Event properties and hydro in small and large systems

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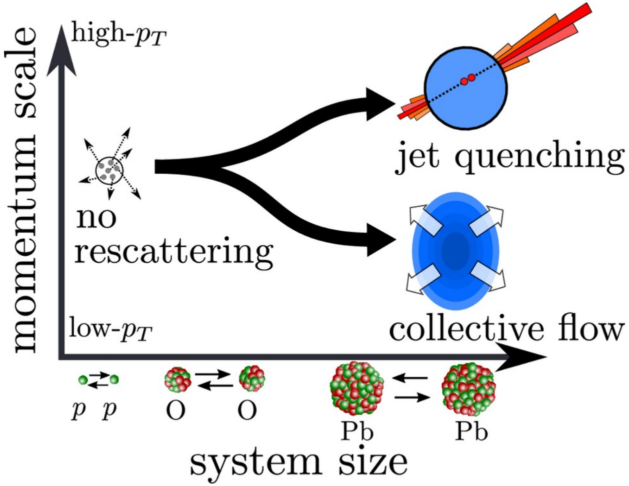
September 2, 2024 QCD Challenges from pp to AA collisions



aleksas.eu

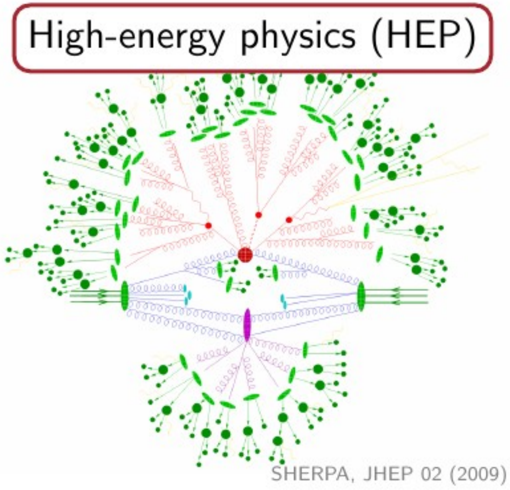
www.isoquant-heidelberg.de

Outline

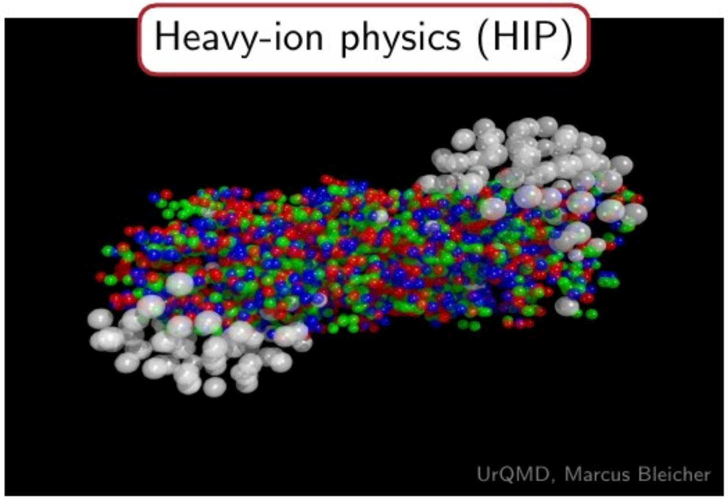
- Emergent phenomena in many-body systems.
- Causality and stability of relativistic hydrodynamics
- Stochastic viscous hydrodynamics
- Collectivity with few ultracold fermionic atoms

Emergent phenomena in complex systems

- HEP: concentrate higher energy in smaller and smaller volume.
- HIP: distribute high energy or high nucleon density over a relatively large volume. – T.D. Lee, 1974, Bear Mountain workshop



more scatterings



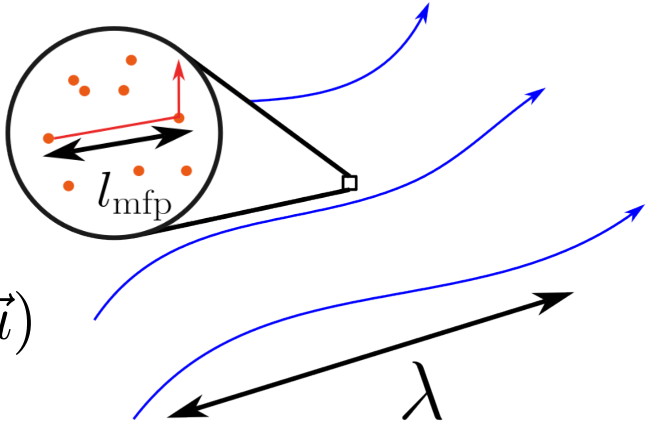
"More is different" – P.W. Anderson (1972)

Hydrodynamics

Navier-Stokes equation for fluid velocity $\vec{u}$

$$\rho\left(\partial_t + \vec{u}\cdot\vec{\nabla}\right)\vec{u} = -\vec{\nabla}p + \eta\nabla^2\vec{u} + \left(\tfrac{1}{3}\eta + \zeta\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{u})$$

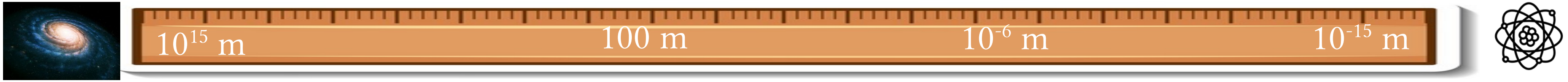
density pressure shear and bulk viscosity

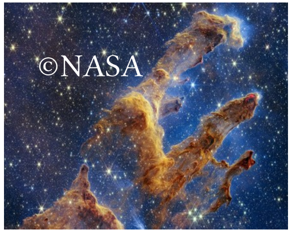


$l_{\rm mfp} \ll \lambda$

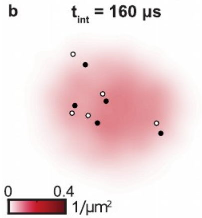
Universal effective theory

Even in small systems!

