RESUMMATION

Anna Kulesza

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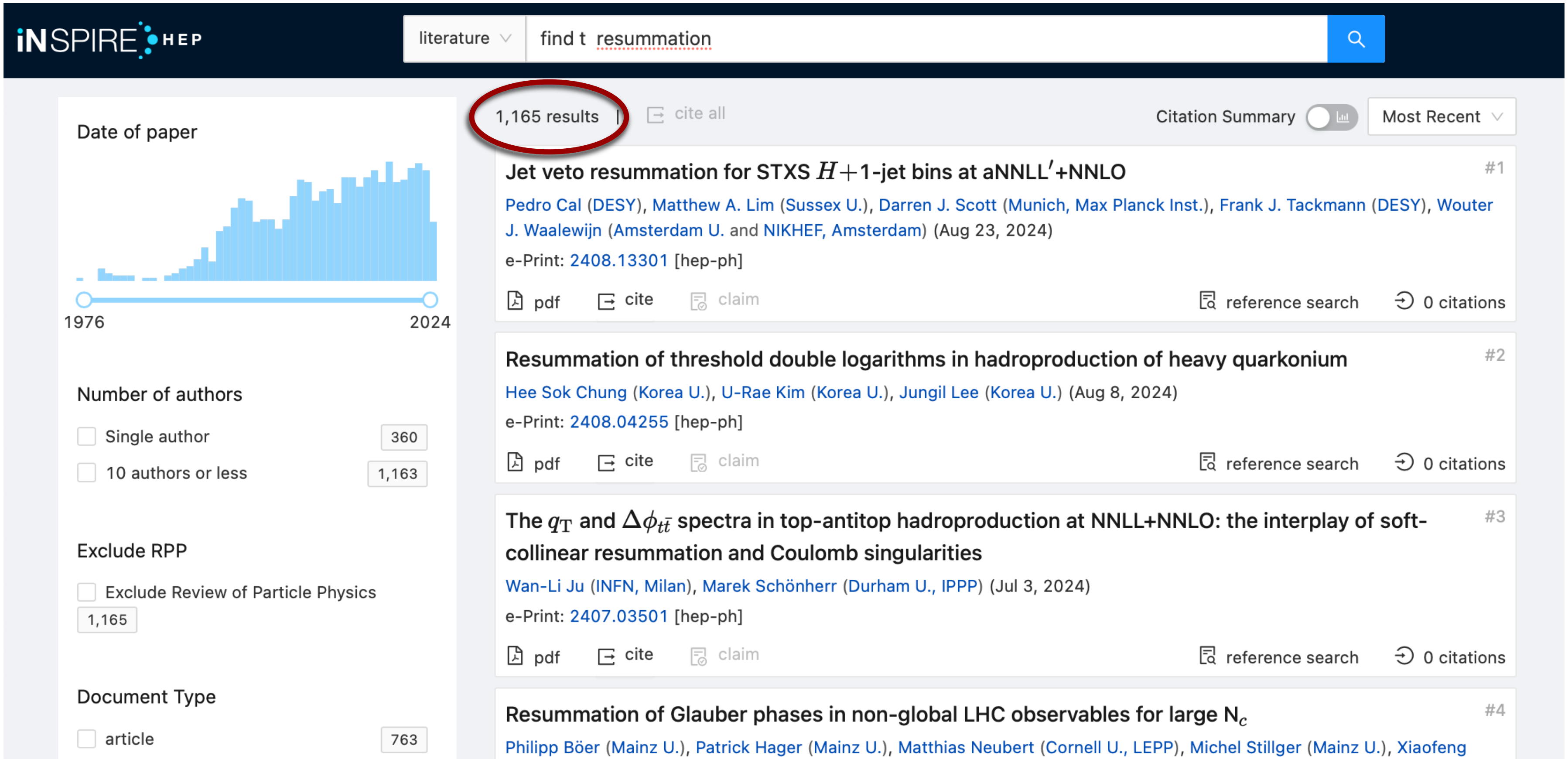
First Look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Jet veto resummation for STXS $H+1$-jet bins at aNNLL$'$+NNLO #1
Pedro Cal (DESY), Matthew A. Lim (Sussex U.), Darren J. Scott (Munich, Max Planck Inst.), Frank J. Tackmann (DESY), Wouter J. Waalewijn (Amsterdam U. and NIKHEF, Amsterdam) (Aug 23, 2024)
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Resummation of threshold double logarithms in hadroproduction of heavy quarkonium #2
Hee Sok Chung (Korea U.), U-Rae Kim (Korea U.), Jungil Lee (Korea U.) (Aug 8, 2024)
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The q_T and $\Delta\phi_{t\bar{t}}$ spectra in top-antitop hadroproduction at NNLL+NNLO: the interplay of soft-collinear resummation and Coulomb singularities #3
Wan-Li Ju (INFN, Milan), Marek Schönherr (Durham U., IPPP) (Jul 3, 2024)
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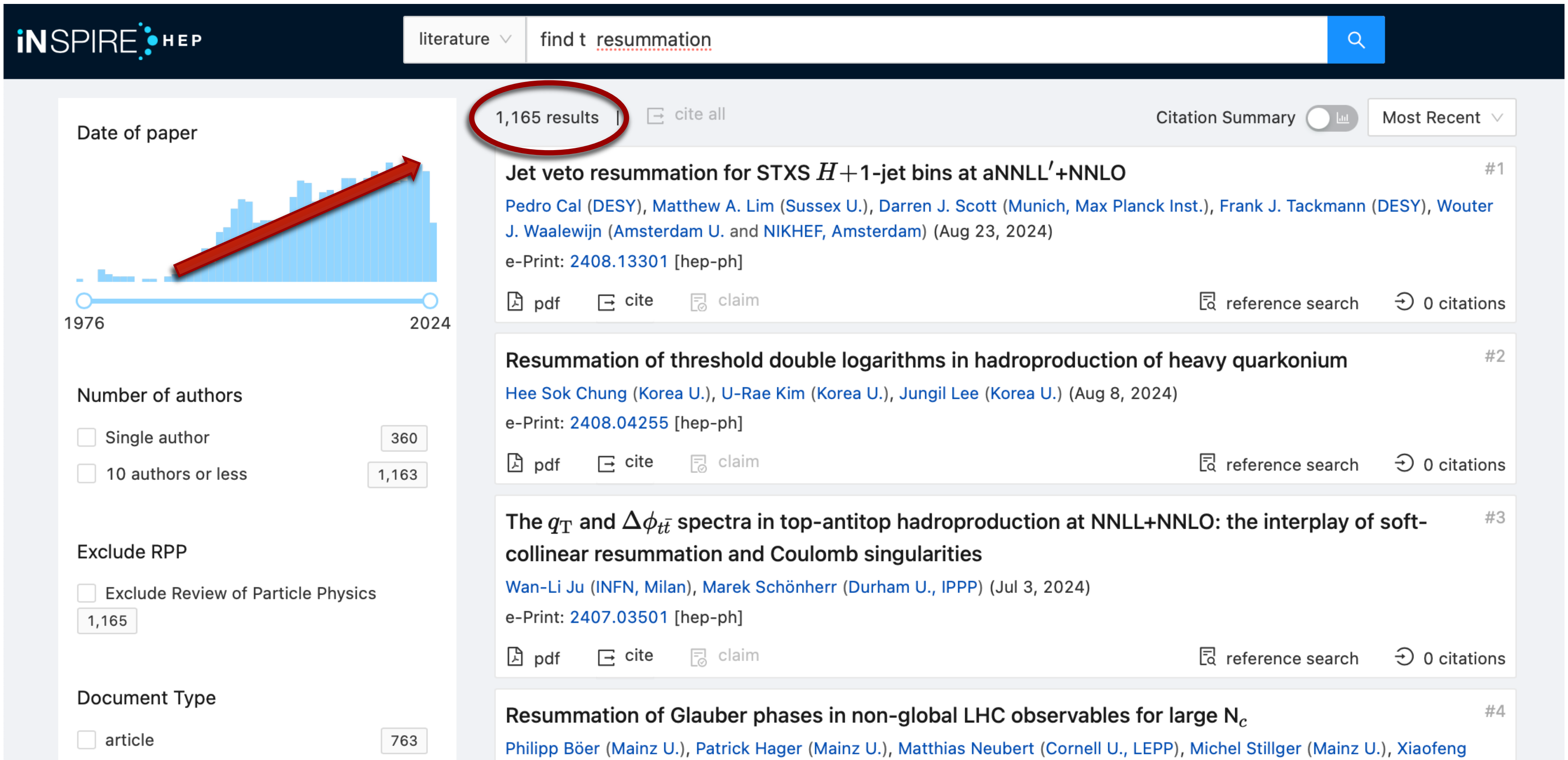
Resummation of Glauber phases in non-global LHC observables for large N_c #4
Philipp Böer (Mainz U.), Patrick Hager (Mainz U.), Matthias Neubert (Cornell U., LEPP), Michel Stillger (Mainz U.), Xiaofeng

First Look



… and that’s just one keyword…

FIRST LOOK CTND.

Disclaimer

- Too little time, too much material…
- Subject can be introduced and discussed in many different ways
- Here only very few aspects can be covered without going too deep into details (apologies for all the omissions!)
- Today's goal: a broad overview of the underlying ideas

Contents

- What is “resummation”?
- Why do we want to resum?
- How can we do it?
- How well can we do it?
- Examples, applications, directions

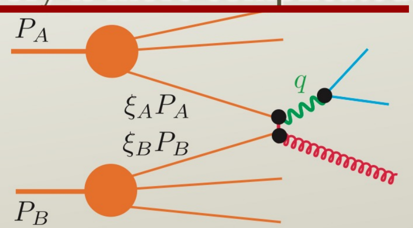
First Sightings,1

- D. Soper's "Basics of QCD perturbation theory" lectures:

Summing logs

- For an infrared-safe process with one hard scale, the theory is simple.
- If there are two hard scales, the theory is more complicated.
- Consider $A + B \to Z + X$

$$\frac{d\sigma}{dP_T\, dy}$$



- If $P_T \sim M_Z$, the theory is simple.
- If $1\ \text{GeV} \ll P_T \ll M_Z$, there are two large scales.
- We need to sum terms of order $\alpha_s^n \log(M_Z/P_T)^{2n-1}$.
- In many cases like this, there are known formulas for summing the logs.

