

Nuclear PDFs

Karol Kovařík

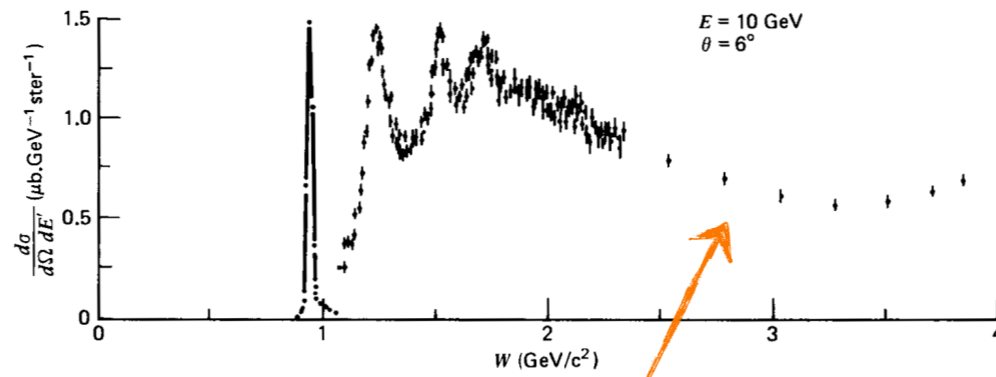
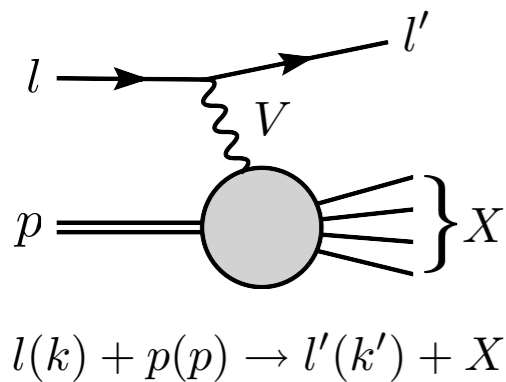
University of Münster



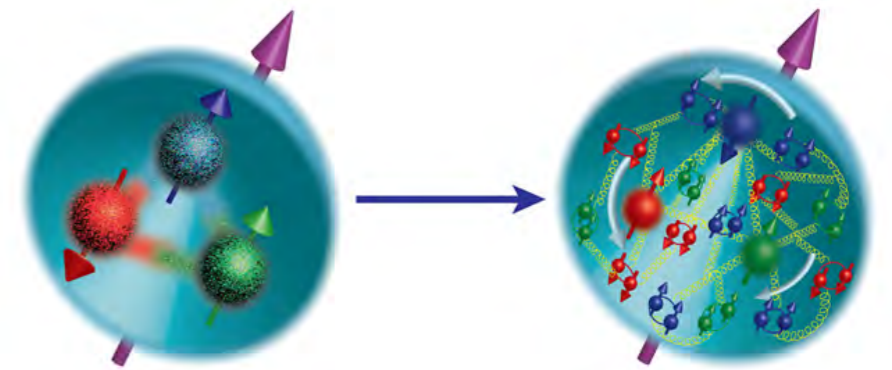
2024 CTEQ summer school on QCD and EW phenomenology

Recap

- Want to discover the inner structure of the proton? BREAK IT UP **HARD**



Deep Inelastic Scattering (DIS)



- Distinguishing between elastic & inelastic scattering

$$W^2 = (p'_1 + p'_2 + \dots + p'_n)^2 = m_p^2 + Q^2 \frac{(1-x)}{x}$$

- if $W^2 = m_p^2$ elastic scattering
- if $W^2 \gg m_p^2$ inelastic scattering

- Kinematic variables either E', θ or x, Q^2

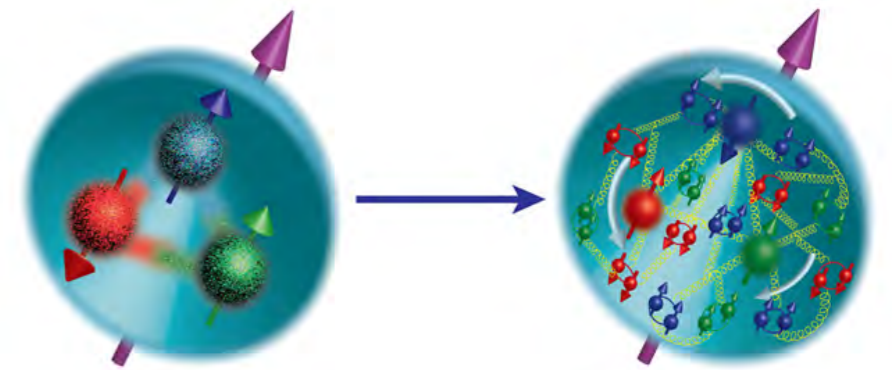
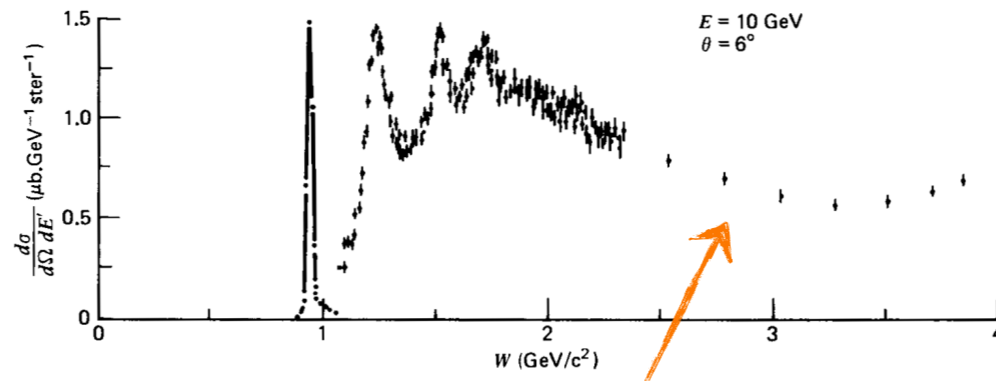
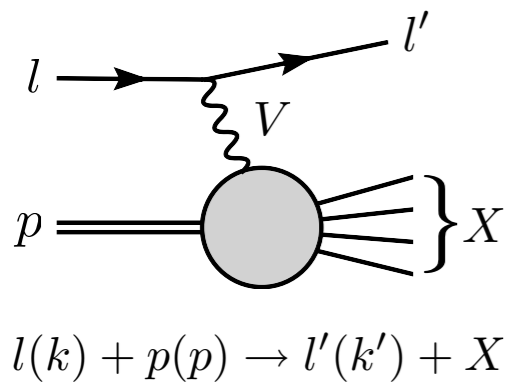
$$Q^2 = -(k - k')^2 \stackrel{\text{lab}}{=} 4EE' \sin^2 \frac{\theta}{2}$$

$$x = \frac{Q^2}{2p \cdot q} \stackrel{\text{lab}}{=} \frac{2EE' \sin^2 \frac{\theta}{2}}{M(E - E')}$$

$$y = \frac{p \cdot q}{p \cdot k} \stackrel{\text{lab}}{=} \frac{E - E'}{E}$$

Recap

- Want to discover the inner structure of the proton? BREAK IT UP **HARD**



Deep Inelastic Scattering (DIS)

- DIS cross-section & structure functions (photon exchange)

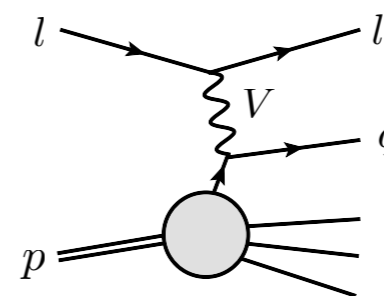
$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{2F_1(x, Q^2)}{M} \sin^2 \frac{\theta}{2} + \frac{F_2(x, Q^2)}{E - E'} \cos^2 \frac{\theta}{2} \right)$$

$$\text{or } \frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{xQ^2} \left[xy^2 F_1(x, Q^2) + (1 - y) F_2(x, Q^2) \right]$$

- Assuming elastic scattering on partons in the proton - inelastic scattering an incoherent sum of elastic scatterings on proton constituents

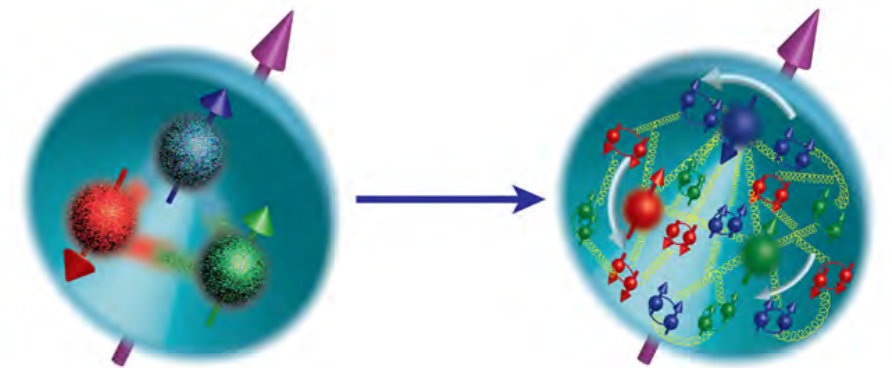
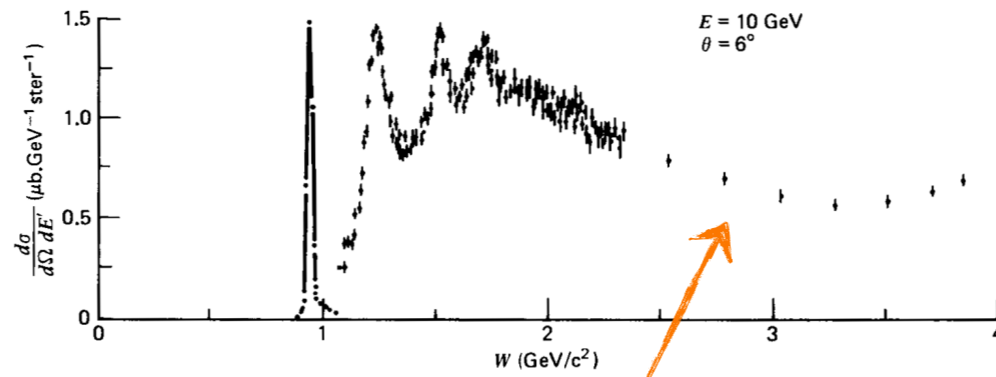
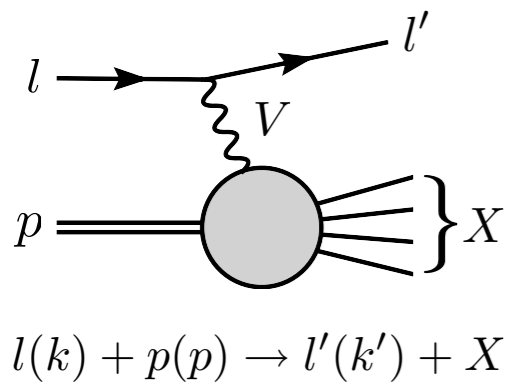
$$\frac{d\sigma}{dx dQ^2} = \sum_q \int d\xi f_q(\xi) \left(\frac{d\sigma^{eq}}{dx dQ^2} \right)_{\text{el.}}$$

parton distribution function



Recap

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Deep Inelastic Scattering (DIS)

- DIS cross-section & structure functions

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{2F_1(x, Q^2)}{M} \sin^2 \frac{\theta}{2} + \frac{F_2(x, Q^2)}{E - E'} \cos^2 \frac{\theta}{2} \right)$$

$$\text{or } \frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{xQ^2} \left[xy^2 F_1(x, Q^2) + (1 - y) F_2(x, Q^2) \right]$$

- Assuming elastic scattering on partons in the proton - the structure function (at LO) is

$$F_2(x, Q^2) = 2xF_1(x, Q^2) = \sum_i e_i^2 \int d\xi \xi f_i(\xi) \delta\left(\xi - \frac{Q^2}{2p \cdot q}\right) = \sum_q e_q^2 x (f_q(x) + f_{\bar{q}}(x))$$

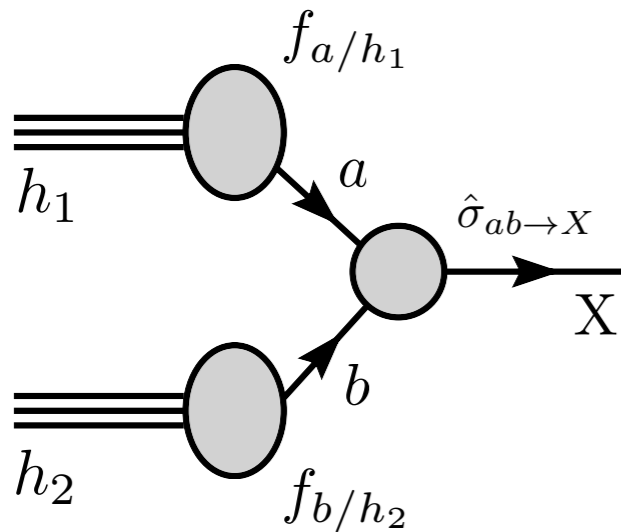
quark have spin 1/2

parton distribution function (PDF)

mom. fraction $\xi = x$ kinematic variable

Recap

- Parton distribution functions are universal

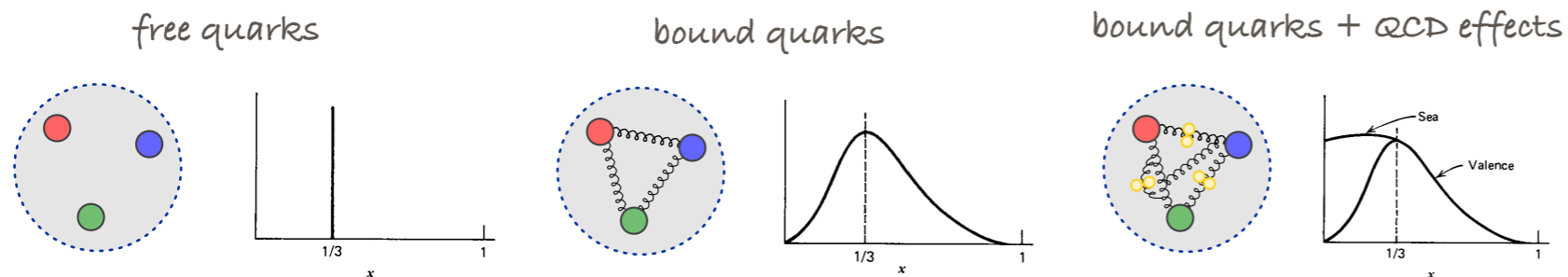


$$\sigma = \sum_{a,b} \int dx_1 dx_2 f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab \rightarrow X}(x_1 x_2 s)$$