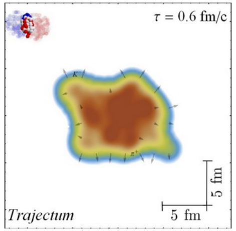




cold atoms



quark gluon plasma



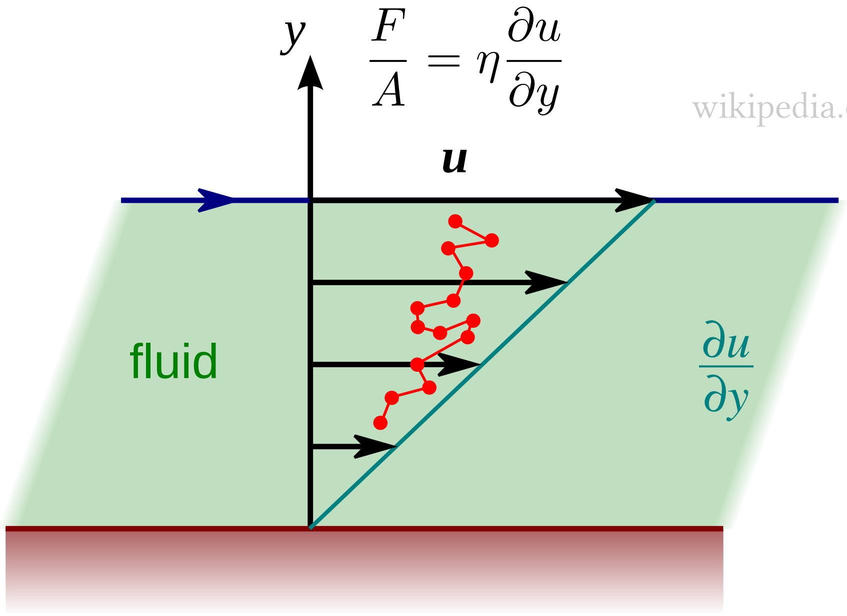
Shear viscosity

$$\eta \sim n \times \langle p \rangle \times l_{\mathrm{mfp}}$$

Large mean free path $\rightarrow$ large viscosity

Small mean free path $\rightarrow$ small viscosity

$$\Delta x \Delta p \geq \hbar/2$$



wikipedia.org

Viscosity in strongly interacting QFT

Kovtun, Son, Starinets, PRL, 2005

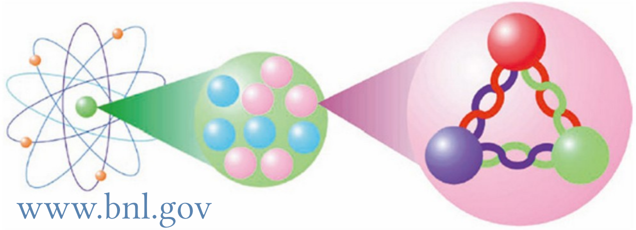
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \approx 0.08 \frac{\hbar}{k_B}$$

"perfect" fluid

entropy $s \sim n k_B$

Strongly interacting quantum systems

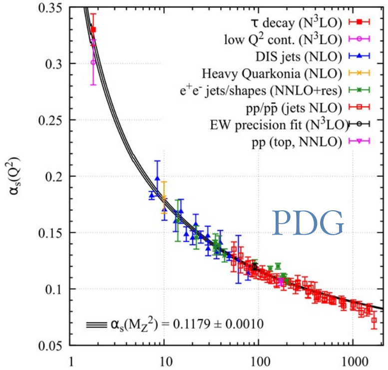
Quark Gluon Plasma



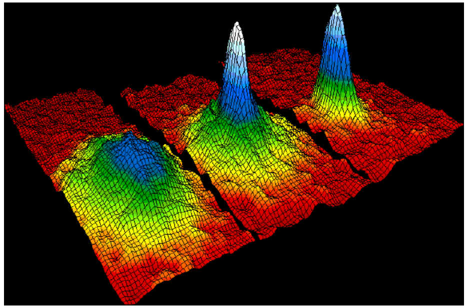
www.bnl.gov

QCD coupling α_s runs with energy scale

→

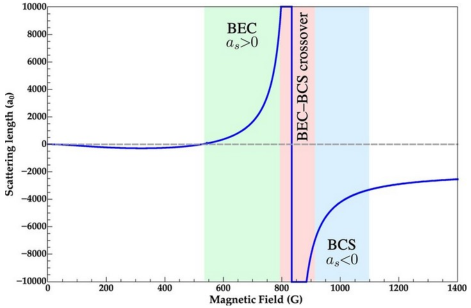


Ultracold quantum gases



Interaction strength diverges near Feschbach resonance

→



Hernández-Rajkov et al., 2008.05046

NIST/JILA/CU-Boulder

How many particles make a fluid?

Sorites ('heap') paradox:
If n grains of sand is a heap, then $n-1$ is also a heap
$\Rightarrow$ a single grain of sand is a heap?

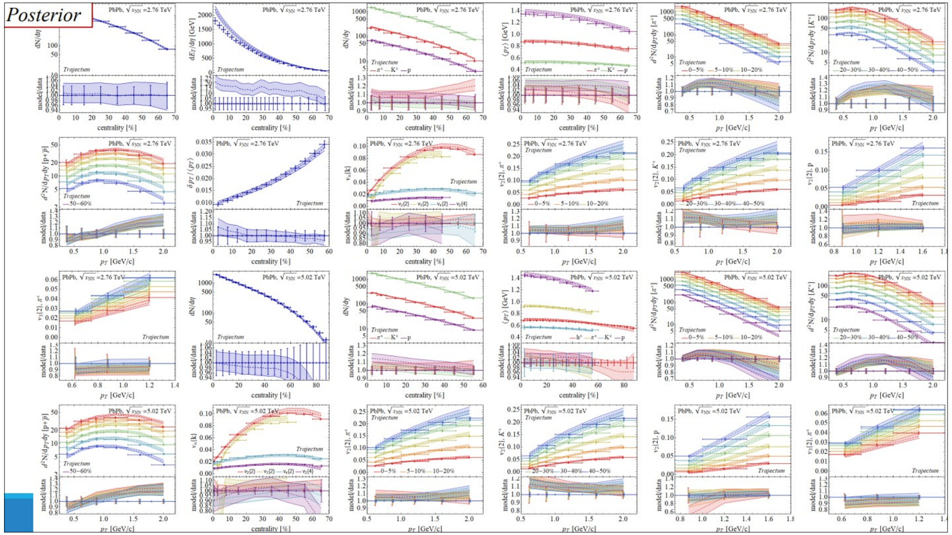
wikipedia.org

Duck test:
If it looks like a duck, swims like a duck,
and quacks like a duck, then it probably is a duck.

Can a few-particle system **behave** like a fluid?

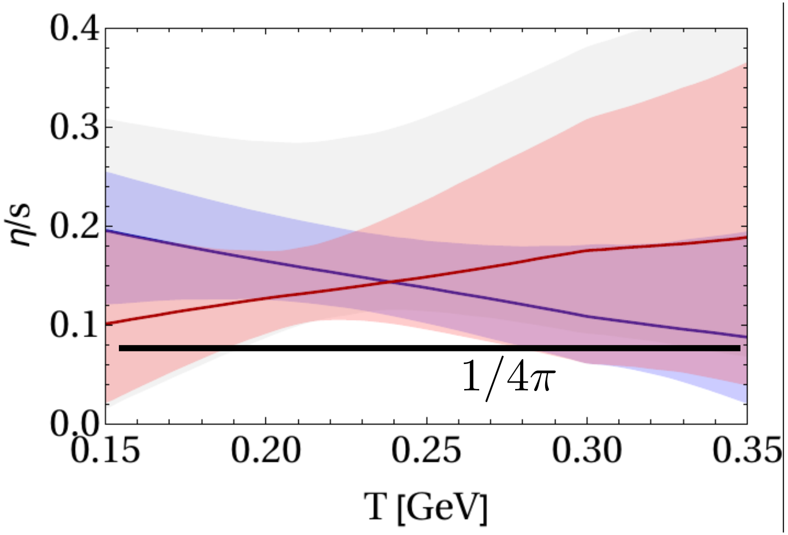
Shear viscosity of Quark Gluon Plasma

Multi-observable Bayesian fits



©Wilke van der Schee

Fitted viscosity to entropy ratio



Nijs, van der Schee, PRL 2022

Quark Gluon Plasma behaves like a nearly "perfect" fluid!

