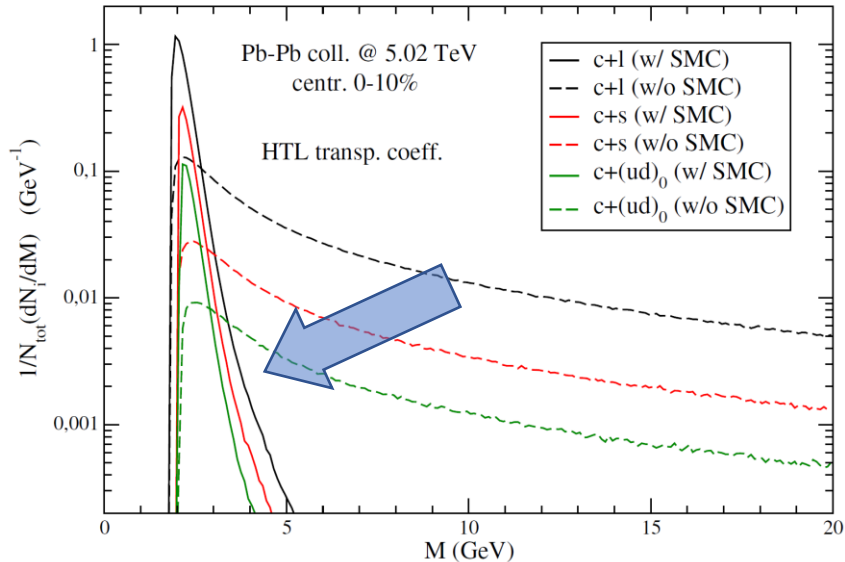
Recombination of the HQ with light antiquark or diquarks:

- Color-singlet clusters with low invariant mass  $M$  ( $M < 4$  GeV) are assumed to undergo an isotropic 2-body decay in their local rest-frame.
- Heavier clusters are instead fragmented as Lund strings.
- Recombination with light diquarks  $\rightarrow$  enhances yields of charmed baryons.
- The local color neutralization  $\rightarrow$  strong space-momentum correlation  $\rightarrow$  enhancement of the collective flow of the final charmed hadrons

*Dense medium (pp & AA)  $\rightarrow$  local color statistical neutralization, qualitatively similar to PYTHIA with local CR  $\rightarrow$  smaller  $M$  qualitatively similar to Coalescence/Resonance Recombination*

*Specific of the approach:*

- Existence of thermal flowing diquarks
- Very strong impact on  $v_2(p_T)$  from  $c \rightarrow D, \Lambda_c$
- Large  $D_s$  production already in pp





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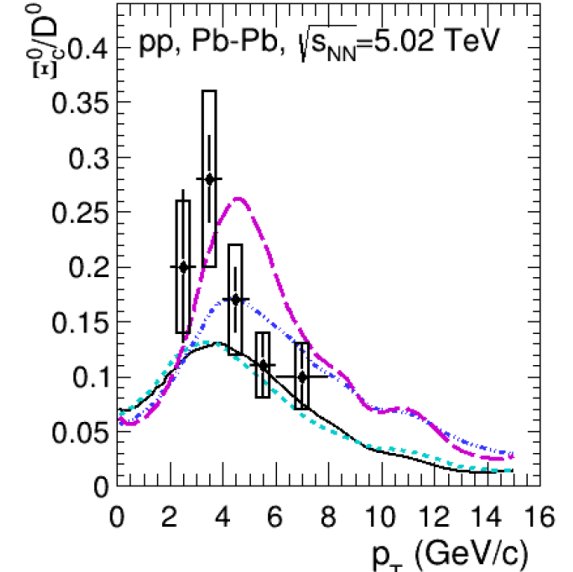
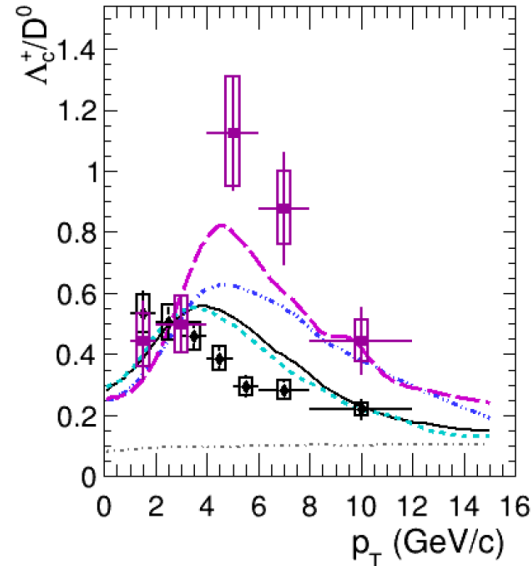
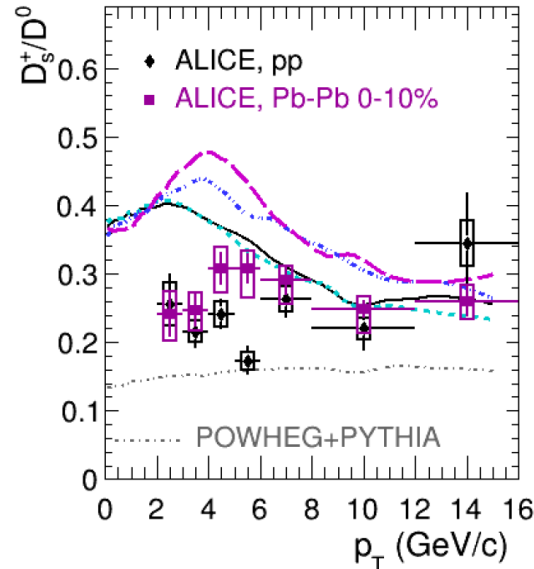
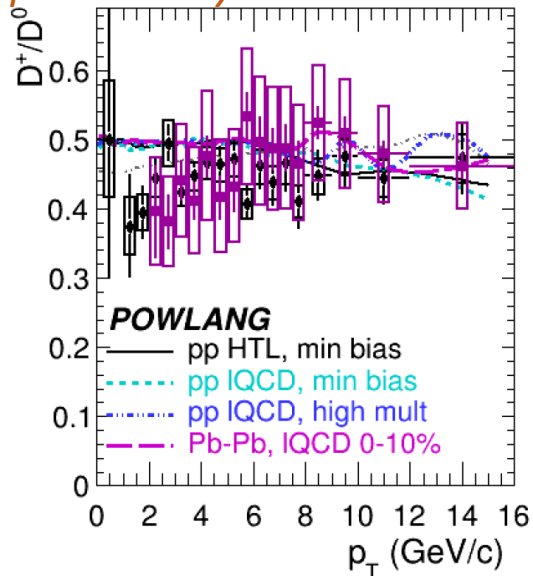
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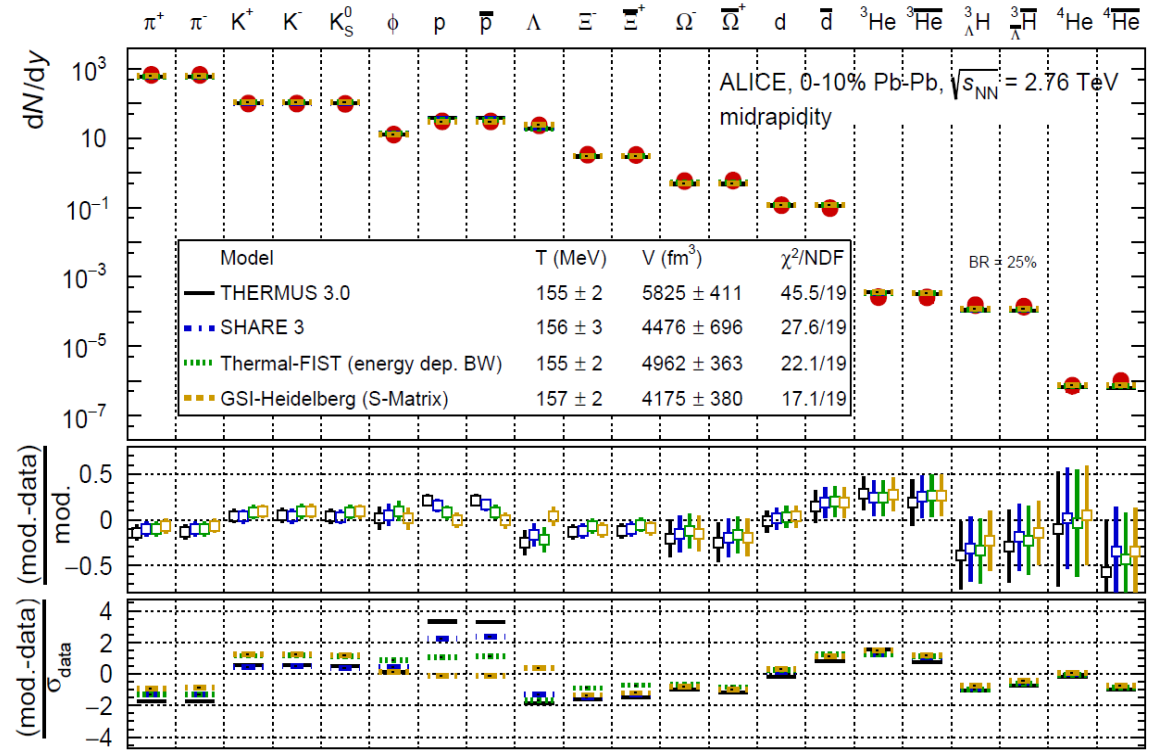
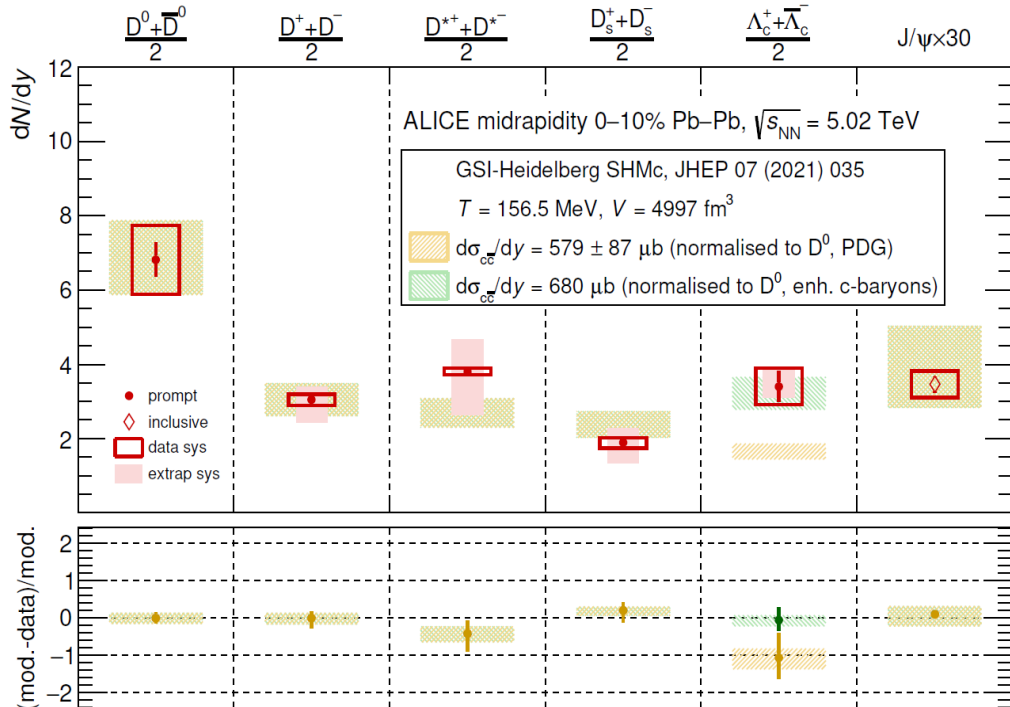




# Statistical hadronization model (SHM)

Multiplicities of light-flavour hadrons well described by different implementation of SHM (Grand Canonical).

- Agreement over 9 orders of magnitude in central AA collisions at midrapidity.
- $T_{\text{chem}} = 156 \text{ MeV}$  close to the pseudo critical temperature from IQCD and very small differences between different implementation of SHM



S. Acharya et al. [ALICE coll], EPJ C 84 (2024) no.8, 813.

GSI-Heidelberg: A. Andronic et al., NPA 772 (2006), A. Andronic et al., PLB 792 (2019) 304, A. Andronic et al., JHEP 07 (2021) 035 [for HF]

Thermal-FIST: V. Vovchenko et al., PRC 98 (2018) 034906, V. Vovchenko et al., Comput. Phys. Commun. 244 (2019) 295

SHARE: G. Torrieri et al., Comput. Phys. Commun. 175 (2006) 635, M. Petran et al., Comput. Phys. Commun. 185 (2014) 2056–2079

THERMUS: S. Wheaton et al., arXiv:1108.4588, J. Cleymans et al. PRC 74 (2006) 034903.

**Light flavour (u,d,s) from p, pA, AA**

# Light baryon-to-meson production in AA

**Catania: Boltzmann+Instantaneous Coal.+Fragm.**

*V. Minissale et al., PRC 92 (2015) 054904*

**CoLBT: linear Boltzmann+hydro bulk+Coal+Fragm.**

*W. Zhao, et al. PRL 128 (2022) 022302*

**EPOS: core (viscous hydro) + statistical hadronization + hadronic cascade.**

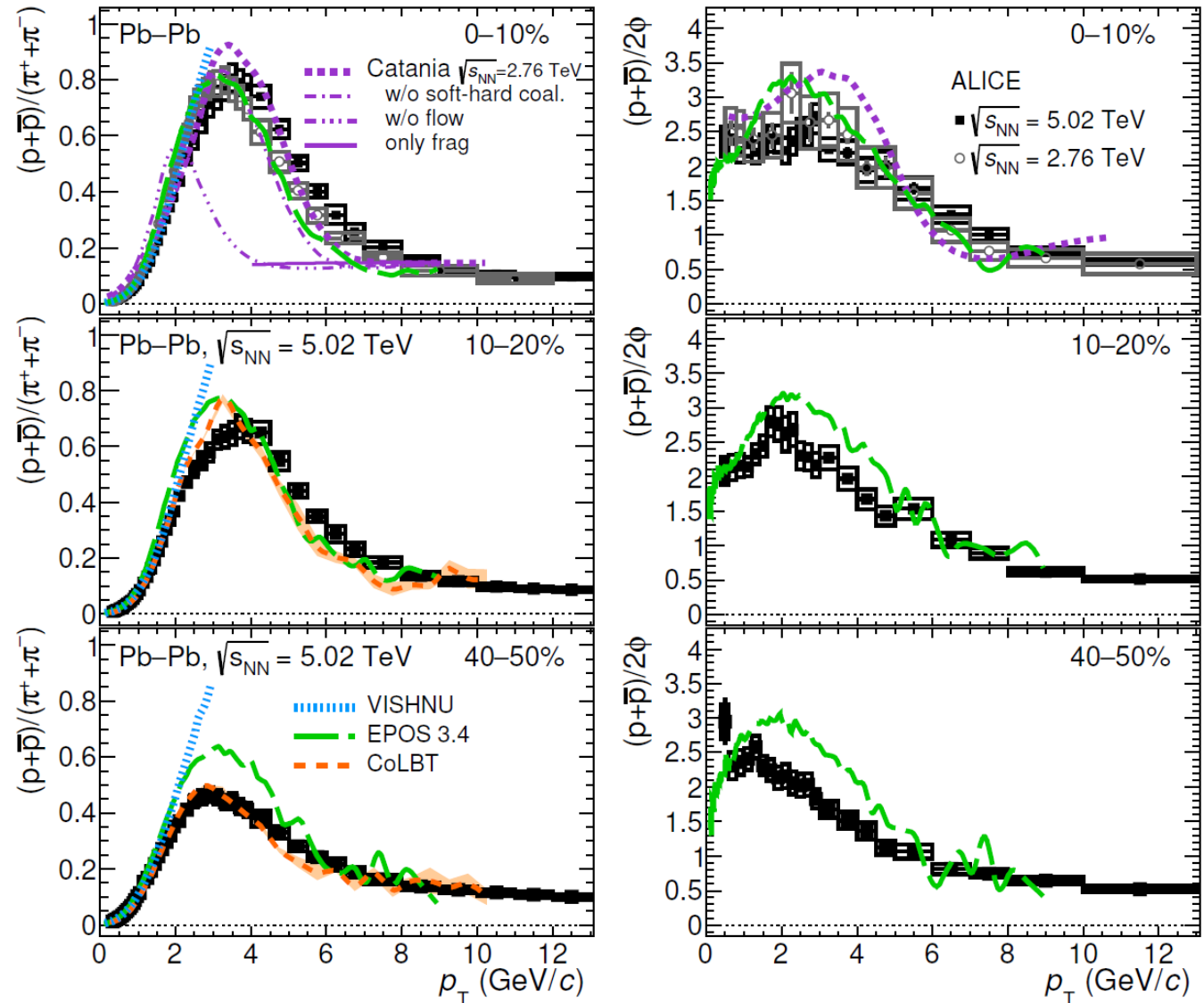
*K. Werner et al., PRC 89 (2014) 064903*

**VISHNU: viscous hydro-> Cooper-Frye -> UrQMD**

*C. Shen et al., Comput. Phys. Commun. 199 (2016) 61*

- high- $p_T$  region ( $>8-10$  GeV/c), particle yield ratios in Pb-Pb collisions match those in pp collisions, fragmentation dominates hadron formation.
- low- $p_T$  ( $<2$  GeV/c), spectra are well described by hydrodynamic models, consistent with an equilibrium evolution of the system.
- Intermediate- $p_T$  region provides insight into hadron formation mechanisms, models with recombination captures key data features.