99

First Sightings,1

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Summing logs

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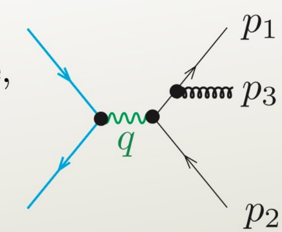
- Since the thrust distribution is infrared safe, we can calculate it in perturbation theory.
- At first order, one finds



$$\frac{d\sigma}{dT} = \left(\frac{4\pi\alpha^2}{Q^2}\sum e_f^2\right)\frac{C_F\alpha_s}{2\pi}$$
$$\times\left[\frac{3(2-T)(2-3T)}{1-T} + \frac{4-6T(1-T)}{T(1-T)}\log\left(\frac{2T-1}{1-T}\right)\right]$$

Note the log.

- $Q^2 = q^2 = (p_1 + p_2 + p_3)^2$.
- $C_F = 4/3$.
- $T \to 1$ corresponds to two narrow jets.
- $d\sigma/dT$ is singular in this limit.

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First Sightings, 2

- S. Marzani's "Jets" lectures:

jet properties

- we want to studies the properties of jets
- hence, we resolve a (high p_t) jet down to a smaller scale, e.g. its mass
- large logarithms appear invalidating the fixed-order expansion
- we need to reorganise the calculation so that we can consider any number of soft/collinear partons: resummation
- vast field with many approaches: dQCD, SCET, etc.

FIRST SIGHTINGS, 2

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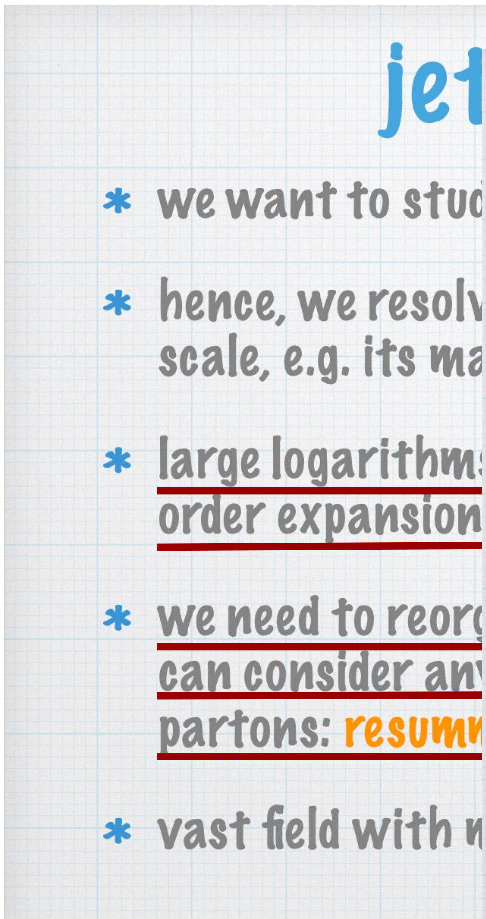

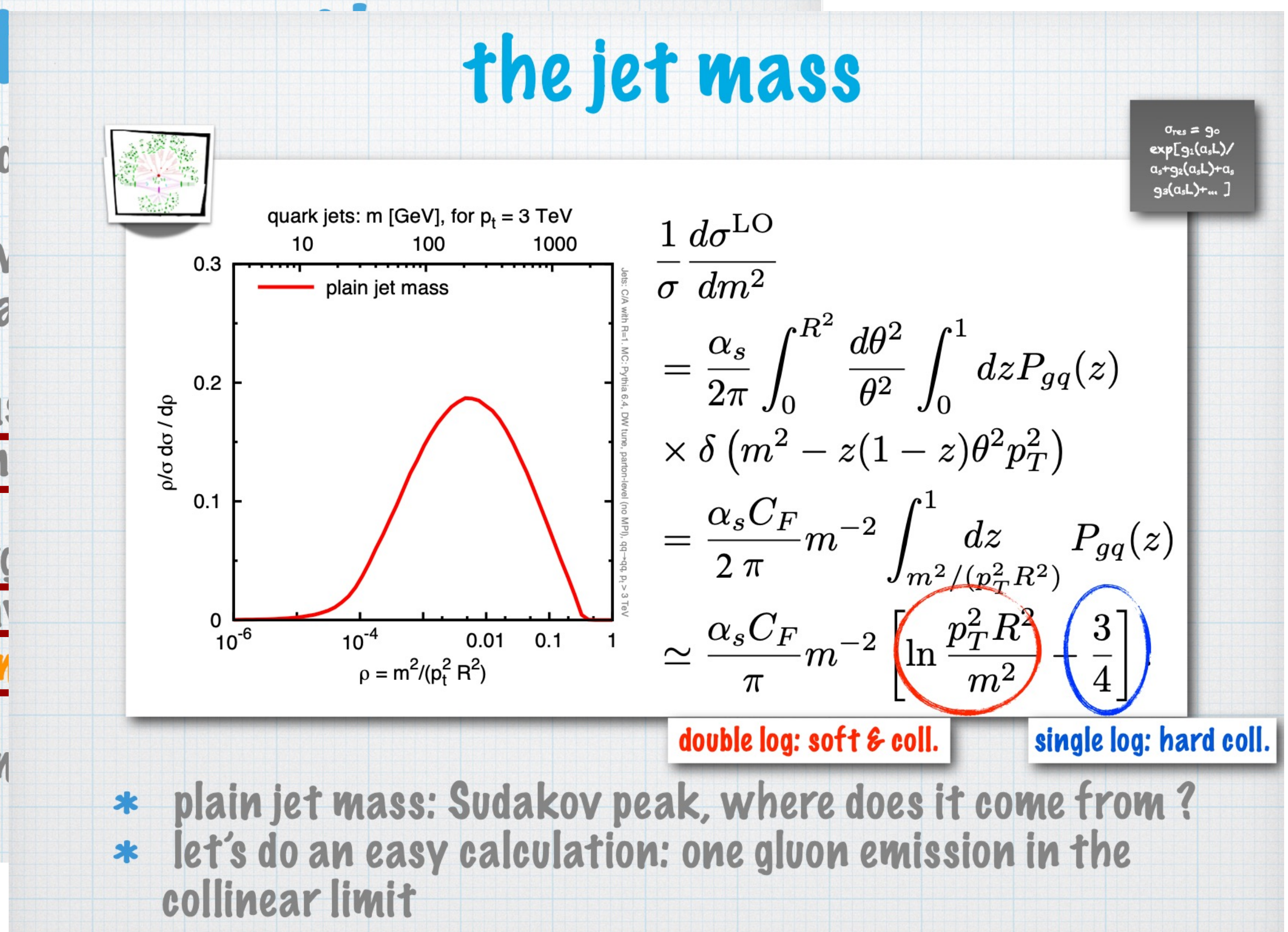

What is Resummation?

- Loosely speaking, resummation means reorganization of the perturbative series in such a way that the subsets of the dominant logarithmic terms are systematically grouped together to all orders.
  - The form of this expression is specific to a given problem and only valid in the region where the logs are large.

- Various types of logarithms:
  - Renormalization and factorization logs: $\alpha_s^n \log^n (Q^2/\mu^2)$ -> running of α_s, DGLAP
  - High energy, small-x logs: $\alpha_s^n \log^{n-1} (s/t)$ -> BFKL equation
  - Sudakov logs: $\alpha_s^n \log^{2n-1} (\zeta)$
    - threshold logs $\zeta=1-z$, $z=M^2/s$, $z=Q^2/s$, …
    - recoil logs $\zeta=p_T^2/Q^2$
    - thrust $\zeta=1-T$

The Goal

Systematic reorganization of perturbative series

$$\hat{\sigma} \sim c_{00} +$$

$$+\alpha_s \left(c_{12} \log^2 (\beta^2) + c_{11} \log (\beta^2) + c_{10} \right) \leftarrow \text{NLO}$$

$$+\alpha_s^2 \left(c_{24} \log^4 (\beta^2) + c_{23} \log^3 (\beta^2) + c_{22} \log^2 (\beta^2) + \ldots \right) \leftarrow \text{NNLO}$$

$$\ldots \qquad \ldots \qquad \ldots$$

↑ $\alpha_s^n \log^{2n}(\beta^2)$ ↑ $\alpha_s^n \log^{2n-1}(\beta^2)$

$\log(\beta^2) \leftrightarrow \log(N) \equiv L$

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$$\ldots \qquad \ldots \qquad \ldots$$

$\uparrow$ $\alpha_s^n \log^{2n}(\beta^2)$ $\uparrow$ $\alpha_s^n \log^{2n-1}(\beta^2)$

$\log(\beta^2) \leftrightarrow \log(N) \equiv L$

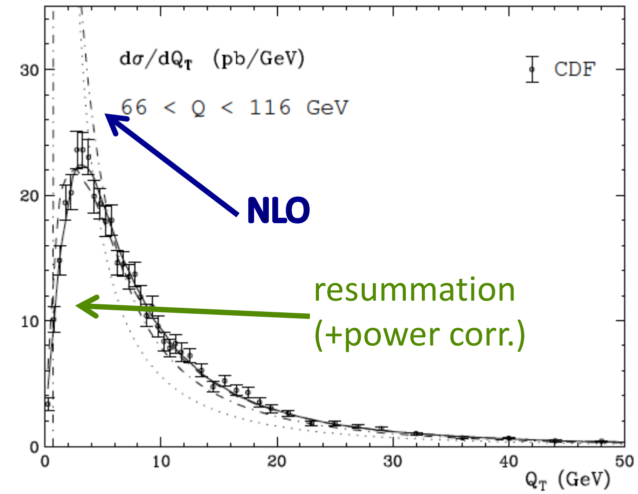
resummation: often in a conjugated space, e.g. a space of Melin moments N, taken wrt. $1-\beta^2$

$$\hat{\sigma}^{(N)} \sim \mathcal{C}(\alpha_s) \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right]$$

sums up LL: $\alpha_s^n \log^{n+1}(N)$ NLL: $\alpha_s^n \log^{n}(N)$ ...