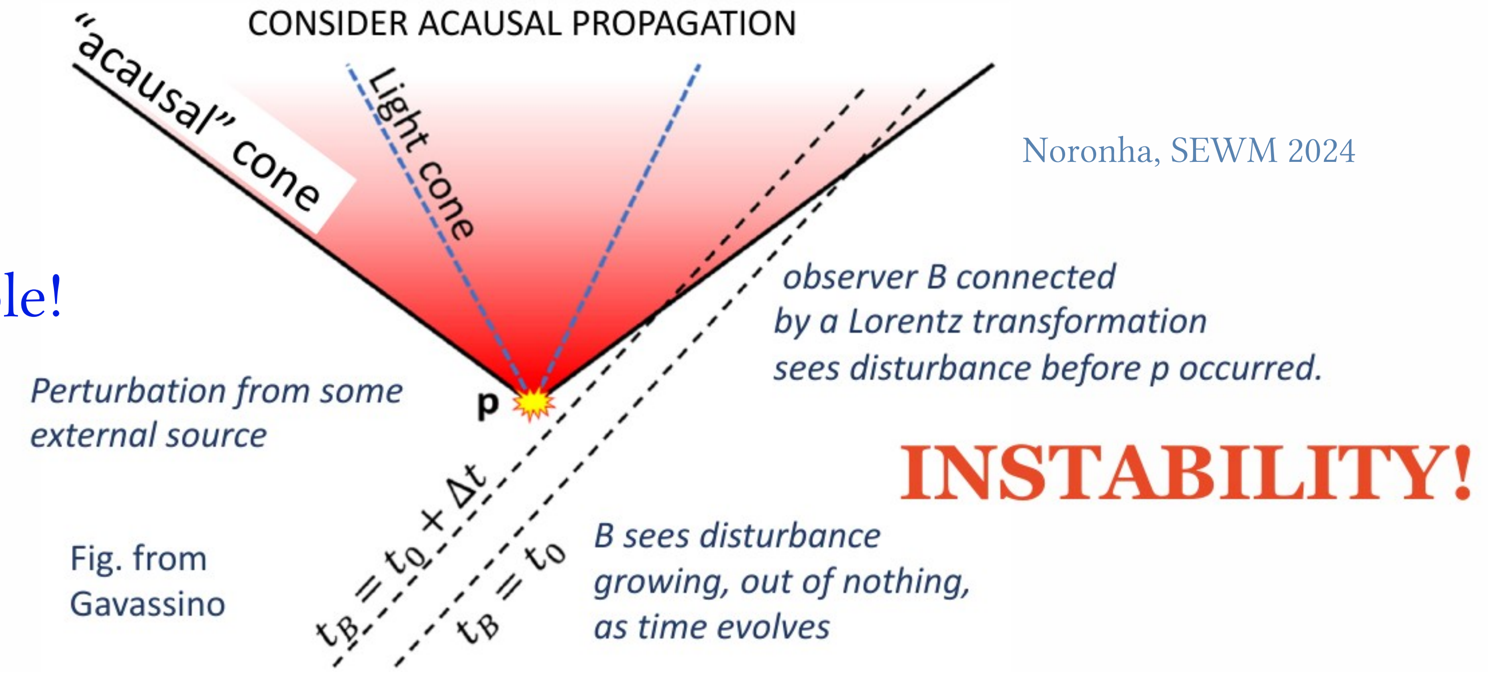
Challenge I: causality and stability

Relativistic Navier-Stokes equation

$$\rho\left(\partial_t + \vec{u}\cdot\vec{\nabla}\right)\vec{u} = -\vec{\nabla}p + \eta\nabla^2\vec{u} + \left(\tfrac{1}{3}\eta + \zeta\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{u})$$

Second-order time-derivatives: $(g^{\alpha\beta} + u^\alpha u^\beta)\partial_\alpha\partial_\beta u^\mu$

Naive formulation
is acausal and unstable!



Noronha, SEWM 2024

Fig. from Gavassino

Muller-Israel-Stewart (2nd order hydro)

from TRAJECTUM, 2010.15134

Relaxation equations for shear and bulk pressures

$$D\Pi = -\frac{1}{\tau_\Pi}\left[\Pi + \zeta\nabla\cdot u + \delta_{\Pi\Pi}\nabla\cdot u\Pi - \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right], \quad (6)$$

New transport coefficients

$$\Delta^\mu_\alpha\Delta^\nu_\beta D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} + \delta_{\pi\pi}\pi^{\mu\nu}\nabla\cdot u - \phi_7\pi^{\langle\mu}_\alpha\pi^{\nu\rangle\alpha} + \tau_{\pi\pi}\pi^{\langle\mu}_\alpha\sigma^{\nu\rangle\alpha} - \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}\right]. \quad (7)$$

Nonlinear constraints for dissipative fluxes

$$c_s^2 + \frac{4}{3}\frac{\eta}{\tau_\pi(e+p)} + \frac{\zeta}{\tau_\Pi(e+p)} \leq 1$$

Bemfica, Disconzi, Noronha, PRL (2019), Bemfica, Disconzi, Hoang, Noronha, Radosz, PRL (2021)

Causality violation in small and large systems

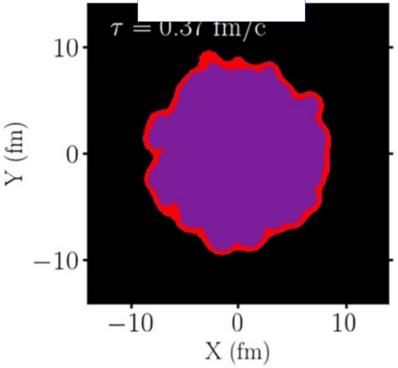
Only small percentage of
total energy in acausal cells

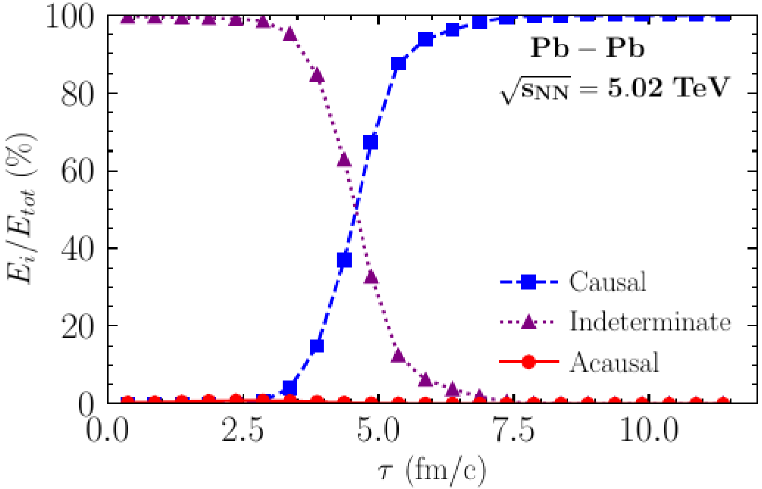
Might still affect viscosity extraction
Especially bulk viscosity

see talk by Renata

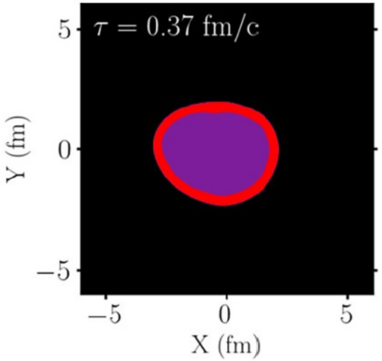
Krupczak et al. (ExTrEMe) PRC 2024

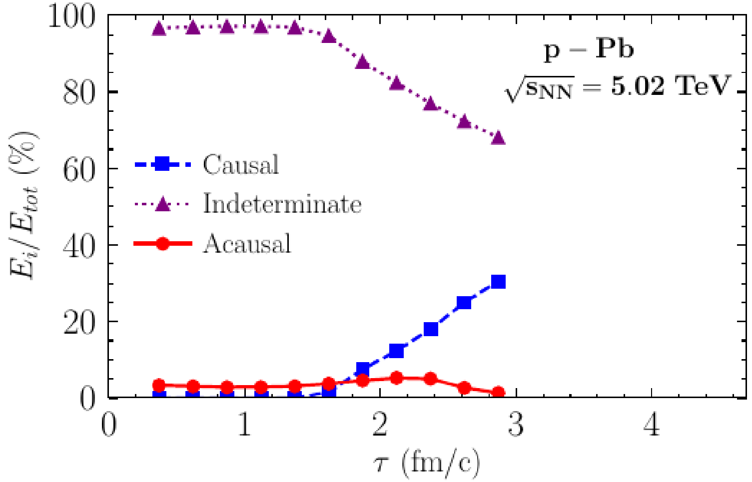
PbPb





pPb





BDNK theory (1^{st} order hydro)