PDFs from experiment

from pQCD

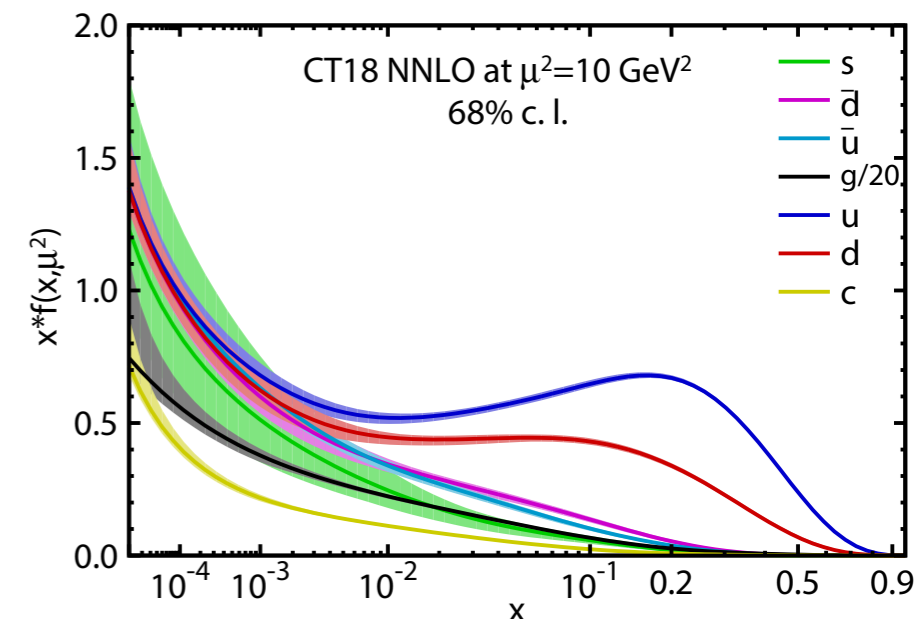
- In parton distribution functions the quark model is connected with QCD



for proton

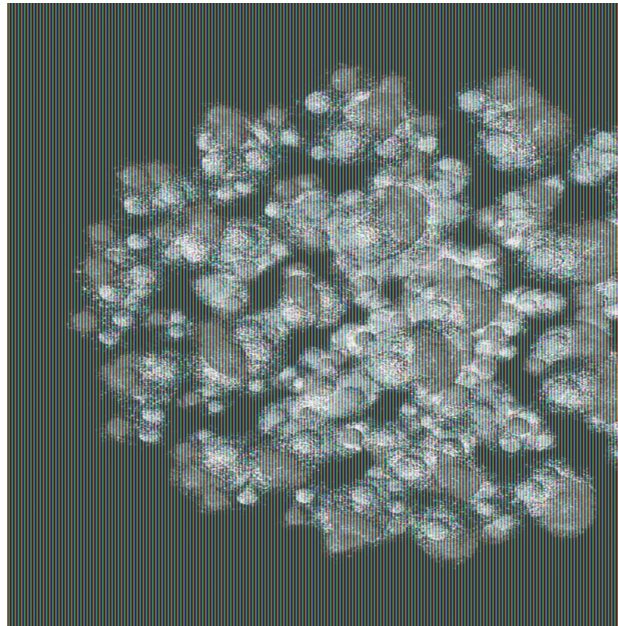
$$\int_0^1 dx [u(x) - \bar{u}(x)] = 2 \quad \int_0^1 dx [d(x) - \bar{d}(x)] = 1$$

valence quark sum rules



Nuclei

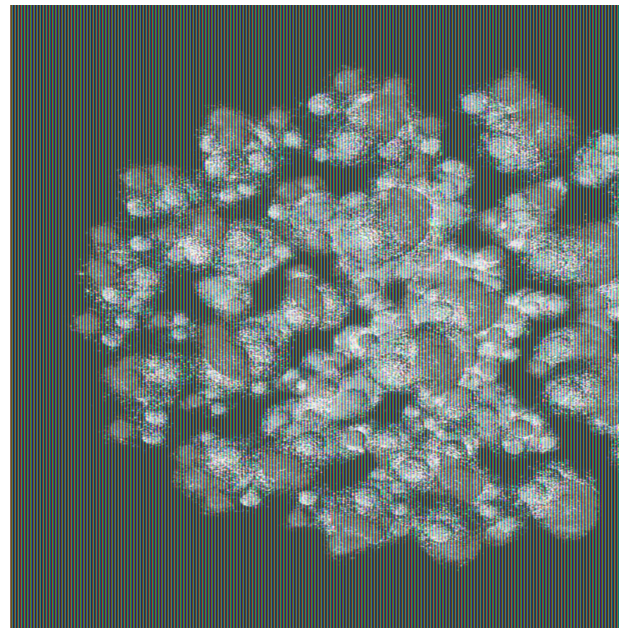
- Explanation of attraction between like-charged protons - strong force - QCD



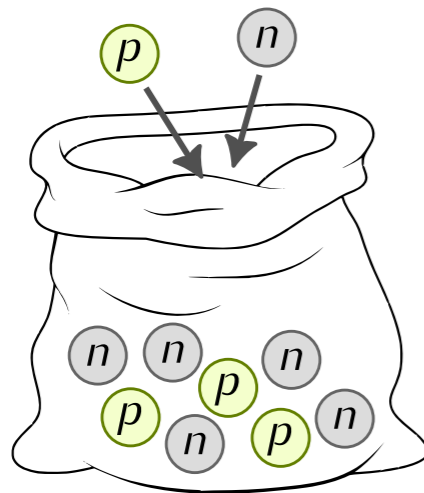
- Can we use the same language of PDFs to talk about the nucleus?

Nuclei

- Explanation of attraction between like-charged protons - strong force - QCD



- Can we use the same language of PDFs to talk about the nucleus? **YES we can**



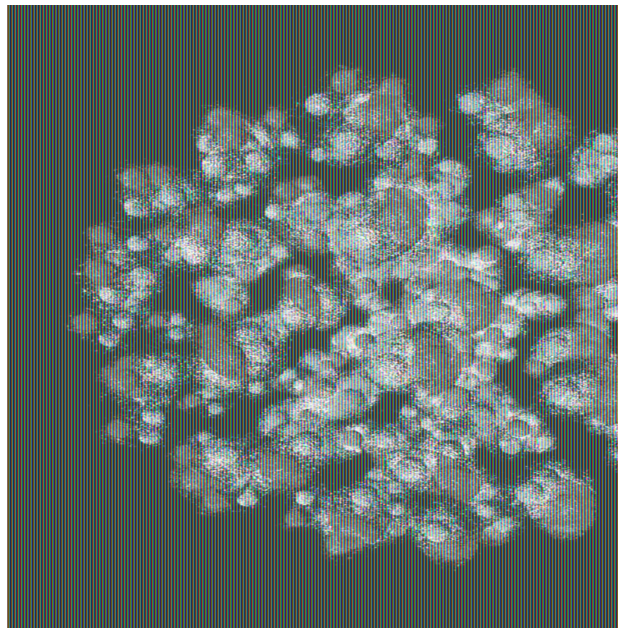
- *Free nucleon approximation* - nuclear binding effects assigned democratically to each nucleon in nucleus (A,Z)

$$f_i^N(x, Q^2)$$

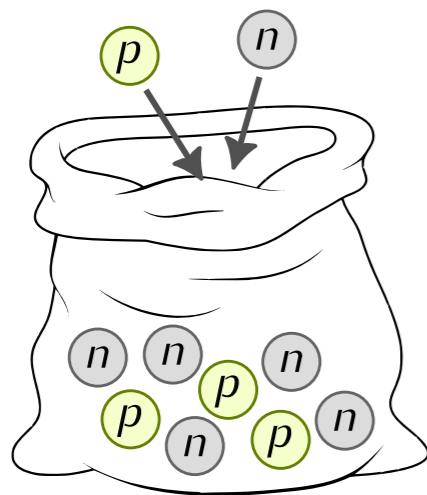
↑
PDF of nucleus

Nuclei

- Explanation of attraction between like-charged protons - strong force - QCD



- Can we use the same language of PDFs to talk about the nucleus? **YES we can**



- *Free nucleon approximation* - nuclear binding effects assigned democratically to each nucleon in nucleus (A,Z)

$$f_i^N(x, Q^2) = A f_i^{(A,Z)}(x, Q^2) = Z f_i^{p/A}(x, Q^2) + (A - Z) f_i^{n/A}(x, Q^2)$$

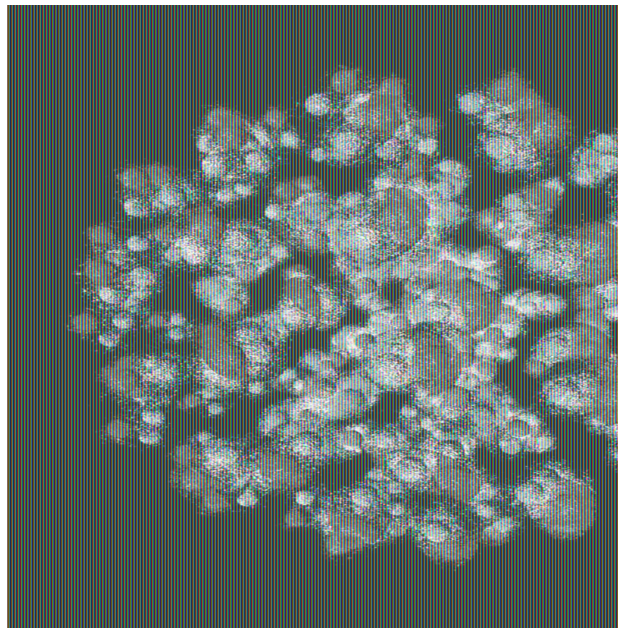
PDF of nucleus

PDF of average nucleon

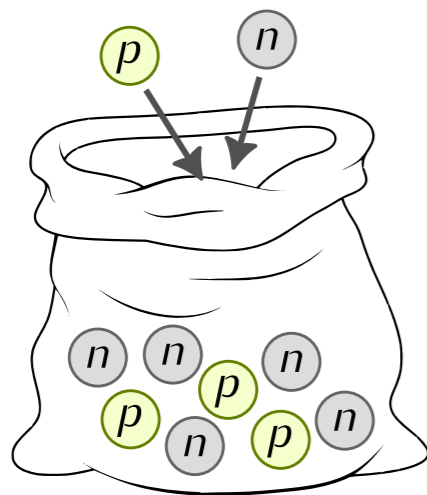
PDF of bound proton

Nuclei

- Explanation of attraction between like-charged protons - strong force - QCD



- Can we use the same language of PDFs to talk about the nucleus? **YES we can**



- *Free nucleon approximation* - nuclear binding effects assigned democratically to each nucleon in nucleus (A,Z)

$$f_i^N(x, Q^2) = A f_i^{(A,Z)}(x, Q^2) = Z f_i^{p/A}(x, Q^2) + (A - Z) f_i^{n/A}(x, Q^2)$$

PDF of nucleus

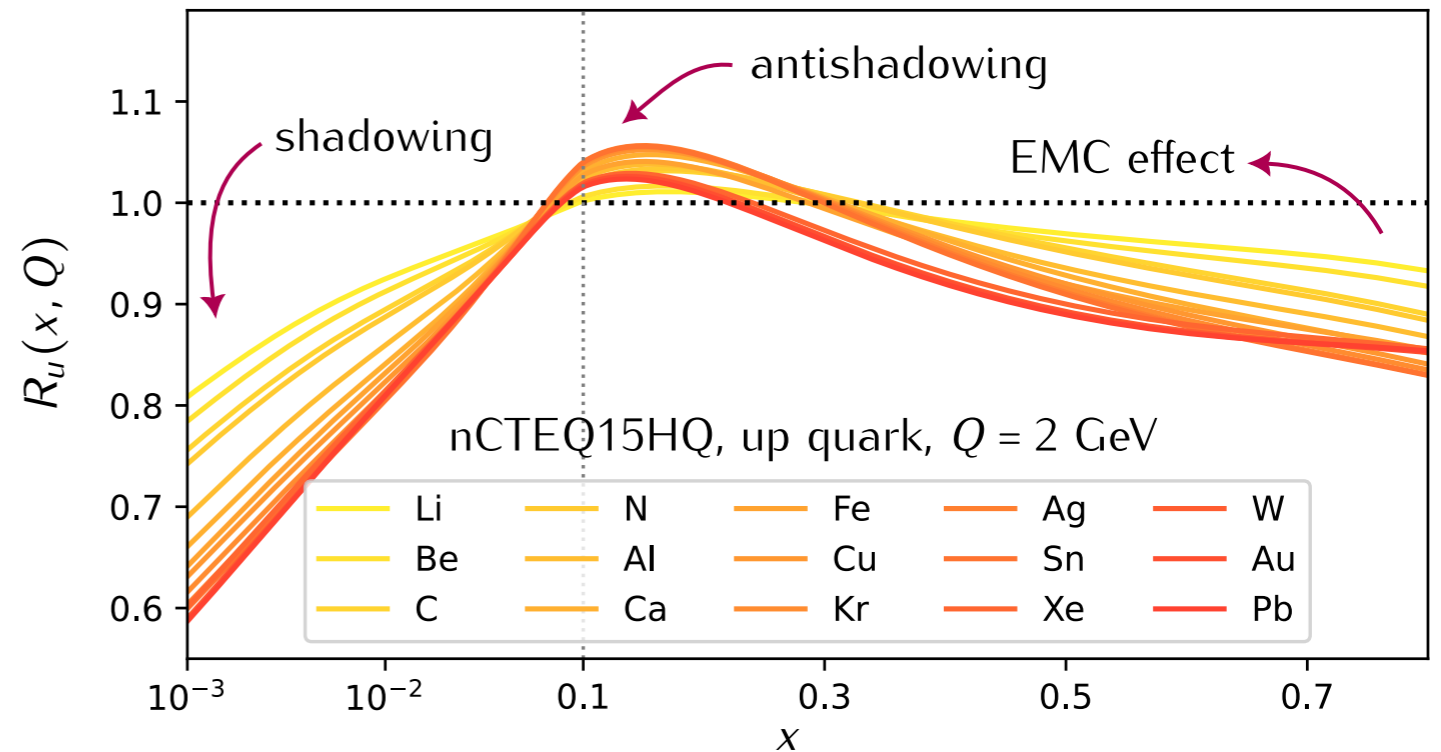
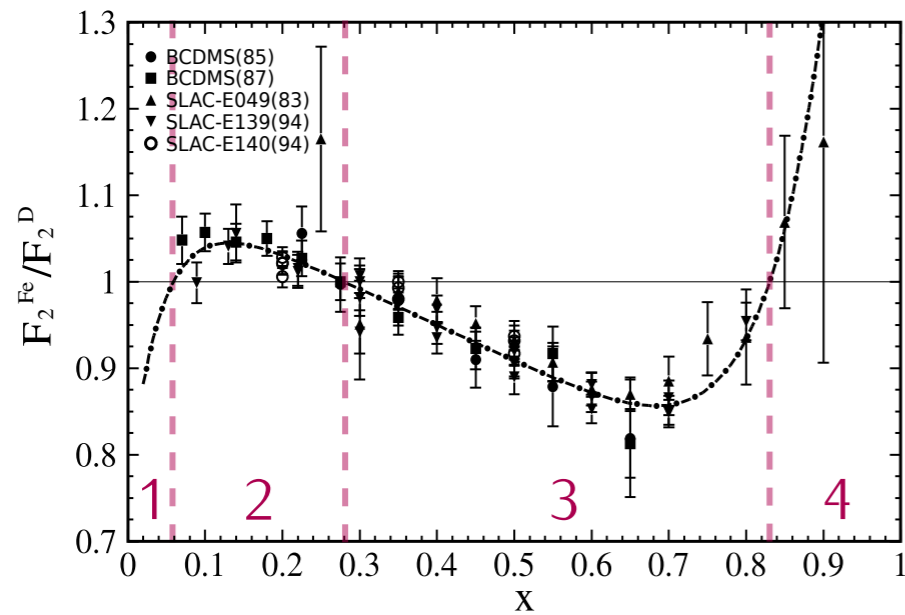
PDF of average nucleon

neutron structure related to proton via isospin

$$(u^{n/A}, d^{n/A}) = (d^{p/A}, u^{p/A})$$

Nuclear binding

- How large are the nuclear binding effects? Are they uniform?



- Nuclear binding effects as correction factors R
 - many different possibilities to define R

$$R_{F_2}(x, Q) = \frac{F_2^A(x, Q)}{F_2^D(x, Q)}$$

$$R_u(x, Q) = \frac{u^{p/A}(x, Q)}{u^p(x, Q)}$$

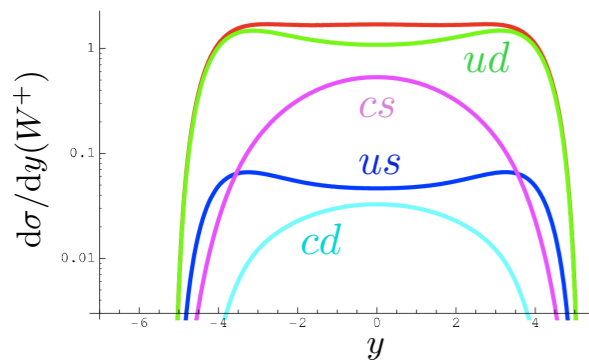
- Different regions in x of the nuclear binding effects were given names
 - Shadowing
 - Anti-shadowing
 - EMC effect
 - Fermi-motion

Nuclear effects

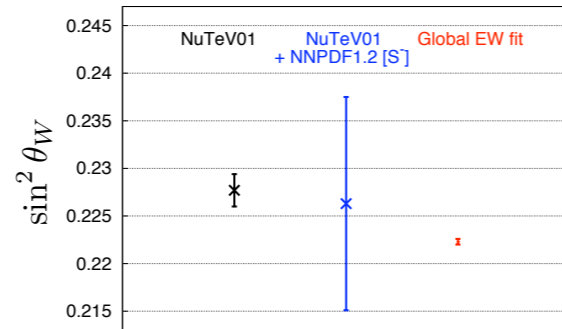
Where/When are nuclear effects relevant/useful ?