BENEFITS

- Resummation extends accuracy of the perturbative prediction beyond fixed-order by adding a systematic treatment of the logarithmic contributions to all orders
  - restoration of predictive power
  - better description of observables -> reduction of the theoretical error (and we need precision for the LHC!)
  - probe of the all-order structure of the perturbation theory



Z boson p_T @Tevatron from *[AK, Sterman, Vogelsang'03]*

from *[Moch, Vermaseren, Vogt'05]*

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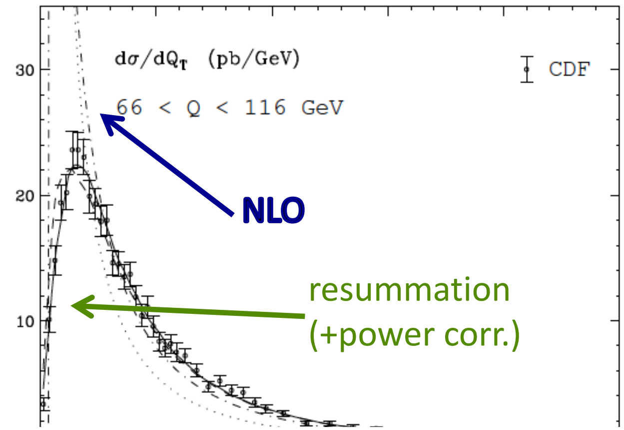
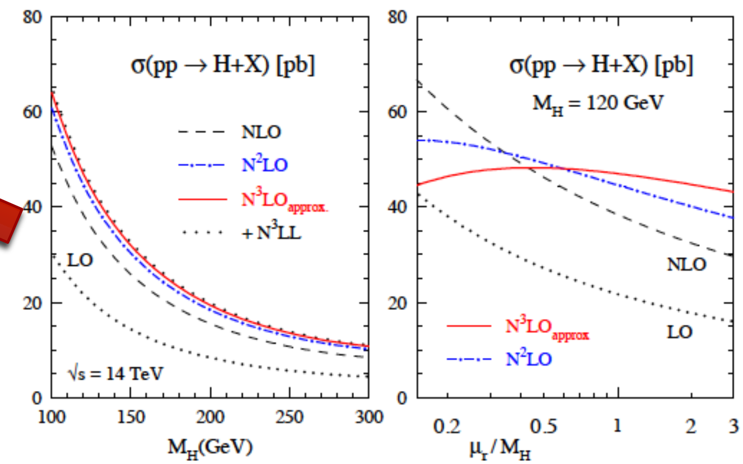
  - by-product: approximation of the fixed-order result from expansion of the all-order resummation





from *[Moch, Vermaseren, Vogt'05]*

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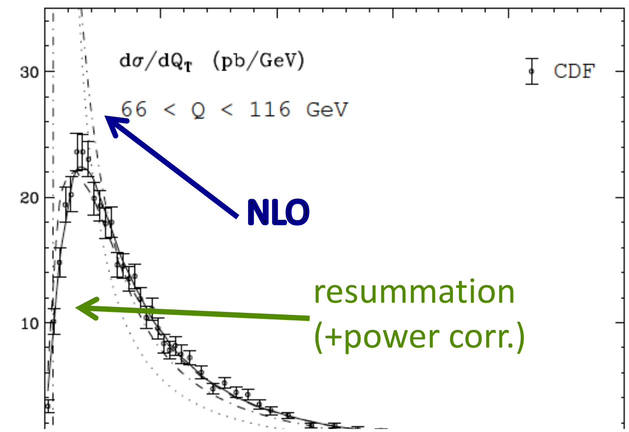

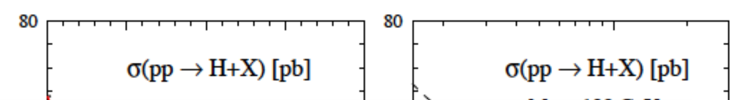

Roughly speaking, fixed-order and resummed calculations are valid in different kinematical regimes (e.g. large p_T vs. small p_T). Therefore, to take benefit of both they should be matched

$$d\sigma_{matched} = d\sigma_{fixed-order} + (d\sigma_{res} - d\sigma_{res\,|\,expanded\ up\ to\ the\ same\ order})$$

resulting in NLO+NLL, NNLO+NNLL etc.

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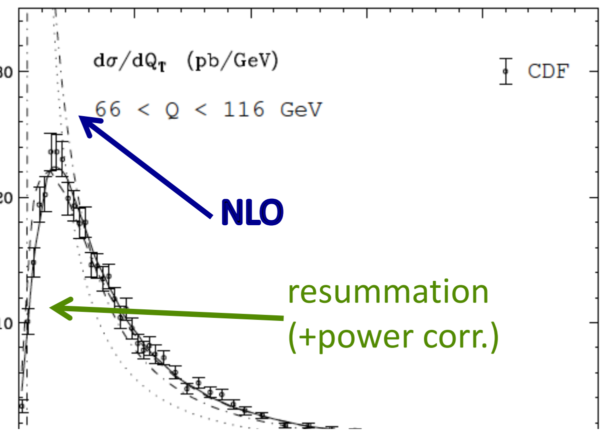

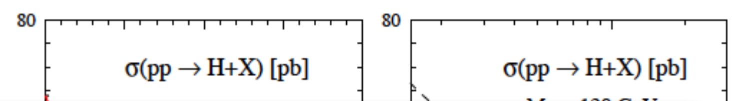

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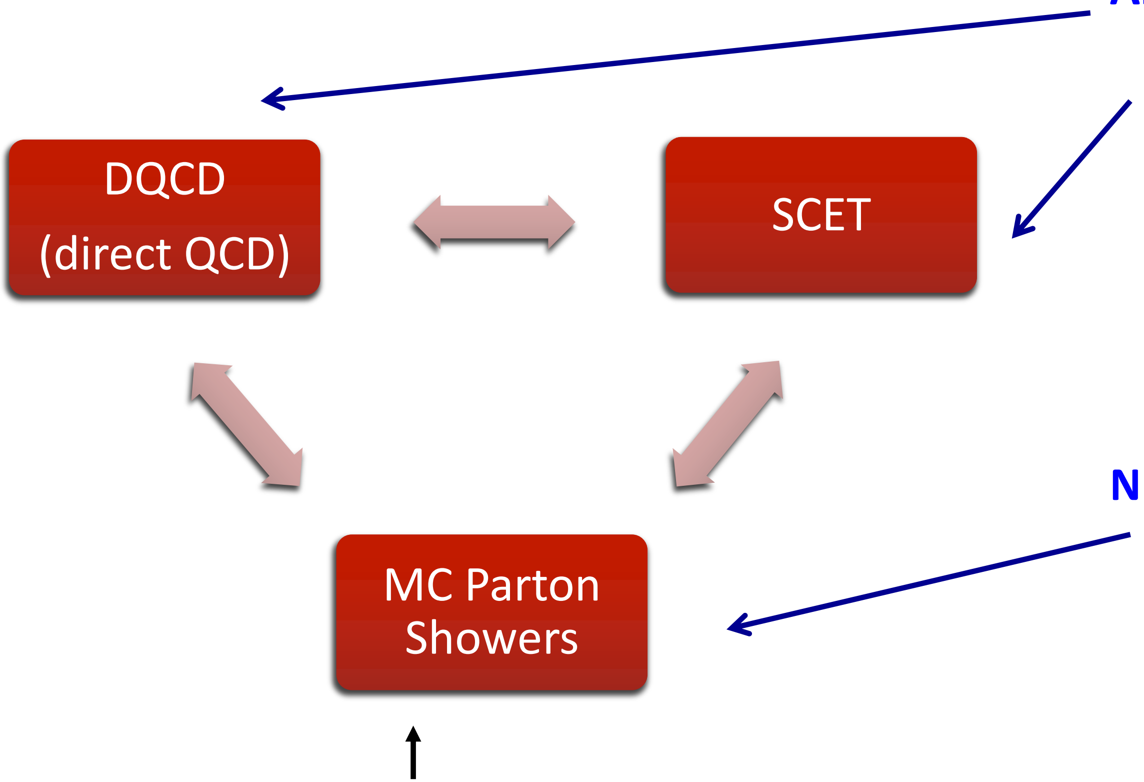
but how to get there?

from [Moch, Vermaseren, Vogt 05]

TAKE YOUR PICK



see lectures by D. Soper and R. Frederix

Analytical methods

- State of the art accuracy from N^4LL processes to NNLL
- Full control over theoretical accuracy
- Separate calculations for each process and observable

Numerical

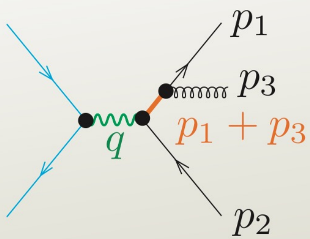
- Automatized tools
- General-purpose and fully differential
- Accuracy (N(N))LL
- Improving accuracy is currently a very active field

Methods

- Historically, various approaches (or "schools") have been formed:
  - based on factorization of matrix elements and cross section *[Catani, Trentadue, Parisi, Petronzio et al.] [Dokshitzer et al.]* "bottom-top"
  - based on renormalization group *[Collins, Soper, Sterman et al.]* "top-bottom"
  - based on the effective field theory approach -> *Soft Collinear Effective Theory (SCET)* dominant mode selection at the Lagrangian level
- Different at technical level, but equally valid
- Independent methods to arrive at the same formal accuracy -> validation of theoretical predictions obtained with various methods
- After all, physics is the same!