Bemfica, Disconzi, noronha PRD (2018), Kovtun JHEP (2019)

Use freedom of hydrodynamic frame to obtain causal theory

→ additional transport coefficients

$$\mathcal{E} = \epsilon + \tau_\epsilon \left[u^c \nabla_c \epsilon + \rho \nabla_c u^c\right]$$

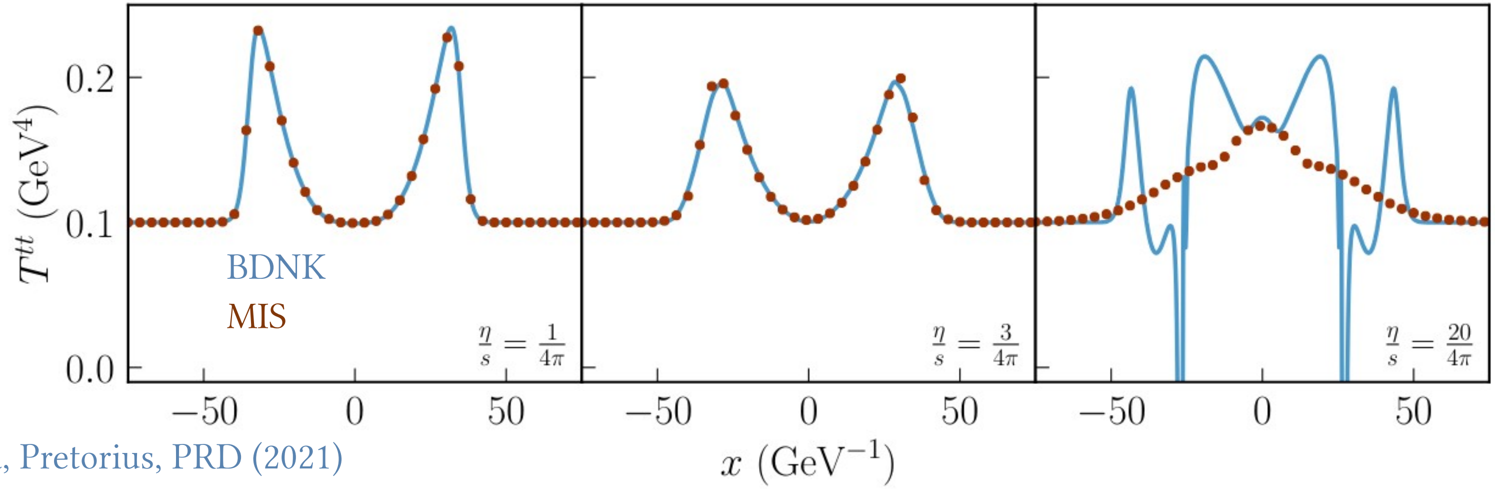
$$\mathcal{P} = P - \zeta \nabla_c u^c + \tau_P \left[u^c \nabla_c \epsilon + \rho \nabla_c u^c\right]$$

$$\mathcal{Q}^a = \tau_Q \rho u^c \nabla_c u^a + \beta_\epsilon \Delta^{ac} \nabla_c \epsilon + \beta_n \Delta^{ac} \nabla_c n$$

$$\mathcal{T}^{ab} = -2\eta\sigma^{ab} \equiv -2\eta \nabla^{<a} u^{b>}$$

First proof of causality and stability

First (1+1D) numerical simulations



Pandya, Pretorius, PRD (2021)

Density frame

Armas, Jain, Scipost (2011), Başar, Bhambure, Singh, Teaney, 2403.04185

Consider relativistic diffusion equation

$$\partial_\mu J^\mu = 0, \quad J^\mu = n u^\mu + j_D^\mu, \quad j_D^\mu = -D\Delta^{\mu\nu}\partial_\nu n$$

density in fluid rest frame

Density frame: $N = J^0$ $\partial_t N + \vec{\nabla}\cdot\vec{J} = 0$

Using ideal equations: $\vec{J} = N\vec{v} + (D/\gamma)(\vec{\nabla}N - \vec{v}(\vec{v}\cdot\vec{\nabla})N)$

density in a lab frame fluid 3-velocity

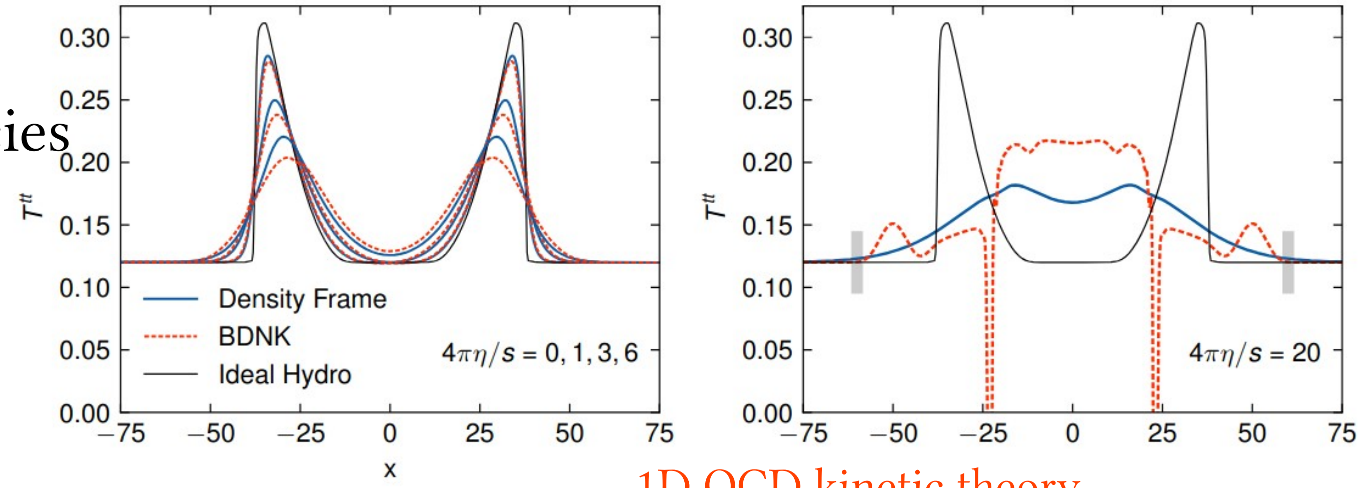
Strictly first order, numerically stable and no new coefficients,

...but does not obey Lorentz causality

Comparison with BDNK and kinetic theory

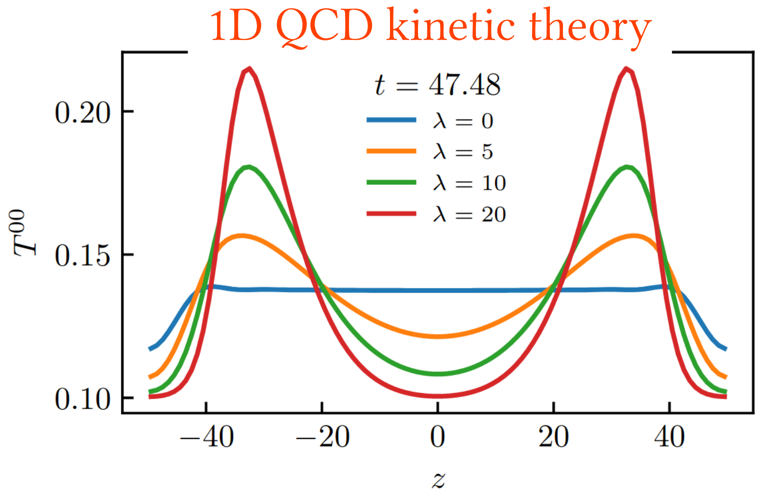
Paquet, Bhambure, AM, Singh, Teaney, Zhou (in preparation)

For $\eta/s \to 0$ agreement with BDNK, but discrepancies for large η/s



Kinetic theory smoothy interpolates to free streaming at $\eta/s \to \infty$.