1. Strange quark content of the proton

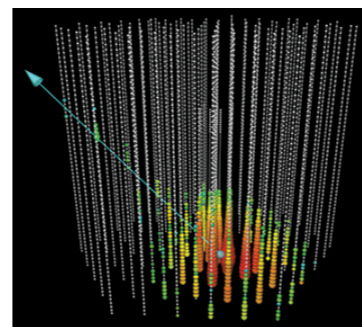
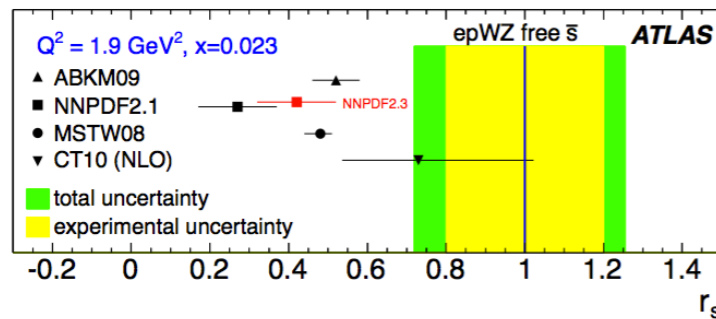
(anti-)strange PDF from (anti-)neutrino DIS with heavy nuclei - nuclear effects important



W-boson production @ LHC

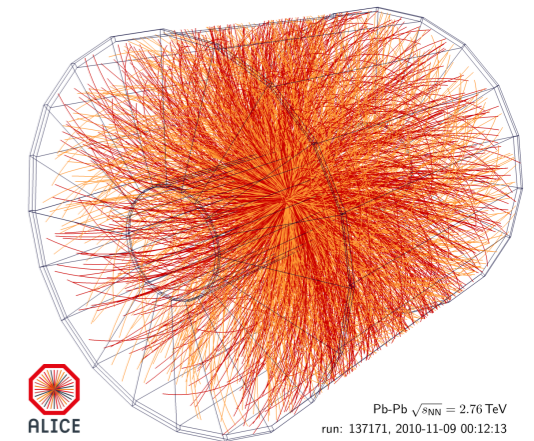
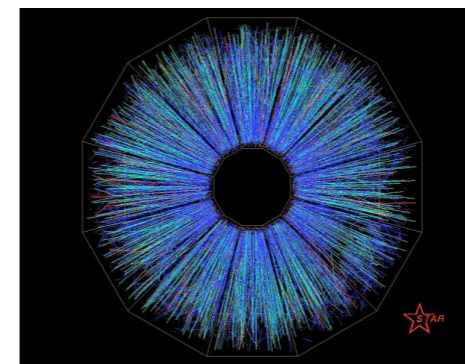


weak mixing angle from NuTeV experiment



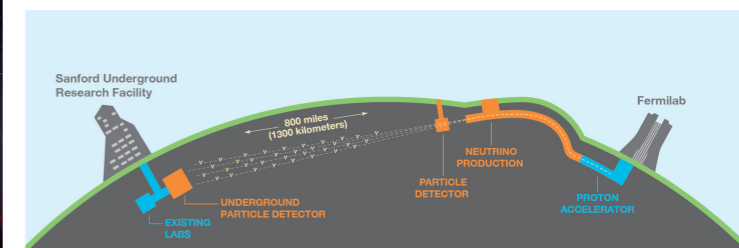
2. Heavy ion collisions @ RHIC, LHC

lead & gold heavy nuclei - nuclear effects in gluon PDF substantial



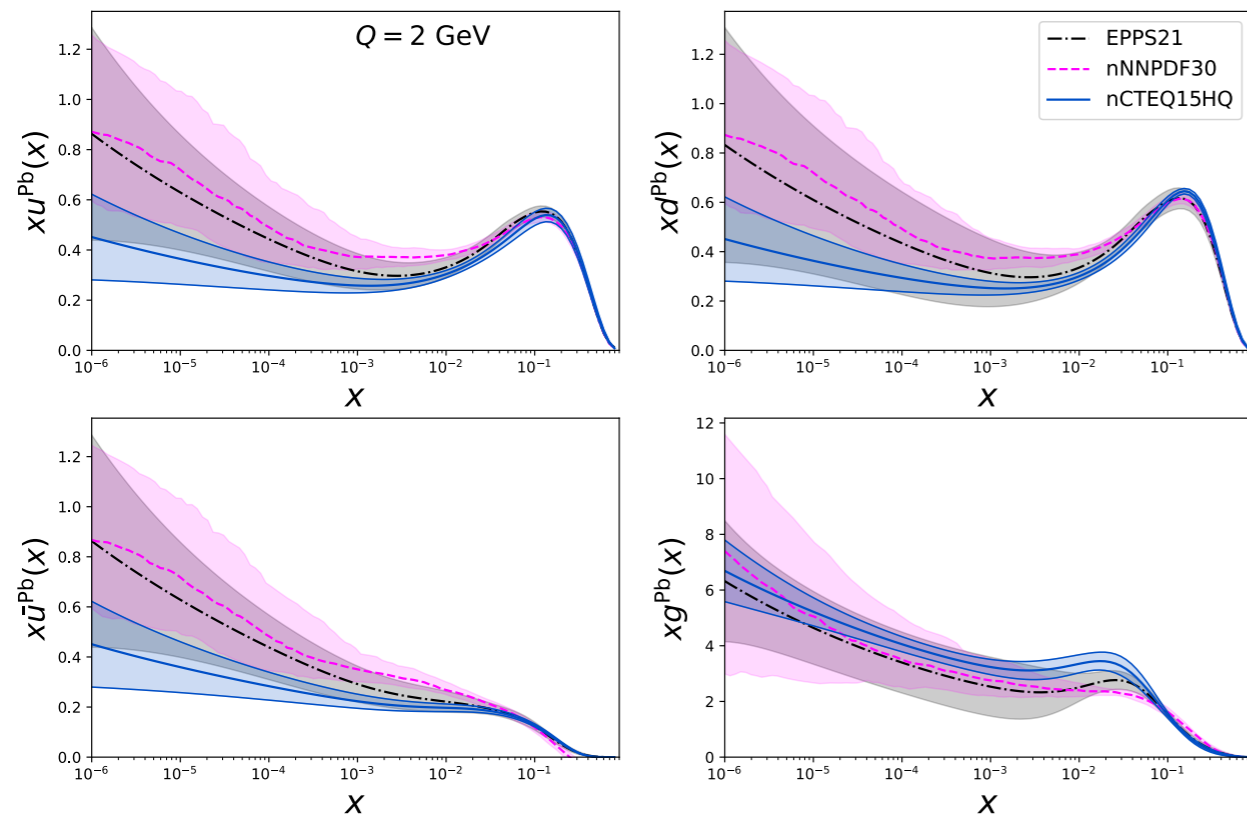
3. Neutrino physics

neutrinos interact only weakly - heavy targets required for sufficient count - IceCube (ice), DUNE (argon)

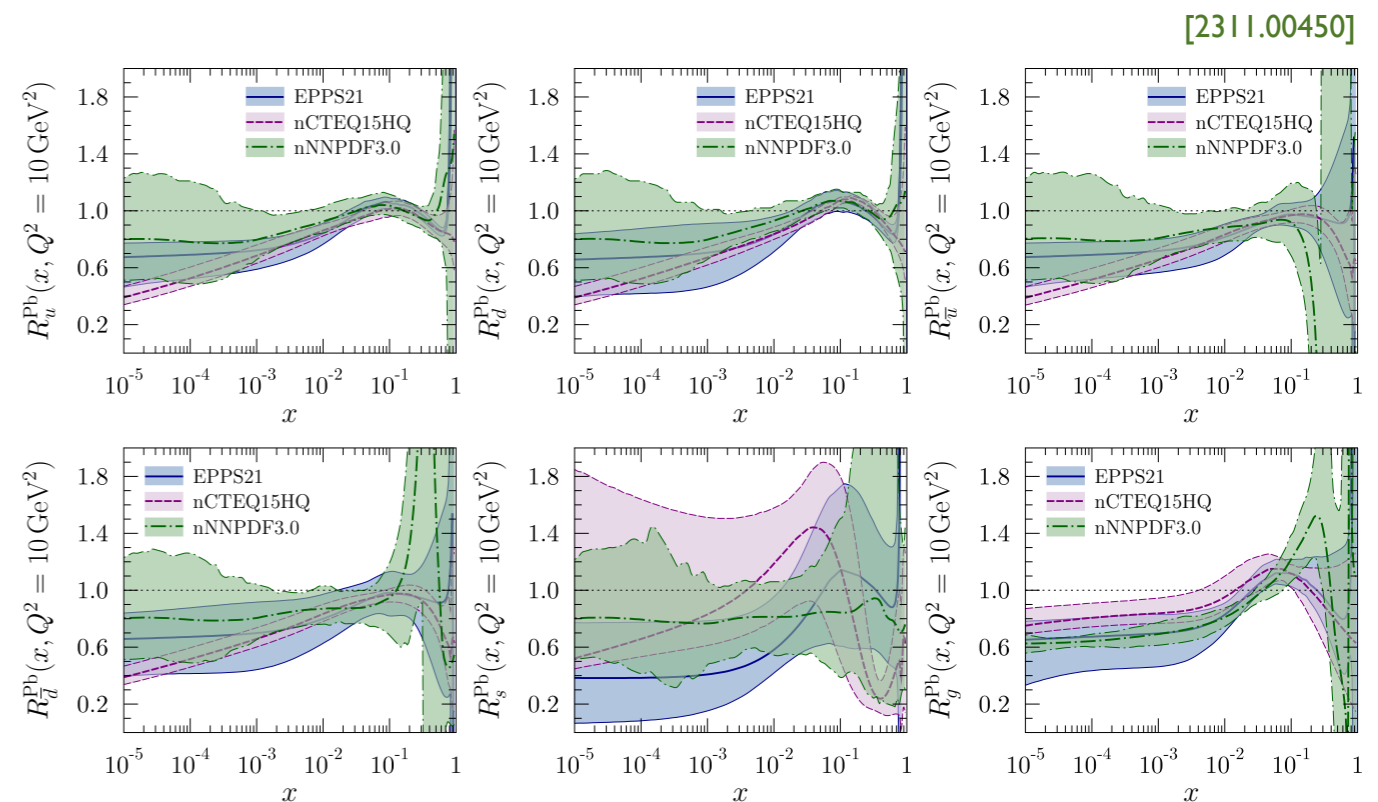


Nuclear PDFs - results

- Nuclear PDFs similar in shape to the proton PDFs



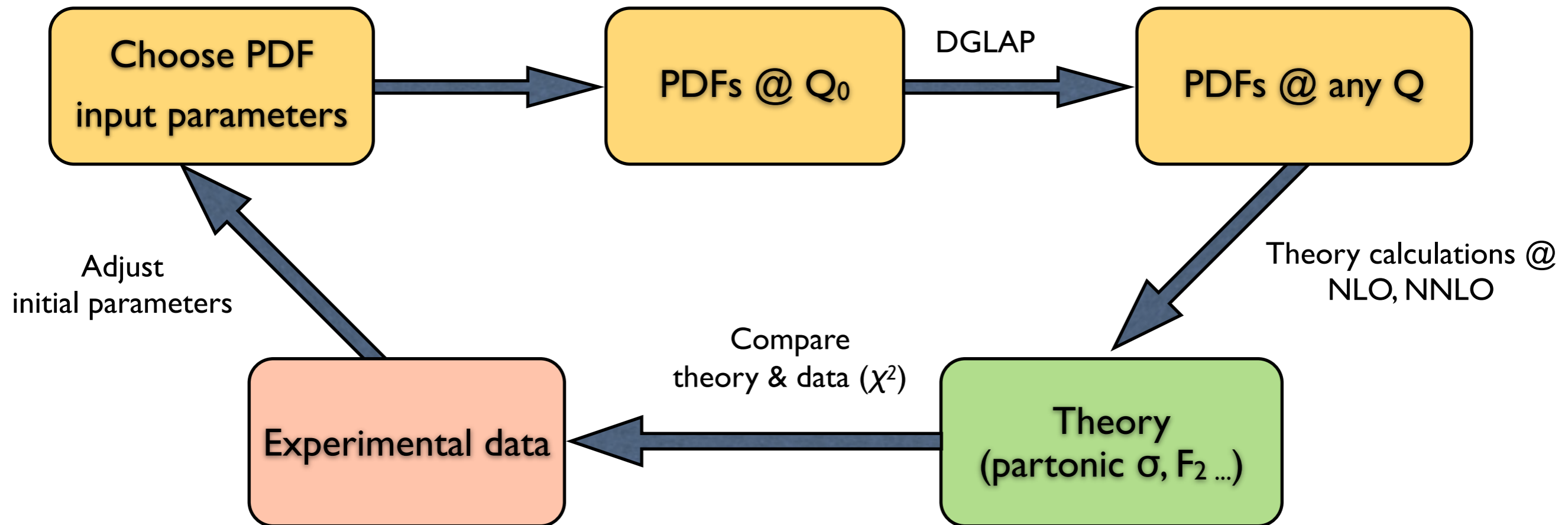
- Nuclear effects are similar in most partons



[2311.00450]

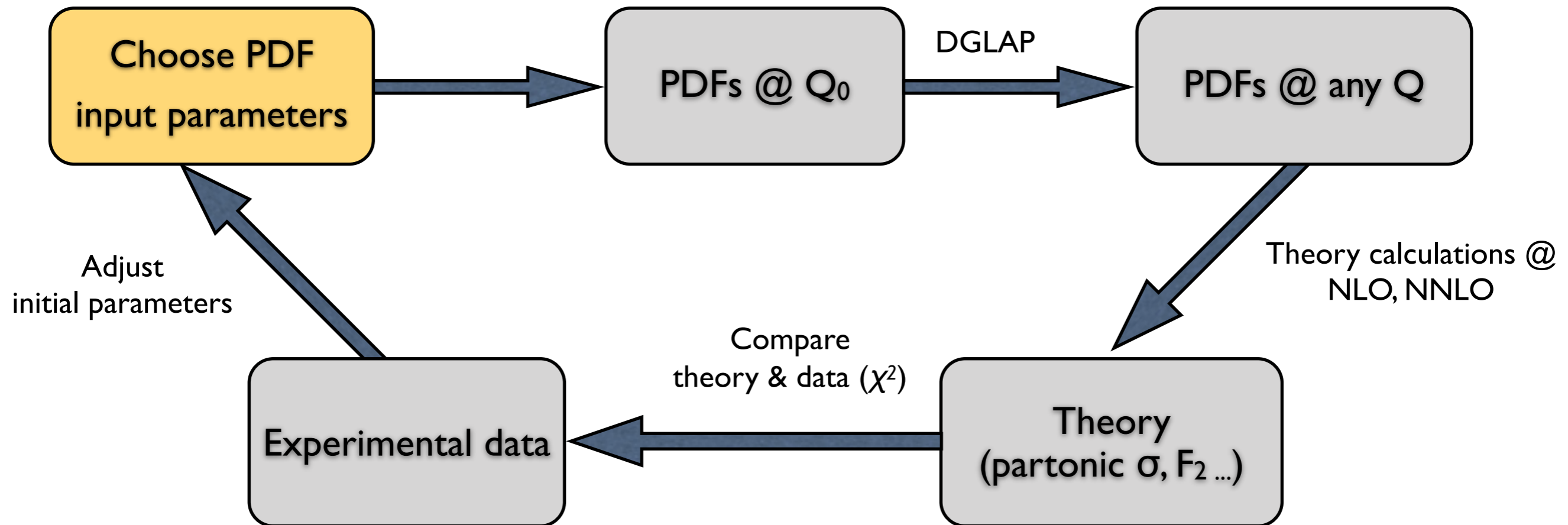
Determining nPDFs

- Determining (nuclear) parton distribution functions
- how do we determine them? What are the moving parts in a typical PDF fitting-machine?



