

# Jets and their structure

Simone Marzani  
Università di Genova &  
INFN Sezione di Genova

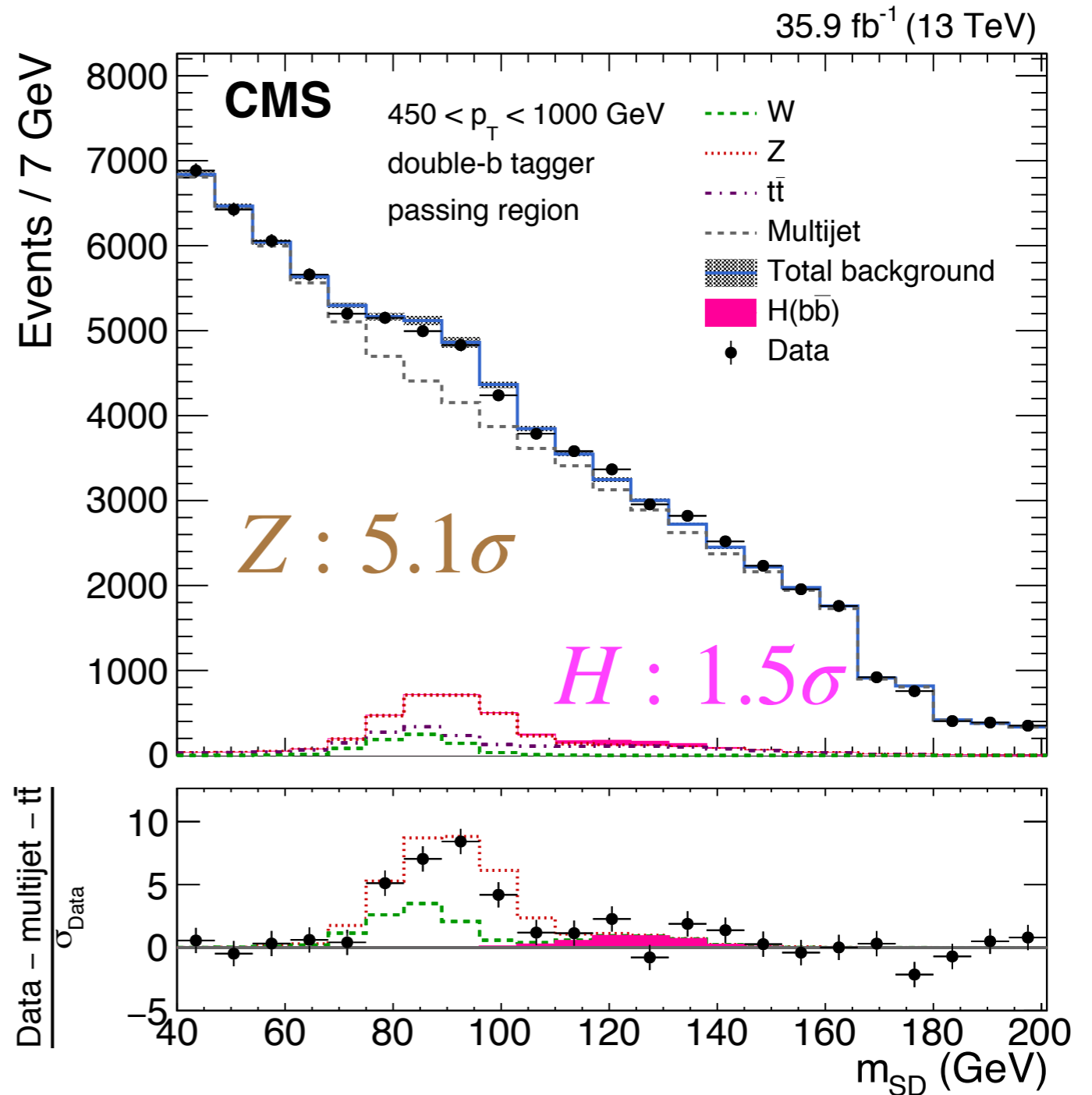


**CTEQ 2024**  
**summer school**



# Lecture 2: jet substructure

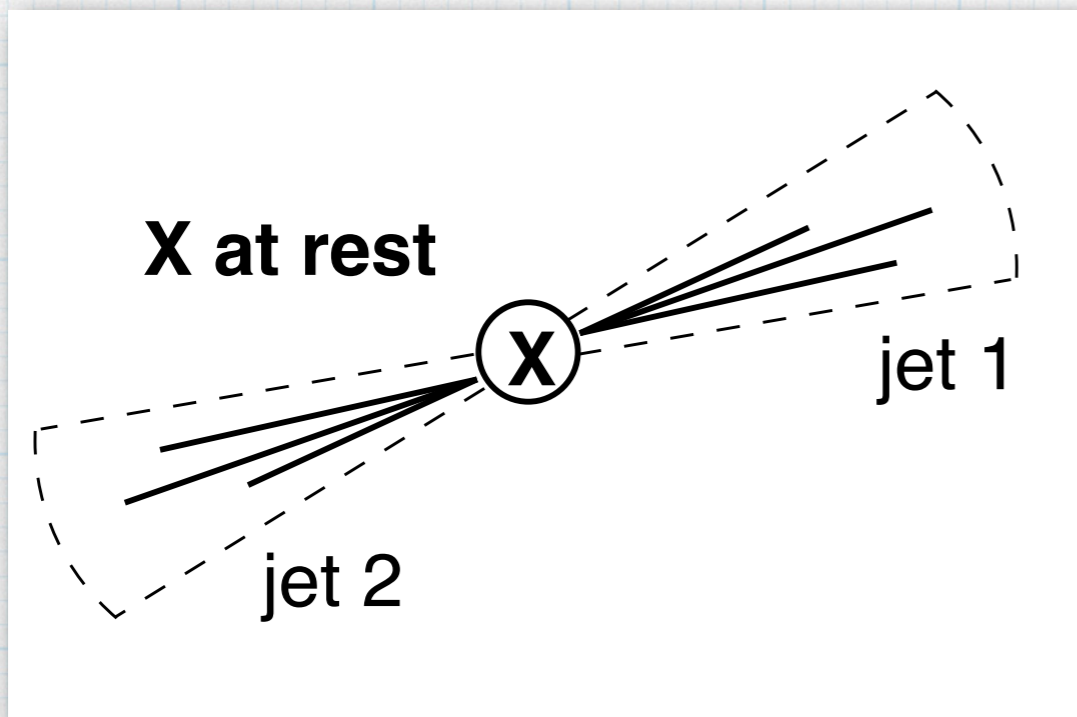