Back to e+e-

General nature of the singularities



- $\mathcal{M}$ contains a factor $1/(p_1 + p_3)^2$.
  $(p_1 + p_3)^2 = 2E_1 E_3 (1 - \cos\theta_{13})$
- This is singular for $\theta_{13} \to 0$ and for $E_3 \to 0$.
- The numerator has a factor θ_{13} for small θ_{13}.
- So

$$|\mathcal{M}|^2 \propto \frac{1}{E_3^2 \theta_{13}^2}, \qquad \theta_{13} \to 0 \text{ or } E_3 \to 0$$

- This gives logarithmically divergent integrals

$$\int d\sigma \sim \int \frac{d^3\vec{p}_3}{E_3} \frac{1}{E_3^2 \theta_{13}^2}$$
$$= \int \frac{E_3^2 dE_3 \, d\cos\theta_{13} \, d\phi}{E_3} \frac{1}{E_3^2 \theta_{13}^2}$$
$$\sim \int \frac{dE_3}{E_3} \frac{d\theta_{13}^2}{\theta_{13}^2} d\phi$$

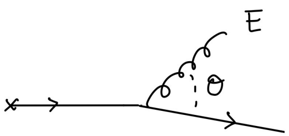
soft

collinear

- IR singularities cancel between real and virtual corrections (KLN theorem) for IR-safe observables
- NLO total cross section: $\sigma_r + \sigma_v = \sigma_0 \left(1 + \frac{\alpha_s}{\pi}\right)$
- However, if phase-space for real emission restricted e.g. by the measurement being exclusive, (double) logarithmic dependence on the phase-space boundary will appear!

Origin of Logs

- Consider a constrain on the phase-space of real emission in the form $E\theta^2 < M$



$$\sigma_r \sim \frac{\alpha_s}{\pi} C_F \int \frac{dE}{E} \int \frac{d\theta^2}{\theta^2}\, \Theta(M - E\theta^2) \qquad \sigma_v \sim -\frac{\alpha_s}{\pi} C_F \int \frac{dE}{E} \int \frac{d\theta^2}{\theta^2}$$

$$\sigma_r + \sigma_v \sim -\frac{\alpha_s}{\pi} C_F \int \frac{dE}{E} \int \frac{d\theta^2}{\theta^2}\, \Theta(E\theta^2 - M) \sim -\frac{\alpha_s}{\pi} C_F \int_M^Q \frac{dE}{E} \int_{M/E}^1 \frac{d\theta^2}{\theta^2} = -\frac{\alpha_s}{2\pi} C_F \log^2\left(\frac{Q}{M}\right)$$

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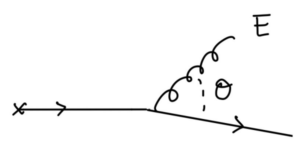

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gets large for *M<<Q*, i.e. when emission forced to be soft and collinear!

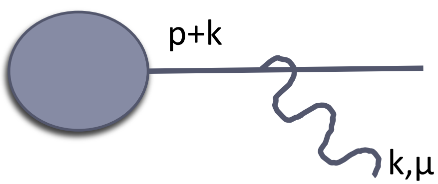
- If restriction on phase-space, real emission inhibited
  - cancellation between real and virtual correction unbalanced
  - finite logarithmic remnants which can get large!
- Same effect, appearing in many disguises, depending on the process, observable etc.
- What about higher-orders?

Factorization of Multiple Emissions

- Factorization of soft and/or collinear emissions is the underlying principle behind resummation
- Intuitive: short distance incoherent with long distance dynamics, long-wavelength soft gluons "insensitive" to short wavelength physics
- Factorization of amplitudes and cross-sections and the exponentiation of single emission probabilities -> "bottom-top"
- All-order factorization separating dynamics at various energy scales (RG approach in dQCD, SCET) "top-bottom"

Eikonal Approximation in QED

- One photon emission in the soft $k^\mu \to 0$ limit



$$M\frac{i(\not{p}+\not{k})}{2p\cdot k+k^2}(-ie\gamma^\mu)\bar{u}(p)\rightarrow eM\frac{p^\mu}{p\cdot k}\bar{u}(p)$$

Eikonal approximation
$k^\mu << p^\mu$
emitting particle keeps
its 4-momenta ➞
"straight line"

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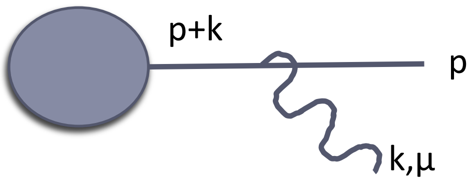

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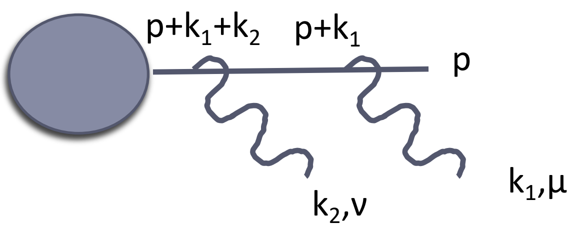
$k^\mu \ll p^\mu$
emitting particle keeps its 4-momenta → "straight line"

- Emission of two soft photons



$$\frac{p^\mu}{p\cdot k_1}\frac{p^\nu}{p\cdot(k_1+k_2)}+(k_1\leftrightarrow k_2)=\frac{p^\mu p^\nu}{p\cdot(k_1+k_2)}\left(\frac{1}{p\cdot k_1}+\frac{1}{p\cdot k_2}\right)=\frac{p^\mu}{p\cdot k_1}\frac{p^\nu}{p\cdot k_2}$$

eikonal identity

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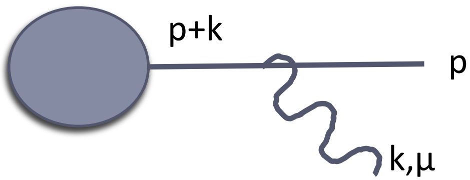

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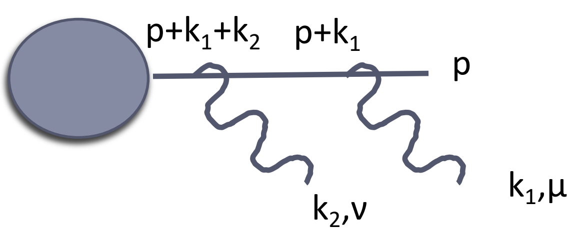
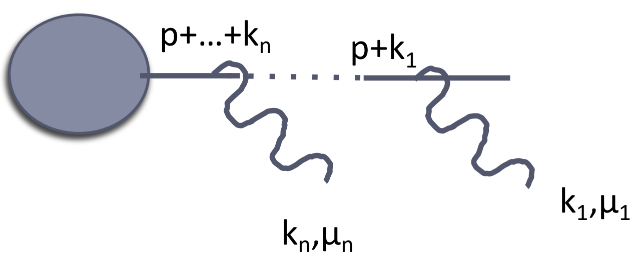


$$\frac{p^\mu}{p\cdot k_1}\frac{p^\nu}{p\cdot(k_1+k_2)}+(k_1\leftrightarrow k_2)=\frac{p^\mu p^\nu}{p\cdot(k_1+k_2)}\left(\frac{1}{p\cdot k_1}+\frac{1}{p\cdot k_2}\right)=\frac{p^\mu}{p\cdot k_1}\frac{p^\nu}{p\cdot k_2}$$

eikonal identity

- Multiple soft photon emission



$$\prod_i \frac{p^{\mu_i}}{p\cdot k_i}$$

Independent, uncorrelated emissions
Factorization at the matrix element level

"Bottom-Top" (1)

Generic hard scattering process:

$$\hat{\sigma}(z) \sim \hat{\sigma}_0 \left[1 + \sum_{n=1}^{\infty} \int_0^1 dz_1 ... dz_n \frac{d\omega_n(z_1,...,z_n)}{dz_1 ... dz_n} \Theta_{PS}^{(n)}(z, z_1, ..., z_n)\right]$$

- In QED, multiple emissions uncorrelated, $\frac{d\omega_n(z_1,...,z_n)}{dz_1...dz_n} = \frac{1}{n!}\prod_{i=1}^{n} \frac{d\omega(z_i)}{dz_i}$
- In QCD need to account for colour charges and non-abelian nature

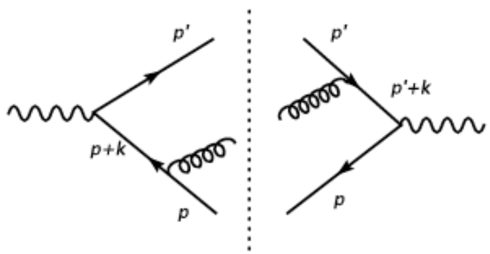
$$M_{eik}(\{p_i\}) = M_{Born}\{p_i\} J^{\mu}\varepsilon_{\mu}$$

$$J^{\mu} = \sum_i e_i \frac{p_i^{\mu}}{p_i \cdot k}$$

eikonal current in QED

$$J^{\mu} = \sum_i g_s T_i \frac{p_i^{\mu}}{p_i \cdot k}$$

in QCD



$$|M_{eik}|^2 = g_s^2 |M_{Born}|^2 J^{\mu}J^{\nu}(-g_{\mu\nu}) = -g_s^2 |M_{Born}|^2 2C_F \frac{p\cdot p'}{(p\cdot k)(p'\cdot k)}$$

In general, for n+1 matrix element:

$$|M_{n+1}|^2 = -g_s^2|M_{Born}|^2 \frac{p\cdot p'}{(p\cdot k)(p'\cdot k)} \langle M_n|T_i \cdot T_j|M_n\rangle$$

- Nevertheless, factorization can be achieved in a more complicated form

"Bottom-Top" (2)

Generic hard scattering process:

$$\hat{\sigma}(z) \sim \hat{\sigma}_0 \left[1 + \sum_{n=1}^{\infty} \int_0^1 dz_1 ... dz_n \frac{d\omega_n(z_1,...,z_n)}{dz_1...dz_n} \Theta_{PS}^{(n)}(z, z_1, ..., z_n)\right]$$

- phase-space factorization $\Theta_{PS}^{(n)}(z, z_1, ..., z_n) = \prod_{i=1}^{n} \Theta_{PS}(z, z_i)$

depends on the process: Θ_{PS} contains kinematical constraints, not always factorizable!