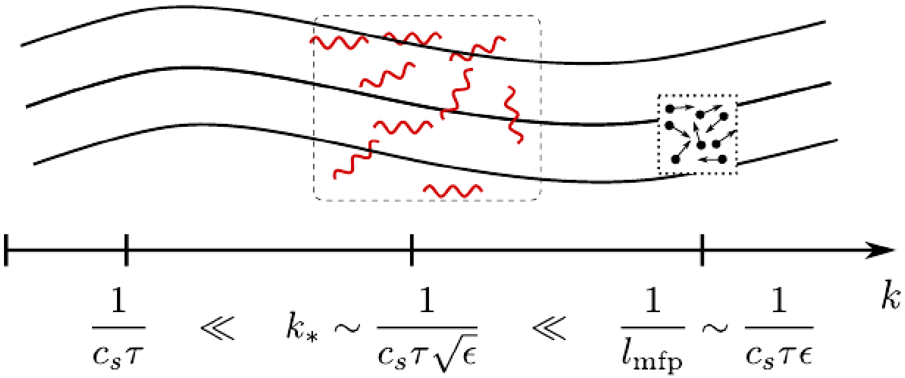
$\longrightarrow$ use to test density-frame results



Challenge II: stochastic noise

Hydrodynamics with noise

No dissipation without fluctuations!



$$T^{\mu\nu} = \underbrace{eu^\mu u^\nu + \mathcal{P}(e)\Delta^{\mu\nu}}_{\text{ideal}} - \underbrace{\eta\sigma^{\mu\nu}}_{\text{visc}} + \underbrace{\xi^{\mu\nu}}_{\text{noise}}$$

$$\langle \xi^{\mu\nu}(x)\xi^{\rho\sigma}(x')\rangle \equiv 2T\eta\delta^4_{xx'}\ \left[\Delta^{\mu\rho}\Delta^{\nu\sigma} + \Delta^{\nu\rho}\Delta^{\mu\sigma} - \tfrac{2}{3}\Delta^{\mu\nu}\Delta^{\rho\sigma}\right]$$

Hydrodynamic noise should be important in small systems

- Thermal fluctuations renormalize EOS and transport coefficients
- Fluctuations grown near critical points

Stickiness of sound and long-time tails

Kovtun, Moore and Romatschke PRD (2011)

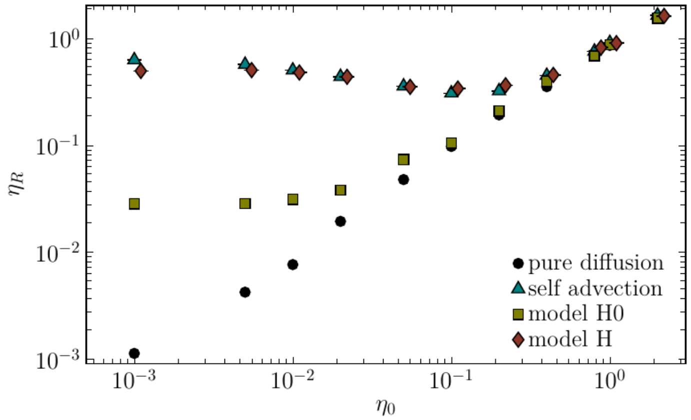
Thermal noise contributes to viscosity

$$\eta_R = \eta + \frac{7}{60\pi^2}\frac{\rho T \Lambda}{\eta}$$

Schaefer SEWM (2024)

Lower limit of viscosity!

Top: Model H



Chattopadhyay, Ott, Schaefer, Skokov, PRL (2024)

Out-of-equilibrium noise creates corrections between 1st and 2nd orders

$$\frac{\langle\langle \tau^2 T^{\eta\eta}(\tau)\rangle\rangle}{e+p} = \frac{p}{e+p} - \frac{4\gamma_\eta}{3\tau} + \frac{1.08318}{s\,(4\pi\gamma_\eta\tau)^{3/2}},$$

Akamatsu, AM, Teaney, PRC (2017,2018)

Stochastic hydrodynamics

Singh, Shen, McDonald, Jeon, Gale NPA (2018)

Adding noise to an update, e.g. Brownian particle

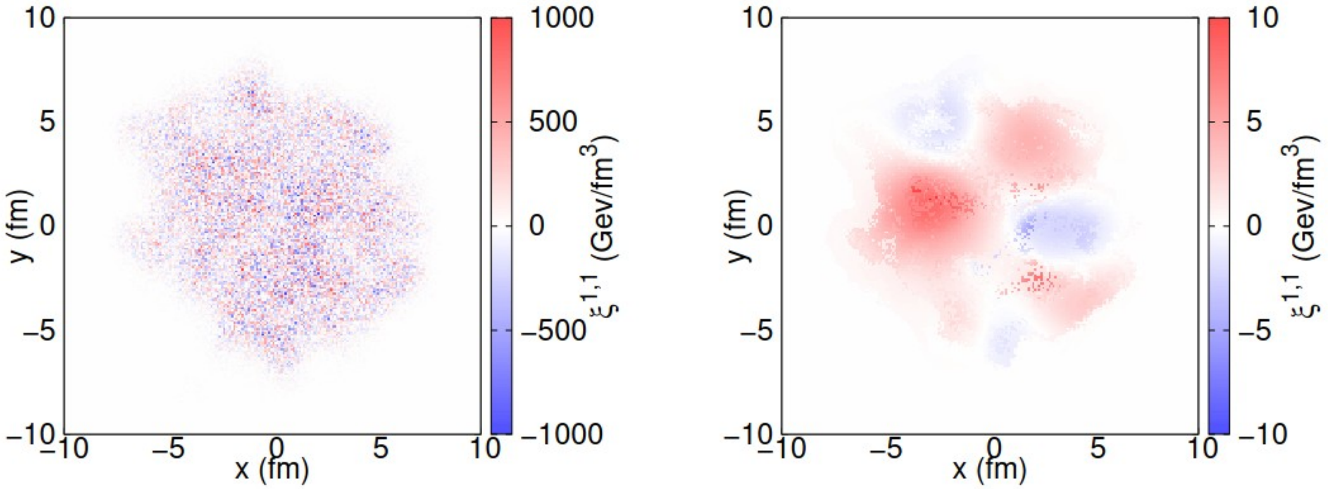
$$\partial_t p = -\gamma p + \xi \qquad \langle \xi\xi \rangle = 2TM\delta(t-t')$$

$$p(t+\Delta t) = p(t) + \Delta t(-\gamma p(t) + \sqrt{2TM\gamma/\Delta t}\theta)$$

Noise contribution increases as $\Delta t \to 0$

Singh, Shen, McDonald, Jeon, Gale NPA (2018)