- \* boosted-objects physics
- \* grooming and tagging
- \* calculations for jet substructure



the (ambitious) target of this lecture is to understand this plot

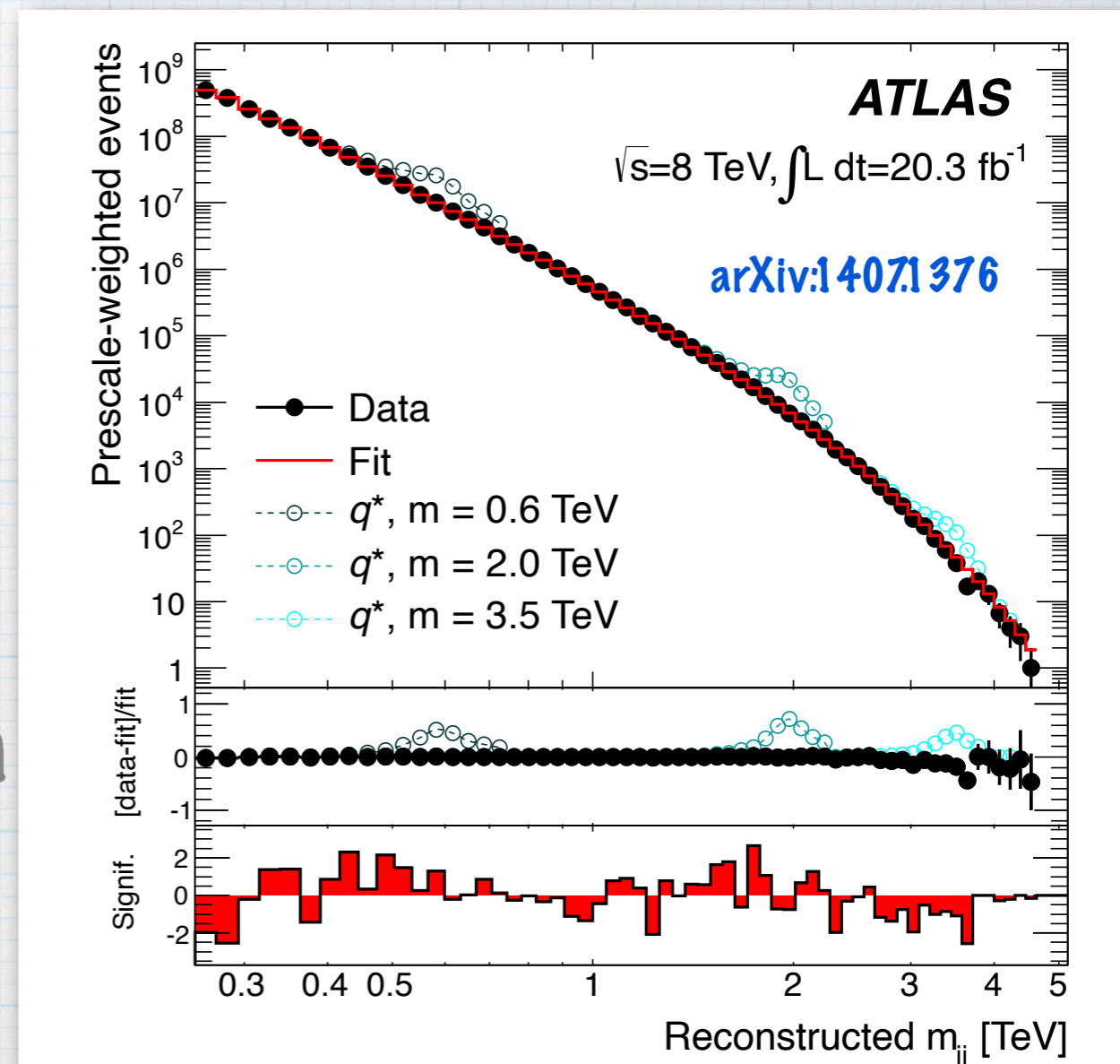
# searching for new particles (I)