- In practice, phase-space factorization often occurs in the space conjugate to the space of kinematic variables
  - Threshold resummation, Mellin transform

$$\delta\left(1 - z - \sum_i z_i\right) = \frac{1}{2\pi i} \int_C dN e^{-N(1-z-\sum_i z_i)} \qquad \ln(1-z) \leftrightarrow \ln N$$

  - Transverse momentum p_T resummation, Fourier transform

$$\delta\left(\mathbf{p_T} - \sum_i \mathbf{k_T^i}\right) = \frac{1}{2\pi^2} \int d^2 b e^{i\mathbf{b}(\mathbf{p_T} - \sum_i \mathbf{k_T^i})} \qquad \ln(Q^2/p_T^2) \leftrightarrow \ln(Q^2 b^2)$$

"Bottom-Top" (3)

Generic hard scattering process:

$$\hat{\sigma}(z) \sim \hat{\sigma}_0 \left[1 + \sum_{n=1}^{\infty} \int_0^1 dz_1 ... dz_n \frac{d\omega_n(z_1, ..., z_n)}{dz_1 ... dz_n} \Theta_{PS}^{(n)}(z, z_1, ..., z_n)\right]$$

- If the amplitude squared and the phase-space factorize

$$\frac{d\omega_n(z_1, ..., z_n)}{dz_1 ... dz_n} = \frac{1}{n!} \prod_{i=1}^{n} \frac{d\omega(z_i)}{dz_i} \qquad \Theta_{PS}^{(n)}(z, z_1, ..., z_n) = \prod_{i=1}^{n} \Theta_{PS}(z, z_i)$$

$$\begin{aligned} \hat{\sigma}(z) &\sim \hat{\sigma}_0 \left\{ 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\int_0^1 dz_i \frac{d\omega(z_i)}{dz_i} \Theta_{PS}(z, z_i) \right]^n \right\} \\ &\sim \hat{\sigma}_0 \exp \left[\int_z^1 dz' \frac{d\omega(z')}{dz'} \Theta_{PS}(z, z') \right] \sim \hat{\sigma}_0 \exp \left[\alpha_S L^2 + \ldots \right] \end{aligned}$$

exponentiation of lowest-order soft corrections

“Top-bottom” Resummation

- Resummation of logarithms of ratios of the scales follows from factorization. In fact all factorizations separating dynamics at different scales result in resummed expressions through RG equation

- Renormalization relations can be seen as factorization of the UV cut-off dependence Λ

$$G_{\text{bare}}(\Lambda, p, g_{\text{bare}}) = Z\left(\Lambda/\mu, g_R(\mu)\right) G_R(\mu, p, g_R(\mu))$$

with μ playing now a role of the factorization scale. From $\frac{dG_{bare}}{d\mu} = 0$ follows the evolution

equation $$\mu \frac{d}{d\mu} \log G_R(\mu, p, g_R(\mu)) = -\mu \frac{d}{d\mu} \log Z(\Lambda/\mu, g_R(\mu)) \equiv \gamma(g_R(\mu))$$

which can be solved

Resummation from Factorization

see E. Laenen, in Pramana 65(2004)1225

- Single log resummation example: moments of the deep inelastic proton structure function factorize as

$$F_{2,P}(N,Q) \equiv \int_0^1 \mathrm{d}x\, x^{N-1} F_{2,P}(x,Q) = C_q(N,Q/\mu)\,\phi_{q/P}(N,\mu)$$

where C_q are IR safe coefficient functions and $\phi_{q/P}$ is the quark distribution function. Since

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\ln F_{2,P}(N,Q) = 0$$

then

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\ln\phi_{q/P}(N,\alpha_s(\mu)) = -\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_q(N,Q/\mu,\alpha_s(\mu)) \equiv \gamma_q(N,\alpha_s(\mu))$$

anomalous dimension

and

$$\gamma_q(N,\alpha_s(\mu)) \simeq \alpha_s(Q)\gamma_q^{(1)}(N)$$

$$\phi_{q/P}(N,Q) = \phi_{q/P}(N,Q_0)\exp\left[\int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_q(N,\alpha_s(\mu))\right] = \phi_{q/P}(N,Q_0)\exp\left[\alpha_s(Q)\gamma_q^{(1)}(N)\ln(Q/Q_0)\right]$$

$$= \left(\frac{Q}{Q_0}\right)^{\alpha_s(Q)\gamma_q^{(1)}}$$

well behaved

Resummation of single logarithms!

Resummation of double logarithms requires additional considerations of dependence on the second variable (gauge vector) via *log(*$p_i \cdot n$*)*

“Top-bottom” Resummation

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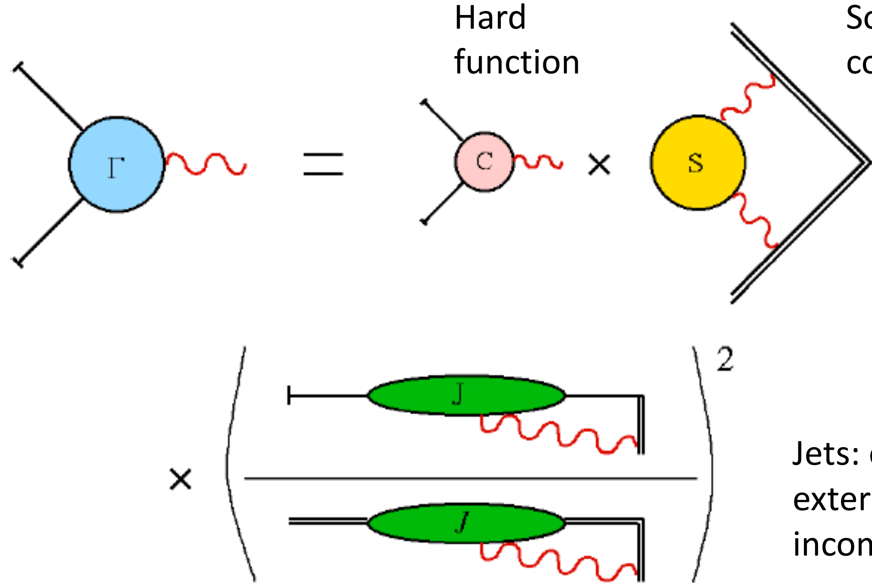
  which can be solved

- Proving factorization is highly non-trivial: requires all-order diagrammatic studies, pioneered in the soft-collinear case by Collins, Soper and Sterman

- Particular variants of the factorized expression from which a corresponding resummed formula is derived are obtained by keeping the appropriate variable (e.g. transverse momentum, energy fraction) fixed

Upshot: "Top-bottom" dQCD

Form factor



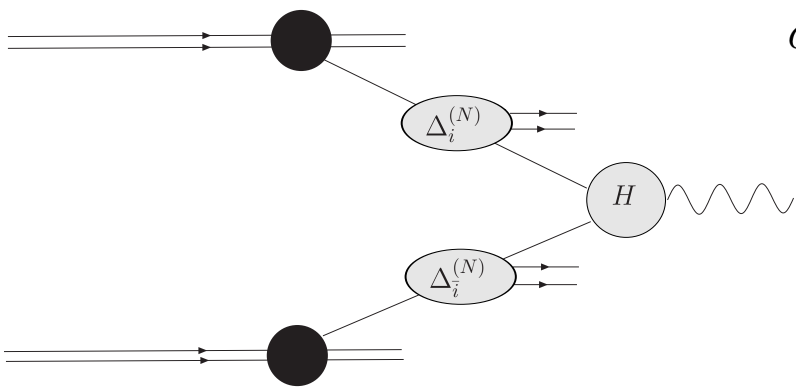
from 0805.3515

$$\Gamma\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) = C\left(\frac{Q^2}{\mu^2},\frac{(p_i\cdot n_i)^2}{n_i^2\mu^2},\alpha_s(\mu^2),\epsilon\right)\times\mathcal{S}\left(\beta_1\cdot\beta_2,\alpha_s(\mu^2),\epsilon\right)$$
$$\times\prod_{i=1}^{2}\left[\frac{J\left(\frac{(p_i\cdot n_i)^2}{n_i^2\mu^2},\alpha_s(\mu^2),\epsilon\right)}{\mathcal{J}\left(\frac{(\beta_i\cdot n_i)^2}{n_i^2},\alpha_s(\mu^2),\epsilon\right)}\right].$$

Threshold Resummed Cross Sections

Schematically, for colour singlet production

[Catani, Trentadue'89][Sterman'87]



$$\hat{\sigma}_{i\bar{i}}^{(N)} = \underbrace{H_{i\bar{i}}^{(N)}}_{\text{hard function}} \times \underbrace{\Delta_i^{(N)}\Delta_j^{(N)}}_{\text{soft-collinear radiation}}$$

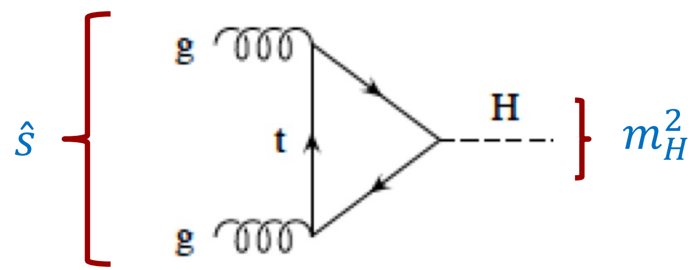
universal factors; KNOWN

exponential functions

$$\ln\left(\Delta_i^{(N)}\Delta_{\bar{i}}^{(N)}\right) = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\int_{\mu_F^2}^{(1-z)^2Q^2}\frac{d\mu^2}{\mu^2}A_i(\alpha_S(\mu^2))$$