Need to filter small wavenumbers to do simulations



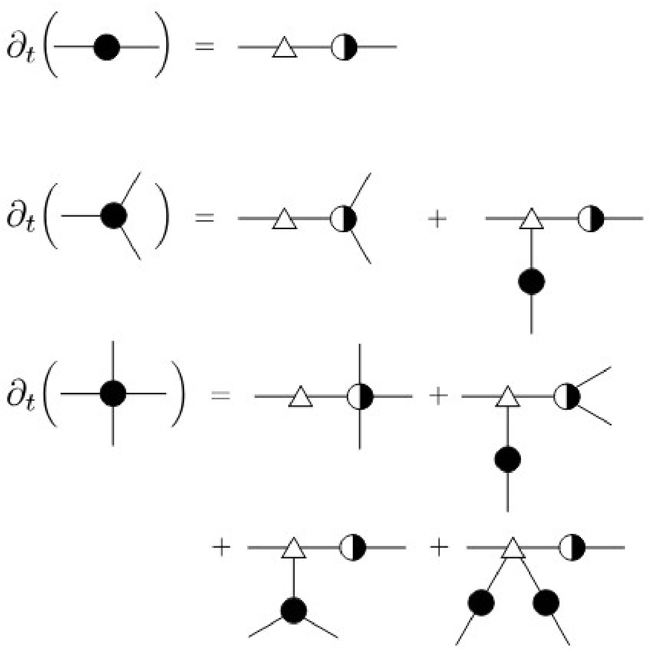
See also Sakai, Murase, Hirano, PRC 2020

Hydro kinetics

Deterministic equations for 2-point correlation functions

$$\partial_t \langle pp \rangle = -2\gamma(\langle pp \rangle - MT)$$

Akamatsu, AM, Teaney, PRC (2017,2018)



Heroic generalization to n-point functions and general frames

An, Basar, Stephanov, Yee, PRL (2021)

So far solutions for simple flows

Akamatsu, AM, Teaney, PRC (2017,2018)

Metropolis algorithm

Başar, Bhambure, Singh, Teaney, 2403.04185

Implement noise with accept-reject algorithm

$$p(t+\Delta t) = p(t) + \sqrt{2TM\gamma\Delta t}\theta \qquad P = \min(1, e^{-\Delta H})$$

dissipation

In one step implement both (on average) dissipation and noise

$$\langle \Delta p \rangle = -\gamma p \Delta t, \quad \langle \Delta p \Delta p \rangle = MT\Delta t^2$$

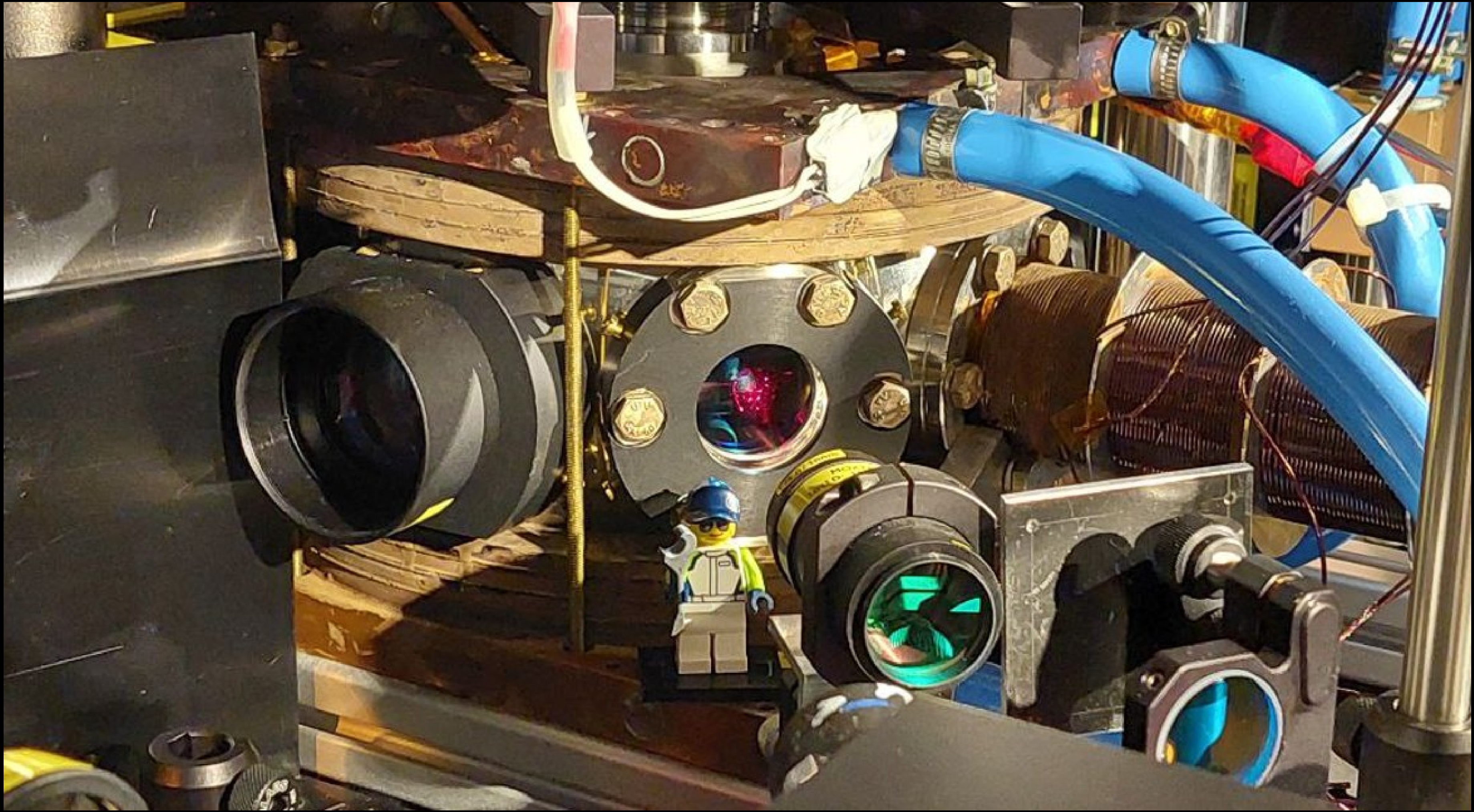
Generalizable to full hydrodynamic with random momentum updates.

Paquet, Bhambure, AM, Singh, Teaney, Zhou (in preparation)

Simulation of fluctuations near critical point (Model G and H)

Florio, Grossi, Soloviev, Teaney, PRD (2022) Chattopadhyay, Ott, Schaefer, Skokov, PRL (2024)