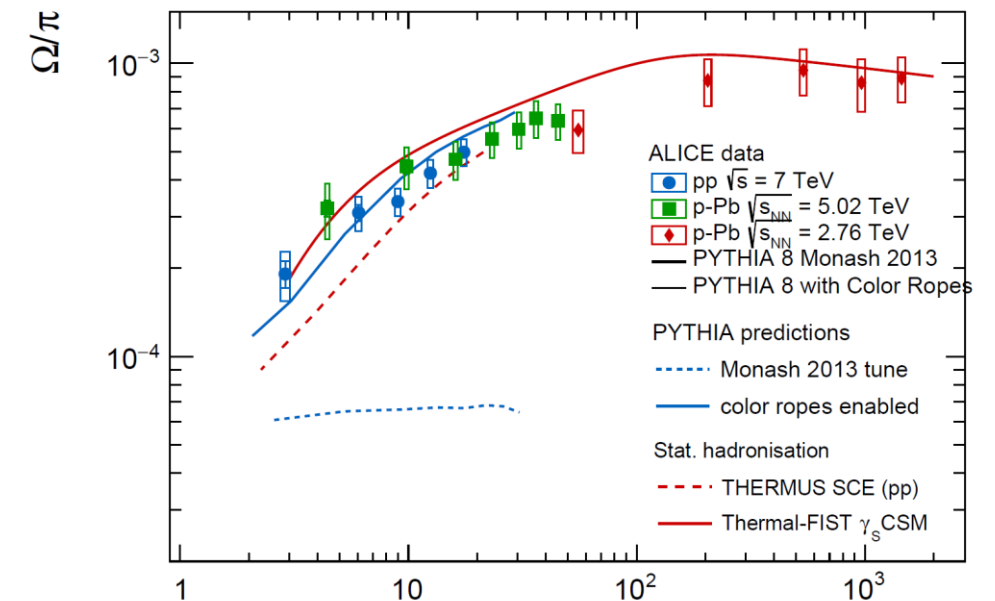
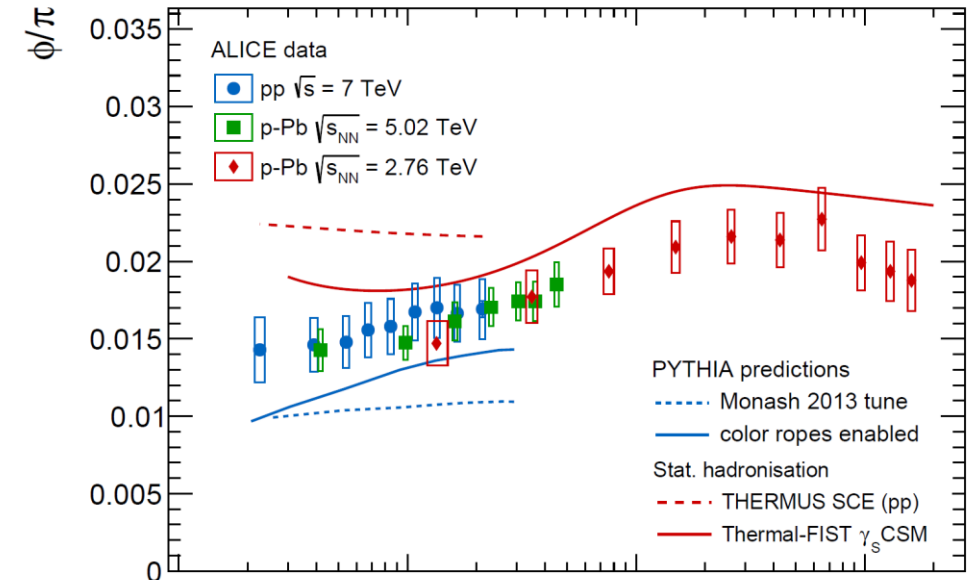
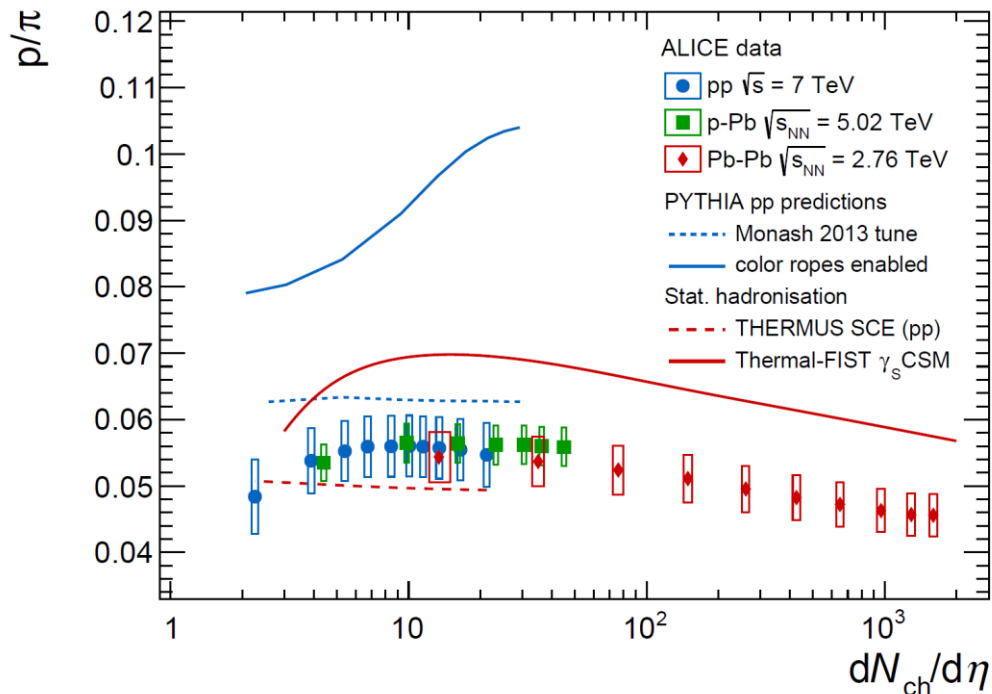
*S. Acharya et al. [ALICE coll], EPJ C 84 (2024) no.8, 813.*



# Light baryon-to-meson production from pp to AA

- color ropes in PYTHIA (enhanced tension color flux tubes) good description of strangeness enhancement in high-multiplicity pp collisions (10% accuracy), but fails to accurately describe the  $p/\pi$ .
- SHM describe the relative increase in strangeness. Thermal FIST (canonical statistical model) describes most particle ratios up to 20% deviation.

*Why does the proton-to-pion ratio show smaller dependence on multiplicity in LHC collisions?*



S. Acharya et al. [ALICE coll], EPJ C 84 (2024) no.8, 813.  $dN_{ch}/d\eta$

J. Adam et al., [ALICE coll] Nat. Phys. 13, 535 (2017).

# Event-by-event production of multistrange hadrons

Measurement of both correlation and normalized second-order cumulants provides strong discriminative power against different model predictions.

$$\frac{\kappa_2(\Delta\Xi)}{\kappa_1(\Xi^+ + \Xi^-)} = \frac{\kappa_2(\Xi^+) + \kappa_2(\Xi^-) - 2\kappa_{11}(\Xi^+, \Xi^-)}{\langle n_{\Xi^+} + n_{\Xi^-} \rangle}, \quad \text{where} \quad \kappa_1(A) = \langle n_A \rangle,$$

$$\rho_{\Delta\Xi\Delta K} = \frac{\kappa_{11}(\Xi^+, K^+) + \kappa_{11}(\Xi^-, K^-) - \kappa_{11}(\Xi^+, K^-) - \kappa_{11}(\Xi^-, K^+)}{\sqrt{\kappa_2(\Delta\Xi)\kappa_2(\Delta K)}}, \quad \kappa_2(A) = \langle n_A^2 \rangle - \langle n_A \rangle^2,$$

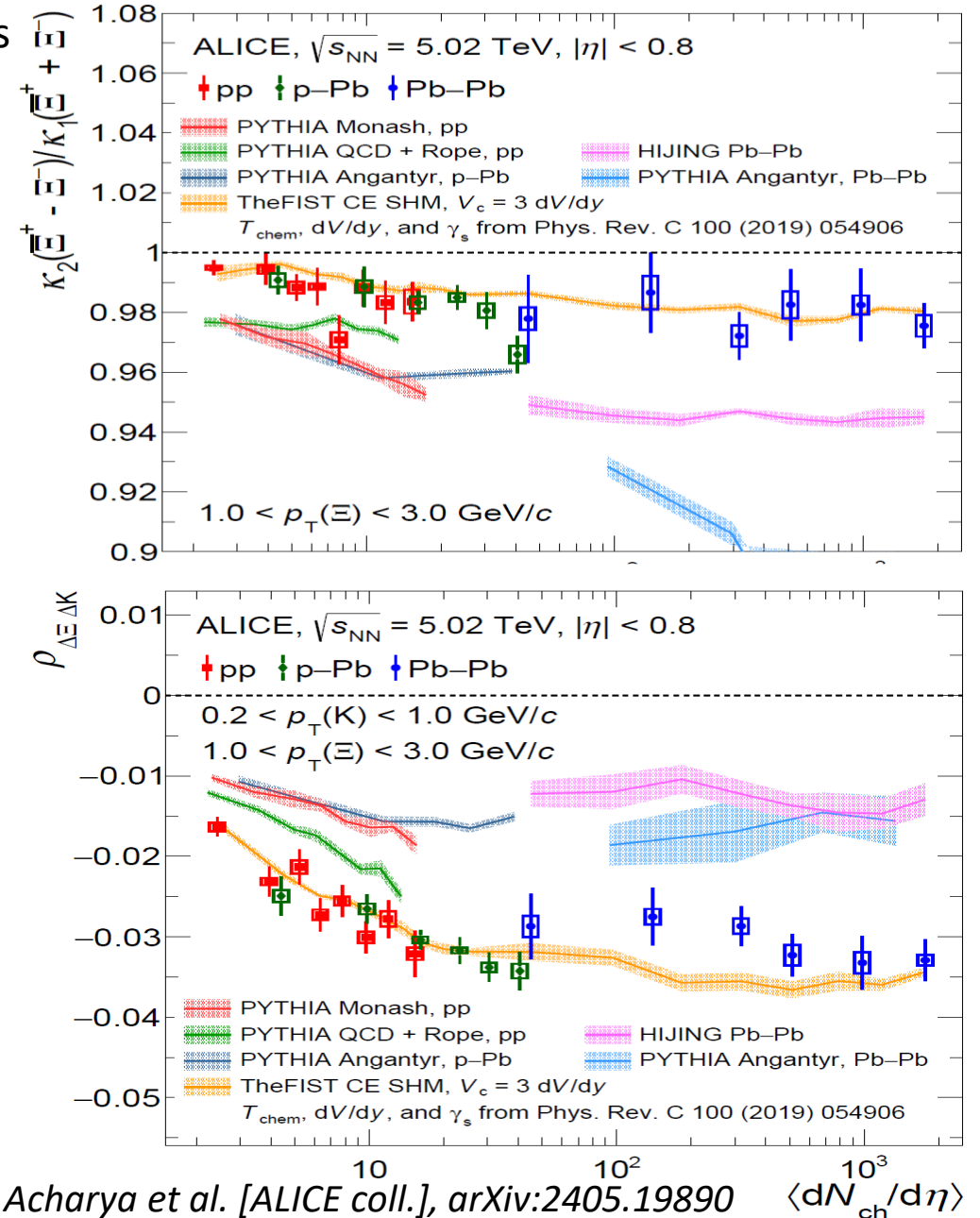
$$\kappa_{11}(A, B) = \langle n_A n_B \rangle - \langle n_A \rangle \langle n_B \rangle$$

- SHM (Canonical Ensemble) Thermal-FIST: The strangeness saturation parameter  $\gamma_s$  is included to account for incomplete strangeness equilibration at low multiplicities. SHM successfully describes both the normalized second-order cumulant and correlation ( $\rho_{\Delta\Xi\Delta K}$ )
- PYTHIA 8 and Rope Hadronization: qualitatively predict negative correlations and second-order cumulant smaller than one, but overestimate it in low- and high-multiplicity regions.

*How does the picture change at higher  $p_T$  where fragmentation is expected to be the dominant hadronization mechanism?*

*How could higher-order cumulants reveal deviations from the thermal baseline? Missing calculation of coal.+fragm. approach*

*A systematic study including  $p$ ,  $\pi$ ,  $\Omega$ ,  $\phi$  ... can give more information about hadronization*





# Light flavour hadron $v_2(p_T)$ in AA

**Catania: Boltzmann+Instantaneous Coal.+Fragm.**

*V. Minissale et al., PRC 92 (2015) 054904*

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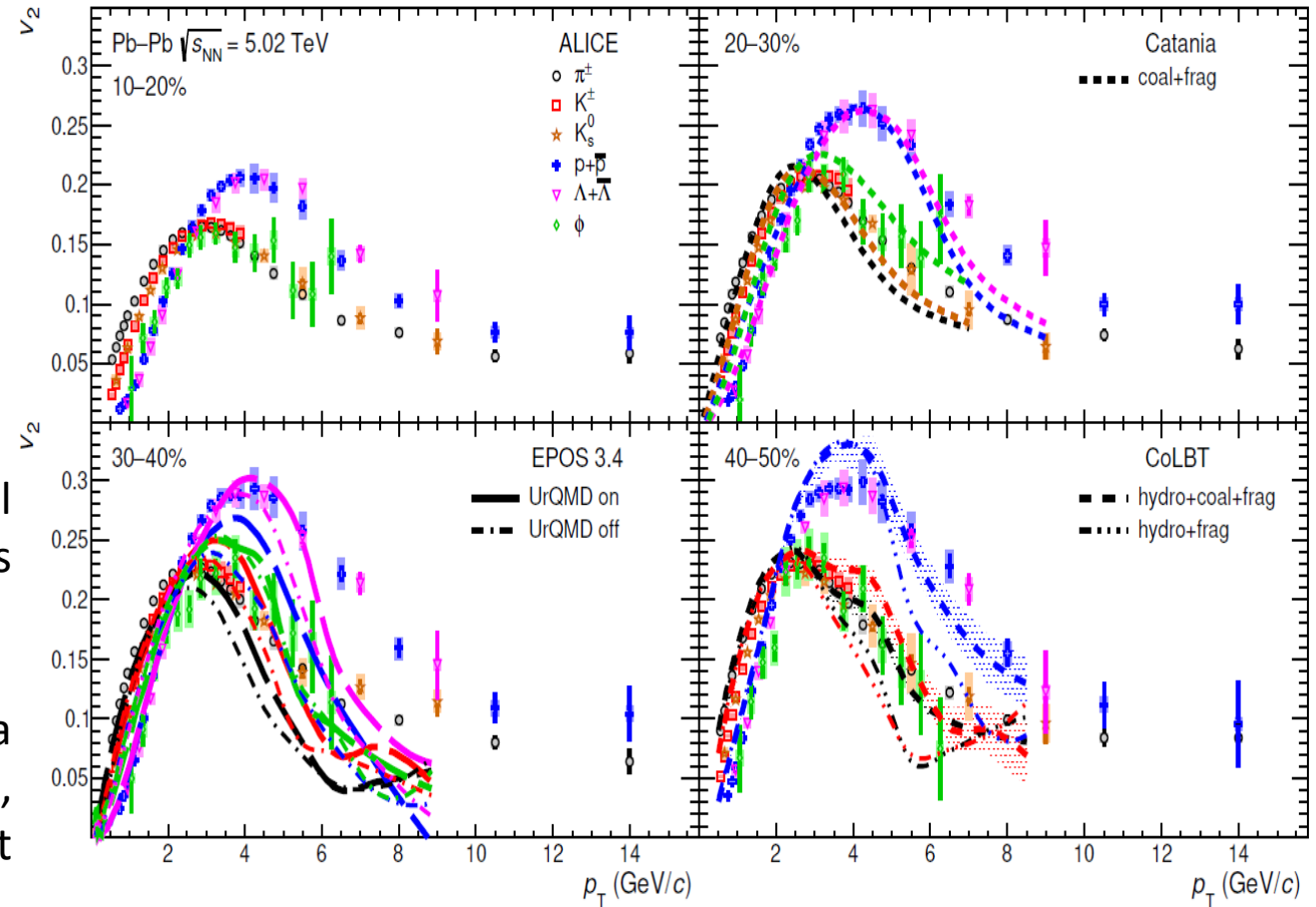
*K. Werner et al., PRC 89 (2014) 064903*

**VISHNU: viscous hydro-> Cooper-Frye -> UrQMD**

*C. Shen et al., Comput. Phys. Commun. 199 (2016) 61*

- EPOS3: hadronic cascade stage (UrQMD), are crucial for description of  $v_2(p_T)$ . EPOS3 successfully reproduces mass-ordering up to  $p_T = 2-3$  GeV.
- Model with recombination and fragmentation (Catania and CoLBT), describe  $v_2(p_T)$  over a broad  $p_T$  range, from hydro like behavior at low  $p_T$  to fragmentation at high  $p_T$ .
- CoLBT simulations that exclude the coalescence process underestimate  $v_2(p_T)$  for  $p_T > 4$  GeV.

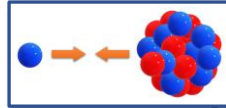
*S. Acharya et al. [ALICE coll], EPJ C 84 (2024) no.8, 813.*



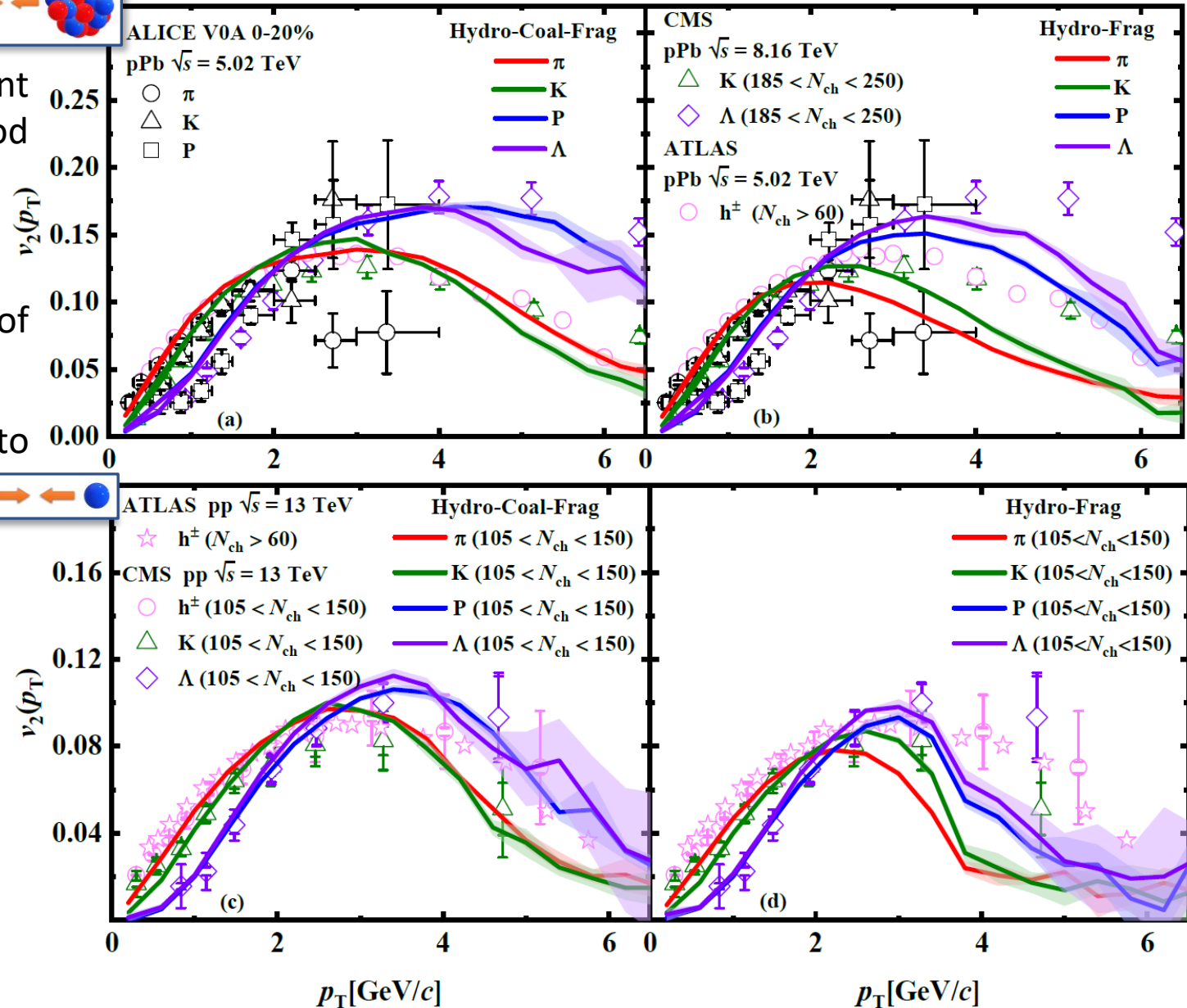


# Impact of Quark Coalescence on $v_2(p_T)$ in pA and pp

Y. Wang et al., arXiv:2401.00913



- Low  $p_T$  ( $p_T < 3$  GeV):
  - Mass ordering hydro behaviour dominant  
-> Coal+fragm and only Fragm scenario good description of data
- Intermediate  $p_T$  ( $3 < p_T < 6$  GeV):
  - Hydro+**Coal+Fragm**: successfully description of baryon/meson grouping and splitting
  - Hydro+**Fragm**: mass ordering -> not enough to reproduce splitting effects



Would PYHTIA-CR predict finite  $v_2$  of  $\pi, k, p$  in pp?  
String shoving?