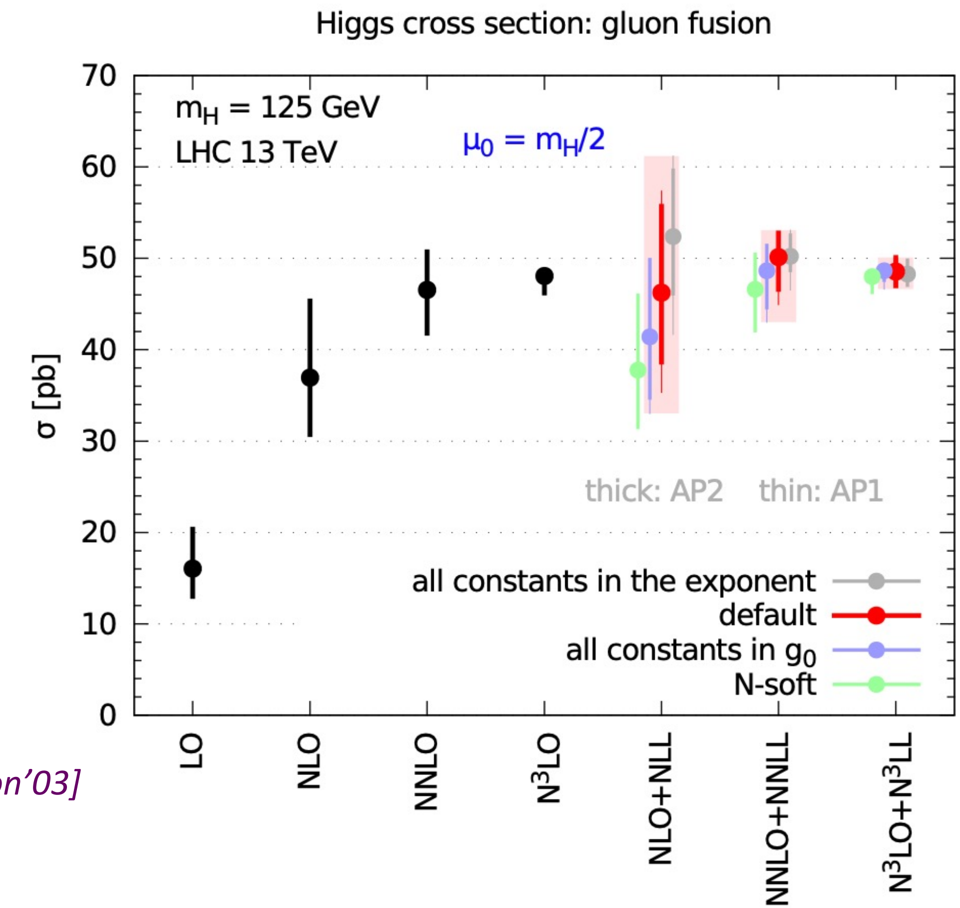
A_i – perturbative function, $A^{(1)}_{q/g}=\alpha_s/\pi\, C_{F/A}$

APPLICATION TO HIGGS PRODUCTION



$\log\,(1 - m_H^2/\hat{s})$

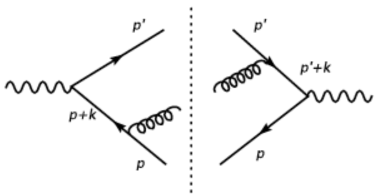
NNLO+NNLL: [Catani, de Florian, Grazzini, Nason'03]



[Bonvini, Marzani, Muselli, Rottoli'16]

Colour Flow

- Two external coloured legs

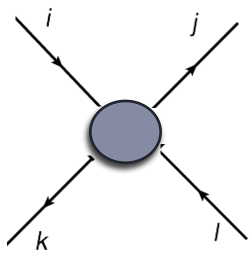


$$|M_{eik}|^2 = -g_s^2 |M_{Born}|^2 \, 2C_F \frac{p \cdot p'}{(p \cdot k)(p' \cdot k)}$$

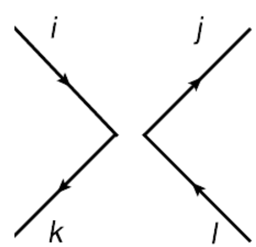
- Result for three coloured legs also involves only simple Casimir factors, but starting from >= four legs objects in colour space
  - gluon emission off quark lines ~ $T_i^a \, T_j^a$
  - in general, decompose amplitudes in a chosen colour basis (colour tensors with indices of external partons)

e.g.

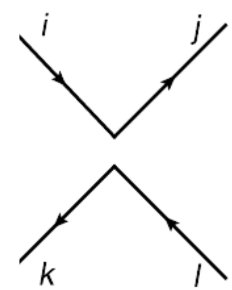
$$q_i \bar{q}_k \rightarrow q_j \bar{q}_l$$



colour basis (possible choice)



$c_1 = \delta_{ik}\delta_{jl}$

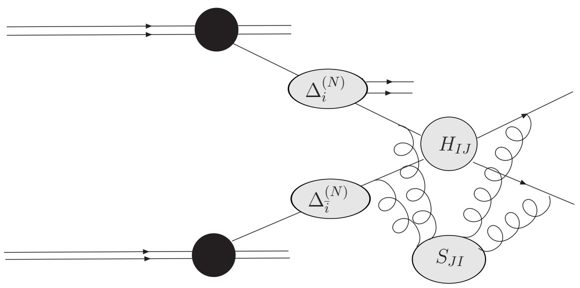


$c_2 = \delta_{ij}\delta_{kl}$

Consequence: soft emission function turns into a matrix in the colour space

Threshold Resummation for 2→2 Processes with colour & mass in the final state

2→2 process with nontrivial colour flow



$$\sigma^{(N)}_{ij\to kl} = \underbrace{H^{(N)}_{ij\to kl,IJ}}_{\text{hard function}} \times \underbrace{\Delta_i^{(N)}\Delta_j^{(N)}}_{\text{soft- collinear radiation}} \times \underbrace{S_{ij\to kl,JI}}_{\text{soft wide-angle emission}}$$

universal factors; KNOWN

process-dependent

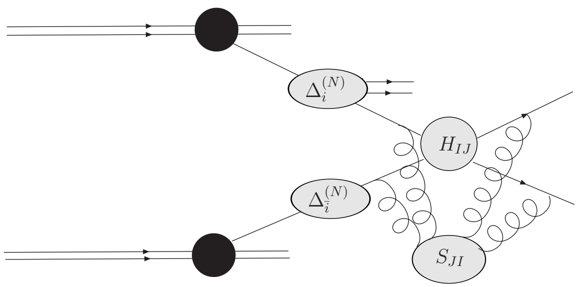
matrices in colour space

S_{JI} from solving the renormalization group equation

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right) S^{(N)}_{JI} = -\Gamma^{\dagger}_{JK} S(N)_{KI} - S(N)_{JL}\Gamma_{LI}$$

Threshold Resummation for 2→2 Processes with colour & mass in the final state

2→2 process with nontrivial colour flow



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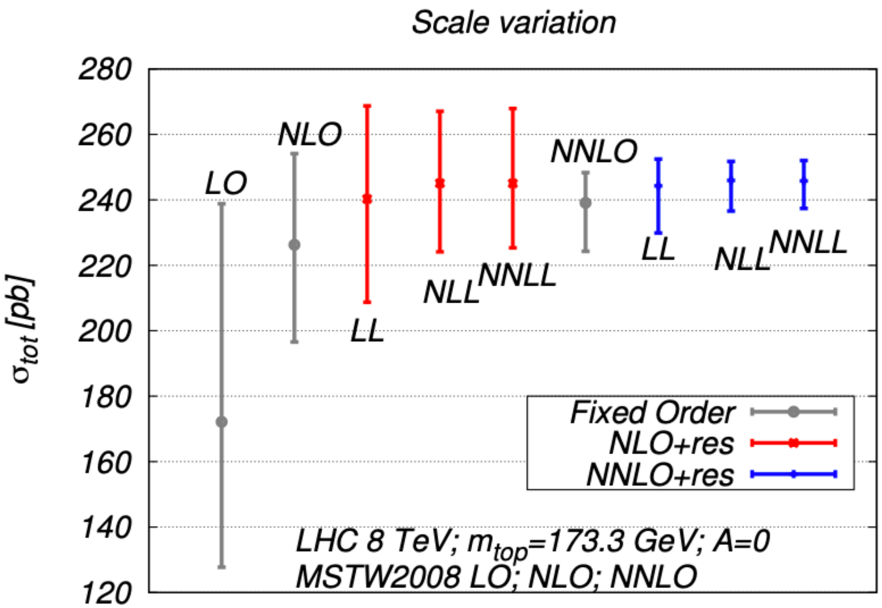
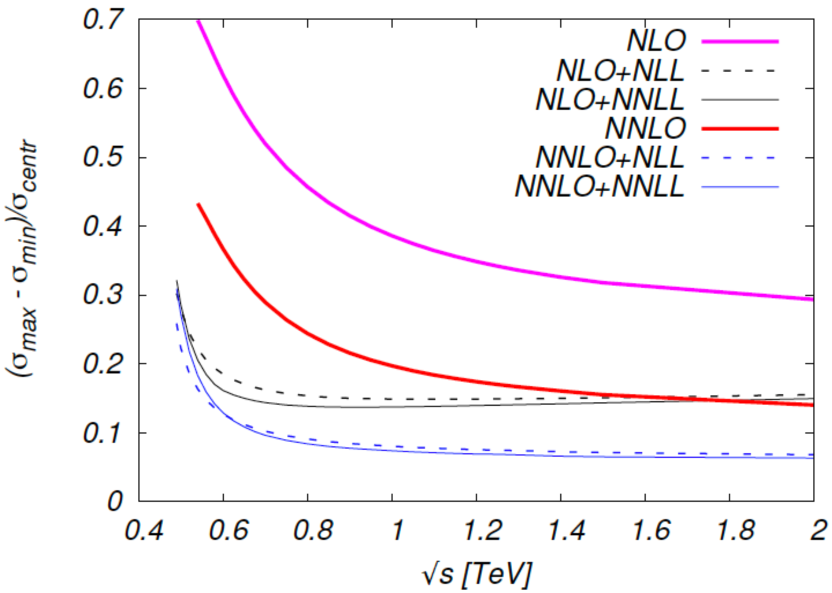
TTBAR CROSS SECTIONS