Challenge III: mesoscopic systems

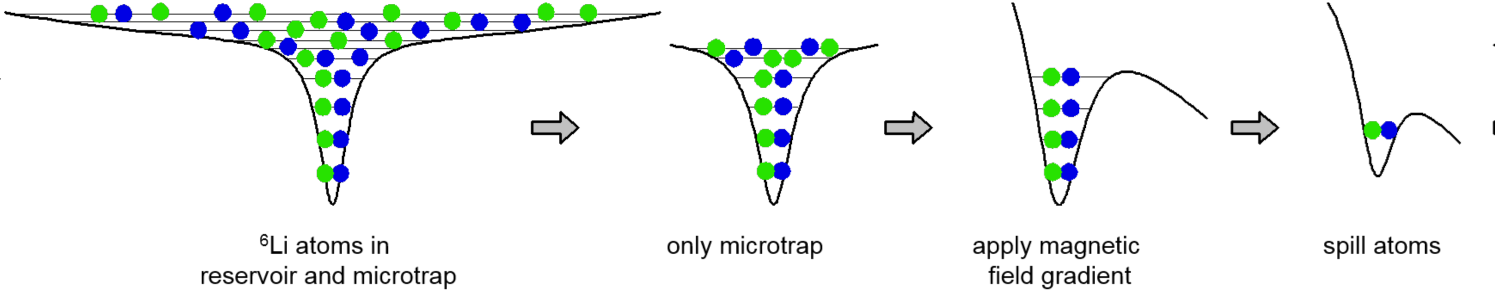


Brandstetter @ Jochim’s lab, Heidelberg University

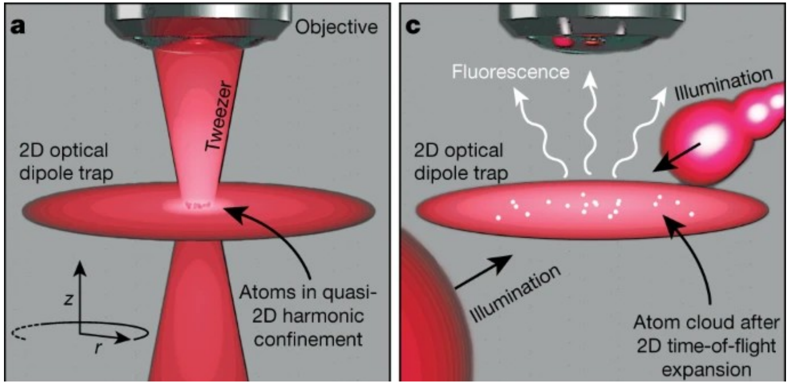
Lithium atoms in 2D harmonic trap

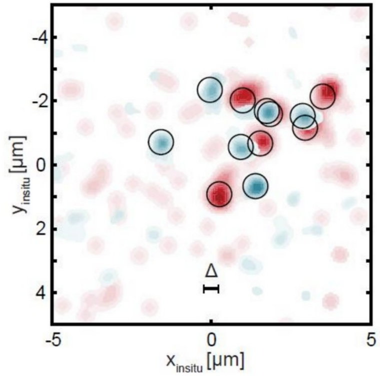
Serwane et al., Science **332** (2011) 6027

Preparation of few atoms in a trap



- time-resolved imaging
- spatial or momentum
- individual atoms
- controllable interactions

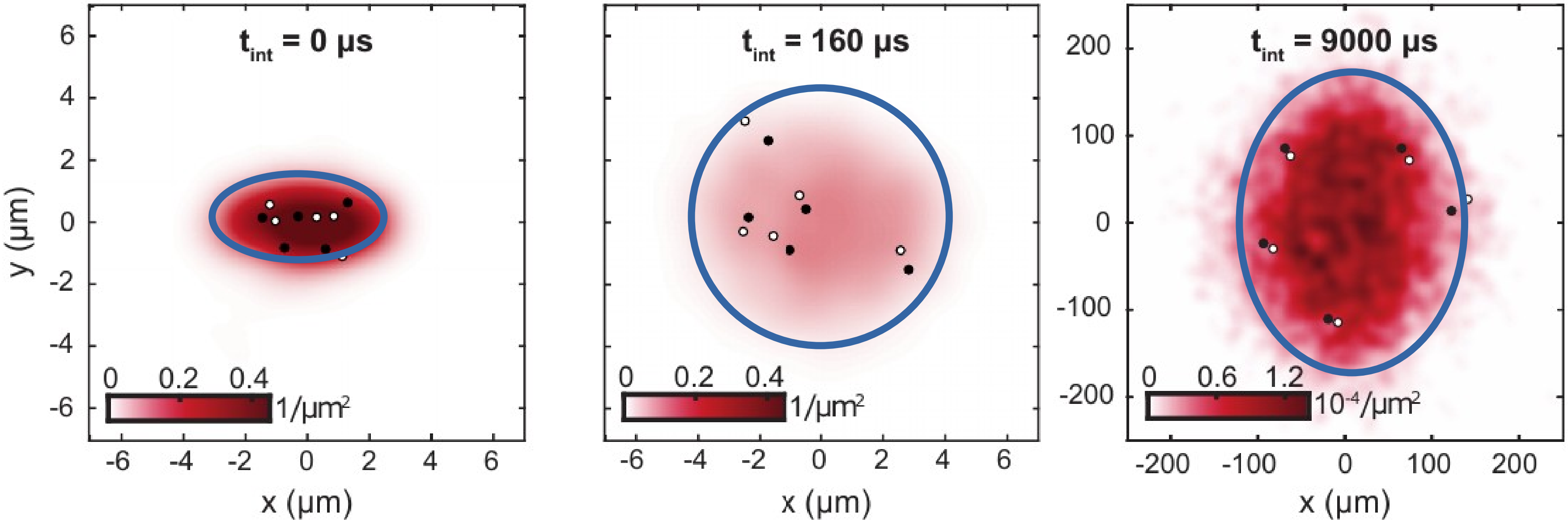




Holten *et al.*, Nature **606**, 287-291 (2022)

Geometry inversion of 10 atoms

Theory proposal: Flörchinger, Giacalone, Heyen, Tharwat, PRC 105 (2022) 4, 044908
Experimenta: Brandstetter, Lunt, Heintze, Giacalone, Heyen, Gałka, Subramanian, Holten, Preiss, Floerchinger, Jochim arXiv:2308.09699