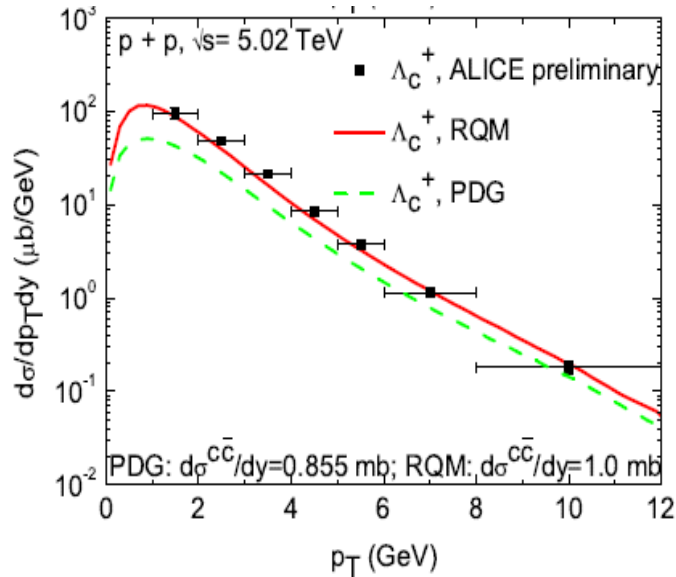
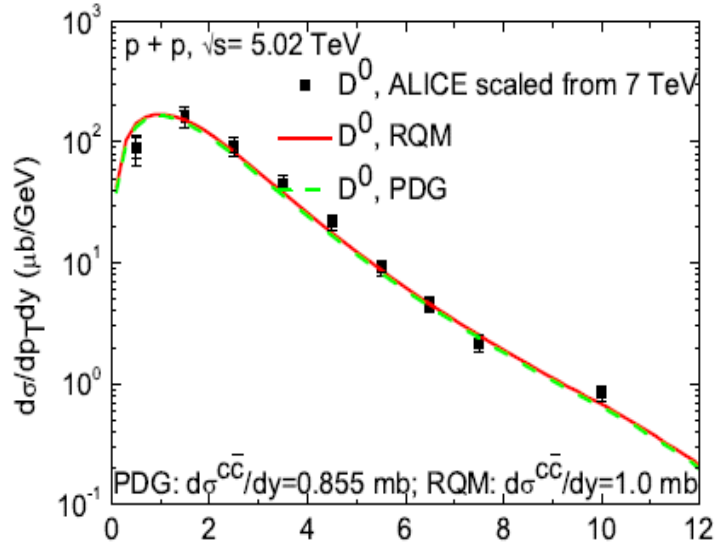
More details in:

W. Zhao et al., PRL 125, no.7, 072301 (2020)

Y. Wang et al., arXiv:2401.00913

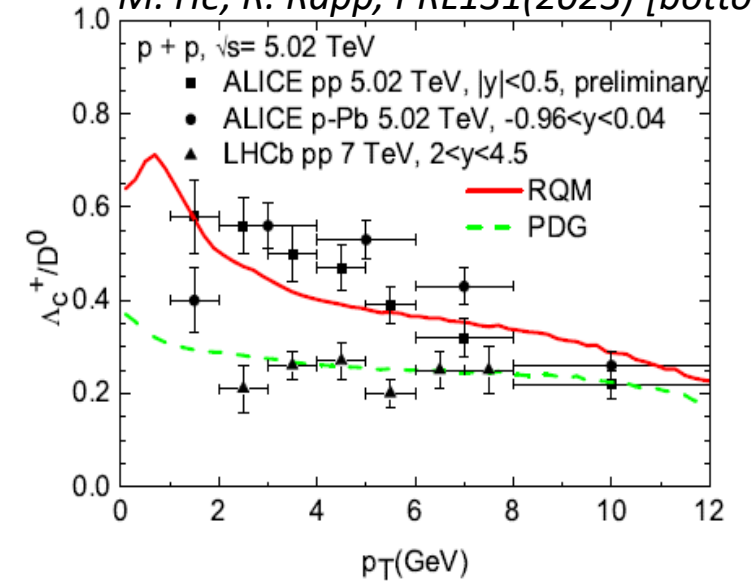
**Heavy flavour (c,b) from p, pA, AA**

# Grand canonical SHM + Frag. In pp



M. He, R. Rapp, PLB795(2019) [charm]

M. He, R. Rapp, PRL131(2023) [bottom]



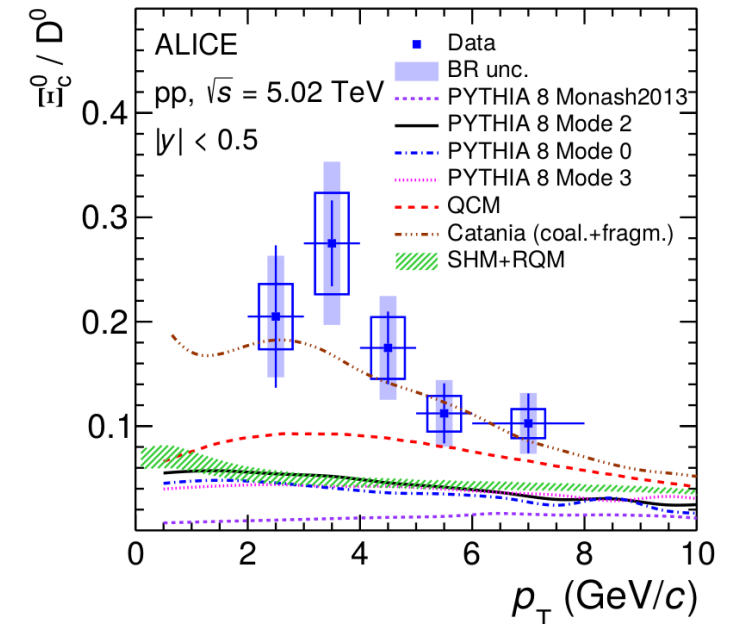
$$n_i = \frac{d_i}{2\pi^2} m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right)$$

- Very good  $\Lambda_c / D$  vs data [ $T_H=170$  MeV, flavor hierarchy?!]
- RQM Resonances not yet seen in e+e-, e-p
- For the yield assumes a thermal distribution, but for comparing data vs  $p_T$  a fragmentation function is exploited
- $\Xi_c / D0$  still lack yields
- Extended to bottom in pp: an explanation of  $\Lambda_b / B0$  evolution
- from e+e- to pp Canonical Suppression [but assuming  $V_{corr}$  with linear evolution with  $N_{tracks}$ ]

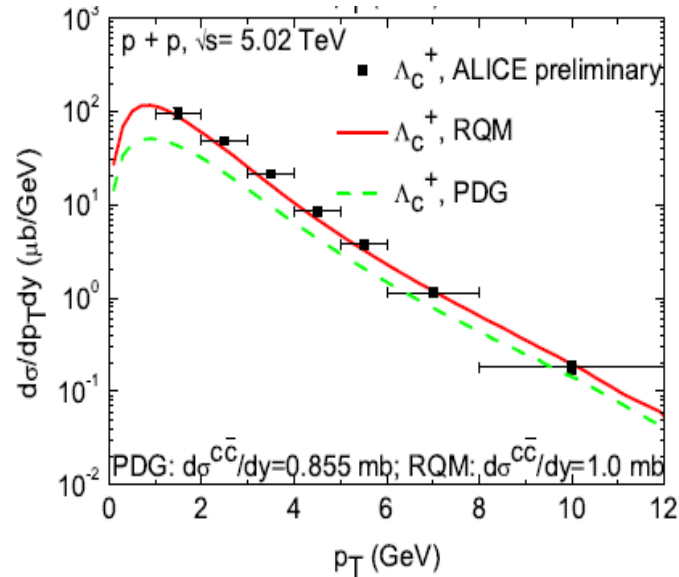
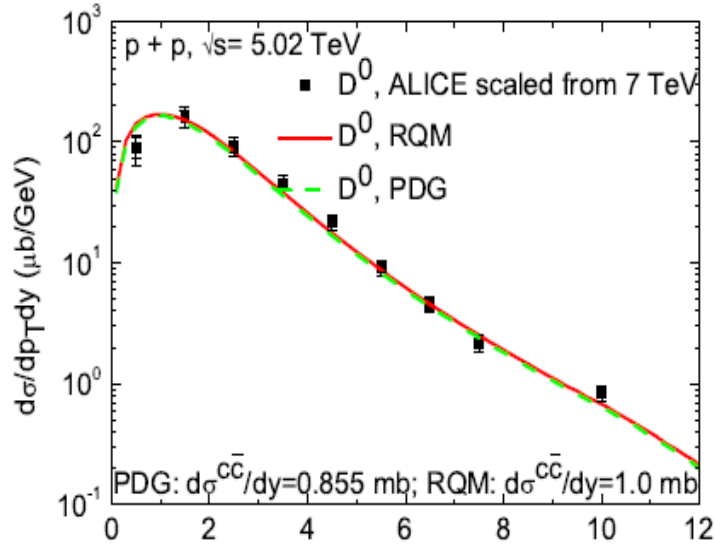
-Increased set of baryons for the  $\Lambda_c$  production:

PDG:  $5\Lambda_c, 3\Sigma_c, 8\Xi_c, 2\Omega_c$

RQM:  $18\Lambda_c, 42\Sigma_c, 62\Xi_c, 34\Omega_c$

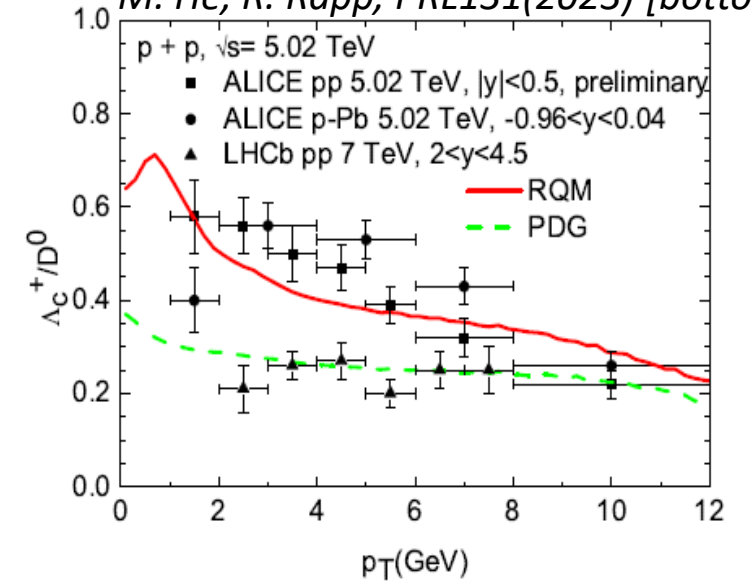


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M. He, R. Rapp, PLB795(2019) [charm]

M. He, R. Rapp, PRL131(2023) [bottom]



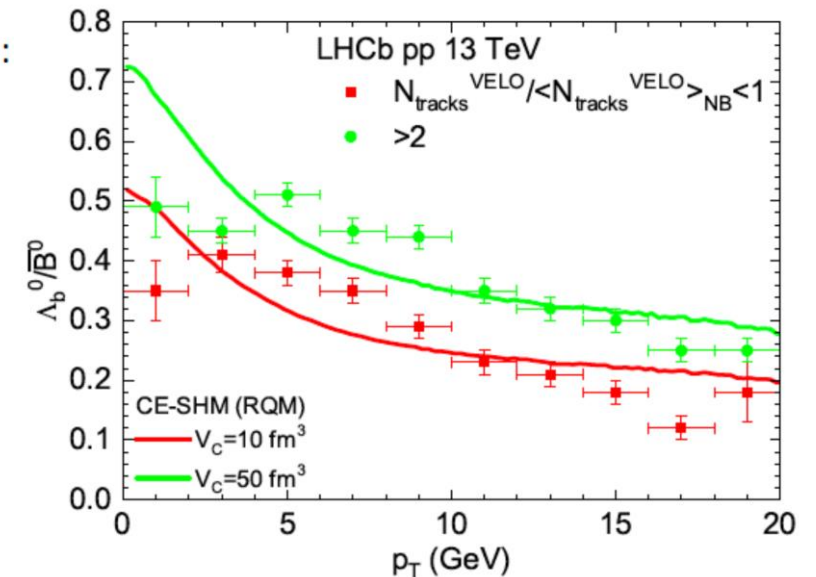
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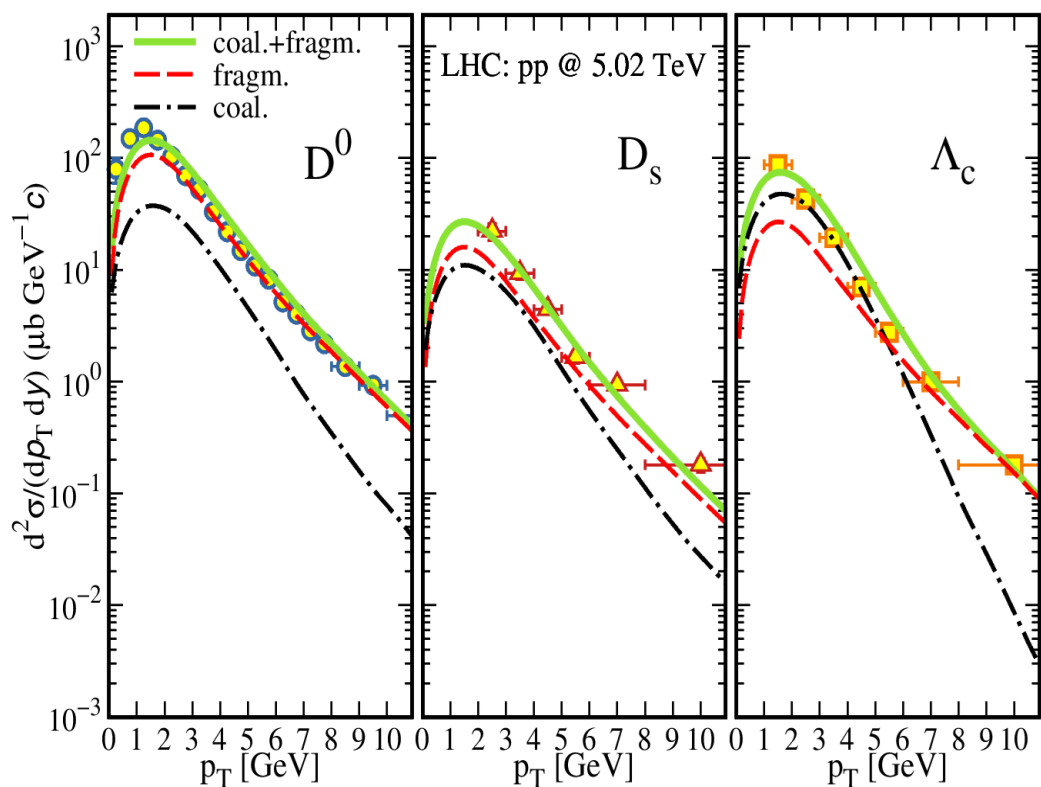
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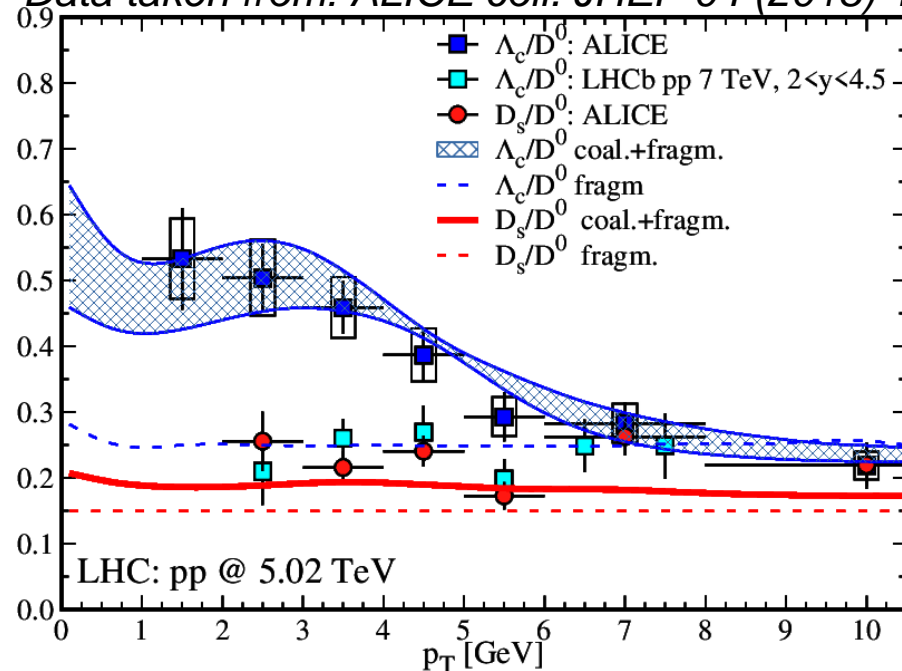
RQM:  $18\Lambda_c, 42\Sigma_c, 62\Xi_c, 34\Omega_c$



# Small systems: Coalescence in pp? (Charm hadrons)



Data taken from: ALICE coll. JHEP 04 (2018) 108



V. Minissale et al., Phys.Lett.B 821 (2021) 136622

Error band correspond to  $\langle r^2 \rangle$  uncertainty in quark model

-Damping of rise-and-fall behaviour in  $\Lambda_c / D^0$  ratio:

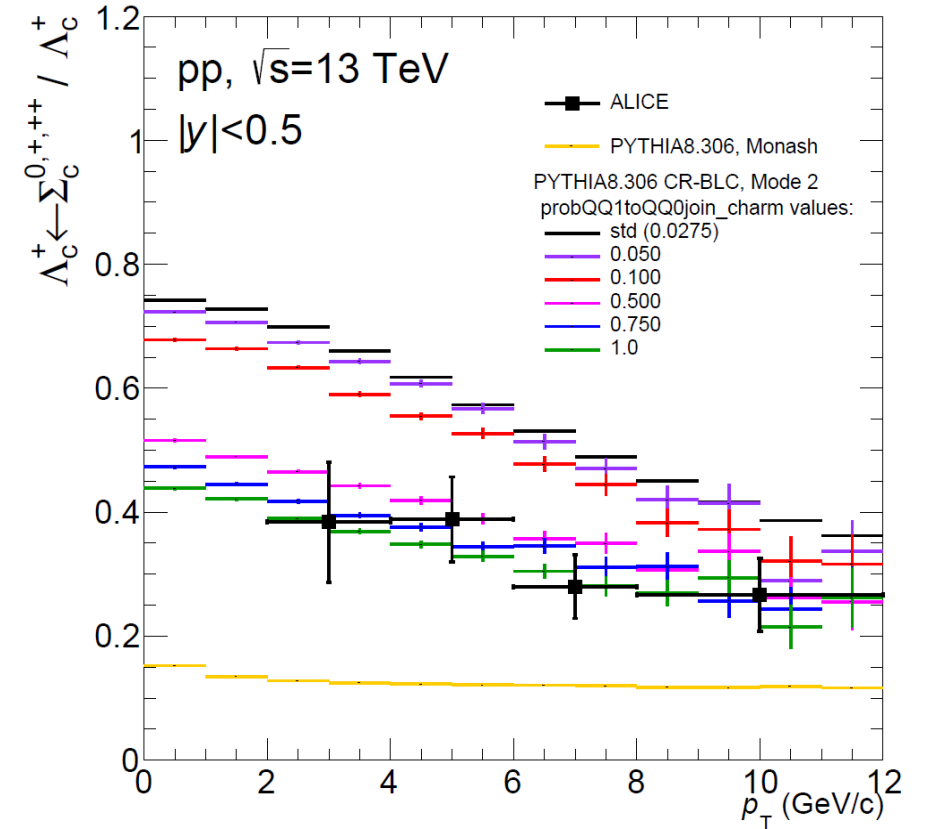
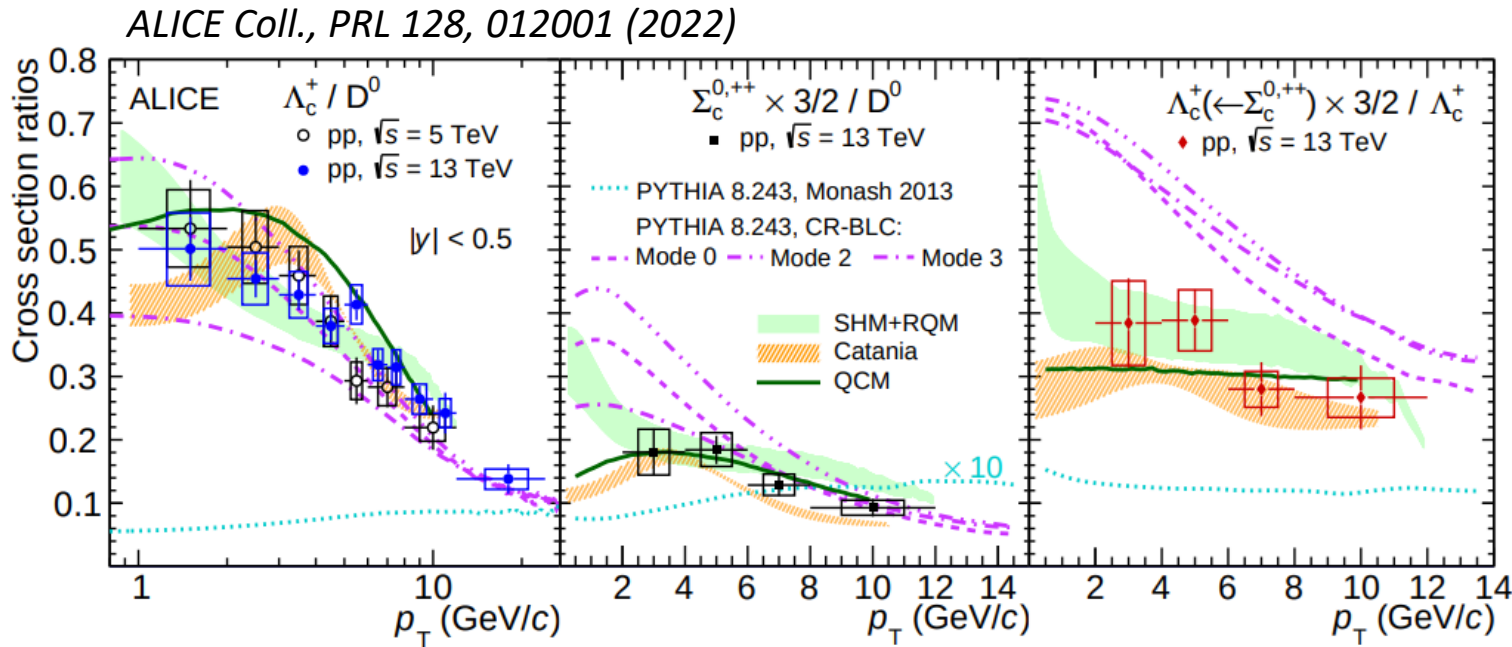
-Comparison with AA: Coal. contribution smaller w.r.t. Fragm.

- Coalescence does not affect significantly  $D^0$  but is dominant for baryons  $\Lambda_c$  and  $\Xi_c$

- More abundant the coalescence contribution for B even in pp, Minissale et al., [2405.19244](#)



# Possible role of diquarks in AA and pp?



- Catania-coal & SHM-RQM/QCM natural good description of  $\Sigma_c / D^0$  and  $\Lambda_c \leftarrow \Sigma_c$
  - PYTHIA-CR too many  $\Sigma_c \rightarrow \Lambda_c / D^0$ ; associated to a suppression of junction **diquark  $l=1$**  (set  $\sim e^+e^-$  for string di-quark). Removing it  $\rightarrow$  Agreement to data of  $\Lambda_c \leftarrow \Sigma_c$
- It goes in the direction of simply recombine according to SU(3)  $\sim$  simple coalescence



# Possible role of diquarks in AA and pp?

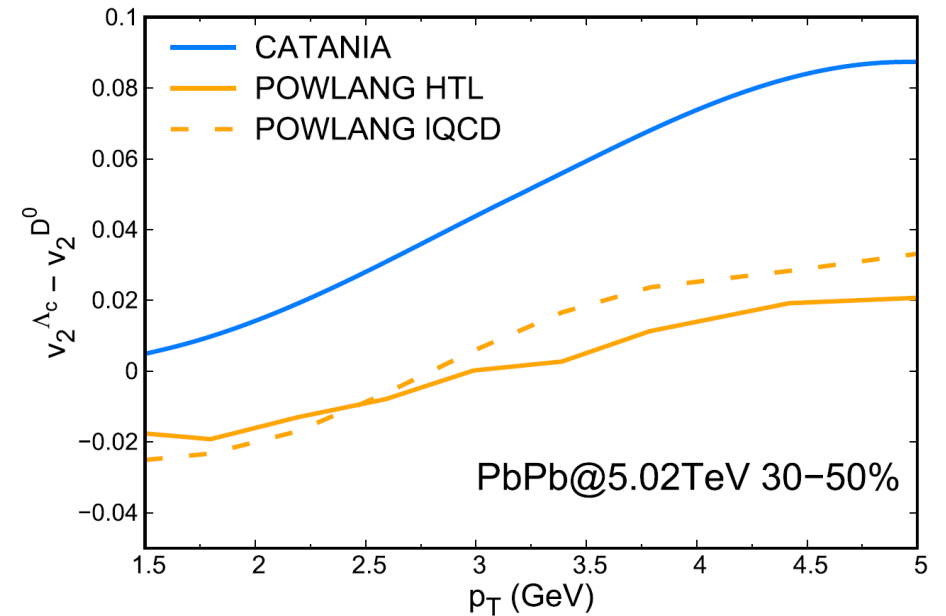
Coal. Approaches (*Catania, LBT, EPOS4HQ... RR-TAMU*)

→  $v_2(\Lambda_c) > v_2(D^0)$  at  $p_T > 2$  GeV

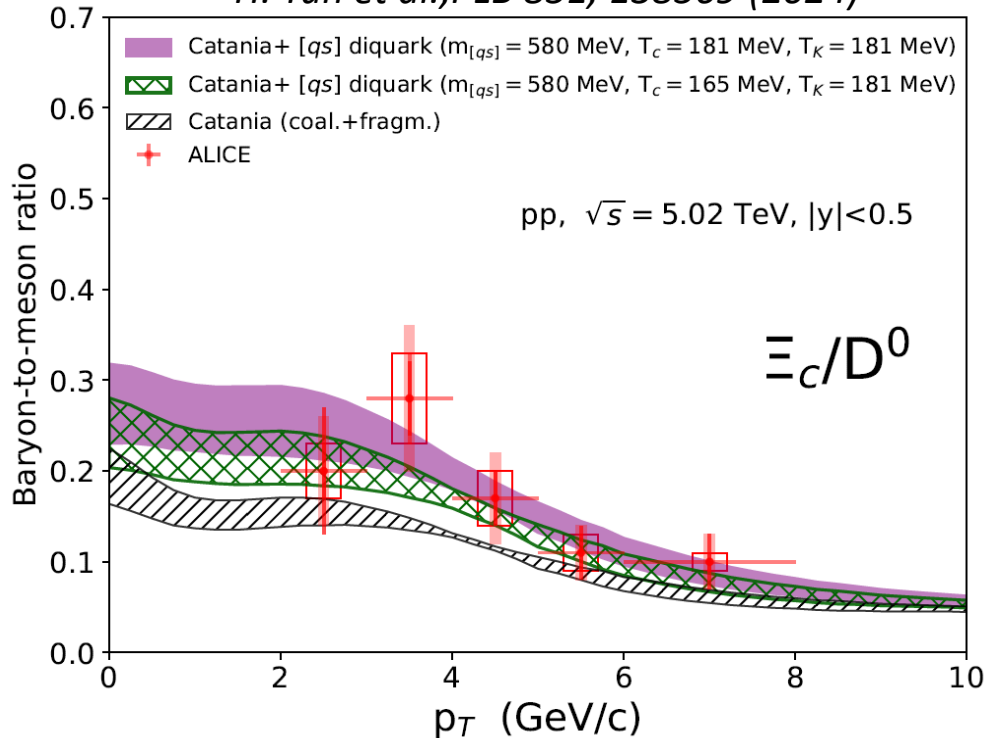
because  $\Lambda_c$  gets flow from 2 light quarks,  $D^0$  from 1+fragm.

POWLANG assume diquark hydrodynamical flow and

$\Lambda_c = (qq) + c \rightarrow v_2(\Lambda_c) \sim v_2(D^0)$  at intermediate  $p_T$

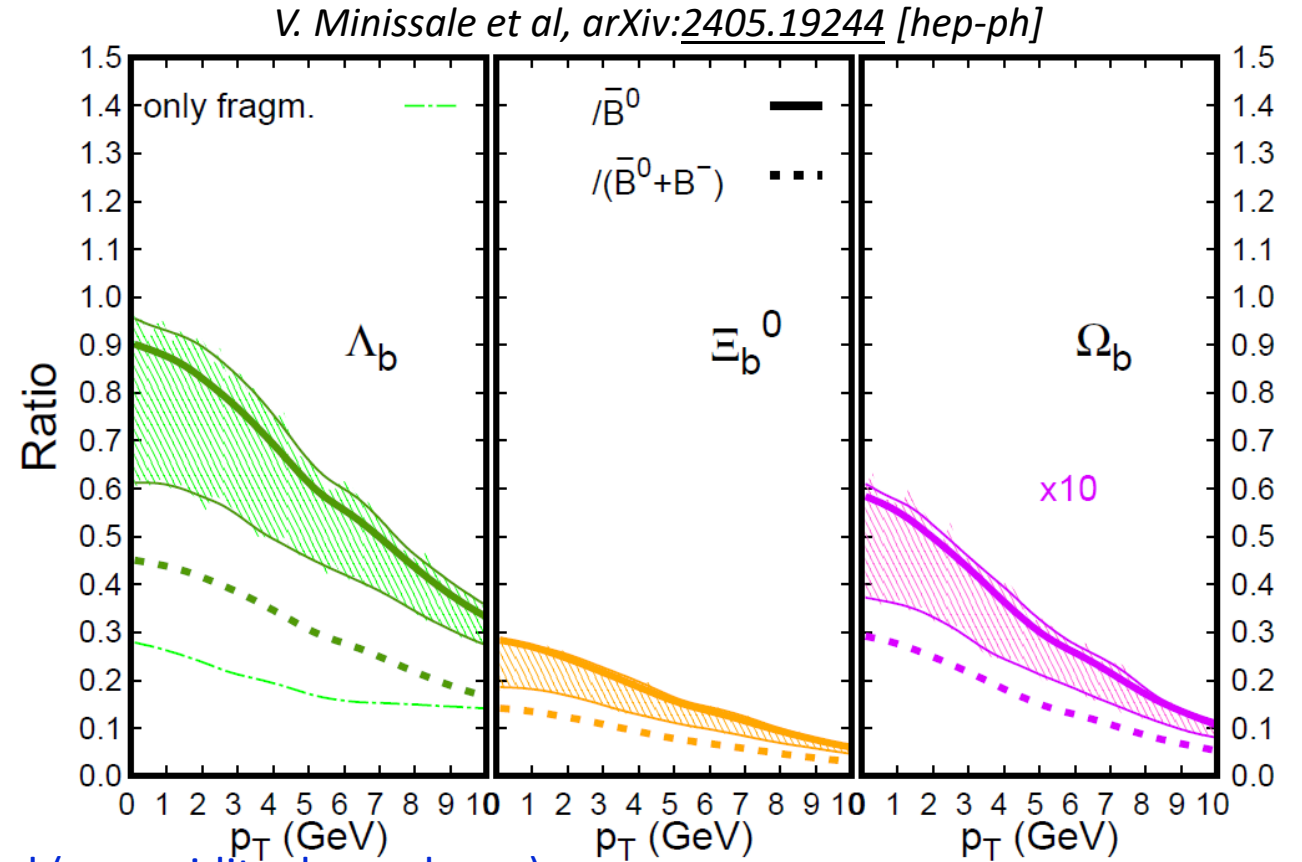
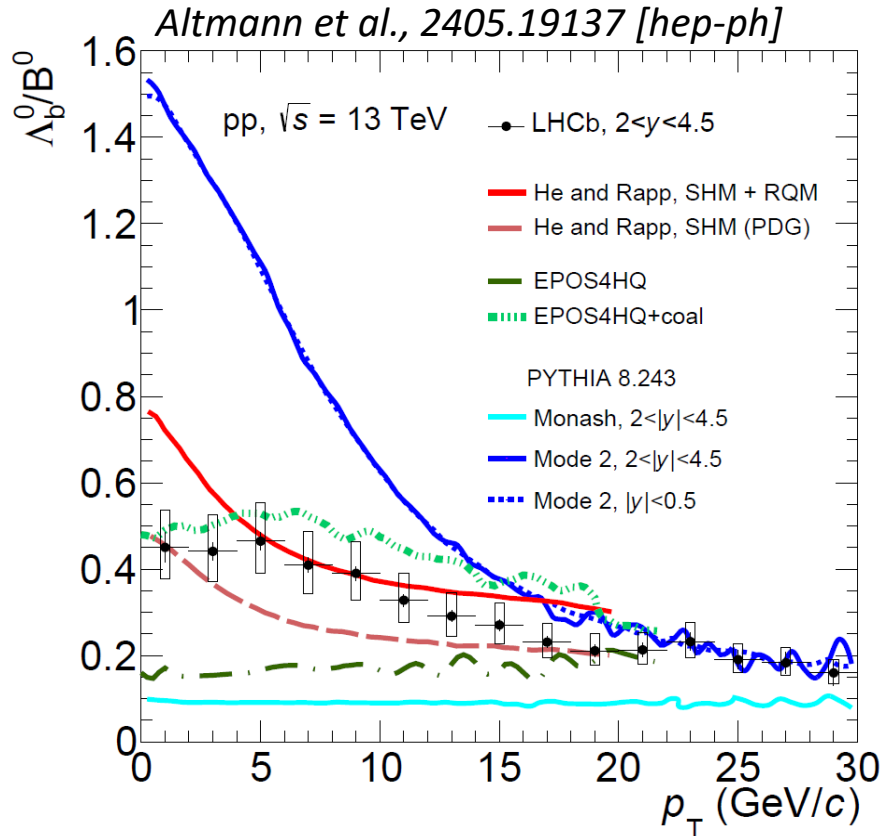


*H. Yun et al., PLB 851, 138569 (2024)*



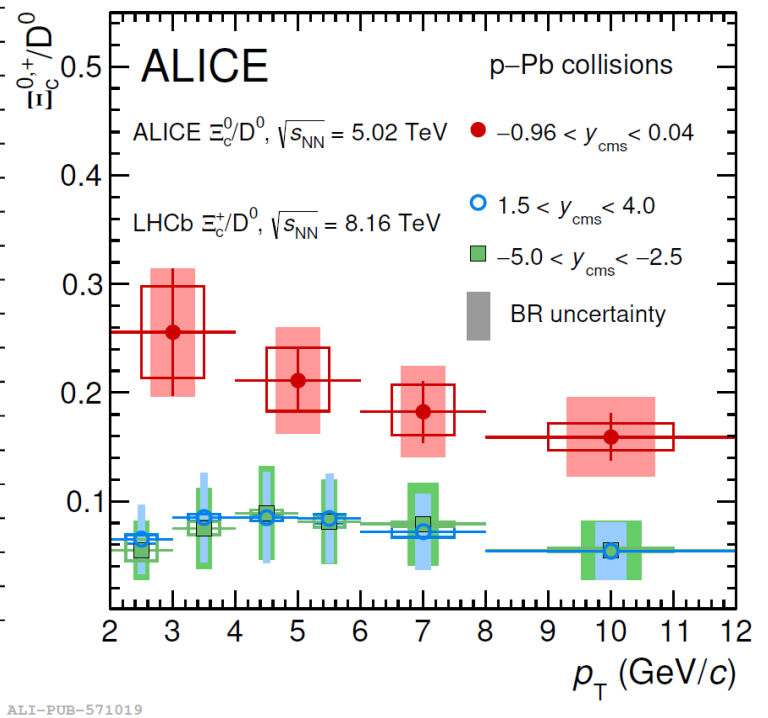
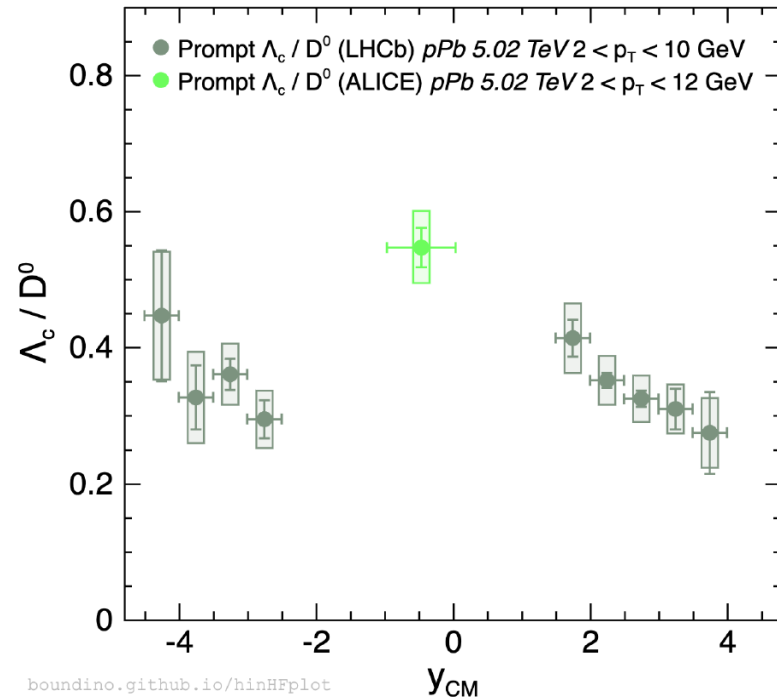
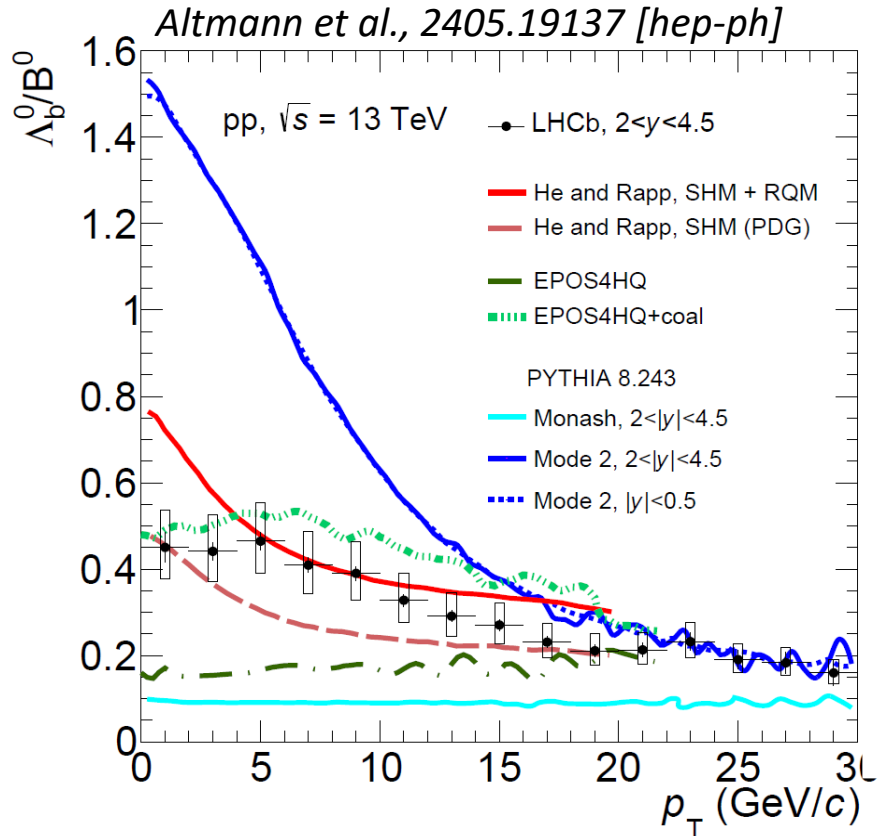
- The (us) and (ds) diquarks are more compact and exhibit stronger binding energy than (ud) diquarks.  
→ enhanced production of  $\Xi_c$  and D particles in high-energy pp → enhancement in  $\Xi_c/D^0$
- Similar enhancements expected in pPb or PbPb collisions?
- Possible impact in the strangeness enhancement for light flavour K,  $\Lambda$ ?