$\log\ (1 - 4m_t^2/\hat{s})$





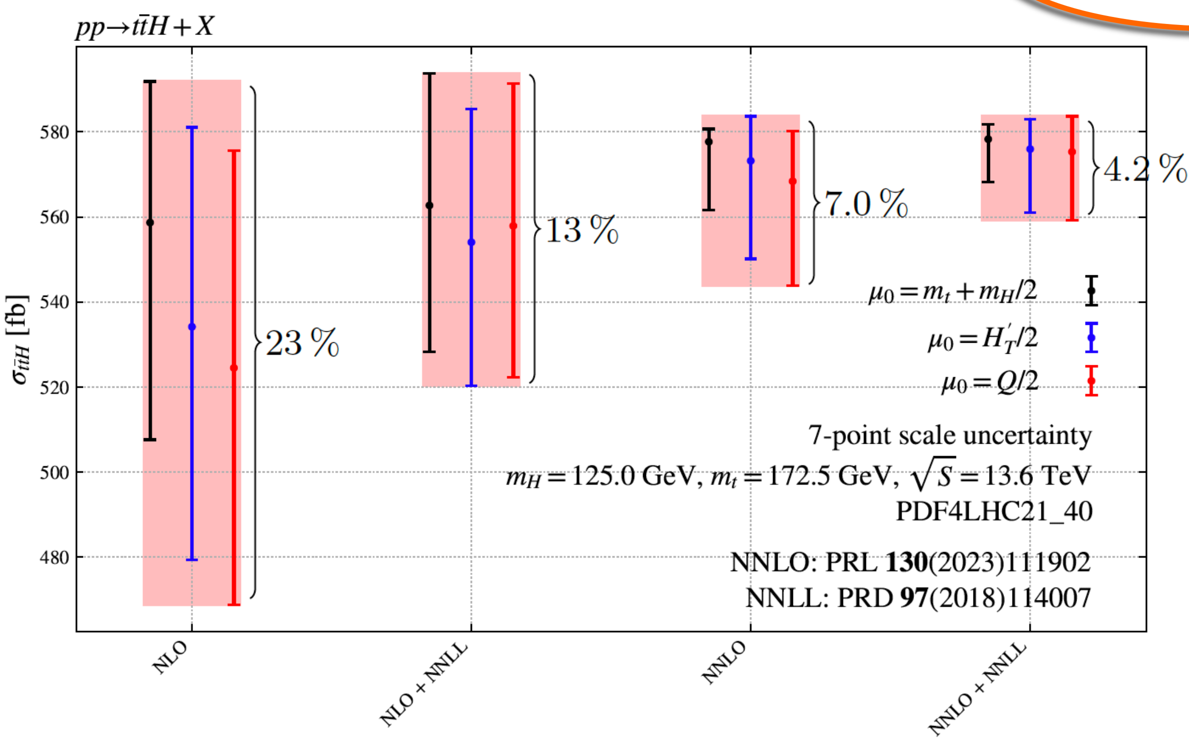
[Fiedler, Mitov, Czakon'13]

TTH@NNLO+NNLL

[Balsach, AK, Motyka, Stebel]

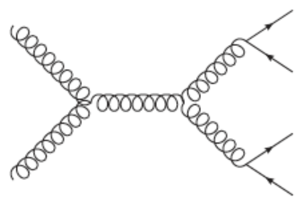
ttH production cross section known at fixed order at NNLO in the "soft-Higgs approximation" [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini'22] . Precision can be further improved by matching with the NNLL soft gluon resummation [AK, Motyka, Stebel, Theeuwes'17]

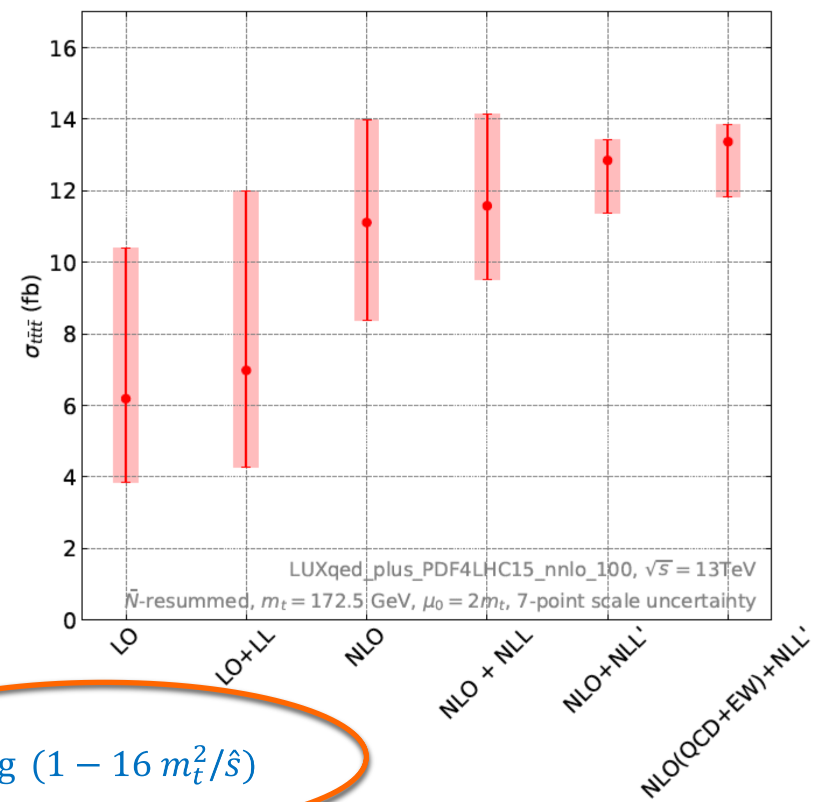
$\log\,(1 - Q^2/\hat{s})$



4-TOP PRODUCTION

[van Beekveld, AK, Moreno Valero, PRL 131 (2023) 21, 211901]





- Resummation of logarithms of *1-(4m$_t$)^2/s* in dQCD
- 4 coloured particles in the final state: highly non-trivial colour flow (14-dim colour space in the gg channel)
- so far, only resummed up to NLL

$$\log\,(1 - 16\, m_t^2/\hat{s})$$

$$d\sigma^{\rm res}_{ij\to t\bar{t}t\bar{t}}(N,\mu_F,\mu_R,\ldots) = {\rm Tr}\left[\mathbf{H}_{ij\to t\bar{t}t\bar{t}}(\mu_F,\mu_R,\ldots)\mathbf{S}_{ij\to t\bar{t}t\bar{t}}(N,\mu_R,\ldots)\right]\Delta_i(N,\mu_F,\mu_R,)\Delta_j(N,\mu_F,\mu_R,\ldots)$$

SCET

- Soft - Collinear Effective Theory (SCET) of QCD *[Bauer, Becher, Beneke, Chapovsky, Diehl, Feldmann, Fleming, Hill, Lee, Luke, Manohar, Neubert, Pirjol, Rothstein, Stewart, …'early 00s]*

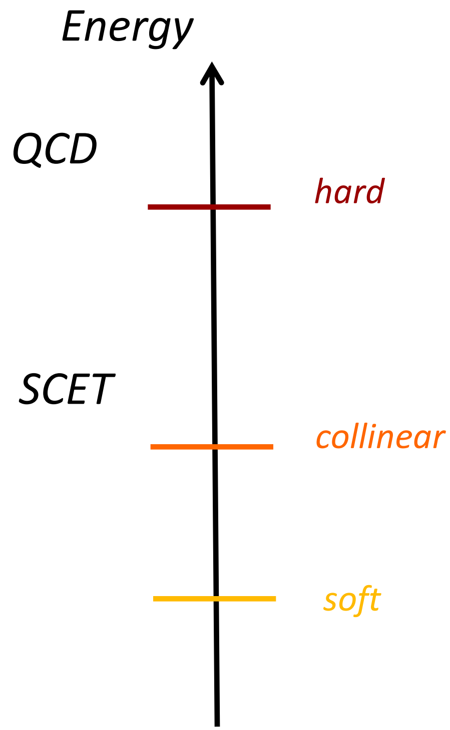
  → dynamics of energetic particles moving close to the light-cone and interacting with soft quanta

  → effective Lagrangian built out of quark and gluon fields with collinear and (ultra)soft momenta

- EFTs provide useful framework for studying multi-scale problems: scale separation → factorization

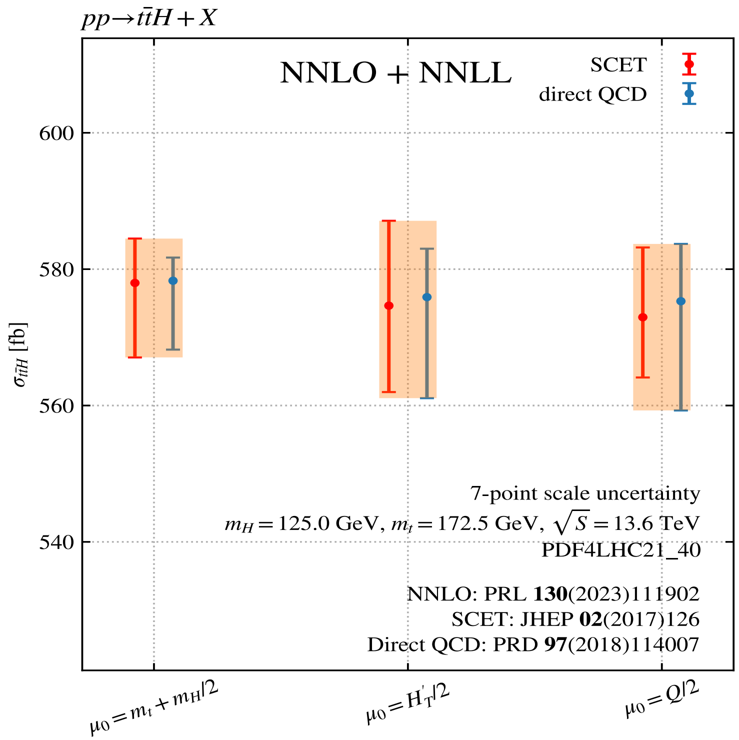
- Resummation from solving RG equations of SCET in momentum space *[Becher,Neubert'06] [Becher,Neubert,Pecjak'07] [Becher,Neubert,Xu'08]*