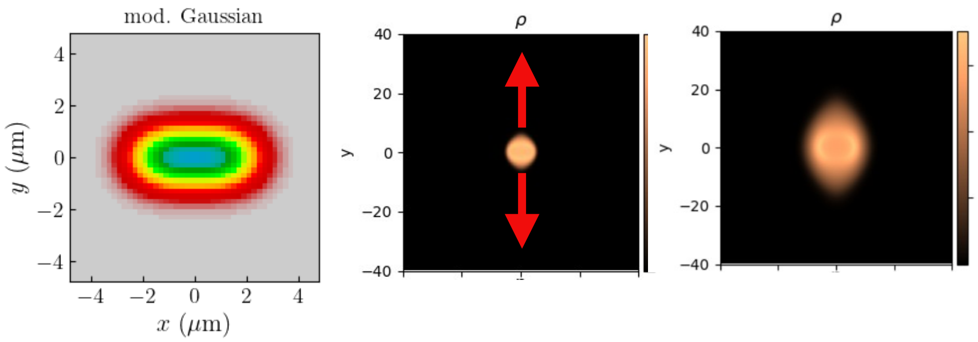
Elliptic flow of 10 Lithium atoms

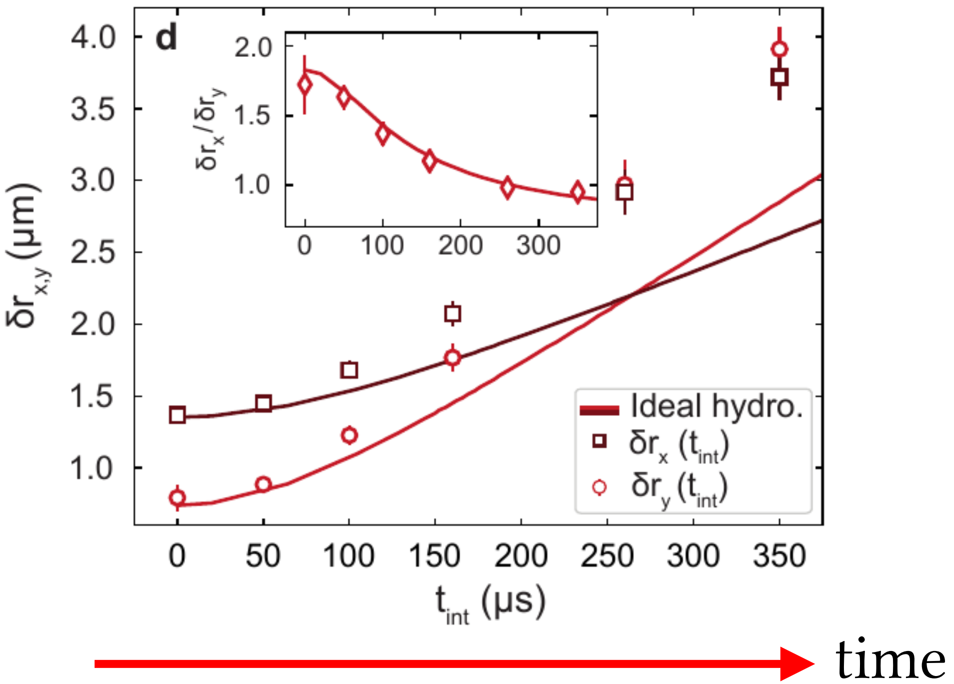
Fluid dynamic description

Brandstetter, Lunt, et al. arXiv:2308.09699

shape vs time

average geometry → expansion → inversion





ideal hydro simulation

initial conditions + equation of state

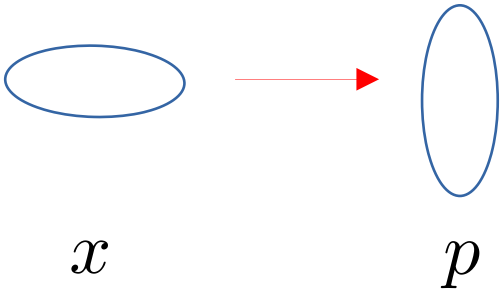
Good hydrodynamic description of 10 atoms!

System size dependence

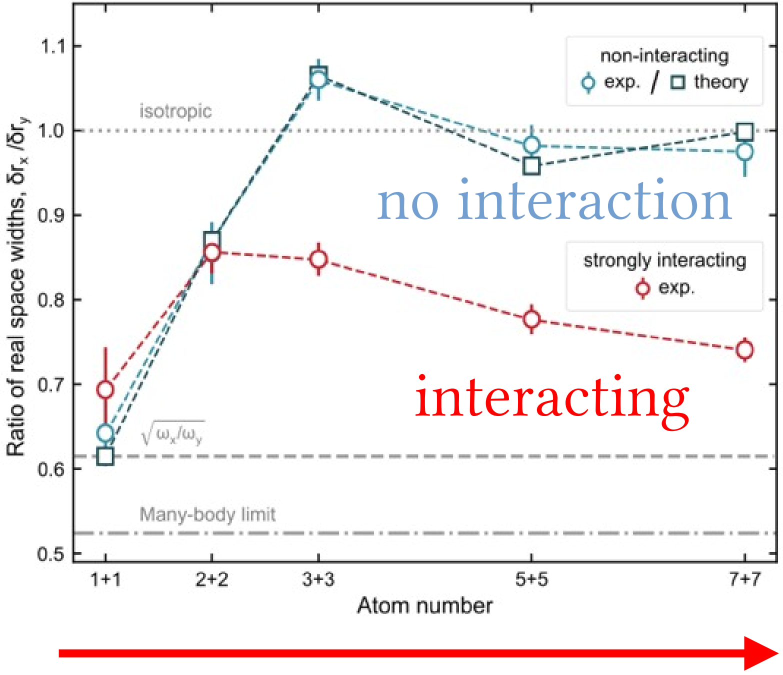
Brandstetter, Lunt, et al. arXiv:2308.09699

$$N = 1$$

$$\Delta x \Delta p \geq \hbar/2$$



final shape vs system size



size

$$N \gg 1$$

round Fermi surface

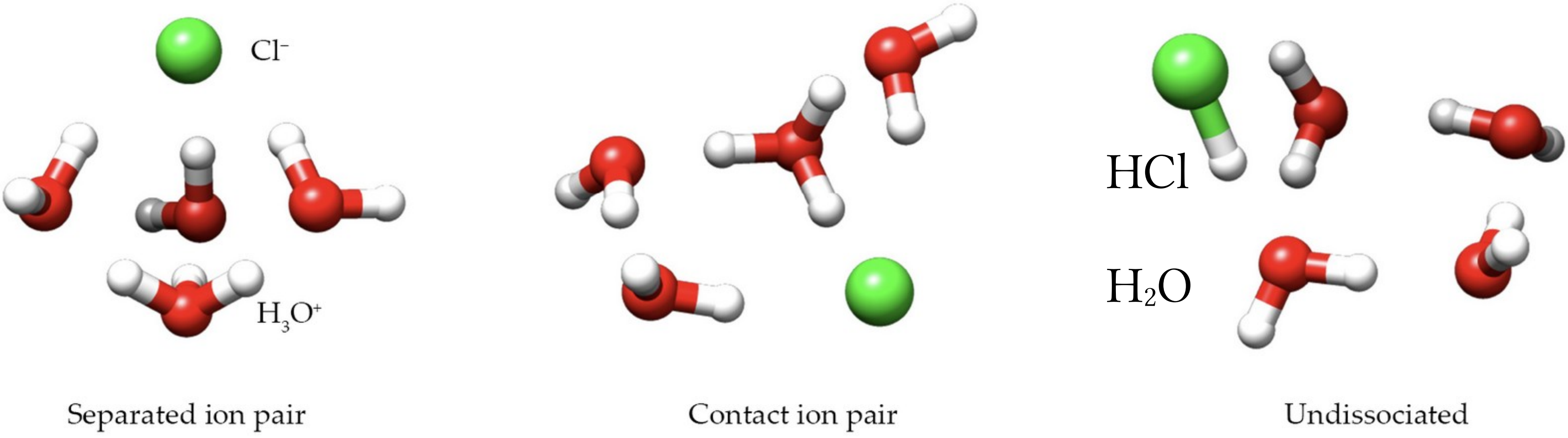
Elliptic flow with > 5 atoms!

Aleksas Mazeliauskas, aleksas.eu

How many H_2O molecules make a droplet?

Miller, Physics Today 77 (8), 10–12 (2024)

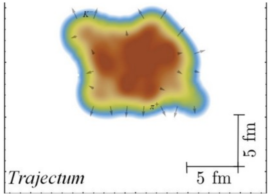
Xie, Tikhonov, Schnell, Science (2024)



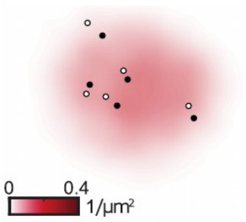
n=5 molecules are enough to dissolve HCl!

Similarities and differences between systems

quark gluon plasma



cold atoms



- Initial conditions: sampling of nucleons/atoms from wavefunction, but no particle production in cold atoms
- Equation of state: many-body equilibrium EOS are used, but cold atoms are at $T=0$.
- Hydrodynamic evolution: show geometry inversion, but no dissipation measured in few-body cold atom systems
- Observables: event-by-event momentum observables in heavy-ions vs ensemble averaged momentum/coordinate observables in cold atoms

Summary and outlook

Summary

QGP passes the "Duck test" as a near perfect fluid in large systems, but there are many challenges to interpretation in small systems

- Matching to microscopic dynamics (not discussed)
  $\longrightarrow$ see talks by Victor and Nicolas
- Systematic uncertainties in transport of hydrodynamics
  $\longrightarrow \eta/s$, but of which hydrodynamics?
- Missing contributions of thermal noise
  $\longrightarrow$ why does average hydrodynamics work so well?
- Sorites paradox
  $\longrightarrow$ what can we learn from mesoscopic cold atom systems?

Postdoc position https://academicjobsonline.org/ajo/jobs/27891