Determining nPDFs

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Nuclear PDFs

- Nuclear effects in PDFs

1. Multiplicative nuclear correction factor

$$f_i^{p/A}(x_N, Q_0^2) = R_i^{p/A}(x_N, Q_0, A, Z) f_i^p(x_N, Q_0^2)$$

$$i = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

bound parton density

free parton density

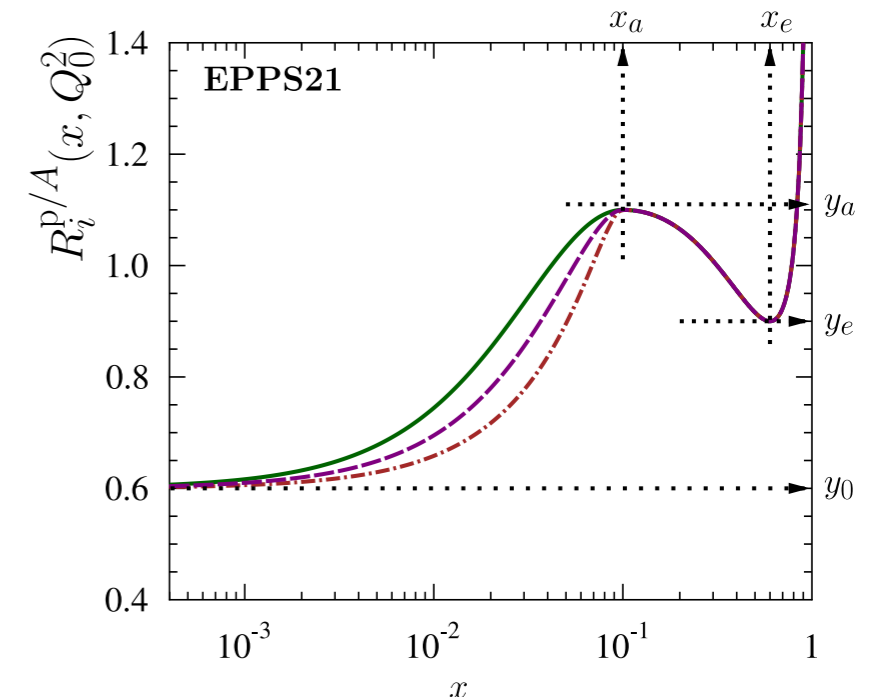
HKN07 - Hirai, Kumano, Nagai [PRC76(2007)065207] arXiv: 0709.0338

EPS09 - Eskola, Paukkunen, Salgado [JHEP0904(2009)065] arXiv: 0902.4154

DSSZ - de Florian, Sassot, Stratmann, Zurita [PRD85(2012)074028] arXiv: 1112.6324

KSASG20 - Khanpour et al. [PRD104(2021)3] arXiv: 2010.00555

EPPS21 - Eskola, Paakkinen, Paukkunen, Salgado [EPJC82(2022)413] arXiv: 2112.12462



- Complicated piecewise-parametrised nuclear correction factor (A-dependence in stitching conditions)

$$R_i^{p/A}(x, Q_0^2) = \begin{cases} a_0 + a_1(x - x_a) [e^{-xa_2/x_a} - e^{-a_2}] & x \leq x_a \\ b_0 x^{b_1} (1 - x)^{b_2} e^{xb_3} & x_a \leq x \leq x_e \\ c_0 + c_1(c_2 - x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

$$y_i(A) = 1 + [y_i(A_{\text{ref}} - 1)] \left(\frac{A}{A_{\text{ref}}} \right)^{\gamma_i}$$

Nuclear PDFs

- Nuclear effects in PDFs

2. Nuclear PDF with neural network

$$f_i^{p/A}(x_N, Q_0^2) = f_i(x_N, A, Q_0^2) \quad f_i^p(x_N, Q_0^2) = f_i(x_N, A = 1, Q_0^2)$$

bound parton density

free parton density

nNNPDF1.0 - Khalek et. al. [EPJC 79(2019)471] arXiv: 1904.00018

nNNPDF2.0 - Khalek et. al. [JHEP 09(2020)183] arXiv: 2006.14629

nNNPDF3.0 - Khalek et. al. [EPJC 82(2022)507] arXiv: 2201.12363

- functional form for bound protons same as for free proton PDF

$$x f_k(x, Q_0) = x^{\alpha_k} (1 - x)^{\beta_k} \text{NN}_k(x, A)$$

$$f_k = \{ \Sigma^{p/A}, T_3^{p/A}, T_8^{p/A} \}$$

$$x f_i(x, Q_0) = B_i x^{\alpha_i} (1 - x)^{\beta_i} \text{NN}_i(x, A)$$

$$f_i = \{ V^{p/A}, V_3^{p/A}, g^{p/A} \}$$

- In the evolution basis - singlet Σ , non-singlet sea quark T_i , valence V_i

$$\Sigma = u + \bar{u} + d + \bar{d} + s + \bar{s}$$

$$V = (u - \bar{u}) + (d - \bar{d}) + (s - \bar{s})$$

$$T_3 = (u + \bar{u}) - (d + \bar{d})$$

$$V_3 = (u - \bar{u}) - (d - \bar{d})$$

$$T_8 = (u + \bar{u} + d + \bar{d}) - 2(s + \bar{s})$$

$$V_8 = (u - \bar{u} + d - \bar{d}) - 2(s - \bar{s})$$

Nuclear PDFs

- Nuclear effects in PDFs

3. Traditional nuclear PDF

$$f_i^{p/A}(x_N, Q_0^2) = f_i(x_N, A, Q_0^2) \quad f_i^p(x_N, Q_0^2) = f_i(x_N, A=1, Q_0^2)$$

bound parton density

free parton density

nCTEQ15 - Kovarik et. al. [PRD93(2016)085037] arXiv: 1509.00792

nCTEQ15WZ, nCTEQ15HQ and other nCTEQ analyses

TUJU21 - Helenius, Walt, Vogelsang [PRD105(2022)9] arXiv: 2112.11904

- functional form for bound protons same as for free proton PDF

$$x f_k(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5}$$

$$k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0) / \bar{u}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}$$

- coefficients with A-dependance (reduces to proton for A=1)

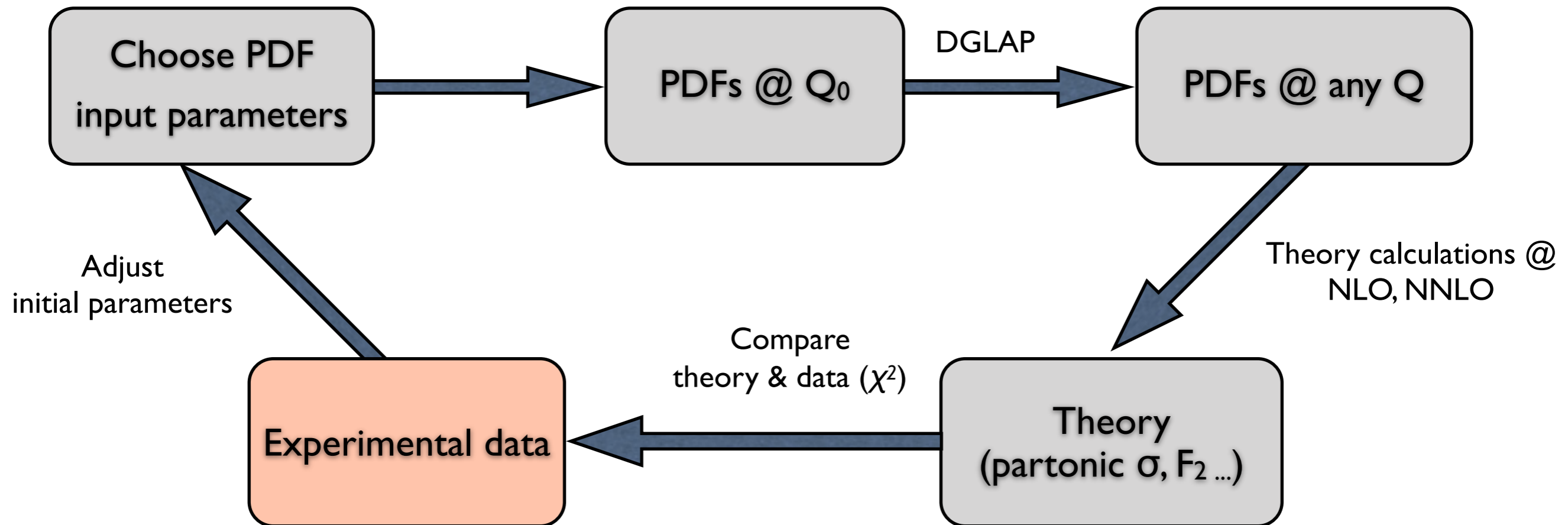
$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1} (1 - A^{-c_{k,2}}), \quad k = \{1, \dots, 5\}$$

Nuclear PDFs

	proton	order	HQ	params	Uncert.
EPPS	CT18A	NLO	GM-VFNS (s-ACOT)	7 indep. PDFs (24 params)	Hessian ($\Delta\chi^2=33$)
nCTEQ	CTEQ6	NLO	GM-VFNS (s-ACOT)	7 indep. PDFs (19 params)	Hessian ($\Delta\chi^2=35$)
nNNPDF	NNPDF4.0	NLO	GM-VFNS (FONLL)	6 indep. PDFs (neural network)	Monte Carlo
TUJU	own	NLO NNLO	GM-VFNS (FONLL)	7 indep. PDFs (16 params)	Hessian ($\Delta\chi^2=50$)
KSASG	CT18	NLO NNLO	GM-VFNS (FONLL)	7 indep. PDFs (18 params)	Hessian ($\Delta\chi^2=20$)

Determining nPDFs

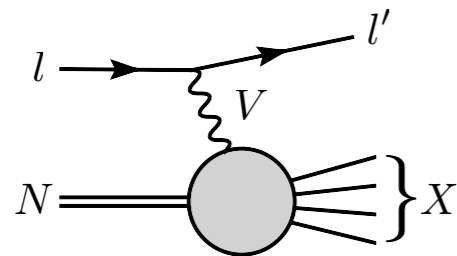
- Determining (nuclear) parton distribution functions
- how do we determine them? What are the moving parts in a typical PDF fitting-machine?



Nuclear data

Data for nuclear PDFs

Deep Inelastic Scattering (NC)



$$l + N \rightarrow l' + X$$

CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

SLAC E-139 & E-049

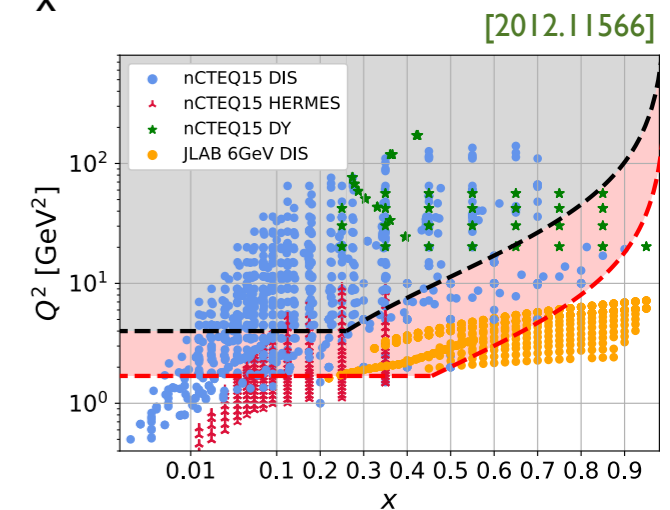
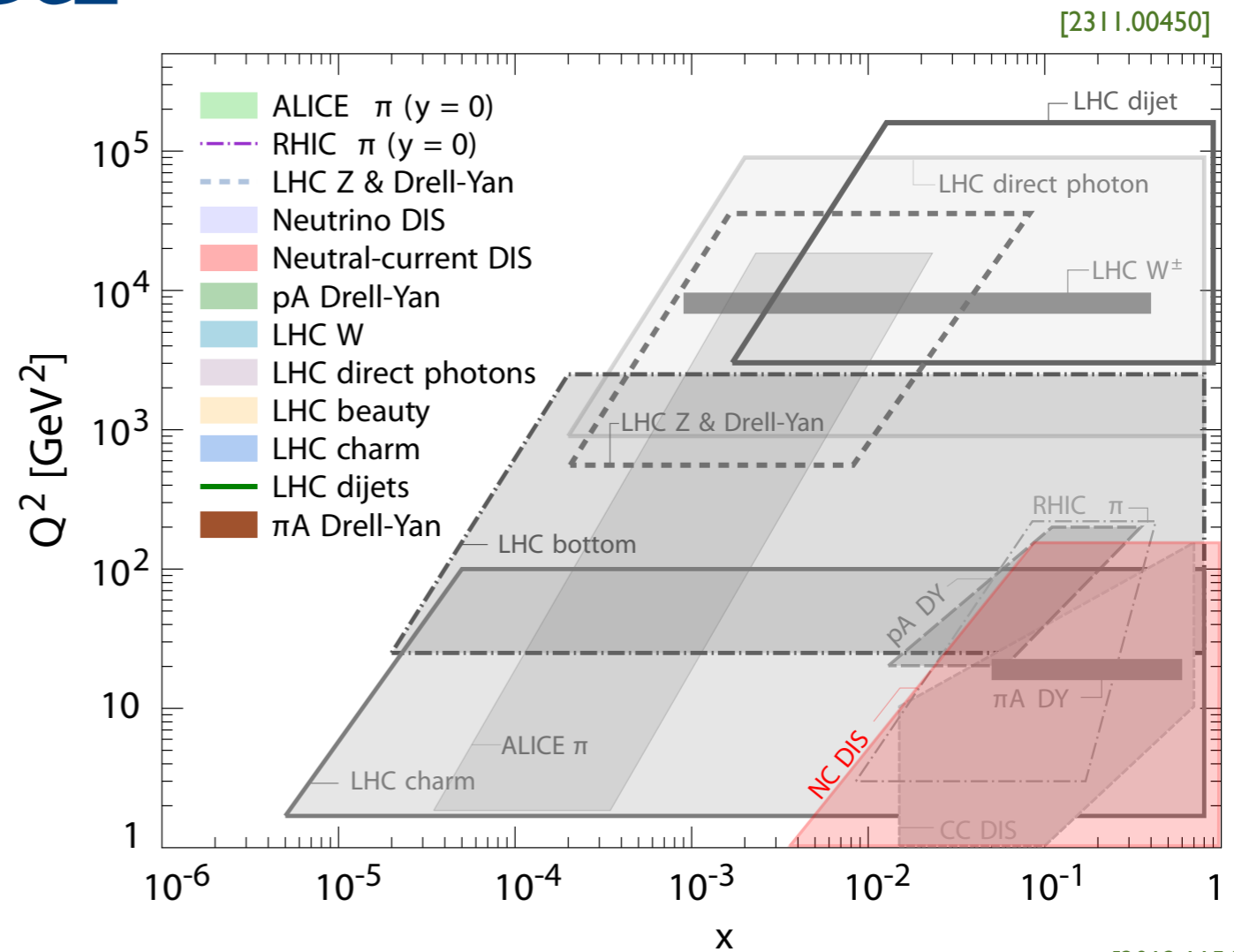
$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

DESY Hermes

$N = (\text{D, He, N, Kr})$

JLab CLAS & Hall C

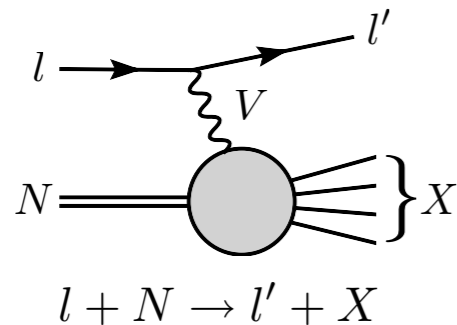
$N = (\text{He, Be, C, Al, Fe, Cu, Au, Pb})$



Nuclear data

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Deep Inelastic Scattering (NC)



CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

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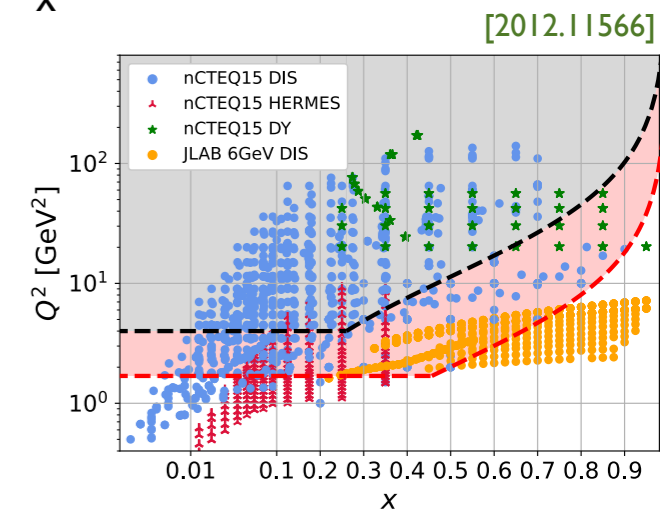
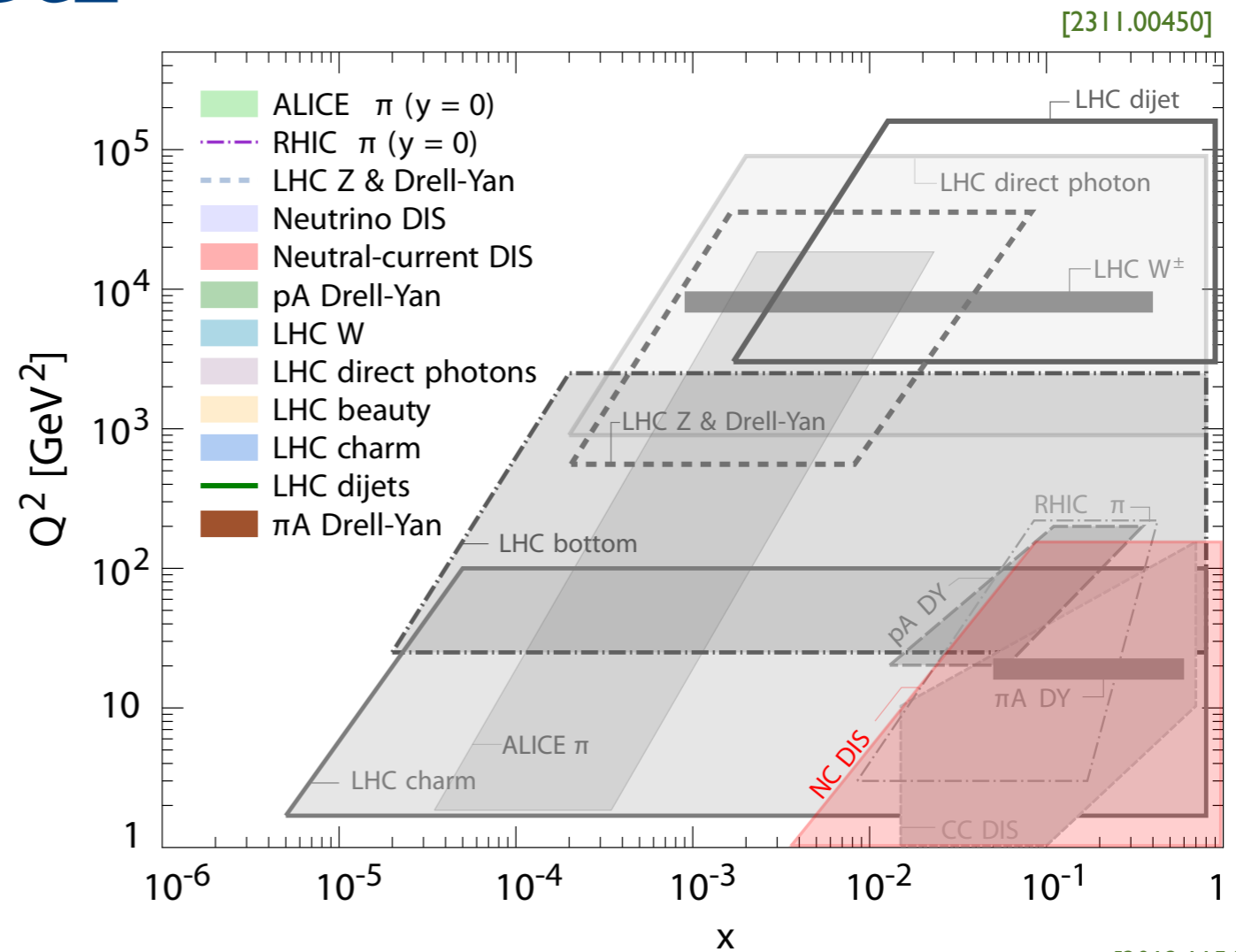
$N = (\text{He, Be, C, Al, Fe, Cu, Au, Pb})$

- Only photon exchange relevant - at leading order the structure function

$$F_2(x, Q^2) = \sum_q e_q^2 x [q(x, Q^2) + \bar{q}(x, Q^2)]$$

- Mainly constrains lin. combinations of (anti-)quark PDFs
- Many different nuclear targets, from 600 to 1350 data points depending on cuts

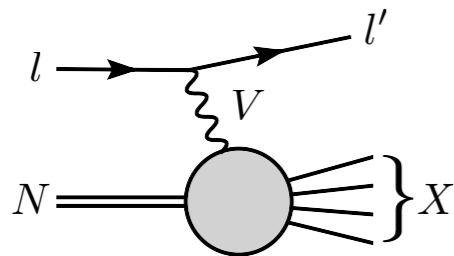
$$Q^2 > 4 \text{ GeV}^2 \quad W > 3.5 \text{ GeV} \quad \text{or} \quad Q^2 > 1.69 \text{ GeV}^2 \quad W > 1.7 \text{ GeV}$$



Nuclear data

- Data for nuclear PDFs

Deep Inelastic Scattering (NC)



$$l + N \rightarrow l' + X$$

CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

SLAC E-139 & E-049

$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

DESY Hermes

$N = (\text{D, He, N, Kr})$

JLab CLAS & Hall C

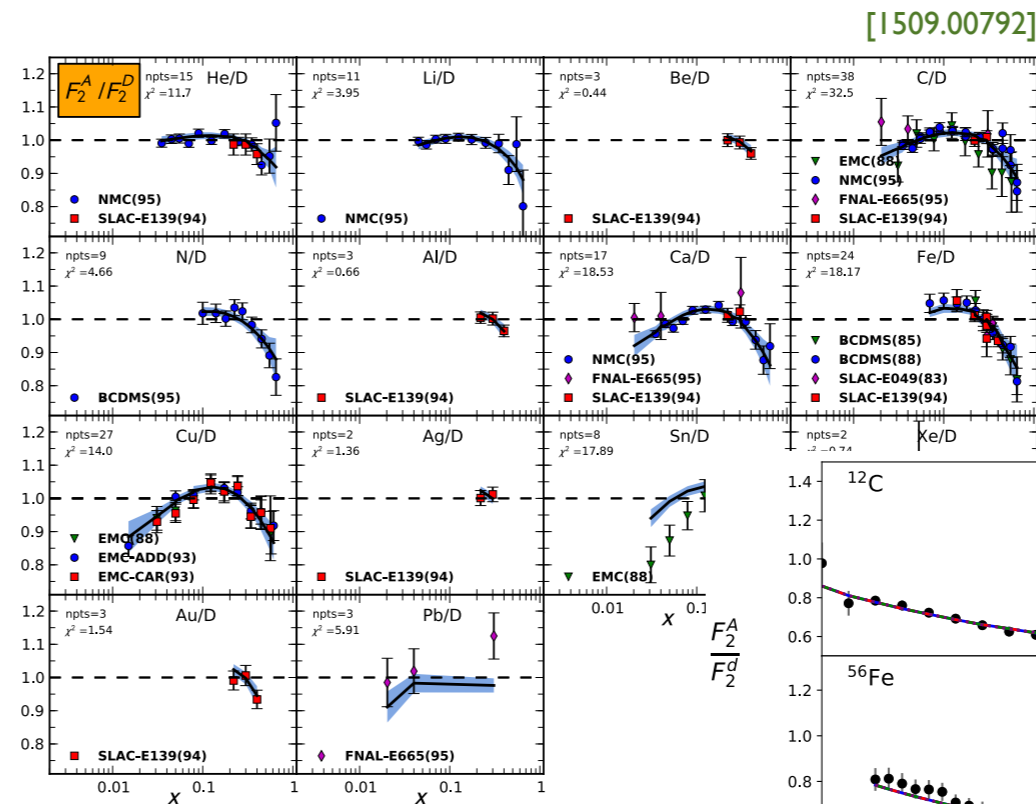
$N = (\text{He, Be, C, Al, Fe, Cu, Au, Pb})$

- Only photon exchange relevant - at leading order the structure function

$$F_2(x, Q^2) = \sum_q e_q^2 x [q(x, Q^2) + \bar{q}(x, Q^2)]$$

- Mainly constrains lin. combinations of (anti-)quark PDFs
- Many different nuclear targets, from 600 to 1350 data points depending on cuts

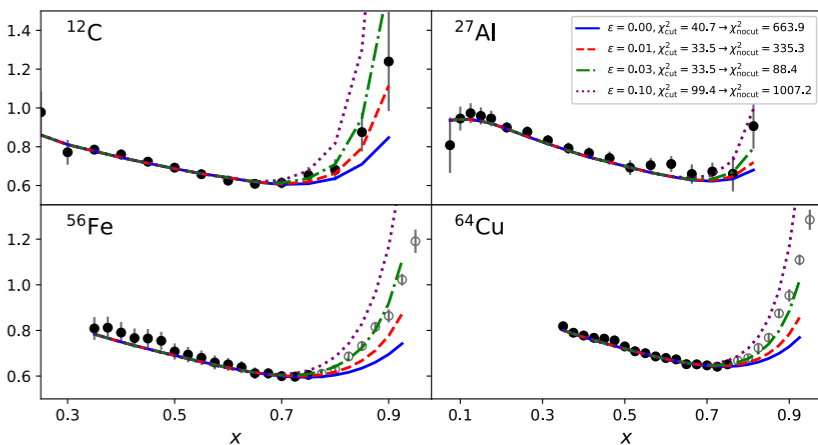
$$Q^2 > 4 \text{ GeV}^2 \quad W > 3.5 \text{ GeV} \quad \text{or} \quad Q^2 > 1.69 \text{ GeV}^2 \quad W > 1.7 \text{ GeV}$$



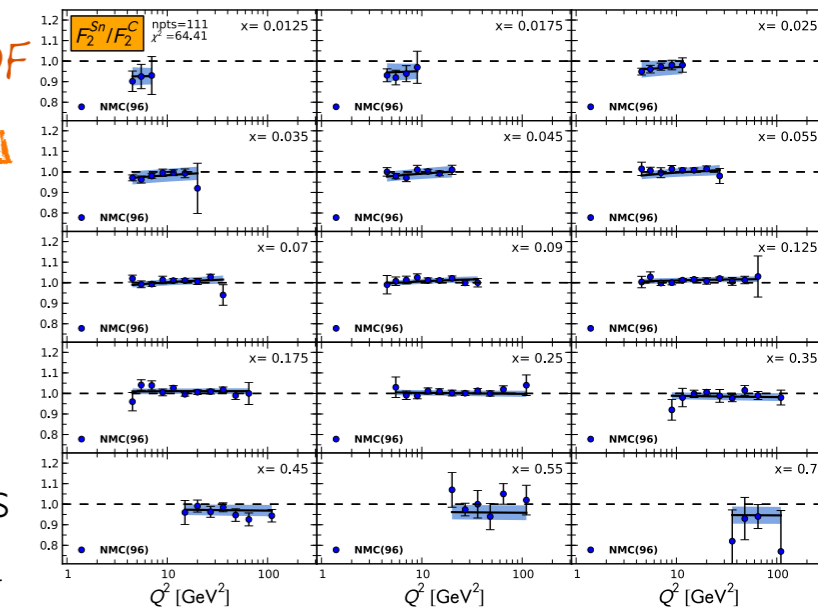
[1509.00792]

high- x JLab data

[2012.11566]



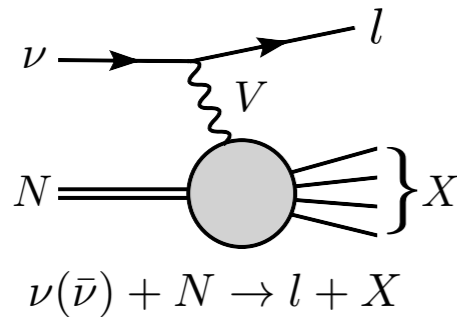
sensitive to gluon PDF



Nuclear data

- Data for nuclear PDFs

Deep Inelastic Scattering (CC)



CDHSW & CCFR & NuTeV

$N = \text{Fe}$

Chorus

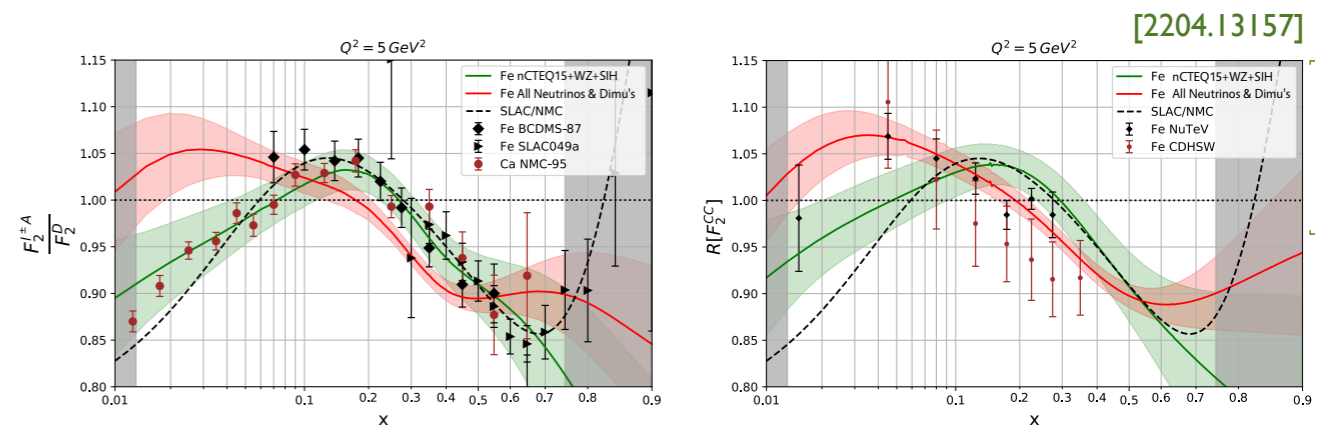
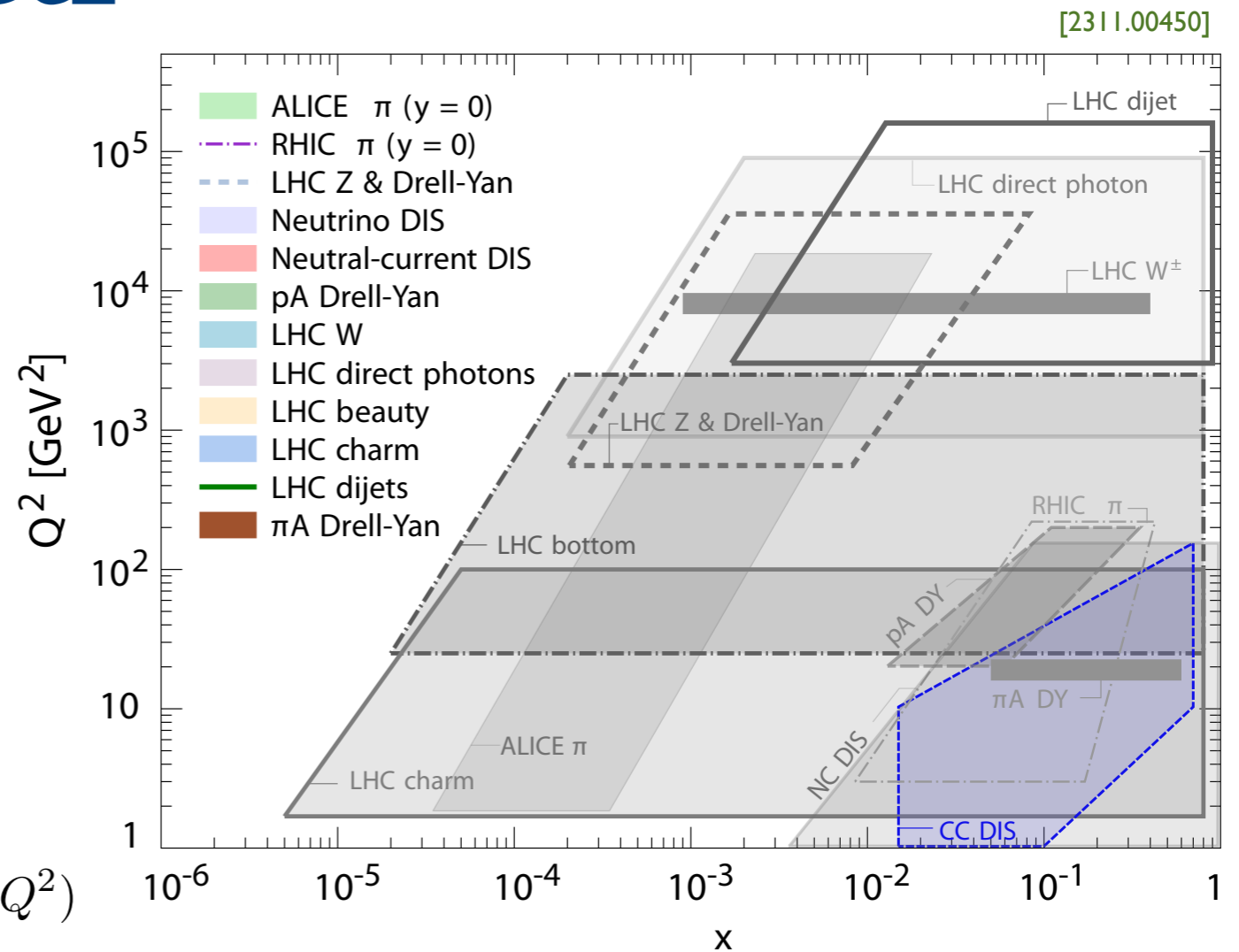
$N = \text{Pb}$