# HF baryon/meson in pp: rapidity dependence



- Again Need CR in PYTHIA → seems too strong at forward (no rapidity dependence)
- EPOS4HQ+coal close to data (rapidity dependence?). At  $y=0$  Catania results
- SHM +RQM about close, less the  $p_T$  shape (Frag.-Function)
- Coal./Fragm. ratio in pp larger for B than D

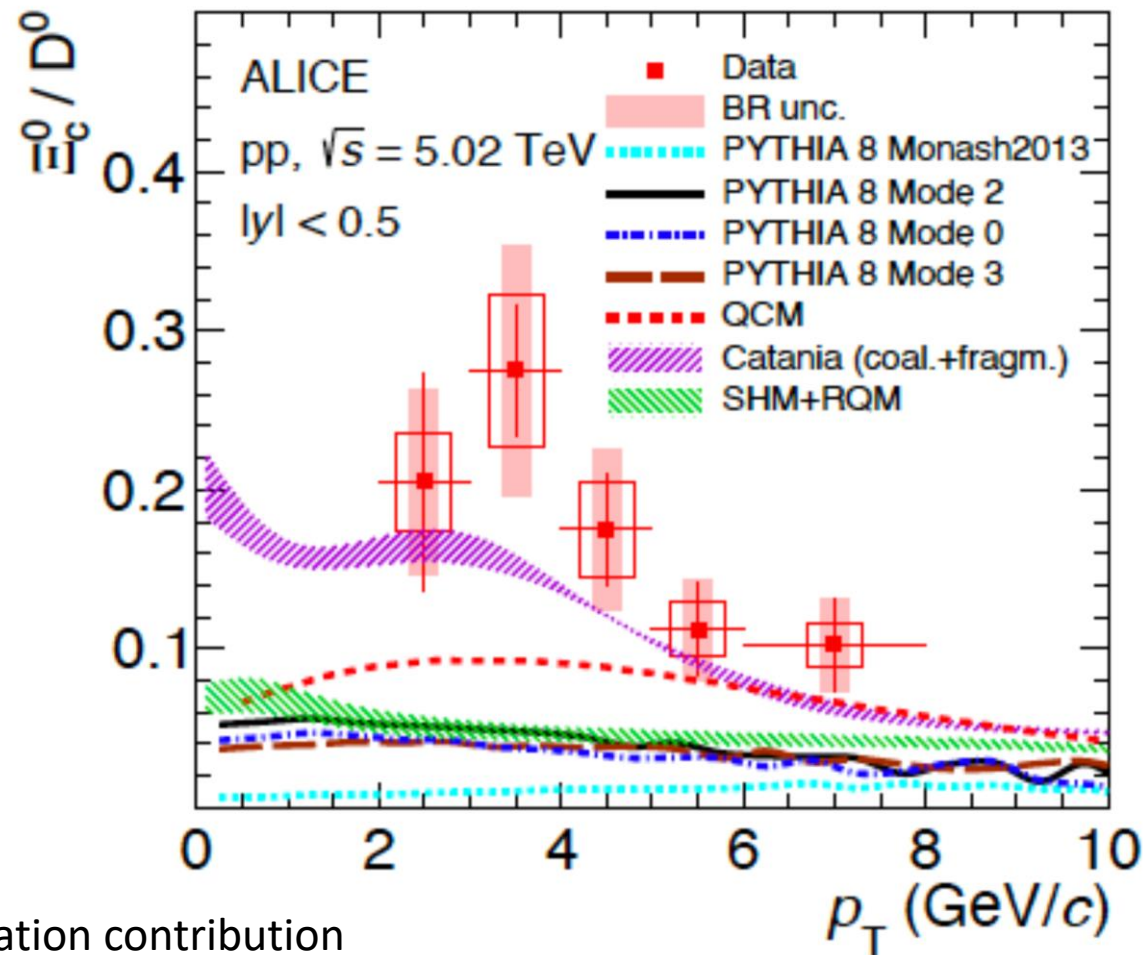
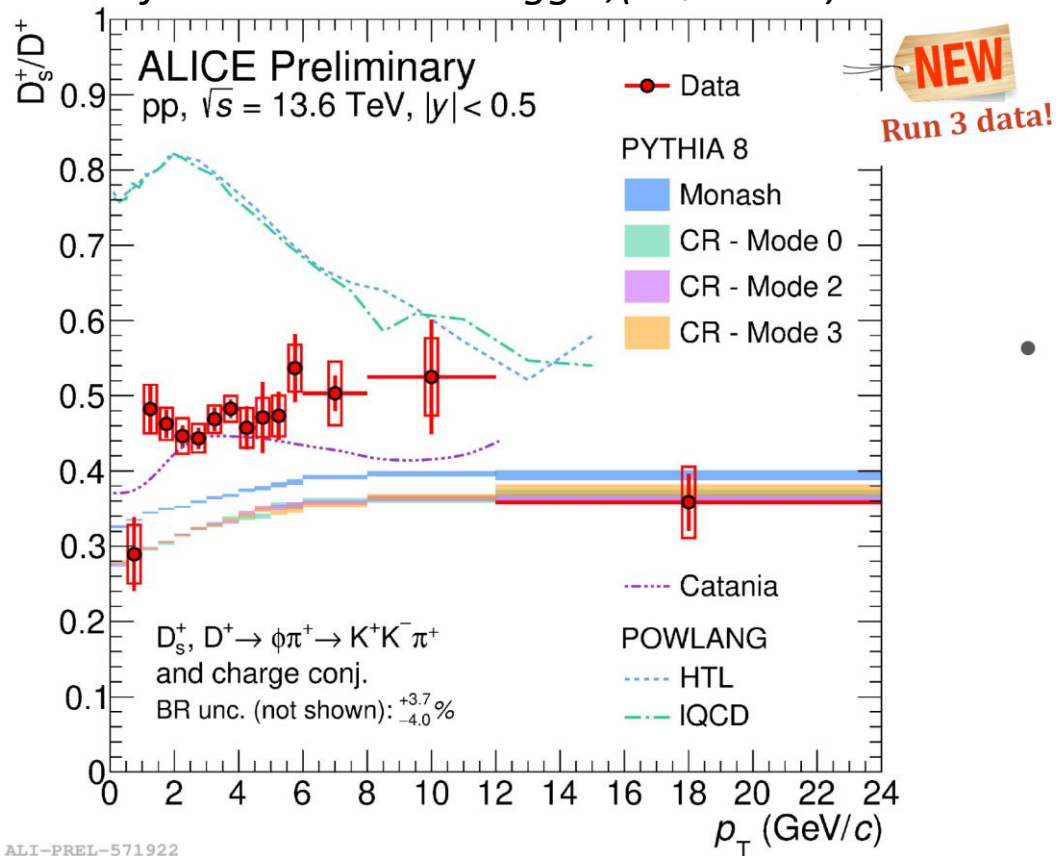
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- Coal./Fragm. ratio in pp larger for B than D
- PYTHIA 8 does not predict rapidity-dependence a discrepancy with LHCb measurements at forward rapidity.
- *Coalescence affect the rapidity dependence of the baryon-to-meson ratio of charm or beauty hadrons?*
- *Do the coal.+fragm. model also describe measurements at forward rapidity? Dominance of fragm.?*

# Strangeness in pp for HF sector

Taken from talk ALICE - Faggin, (SQM2024) 4 Tue 11:00



- Catania Coalesc.+Frag. quite ok, but it is large the fragmentation contribution
- POWLANG/LCN too high, but the approach has only recombination also for mesons
- PYTHIA-CR seems to have a lack of strangeness [see also  $\Xi_c$ ]

# HF baryon/meson in pp pA AA

Advantages of implementing coal. in **EPOS4**:

- Full dynamical realistic dynamics from ep, pp to AA

*J. Zhao et al., PRD 109, no.5, 054011 (2024)*

*J. Zhao et al., PRC 110, no.2, 024909 (2024)*

*difference in coal. wrt Catania: Assume RQM states like in SHM*

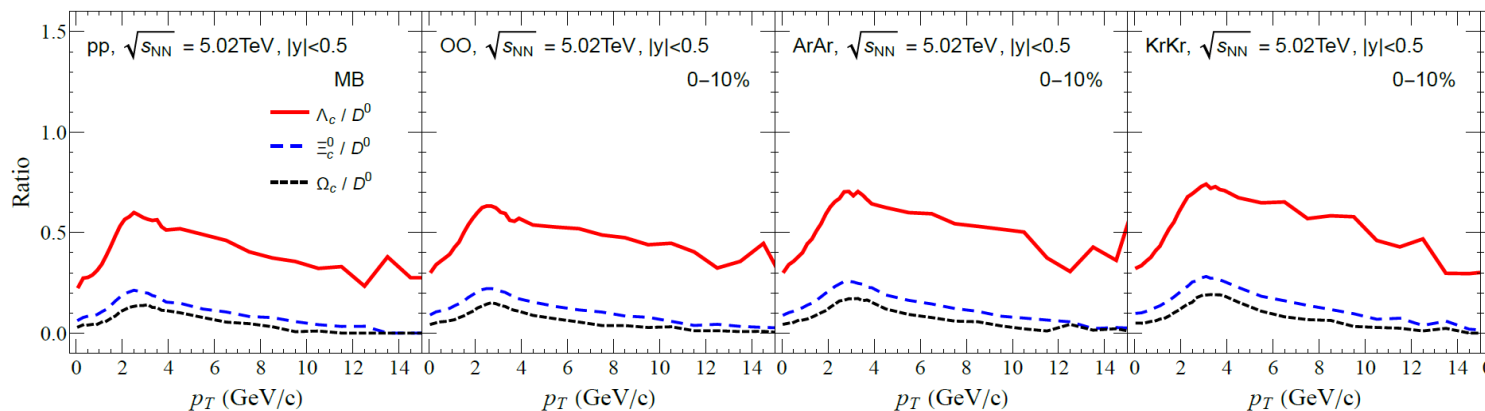
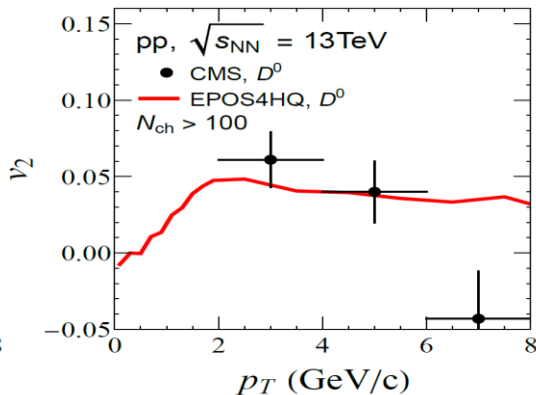
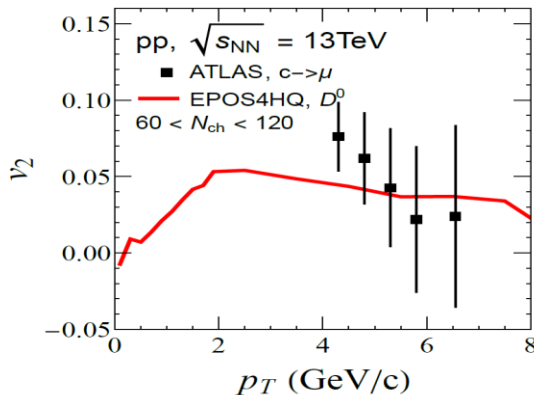
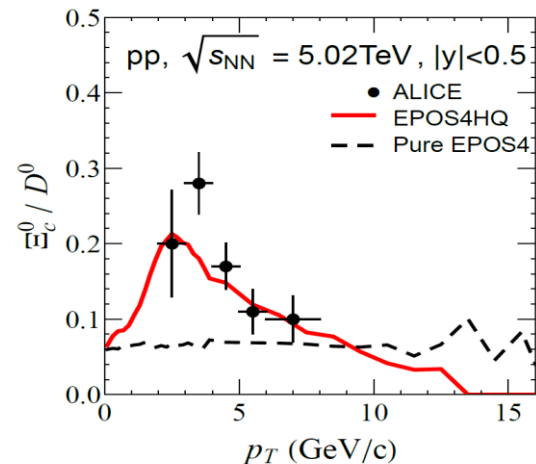
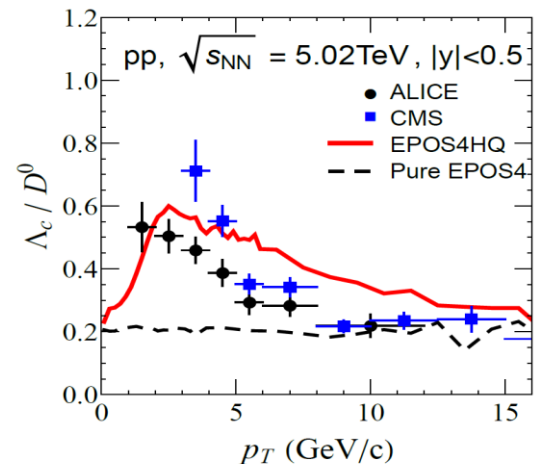
- **Able to predict also a sizeable elliptic flows in pp**

→ more solid constraints to hadronization and the properties of the pp QCD matter created.

→  $v_2(\Lambda_c)/v_2(D^0)$  would give more insight into coal.

*Would PYHTIA-CR predict finite  $v_2$  of  $D$ ,  $\Lambda_c$  in pp?  
String shoving?*

*J. Zhao et al., arXiv:2407.20919*



**Multi-charm in PbPb - KrKr – ArAr -00**



# Statistical Thermal Model (SHM) + charm(SHMc)

## grand canonical partition function

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty p^2 dp \ln [1 \pm \exp(- (E_i - \mu_i) / T)]$$

chemical potential  $\leftrightarrow$   
conservation quantum numbers  
( $N_B, N_s, N_c$ )

## Equilibrium + hadron-resonance gas + freeze-out temperature.

Production depends on hadron masses and degeneracy, and on system properties.

## Charm hadrons according to thermal weights

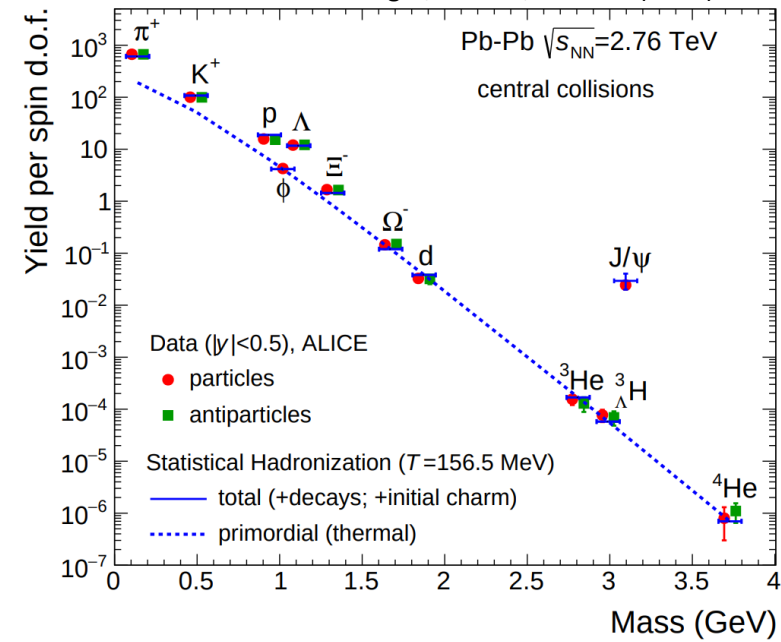
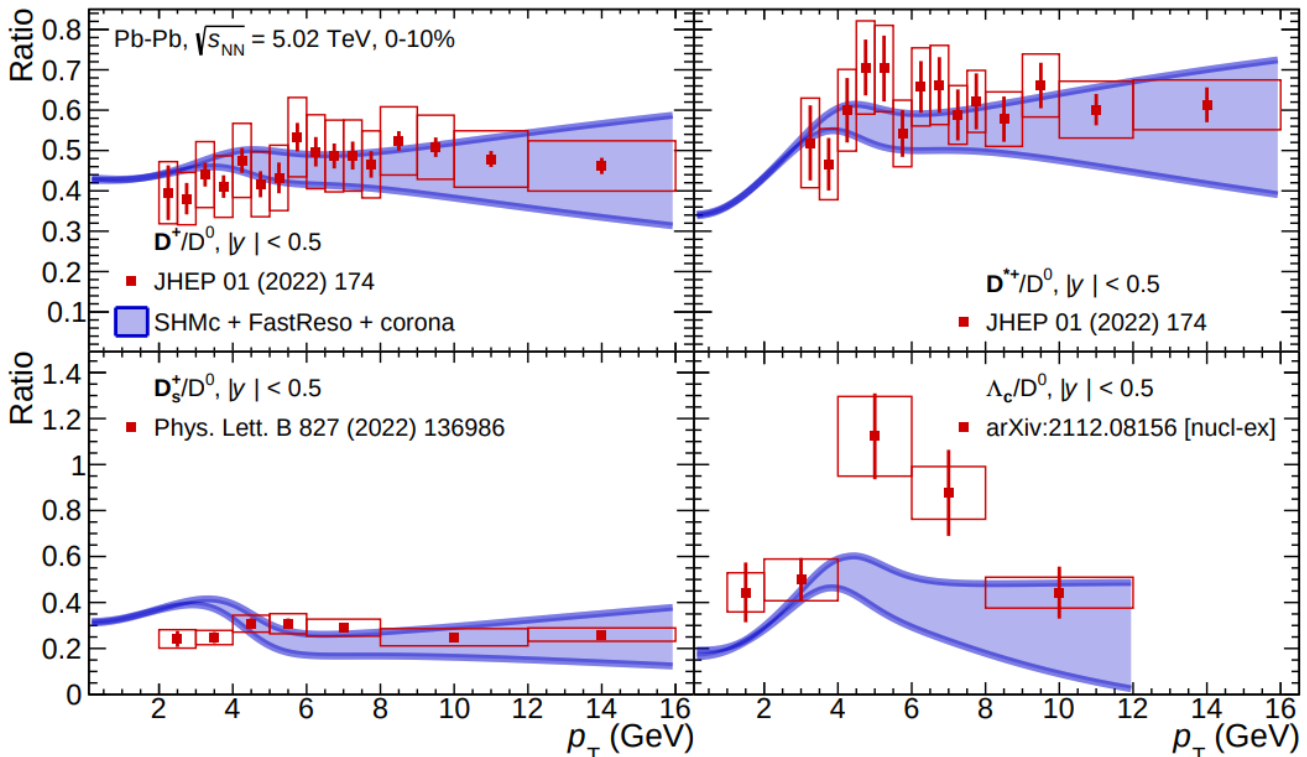
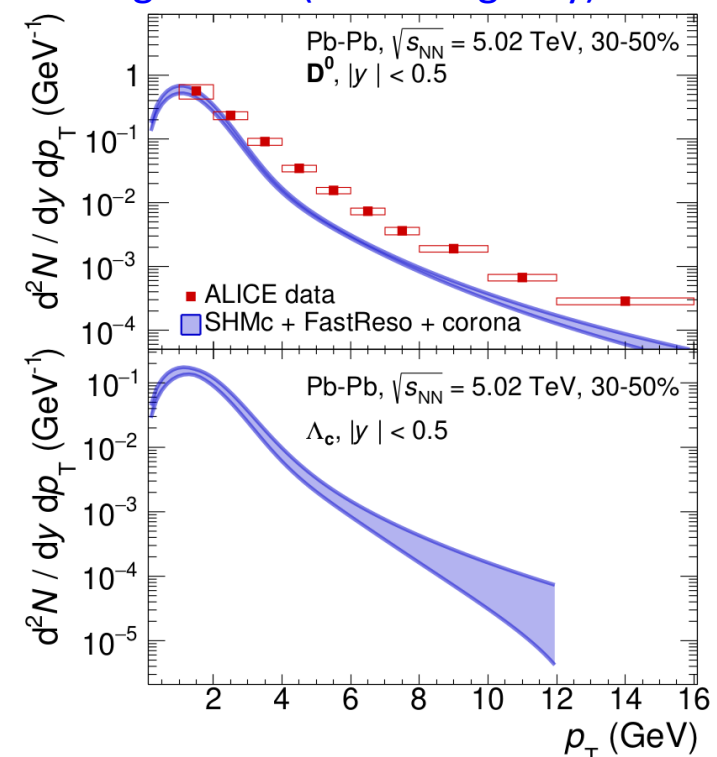
the total charm content of the fireball is fixed by the measured open charm cross section.