- Relation to direct QCD studied extensively *[Bonvini, Forte, Ghezzi, Ridolfi'12][Sterman, Zheng'13], [Almeida et al.'14]*



TTH@NNLO+NNLL

- Comparison with the NNLL+NNLO result based on SCET *[Broggio, Ferroglia, Pecjak, Yang'16]* within the framework of the ttH LHCHWG subgroup

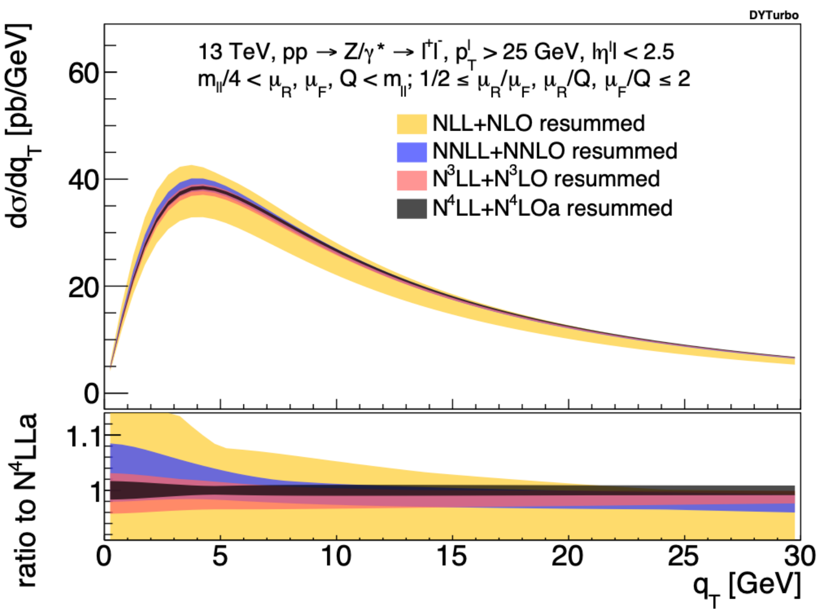


- Two **very different frameworks**: perturbative "full" theory (QCD) vs effective theory (SCET)
- Analytical formulas agree at NNLL
- Different subsets of subleading terms are included beyond NNLL → small numerical differences
- Results for central scale choices agree within a few permille

[NNLL dQCD: Balsach, AK, Motyka, Stebel] [NNLL SCET: Broggio, Ferroglia, Pecjak] [NNLO: Devoto, Grazzini, Kallweit, Mazzitelli, Savoini]

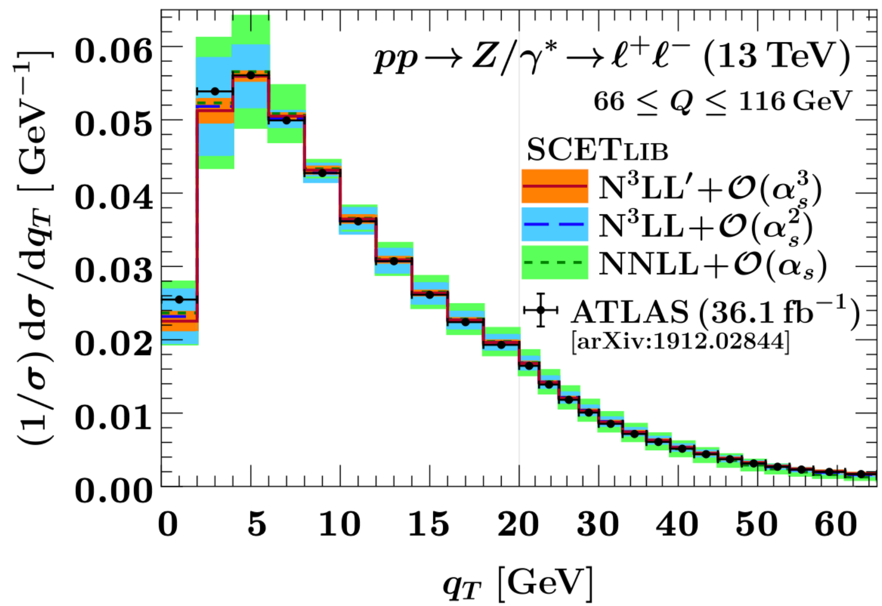
OTHER LOGS: Z PT

dQCD



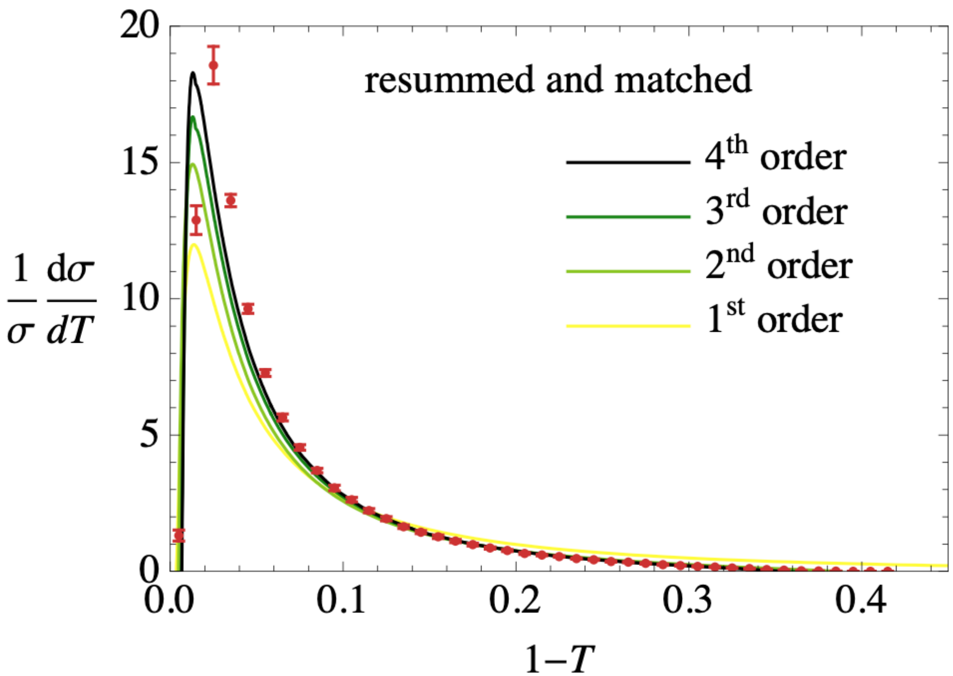
[Camarda, Cieri, Ferrera'23]

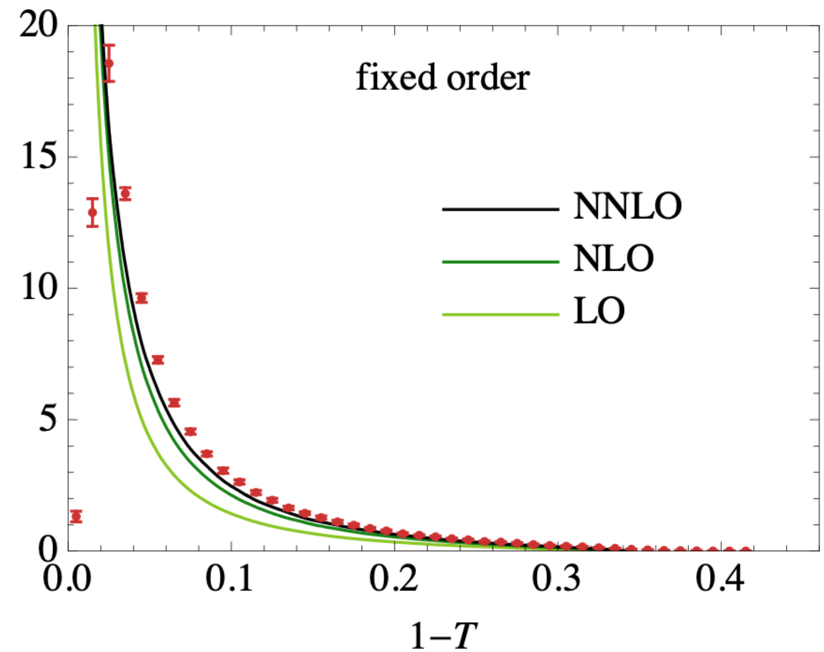
SCET



[Billis, Ebert, Michel, Tackmann'22]

OTHER LOGS: THRUST





[Bechert, Schwartz'08], see also [Abate et al.]'10'12

resummed: N^3LL

fixed-order: NNLO

Further Systematic Improvements

- Analytical resummation techniques lend themselves to further systematic improvements w.r.t. logarithmic accuracy
  - calculations of anomalous dimensions at higher orders increase accuracy of resummation exponents
  - apart from exponents also hard function needed: fixed-order loop calculations keeping info on the colour structure
  - consistent matching with fixed order, resulting in N^{x}LO+N^{y}LL predictions

- Next-to-eikonal approximation *[Laenen, Magnea, Stavenga, White, Bonocore, Vernazza,Larkoski, Neill,Stewart, Kolodrubetz, Moult, Stewart, del Duca, van Beekveld, Beneke, Jaskiewicz, Szafron, …]*

$$\frac{d\sigma}{d\xi} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} \left[c_{nm}^{(-1)} \left(\frac{\log^m \xi}{\xi}\right)_+ + c_{nm}^{(\delta)}\,\delta(\xi) + c_{nm}^{(0)} \log^m \xi + \ldots \right]$$

treated in standard resummation

loops + real phase-space

next-to-leading power logarithms, factorization, exponentiation?

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Other topics of research: resummation of EW logs, combined QCD and EW resummation, non-global logarithms, small-x resummation, (semi-)numerical approaches, combined resummations, …

Summary

- Resummation crucial for proper description of multiple classes of observables probing the IR dynamics
- Well established techniques: direct QCD and SCET
- As the ability and accuracy of higher-order calculation grows, so does the accuracy of resummation
- NNLL pretty much standard now, some quantities already known at N^3LL and N^4LL
- Higher logarithmic accuracy than those offered by parton showers, systematically improvable
- Very dynamical field, many new developments