- neutrino DIS contributes to $F_2(x, Q^2)$ and $F_3(x, Q^2)$

- different PDF combinations contribute to flavor separation together with NC DIS

$$F_2(x, Q^2) = x \sum_q [q(x, Q^2) + \bar{q}(x, Q^2)]$$

$$xF_3(x, Q^2) = x \sum_q [q(x, Q^2) - \bar{q}(x, Q^2)]$$

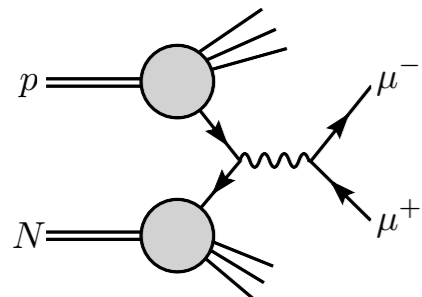


Nuclear data

[2311.00450]

- Data for nuclear PDFs

Drell-Yan process



$$p + N \rightarrow \mu^+ \mu^- + X$$

FNAL E-665

$N = (D, C, Ca, Pb, Xe)$

LHC Run I - ATLAS, CMS Z^0, W^+, W^-

- ALICE W^+, W^-

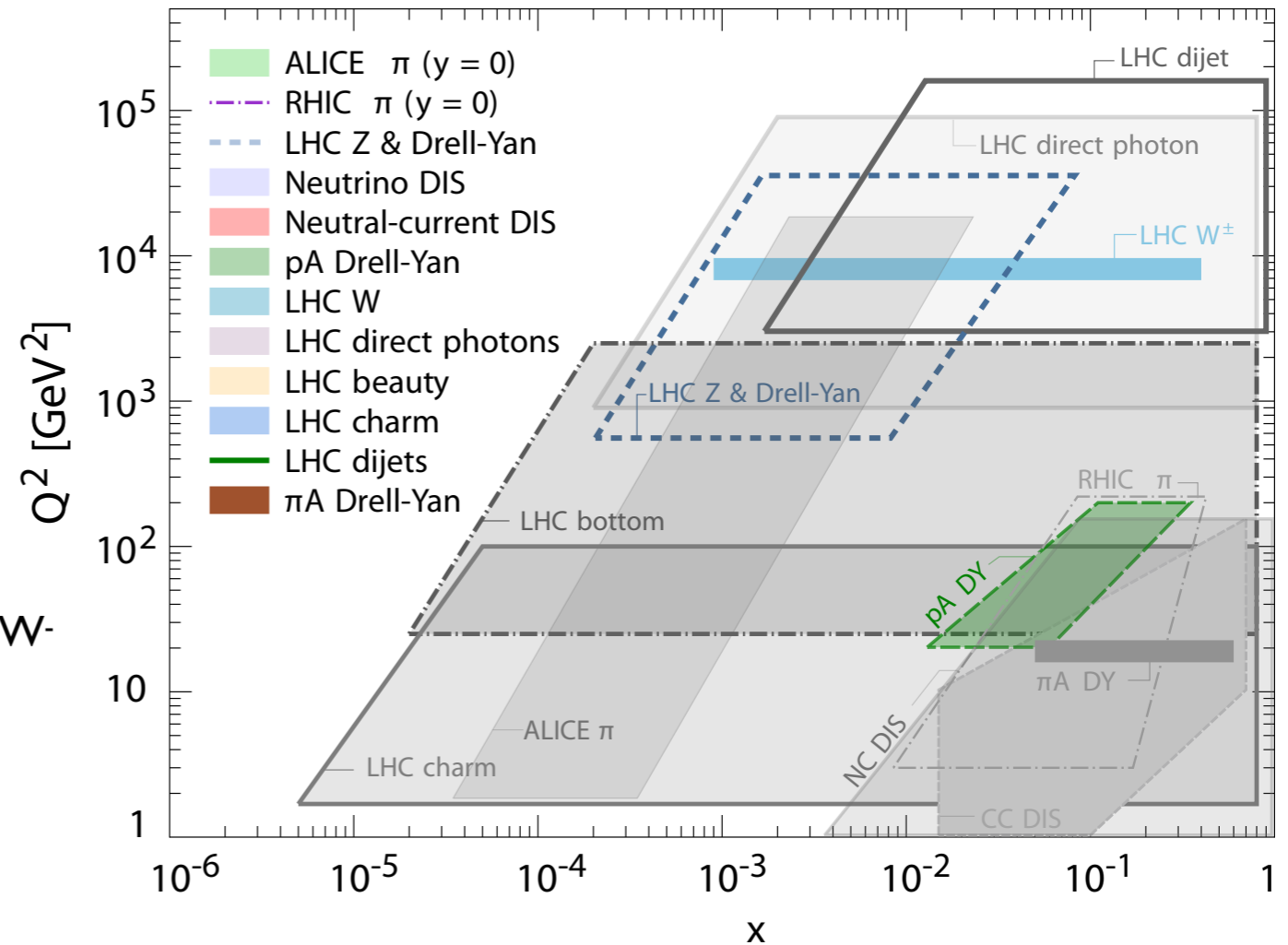
FNAL E-772 & E-886

$N = (D, C, Ca, Fe, W)$

- LHCb Z^0

LHC Run II - CMS W^+, W^-

$N = Pb$



- DY dominated by photon exchange away from W & Z resonances

$$\frac{d\sigma}{dQ^2 dy} = \frac{4\pi\alpha^2}{9Q^2 s} \sum_i e_i^2 [q_i(x_a, Q^2) \bar{q}_i(x_b, Q^2) + a \leftrightarrow b]$$

$$Q^2 = (p_l + p_{\bar{l}})^2$$

$$x_{a,b} = \frac{Q}{\sqrt{s}} \exp(\pm y)$$

- DY at the W & Z resonances - different PDF combinations

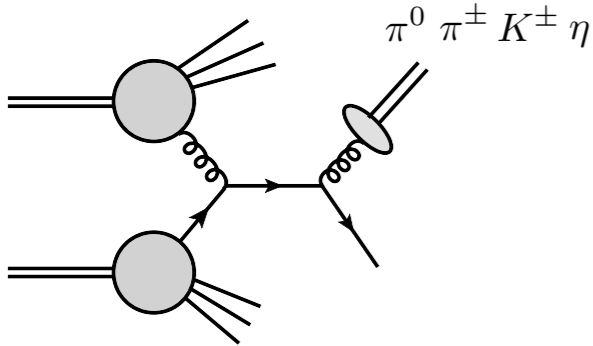
$$\frac{d\sigma^W}{dy} = \frac{\sqrt{2}\pi G_F m_W^2}{3s} \sum_{i,j} |V_{ij}^{CKM}| [q_i(x_a, Q^2) \bar{q}_j(x_b, Q^2) + a \leftrightarrow b]$$

$$\frac{d\sigma^Z}{dy} = \frac{\sqrt{2}\pi G_F m_Z^2}{3s} \sum_i (V_i^2 + A_i^2) [q_i(x_a, Q^2) \bar{q}_i(x_b, Q^2) + a \leftrightarrow b]$$

Nuclear data

• Data for nuclear PDFs

Single hadron production



RHIC - PHENIX $\pi^0 \pi^\pm K^\pm \eta$

$N = \text{Au}$

RHIC - STAR $\pi^0 \pi^\pm \eta$

$N = \text{Au}$

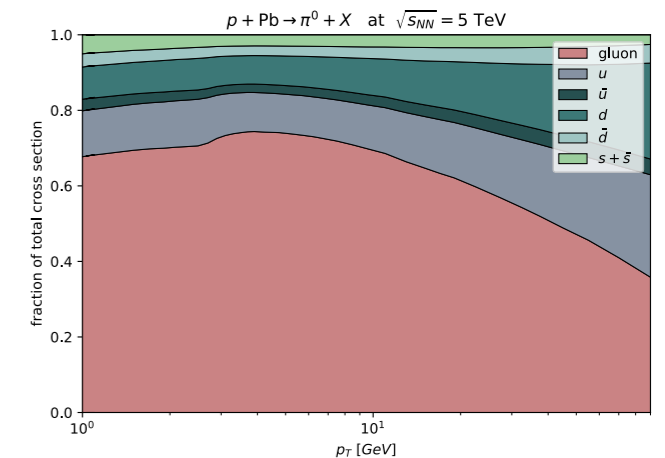
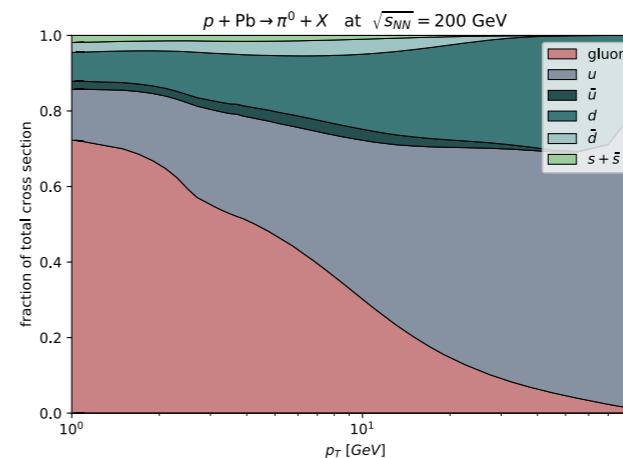
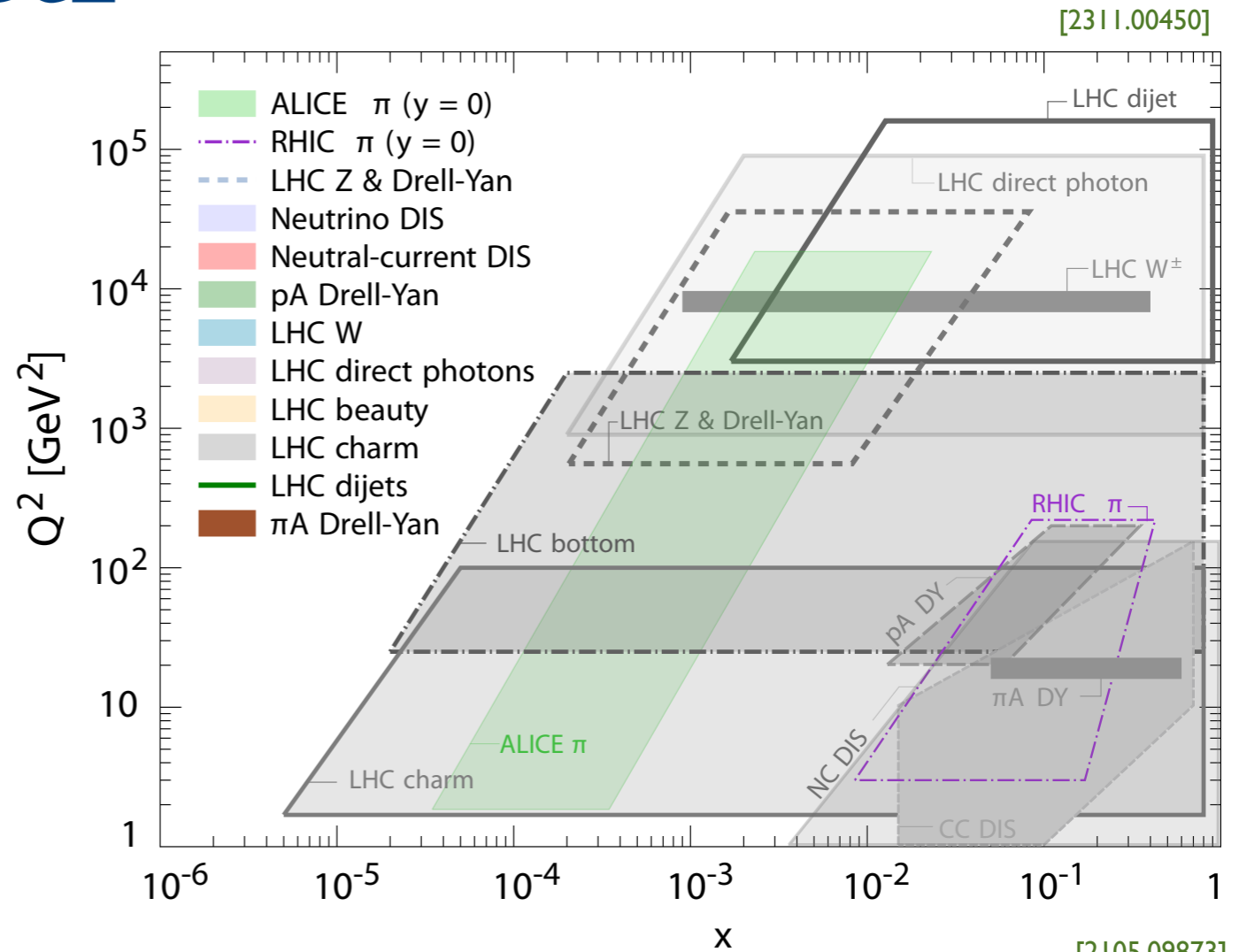
LHC - ALICE $\pi^0 \pi^\pm K^\pm \eta$

$N = \text{Pb}$

• can constrain gluon PDF (for lower p_T RHIC, any p_T LHC), abundant data

• pQCD cannot be used for very low p_T ($p_T < 3$ GeV)

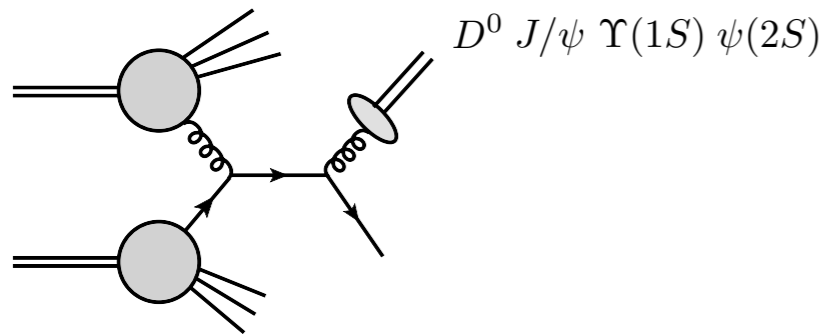
• fragmentation functions necessary for any prediction