$$N_{c\bar{c}}^{dir} = \frac{1}{2} g_c V \left( \sum_i n_{D_i}^{th} + n_{\Lambda_c}^{th} \right) + g_c^2 V \left( \sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th} \right)$$

pQCD production  $N_{c,anti-c} = 9.6 \rightarrow g_c = 30.1$  (charm fugacity)

Andronic et al.,  
JHEP 07 (2021) 035

SHMc yields+blast wave  
 $\rightarrow p_T$  spectra



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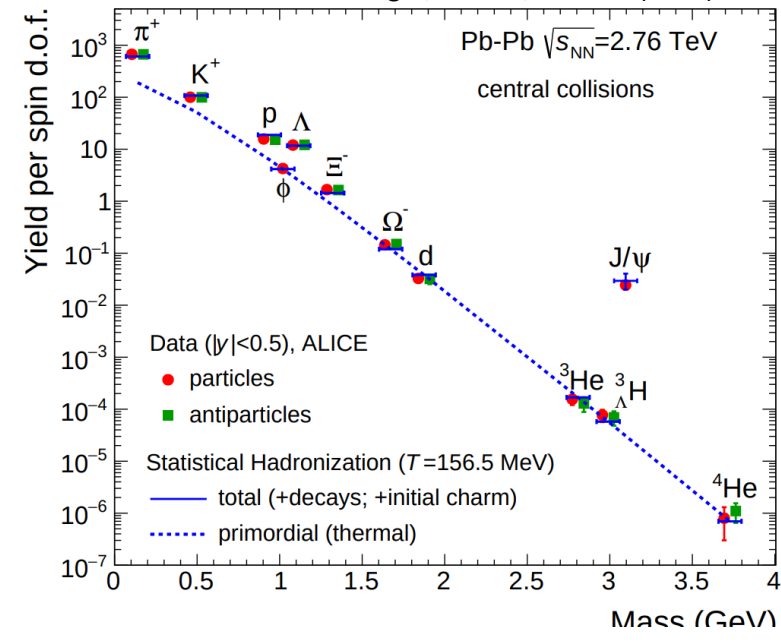
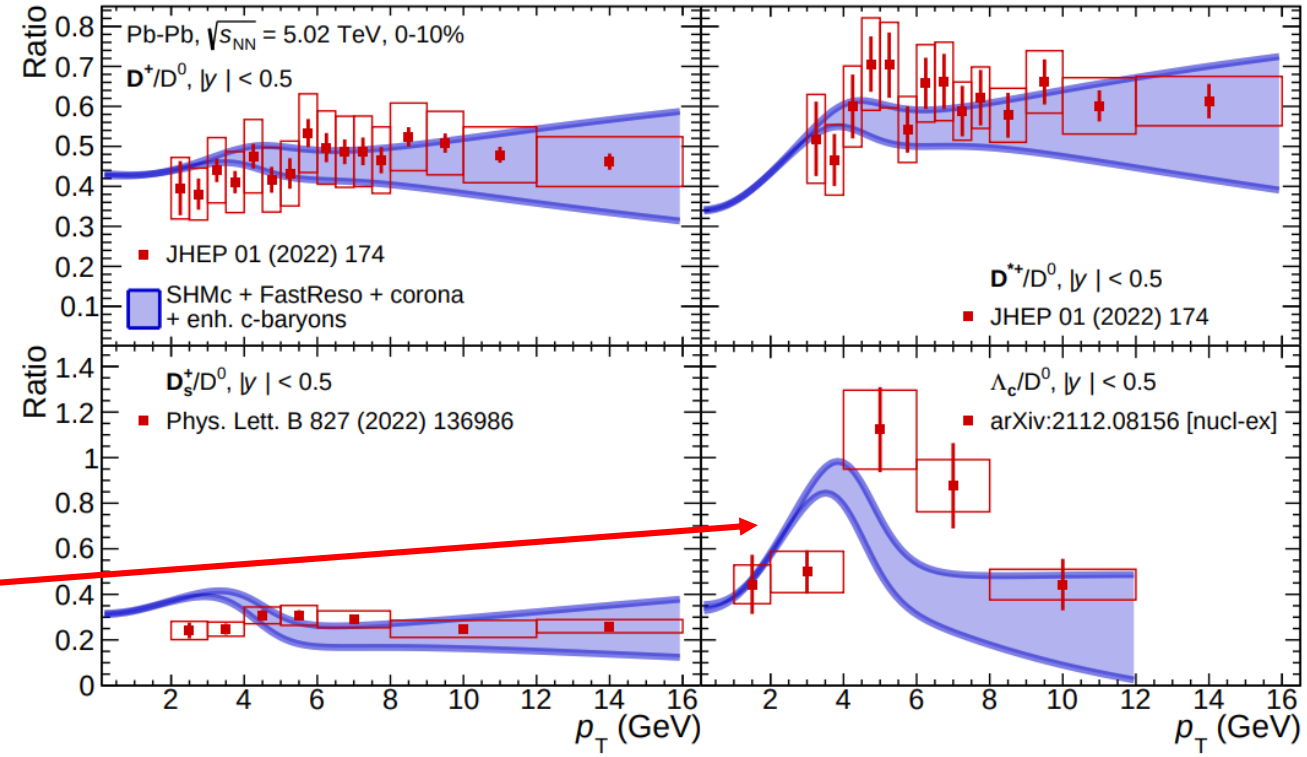
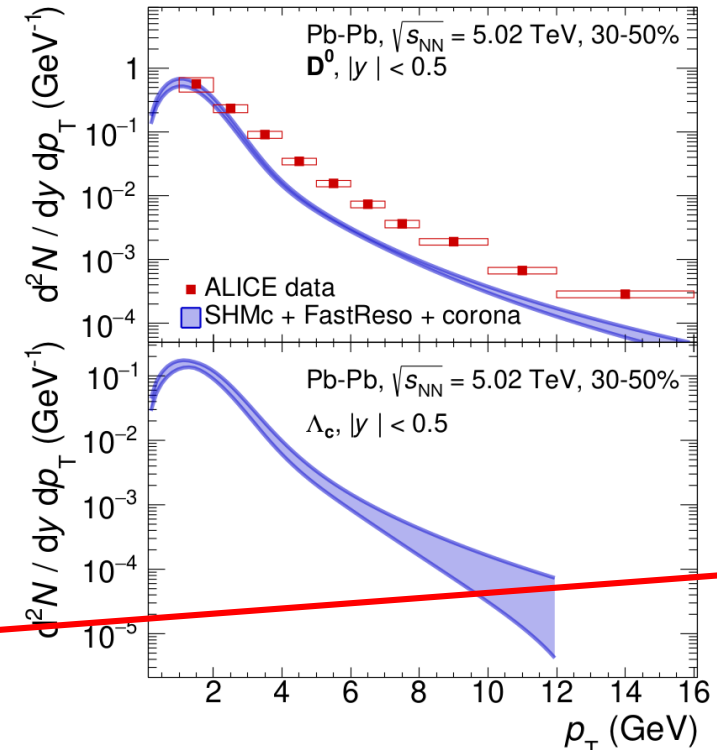
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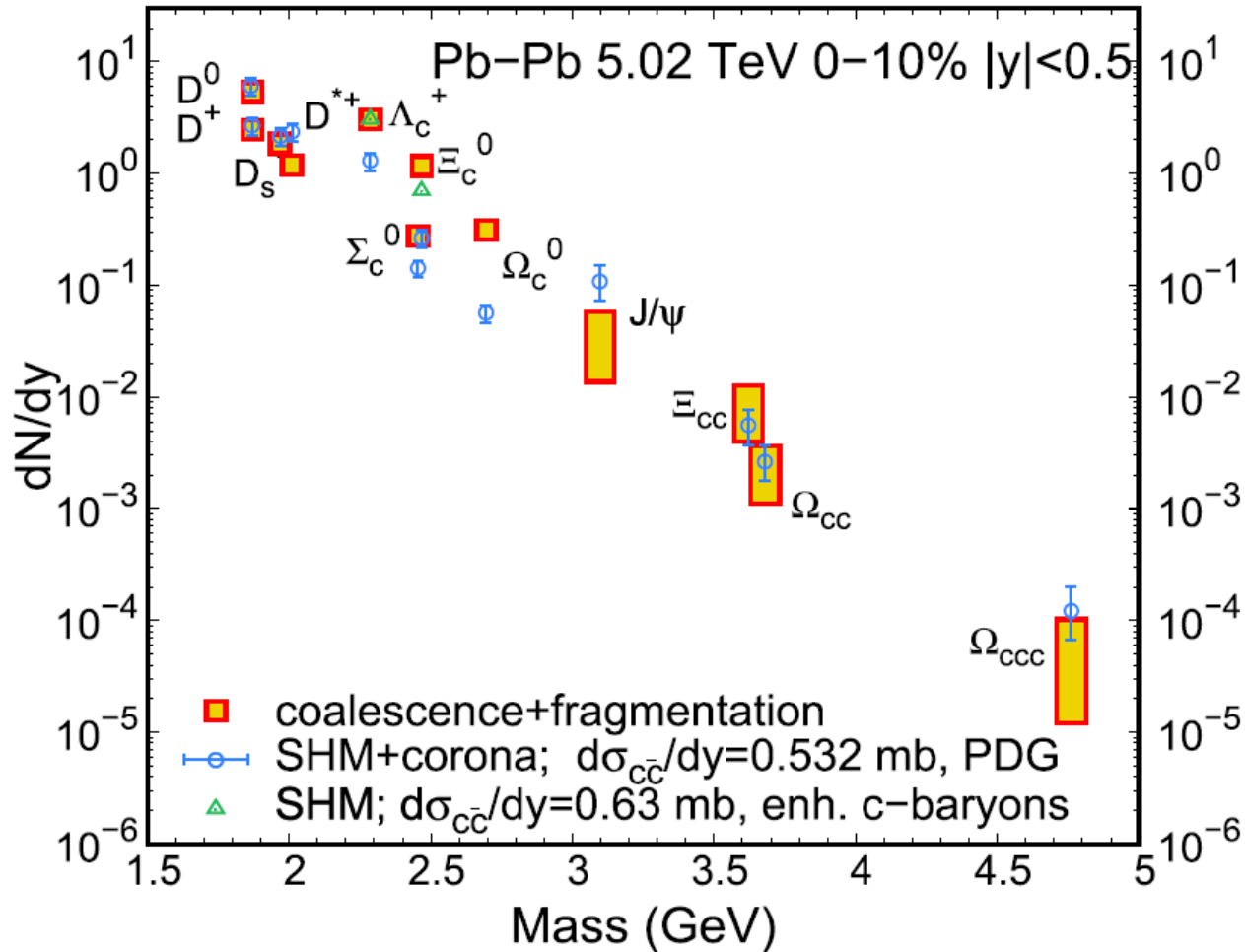
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 $\rightarrow p_T$  spectra

With enhanced set  
of charmed baryons



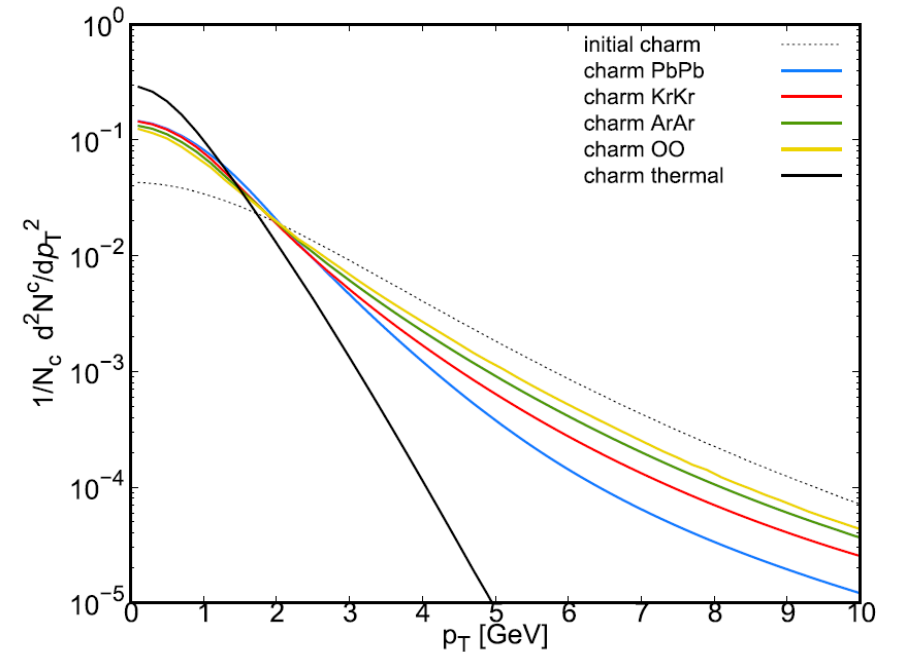
# Yields in PbPb: coalescence vs SHM

V. Minissale, S. Plumari, Y. Sun and V. Greco, *Eur. Phys. J. C* 84, no.3, 228 (2024)



→ upper limit: charm thermal distribution

→ lower limit: PbPb distribution with widths rescaled as standard Harm. Oscill. ( $\omega$  from  $\Omega_c^0$ )



We employ same volume in SHM

A. Andronic JHEP (2021) 035

	OO	ArAr	KrKr	PbPb
$R_0$ (fm)	2.76	3.75	4.9	6.5
$R_{max}$ (fm)	5.2	7.65	10.1	14.1
$\tau$ (fm)	4	5	6.2	8
$\beta_{max}$	0.55	0.6	0.64	0.7
$V_{ y <0.5}$ (fm <sup>3</sup> )	345	920	2000	5000

$\Sigma_c^0, \Xi_c^0, \Omega_c^0$ , widths from quark model

$\Xi_{cc}, \Omega_{cc}$  widths obtained rescaling with harm. oscillator

# Yields in PbPb: coalescence

V. Minissale, S. Plumari, Y. Sun and V. Greco, *Eur. Phys. J. C* 84, no.3, 228 (2024)

$D^0$  and  $\Lambda_c$  determine the yield, the radius variation is compensated by the constraint on the charm hadronization

A  $\pm 50\%$  in the radius of  $\Omega_{ccc}$  induces a change in the yield by about 1 order of magnitude

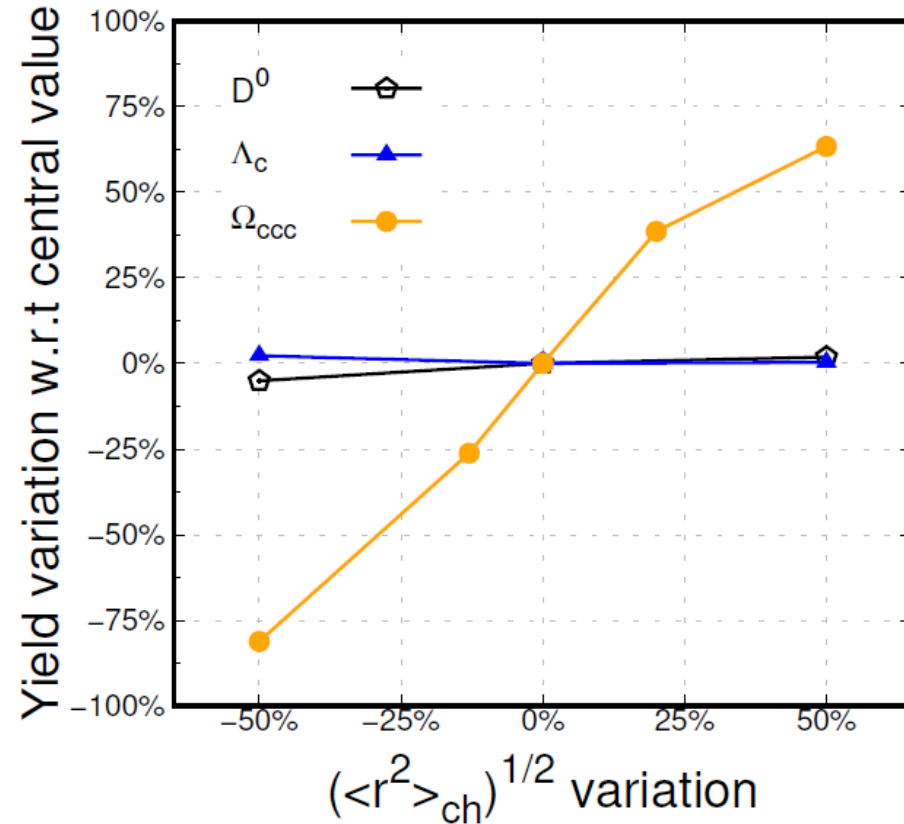
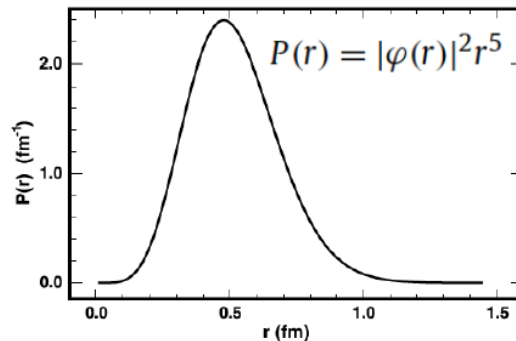
$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{i < j} V_{cc}(\mathbf{r}_i, \mathbf{r}_j). \quad V_{c\bar{c}}(\mathbf{r}_i, \mathbf{r}_j) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma |\mathbf{r}_{ij}|,$$

Solve the 3-body problem by a 1-body in higher dimensions hyperspherical coordinates method

$$\left[ \frac{1}{2m_c} \left( -\frac{d^2}{dr^2} - \frac{5}{r} \frac{d}{dr} \right) + v(r) \right] \varphi(r) = E\varphi(r)$$

$$W(\mathbf{r}, \mathbf{p}) = \int d^6\mathbf{y} e^{-i\mathbf{p}\cdot\mathbf{y}} \psi\left(\mathbf{r} + \frac{\mathbf{y}}{2}\right) \psi^*\left(\mathbf{r} - \frac{\mathbf{y}}{2}\right)$$

$$W(r, p, \theta) = \frac{1}{\pi^3} \int d^6\mathbf{y} e^{-i\mathbf{p}\cdot\mathbf{y}} \varphi(r_y^+) \varphi^*(r_y^-),$$

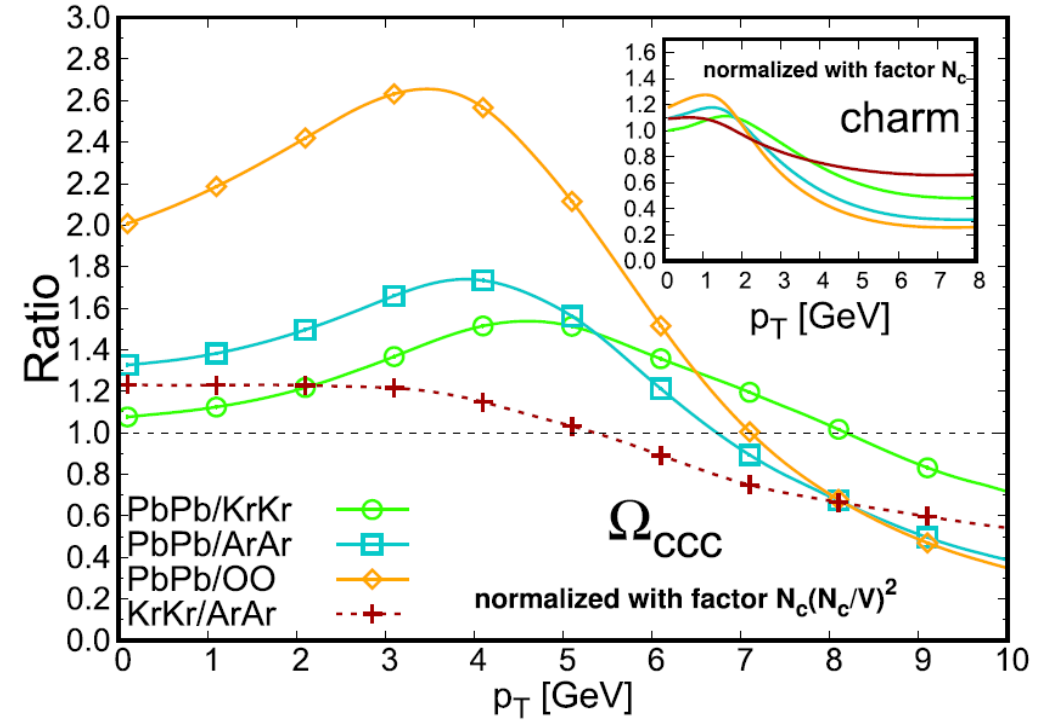
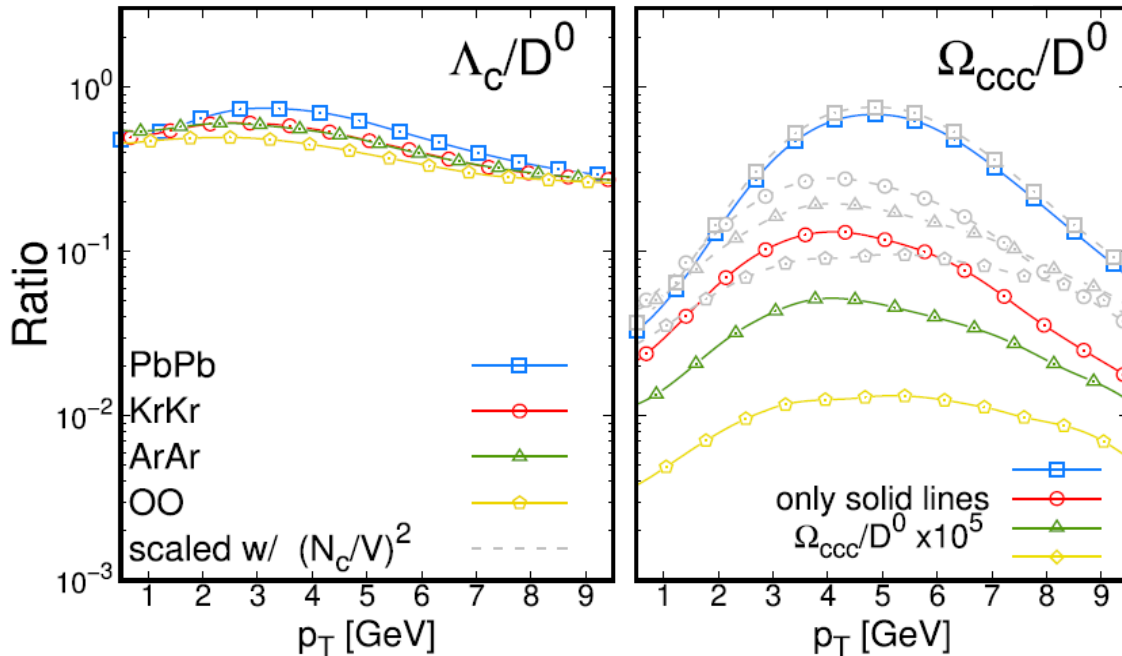
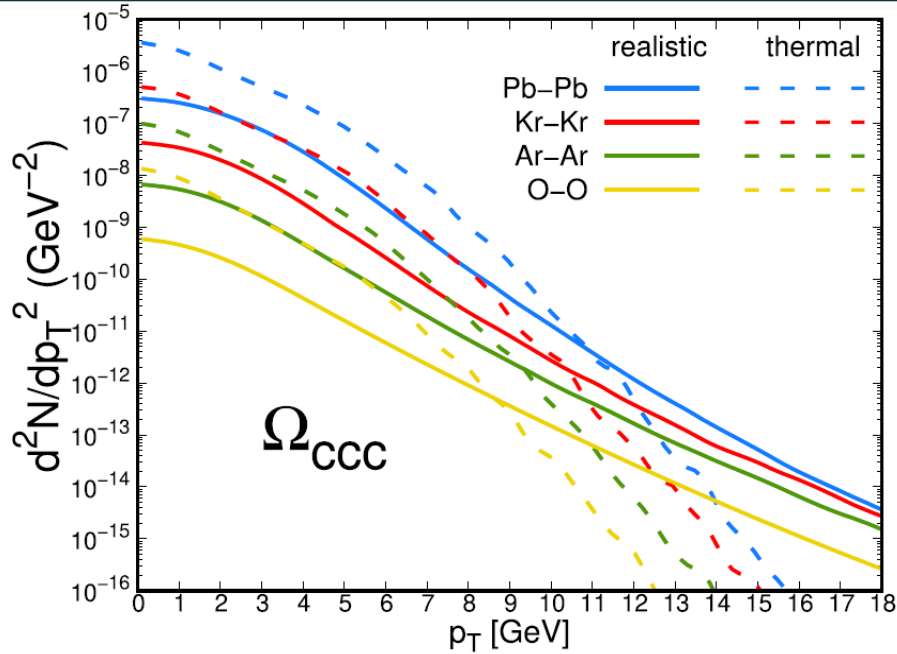


$$\frac{dN}{d^2\mathbf{P}_T d\eta} = C \int_{\Sigma} \frac{P^\mu d\sigma_\mu(R)}{(2\pi)^3} \int \frac{d^4r_x d^4r_y d^4p_x d^4p_y}{(2\pi)^6} \times F(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{p}_1, \tilde{p}_2, \tilde{p}_3) W(r_x, r_y, p_x, p_y),$$

$\Omega_{ccc}$   $\langle r \rangle = 0.5$  fm &  $\sigma_r \cdot \sigma_p \approx 1.5$   
similar to Tsinghua PLB746 (2015)

# Ratios of $p_T$ distribution $\Omega_{ccc}$ in PbPb/KrKr/ArAr/OO

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- It can be a meter of non-equilibrium.
- Translation of features of charm spectra at low  $p_T$  in to higher momentum region.
- More sensitive of multicharm  $\Omega_{ccc}/D^0$  with respect to  $\Lambda_c/D^0$



# Assessment of some of open issues

- Measurement of correlations, second-order and high order cumulants of Strange, multi-strange and charmed hadrons provides strong discriminative power against different model.
- Rapidity evolution of light and heavy flavor baryon/meson,  $v_n(p_T)$  [most work at  $y \sim 0$ ]
- Polarization possible new window for further tests of coalescence and fragmentation
- Extension to bottom + reduced data error bars, will show similar agreement?  
 $\Lambda_b/B^0$ ,  $\Xi_b/B^0$ ,  $\Omega_b/B^0$  further constraint to the hadronization mechanism
- Coal./Fragm. dominance of coal. in  $\Lambda_c$ ? a probe large  $v_2(D) / v_2(\Lambda_c)$  vs  $p_T$
- di-quark role in  $\Xi_c/D^0$ ? need smaller error at low  $p_T$   
System size scan of  $\Xi_c/D^0$ ,  $\Omega_c/D^0$  further constraint to the hadronization mechanism?  
hydro flowing diquarks?  $v_2(\Lambda_c) - v_2(D)$  at intermediate  $p_T$
- PDG/RQM resonances ...
- **Multicharm baryon** production (ALICE3):  $\Omega_{ccc}$  yield large sensitivity to charm kinetic equilibration and its wave function width





# Polarization

Recombination

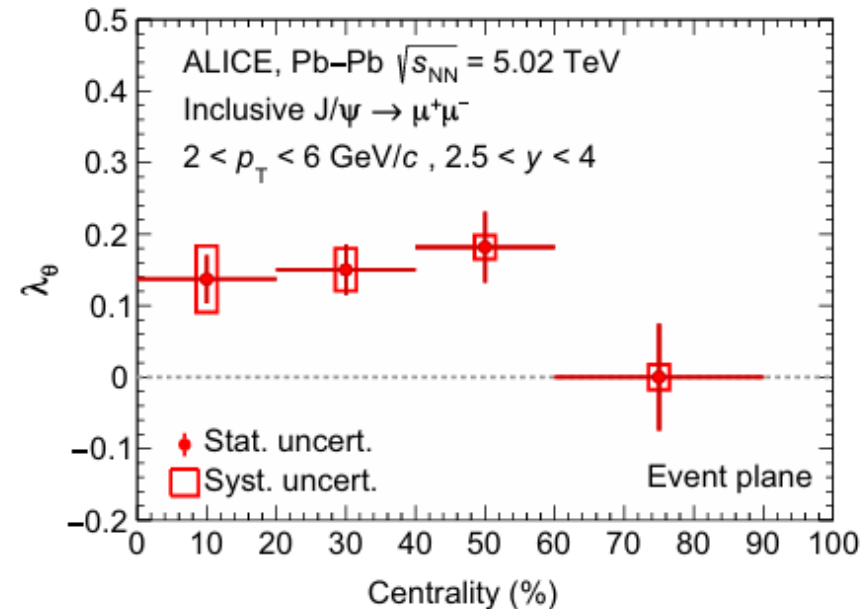
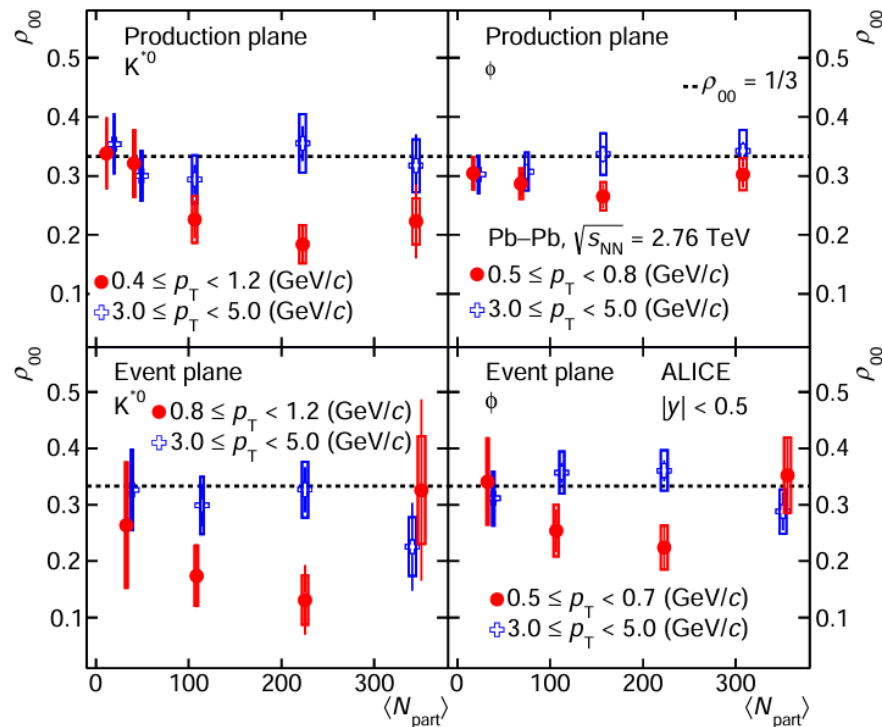
Fragmentation

$$\rho_{00} = \frac{1 - P_q \cdot P_{\bar{q}}}{3 + P_q \cdot P_{\bar{q}}} = \begin{cases} < \frac{1}{3}, & \text{charged meson} \\ > \frac{1}{3}, & \text{neutral mesons} \end{cases}$$

$$\rho_{00} = \frac{1 + \beta P_q^2}{3 - \beta P_q^2} > \frac{1}{3}$$

Hadronization underlying the interpretation for light hadrons, open HF and quarkonia

S. Acharya et al. [ALICE coll.], PRL 125 (2020) no.1, 012301

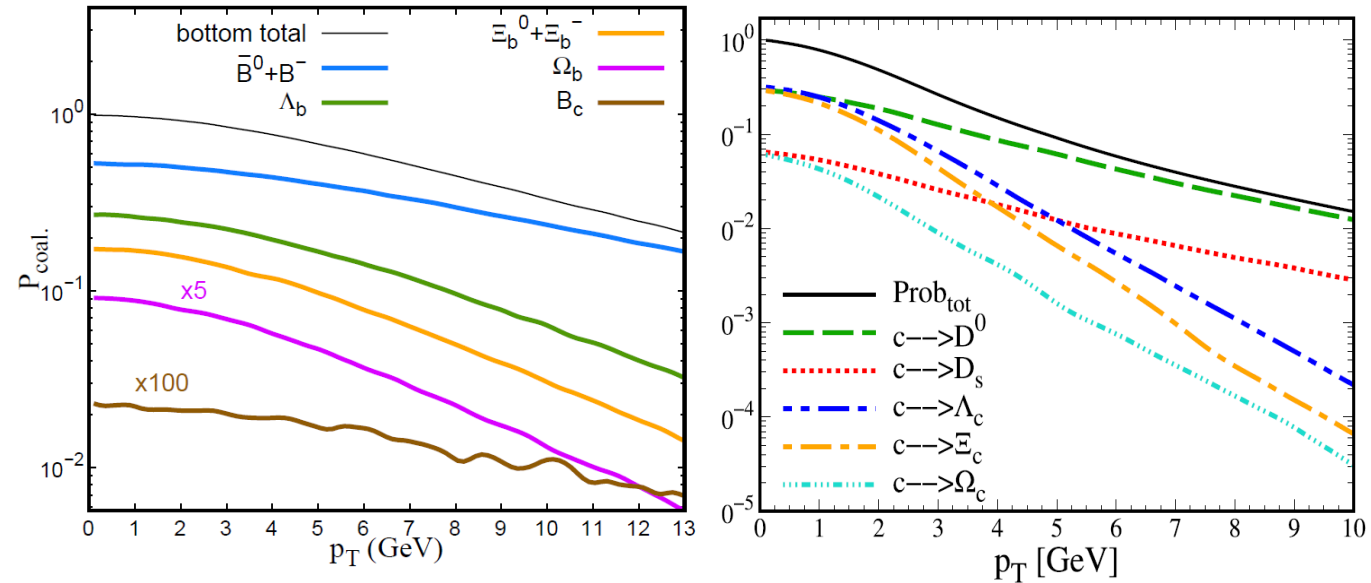
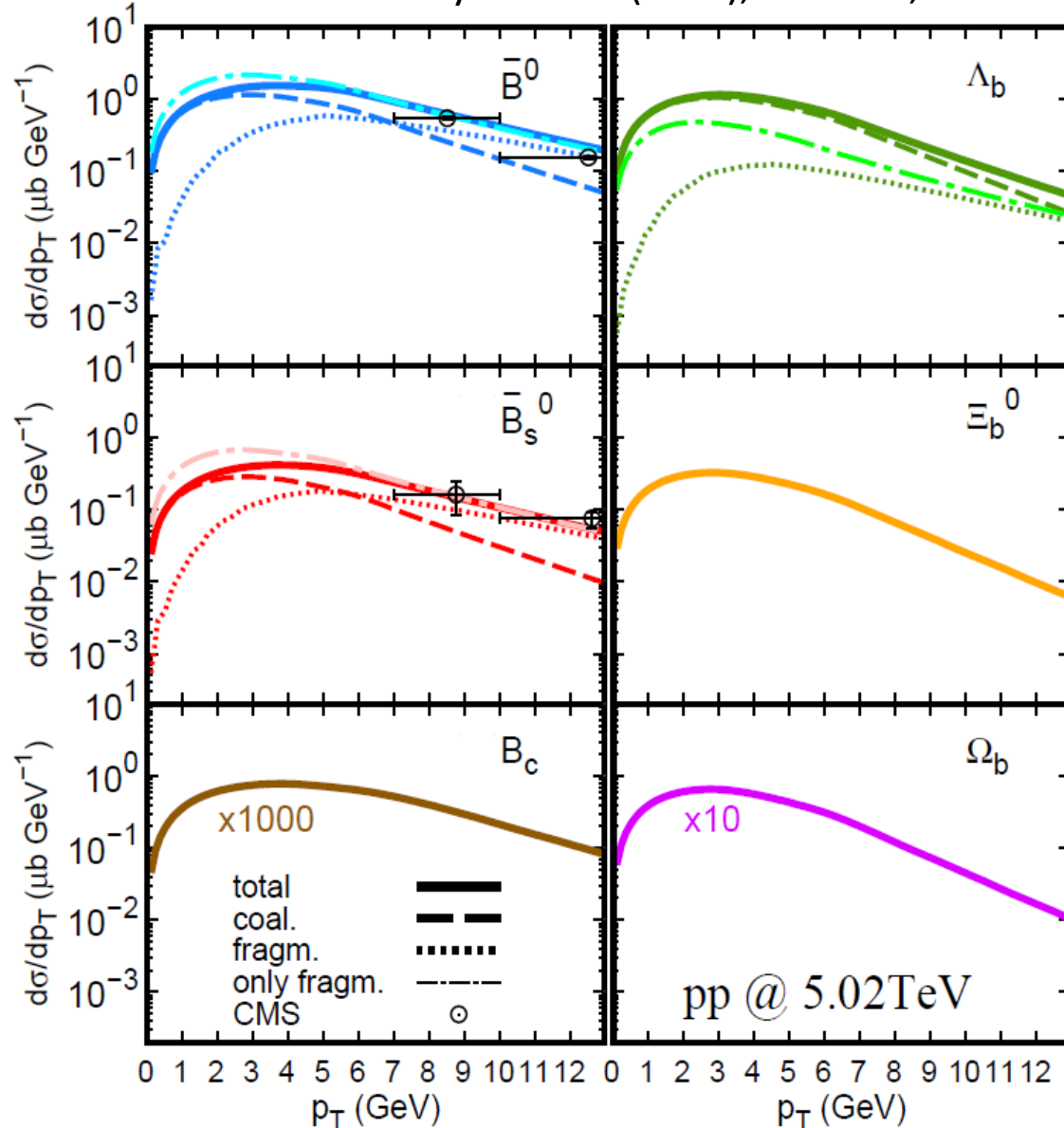


S. Acharya et al. [ALICE coll.], PRL 131 (2023) no.4, 042303

# Small systems: Coalescence in pp? (Bottom hadrons)

Data from: A. M. Sirunyan et al. (CMS), PRL 119, 152301 (2017).