Nuclear data

Data for nuclear PDFs

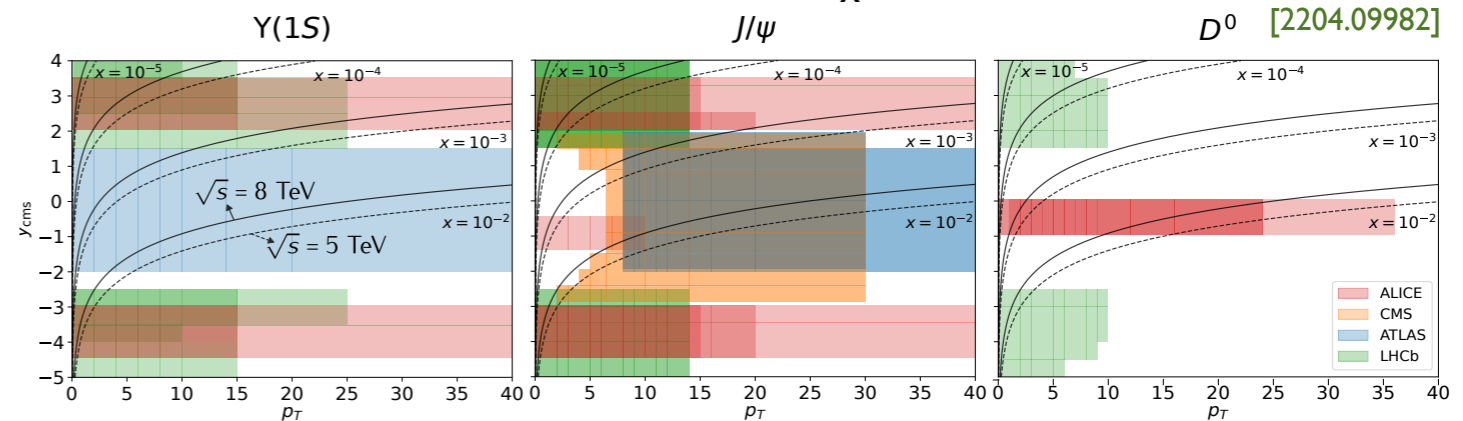
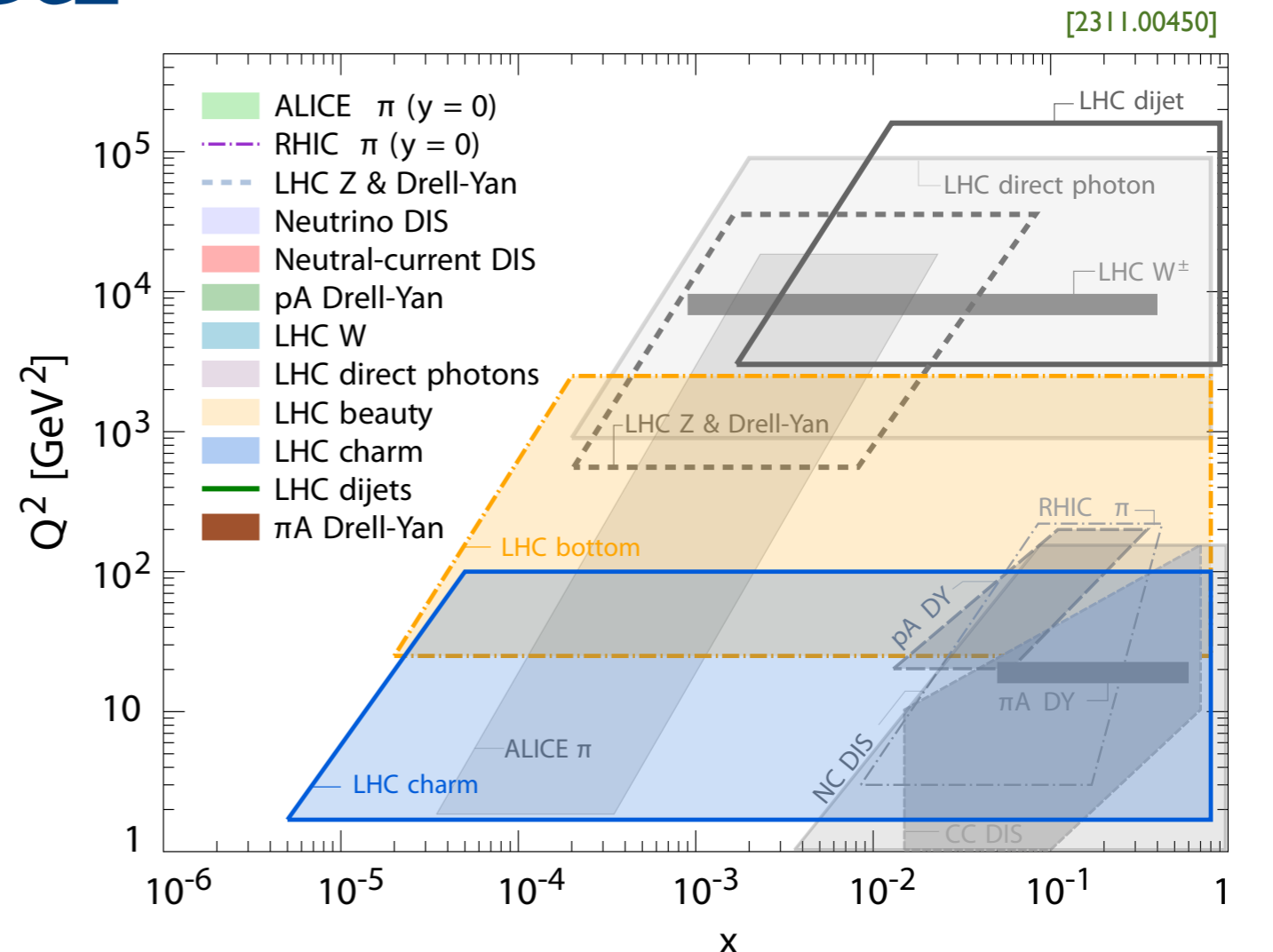
Heavy quark production



- LHC - ATLAS $J/\psi \ \Upsilon(1S) \ \psi(2S)$
- CMS $J/\psi \ \Upsilon(1S) \ \psi(2S)$
- LHCb $D^0 \ J/\psi \ \Upsilon(1S)$
- ALICE $D^0 \ J/\psi \ \Upsilon(1S) \ \psi(2S)$

$N = \text{Pb}$

- sensitive to gluon PDF down to extremely low x
- pQCD cannot be used for quarkonia
 - NRQCD eff. field theory required
- fragmentation functions necessary for open HQ predictions



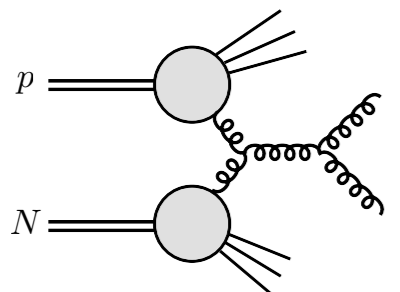
quarkonia $Q\bar{Q}$ -bound states

open HQ - $Q\bar{q}$ states

Nuclear data

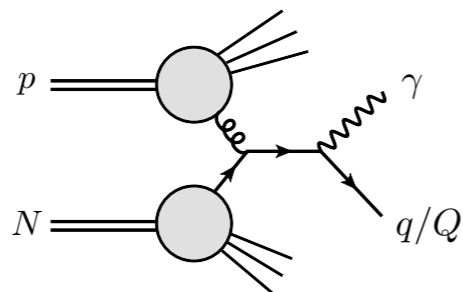
Data for nuclear PDFs

Di-jet production



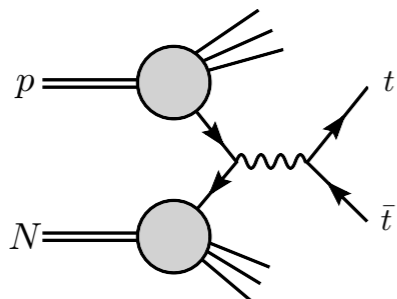
$$p + N \rightarrow 2 \text{ jets} + X$$

Direct photon production



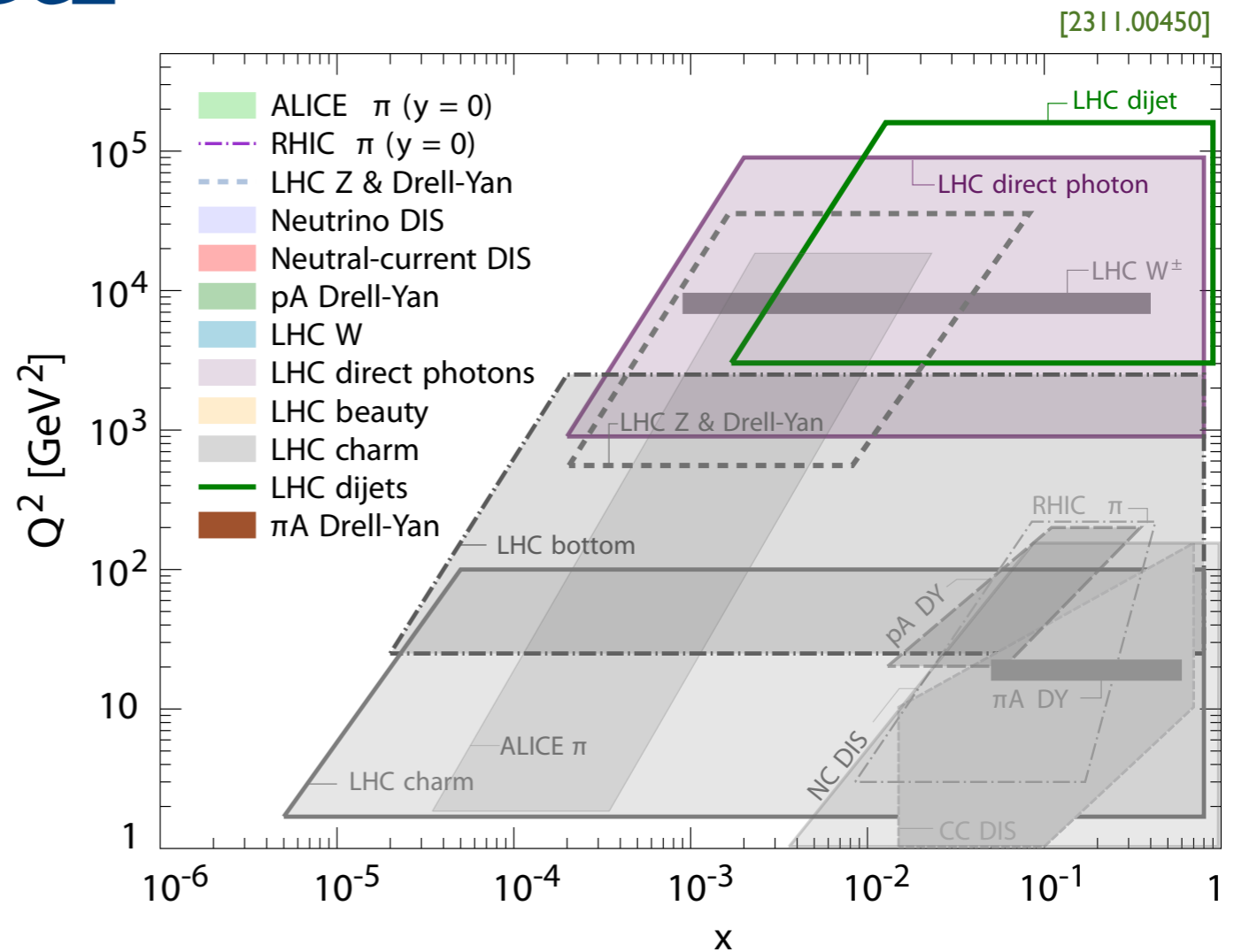
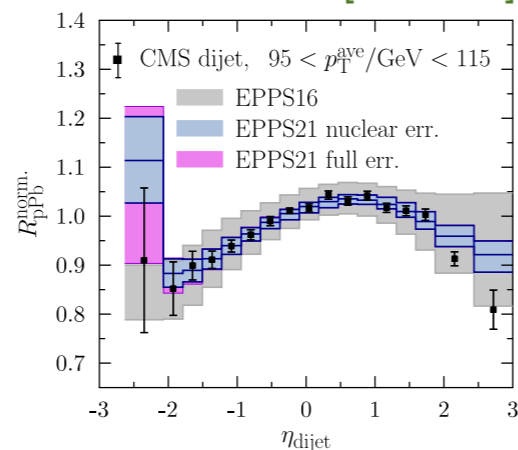
$$p + N \rightarrow \gamma q/Q + X$$

Top-quark production



$$p + N \rightarrow t \bar{t} + X$$

[2112.12462]

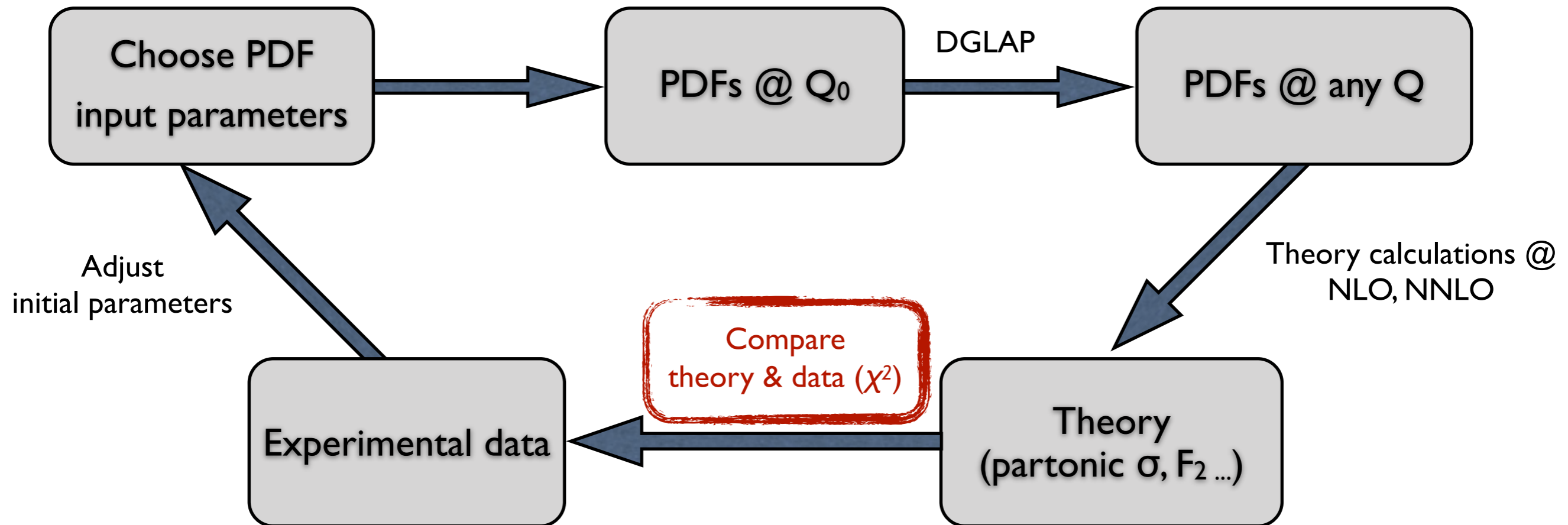


[2311.00450]

- Di-jet data - promising - can reduce gluon PDF uncertainty **BUT** pp and pPb data not described well (only ratios)
- Direct photon data - additional constraint of the gluon PDF **BUT** precision of the data cannot compete with HQ
- Top quark data - ultimate constraint of the gluon PDF **BUT** only 2 data points - total cross-section

Determining nPDFs

- Determining (nuclear) parton distribution functions
- how do we determine them? What are the moving parts in a typical PDF fitting-machine?



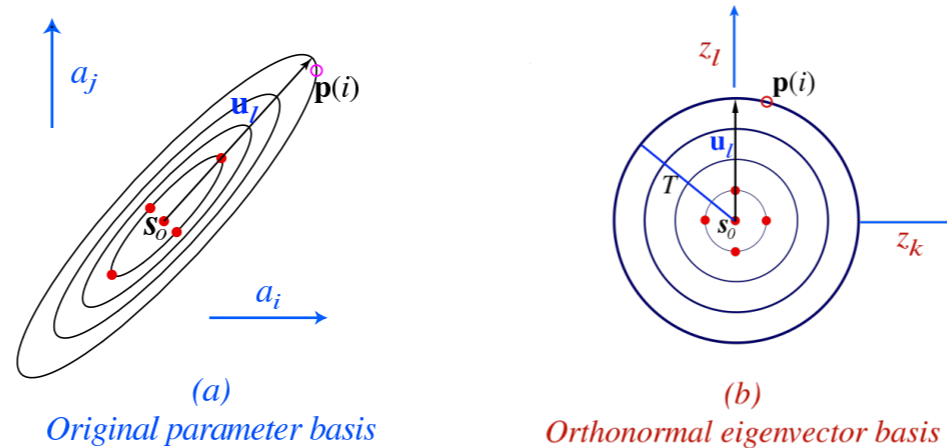
Uncertainties

- Uncertainties of (nuclear PDF)

- Hessian approach to uncertainties

$$\chi^2(a) = \chi_0^2 + \frac{1}{2} \frac{\partial^2 \chi^2}{\partial a_i \partial a_j} (a - a_0)_i (a - a_0)_j + \dots \rightarrow \chi_0^2 + \sum_i z_i^2$$