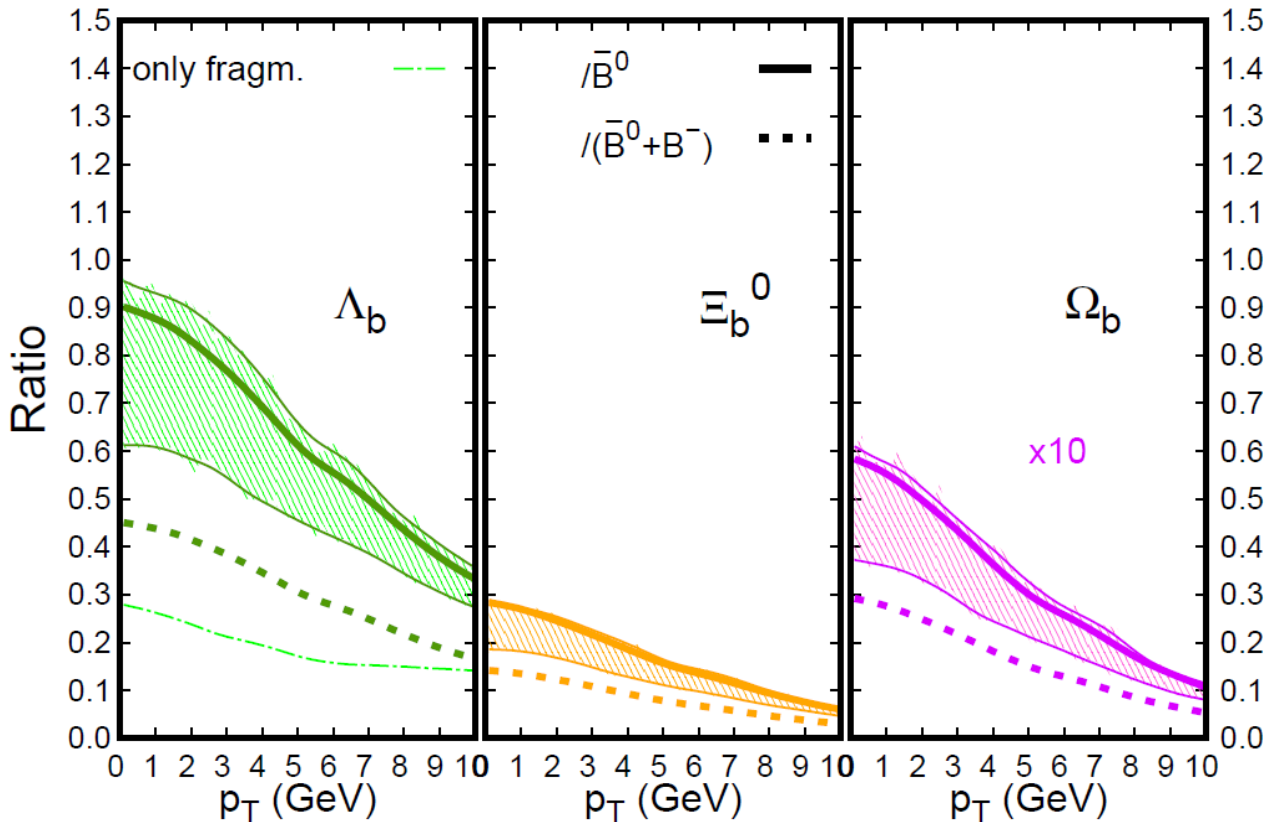
V. Minissale et al. arXiv:2405.19244 [hep-ph]



- $P_{\text{coal}}$  of bottom is flatter than  $P_{\text{coal}}$  of charm  
 -> Coal. greater impact on bottom hadron production
- B meson production at  $p_T < 5$  GeV mainly from Coal
- $\Lambda_b$  production mainly from Coal. for  $p_T < 10$  GeV

# Small systems: Coalescence in pp? (Bottom hadrons)

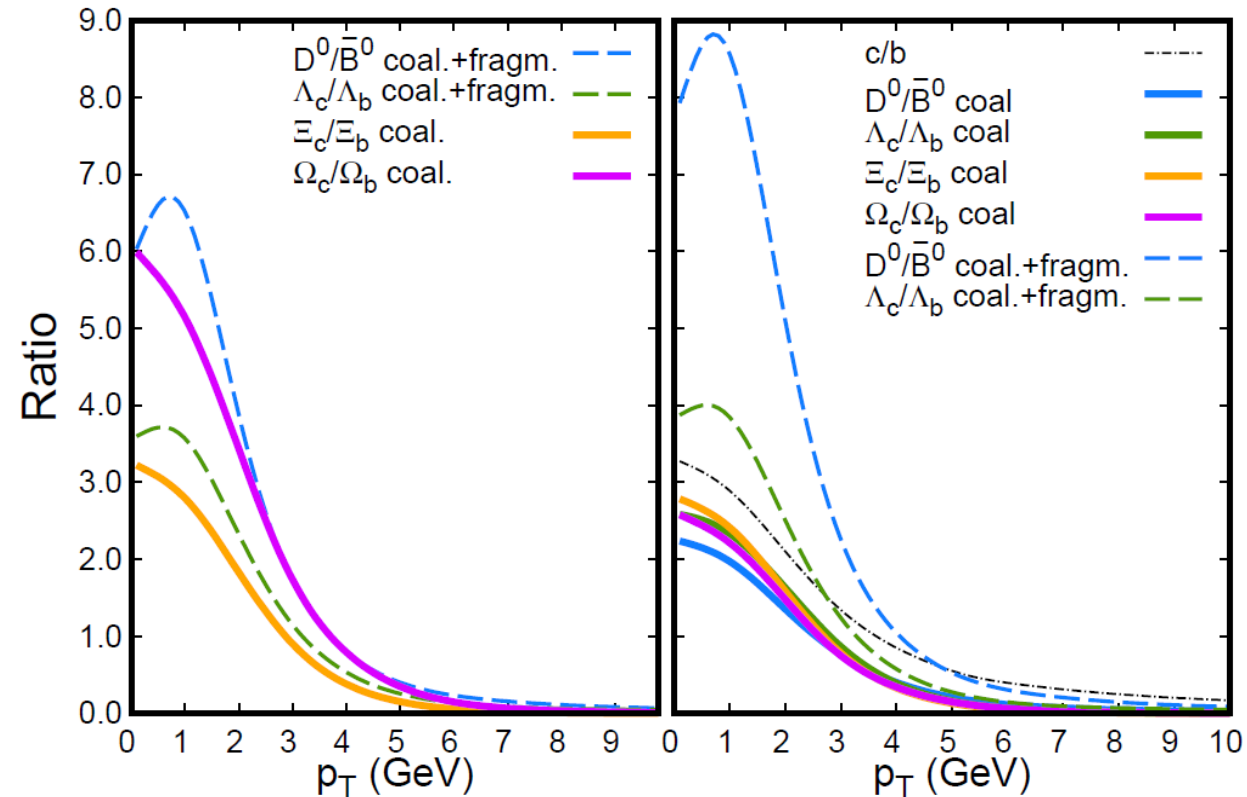
V. Minissale et al, arXiv:2405.19244 [hep-ph]



Coal gives enhancement of Baryon/meson ratio

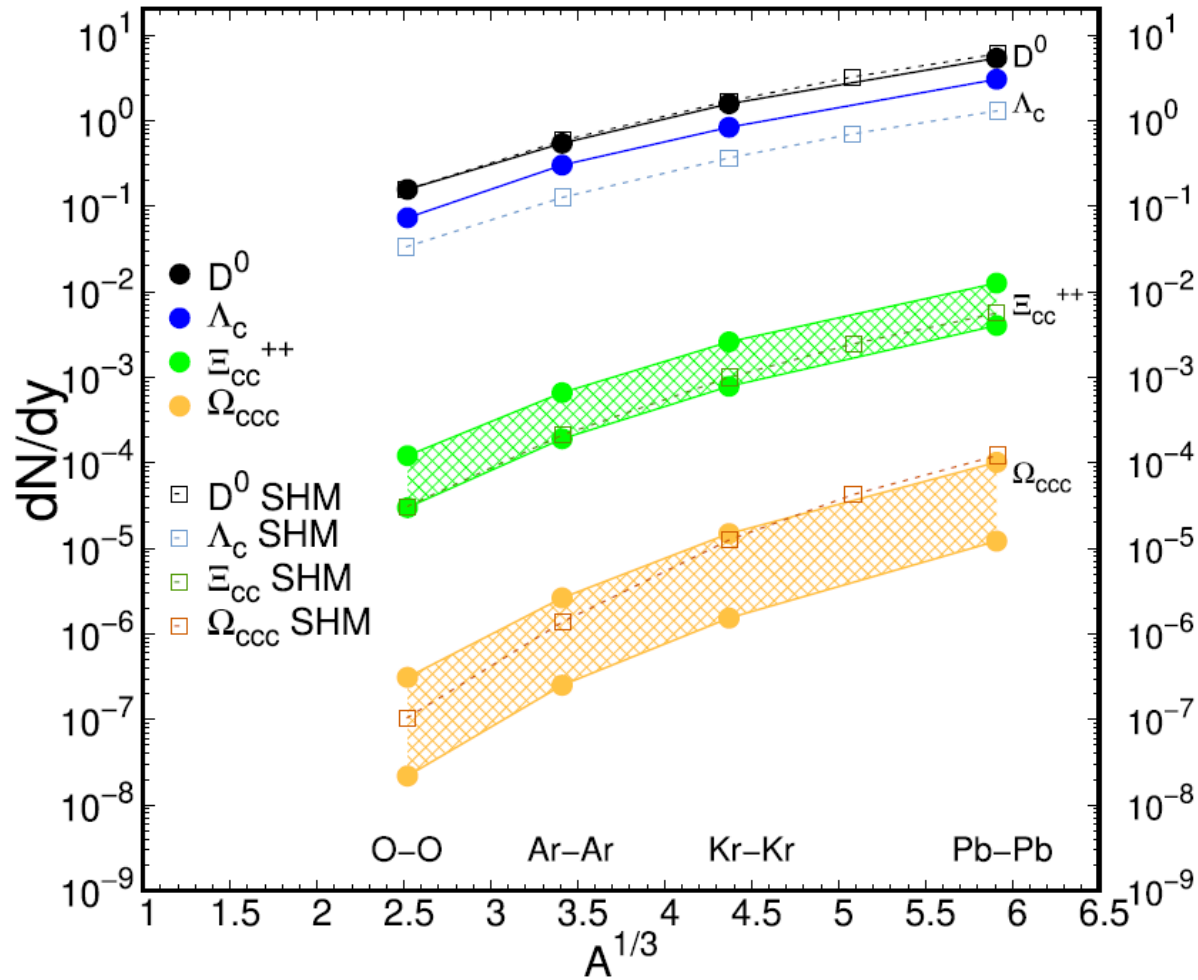
Error band correspond to  $\langle r^2 \rangle$  uncertainty in quark model

- $D/B$ ,  $\Lambda_c/\Lambda_b$ ,  $\Xi_c/\Xi_b$ ,  $\Omega_c/\Omega_b$  provide information about hadronization and  $f(c)/f(b)$
- Scaling if only coal. is assumed



# Yields in PbPb: coalescence vs SHM

V. Minissale, S. Plumari, Y. Sun and V. Greco, *Eur. Phys. J. C* 84, no.3, 228 (2024)



$\Sigma_c^0, \Xi_c^0, \Omega_c^0$ , widths from quark model

$\Xi_{cc}, \Omega_{ccc}$  widths obtained rescaling with harm. oscillator

$$\sigma_{ri} = \frac{1}{\sqrt{\mu_i \omega}} \quad \mu_1 = \frac{m_1 m_2}{m_1 + m_2}; \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}$$

→ upper limit: charm thermal distribution

→ lower limit: PbPb distribution with widths rescaled as standard Harm. Oscill. ( $\omega$  from  $\Omega_c^0$ )

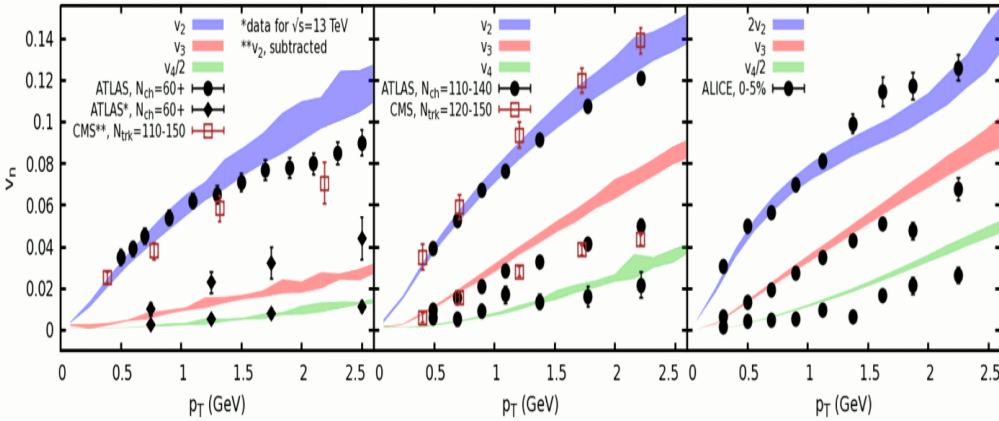
	$D^0$	$\Lambda_c$	$\Xi_{cc}^{+,++}$	$\Omega_{ccc}$
<i>OO</i>	0.156	0.0732	$3 - 12.1 \cdot 10^{-5}$	$2.2 - 29.2 \cdot 10^{-8}$
<i>ArAr</i>	0.543	0.301	$1.9 - 6.6 \cdot 10^{-4}$	$2.5 - 26.3 \cdot 10^{-7}$
<i>KrKr</i>	1.564	0.835	$0.78 - 2.6 \cdot 10^{-3}$	$1.5 - 14.9 \cdot 10^{-6}$
<i>PbPb</i>	5.343	3.0123	$4 - 12.5 \cdot 10^{-3}$	$0.12 - 1.01 \cdot 10^{-4}$

# Small systems

## Traditional view:

- QGP in Pb+Pb
- no QGP in p+p (“baseline”)

superSONIC for p+p,  $\sqrt{s}=5.02$  TeV, 0-1%    superSONIC for p+Pb,  $\sqrt{s}=5.02$  TeV, 0-5%    superSONIC for Pb+Pb,  $\sqrt{s}=5.02$  TeV, 0-5%



R. D. Weller, P. Romatschke PLB 774 (2017) 351-356

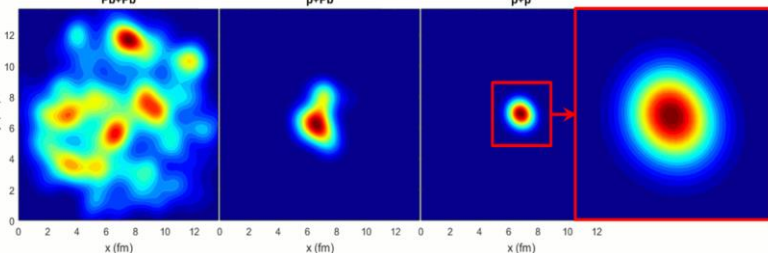
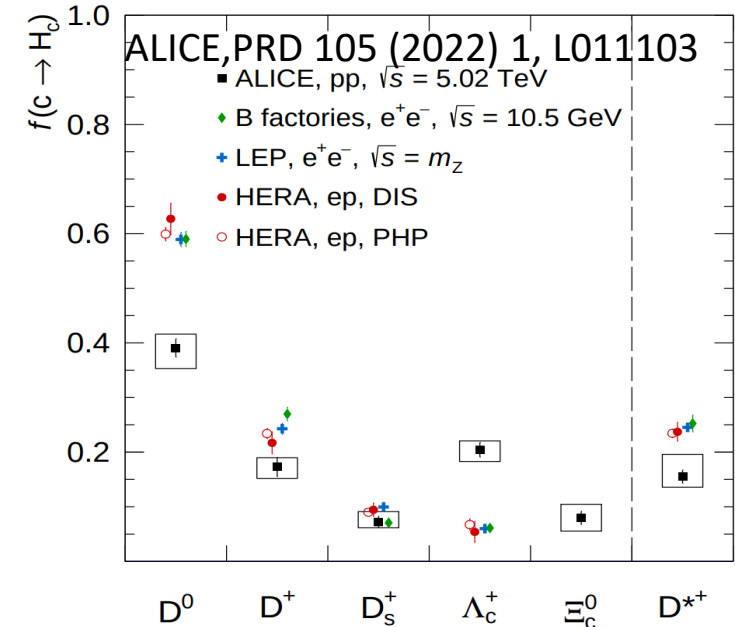
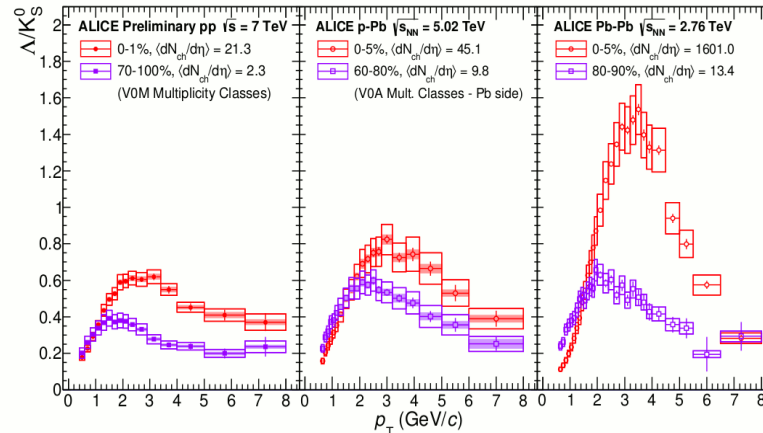
## Fragmentation:

production from hard-scattering processes (PDF+pQCD).

Fragmentation functions assumed “universal”

- Indication that fragmentation depends on the collision system
- Assumption of their universality not supported by the measured cross sections

ALICE Coll., PRL 111 (2013) 222301  
ALICE Coll., J. Phys.: Conf. Ser. 509 (2014) 012091  
ALICE Coll. NPA 956 (2016) 777-780.



## Objections to apply hydro in pp

- Too few particles, cannot be collective
- System not in equilibrium