Hessian diagonal Hessian



- Choice of $\Delta\chi^2 = \chi^2 - \chi_0^2$: $\Delta\chi^2 \sim 20 - 50$

- Construct error PDFs for each parameter in 2 directions (#error PDF sets = $2N_{\text{par}}$):

$$z_i = \pm \sqrt{\Delta\chi^2} \quad i = 1, \dots, N_{\text{par}} \quad X_i^\pm(z) = X_i^\pm(0, 0, \dots, \pm \sqrt{\Delta\chi^2}, \dots, 0, 0) \quad X = f_k(x, Q_0^2)$$

error PDFs

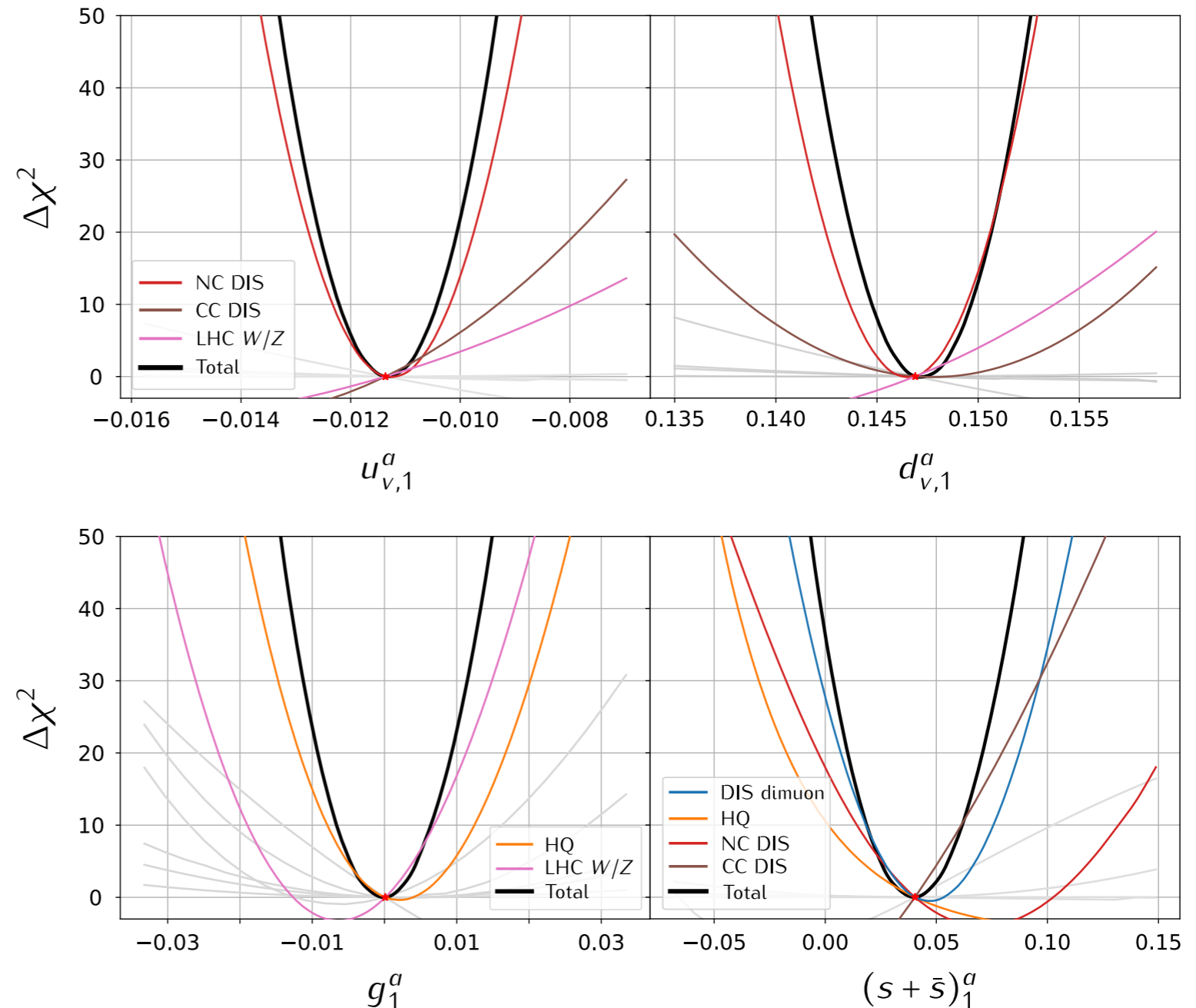
- Calculate PDF uncertainty of cross-section

$$(\Delta\sigma)^2 \approx \frac{1}{4} \sum_i^{N_p} \left(\sigma(X_i^+) - \sigma(X_i^-) \right)^2$$

Uncertainties

• Uncertainties of (nuclear PDF)

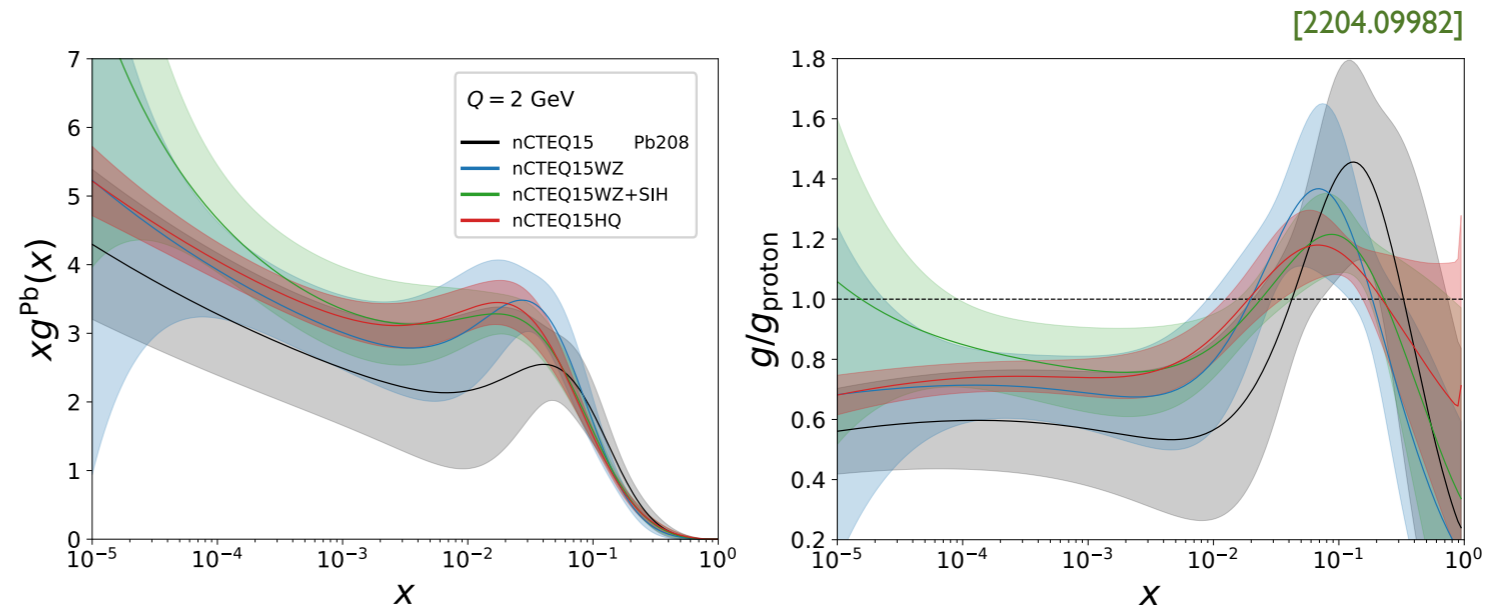
- subsets of data constrain different PDFs at different x
- sensitivity can be made visible by χ^2 -scans for single experiments
- NC & CC DIS constrain up & down quark PDFs
- W/Z Drell-Yan and HQ data constrain gluon PDFs
- di-muon CC DIS are very sensitive to the strange PDF



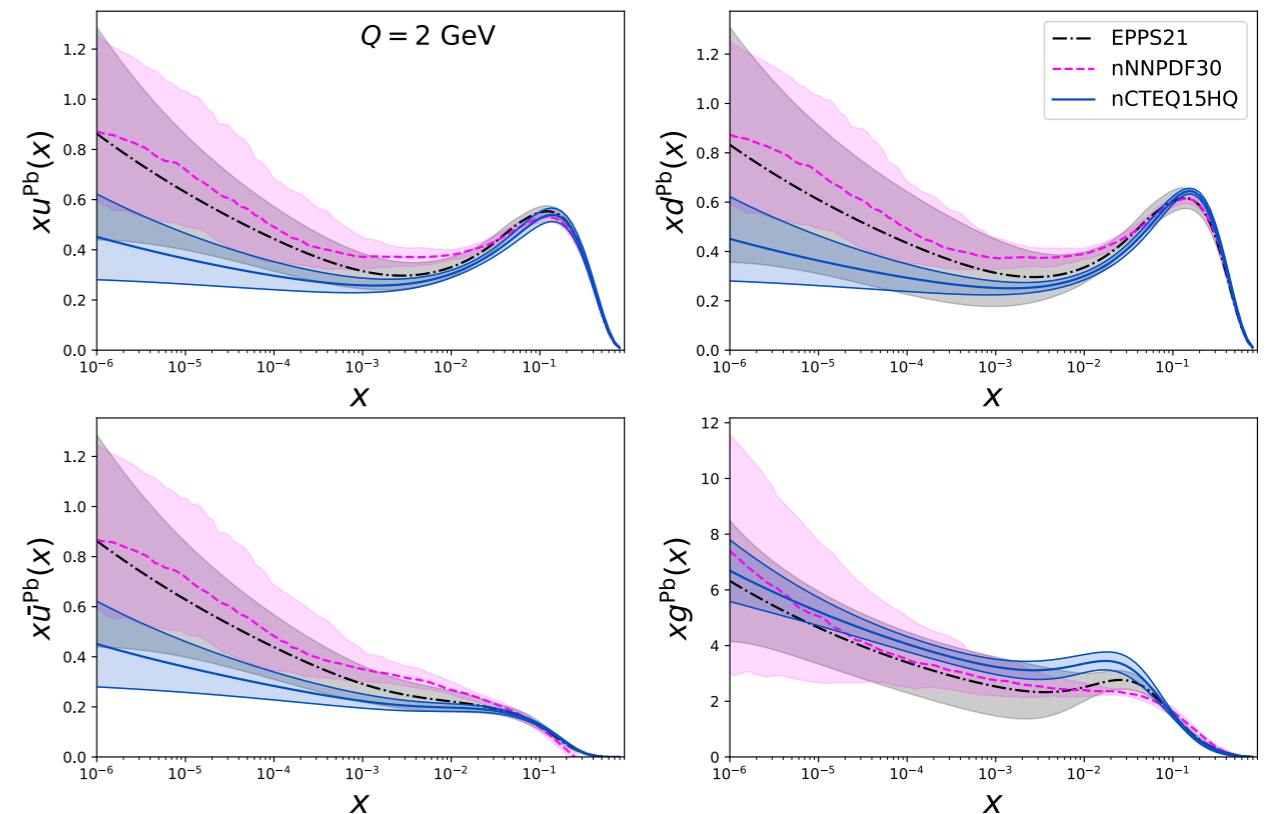
Nuclear PDFs

Current nPDF analyses

- adding more precise LHC pPb data improves the uncertainty
- framework, fitting approaches and also data selection very different - different nPDF analyses can seem incompatible

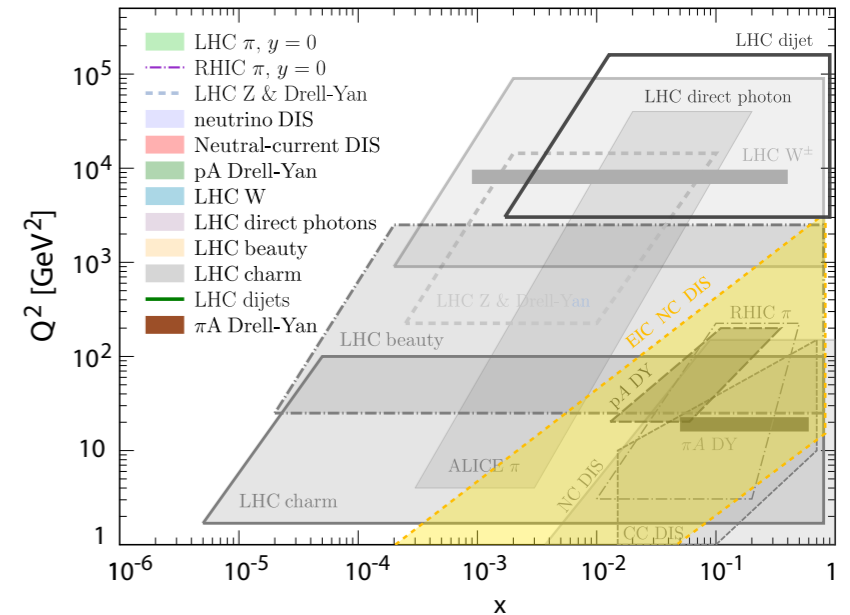
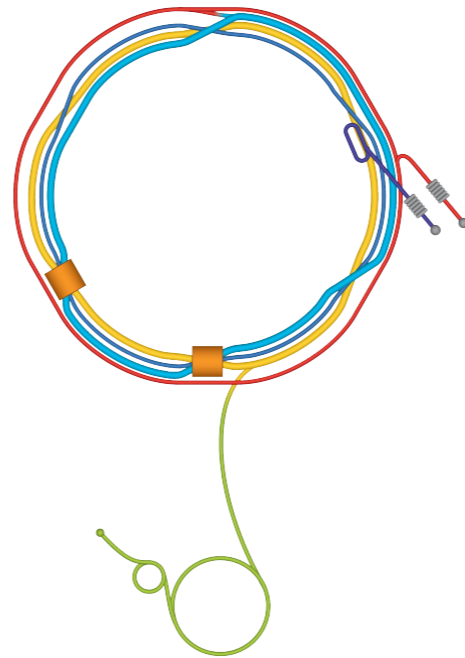


more to come from LHC
but above all ...

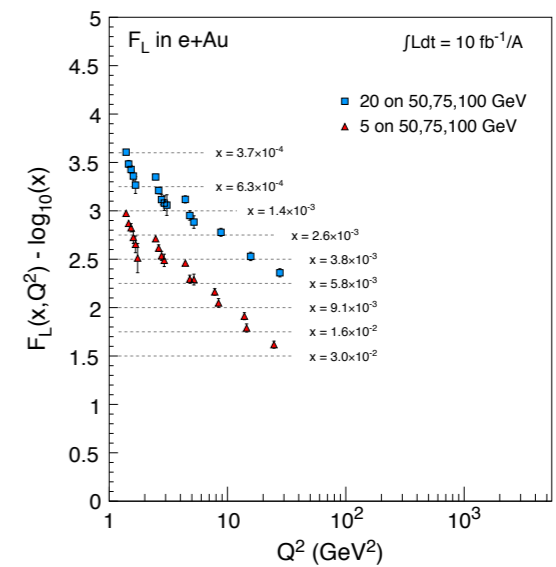
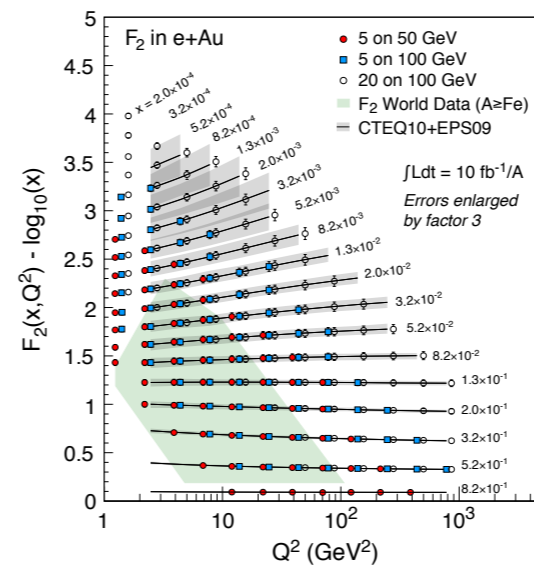
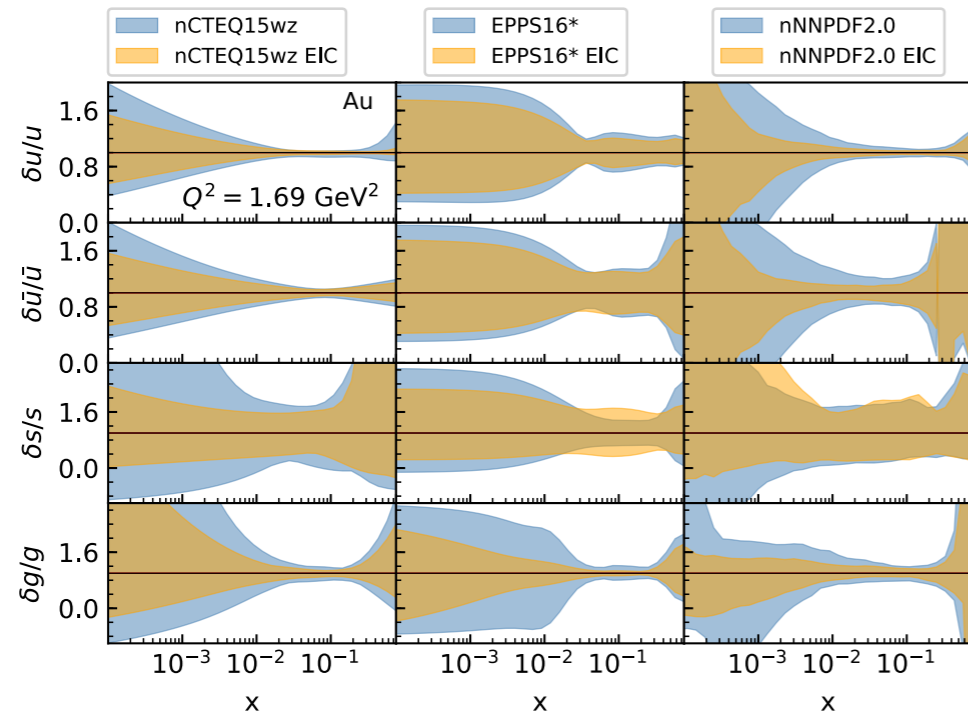


Future of nPDFs

- Electron-Ion collider (EIC)
 - in preparation in BNL - start ~ 2030
 - main goal - spin & flavor structure of nucleon
 - center-of-mass energy ~ 100 GeV
 - high-luminosity ~ $10^{34} \text{ cm}^{-2}\text{s}^{-1}$



- very precise DIS on nuclei - same as HERA for protons



Thanks