- \* Standard analysis: the heavy particle  $X$  decays into two partons, reconstructed as two jets



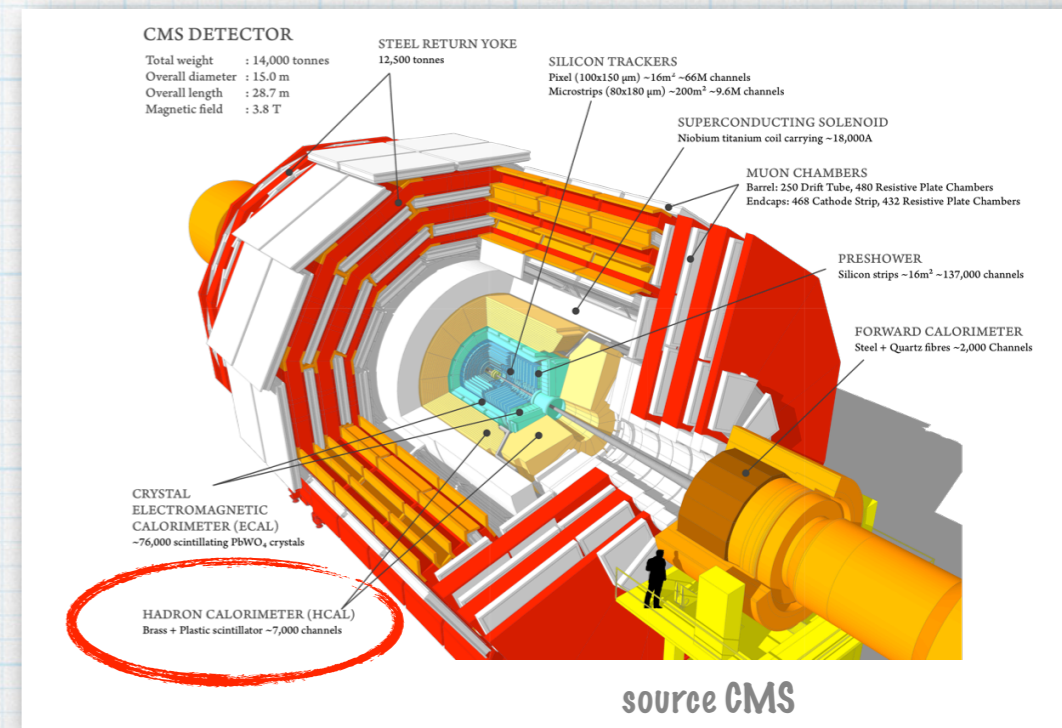
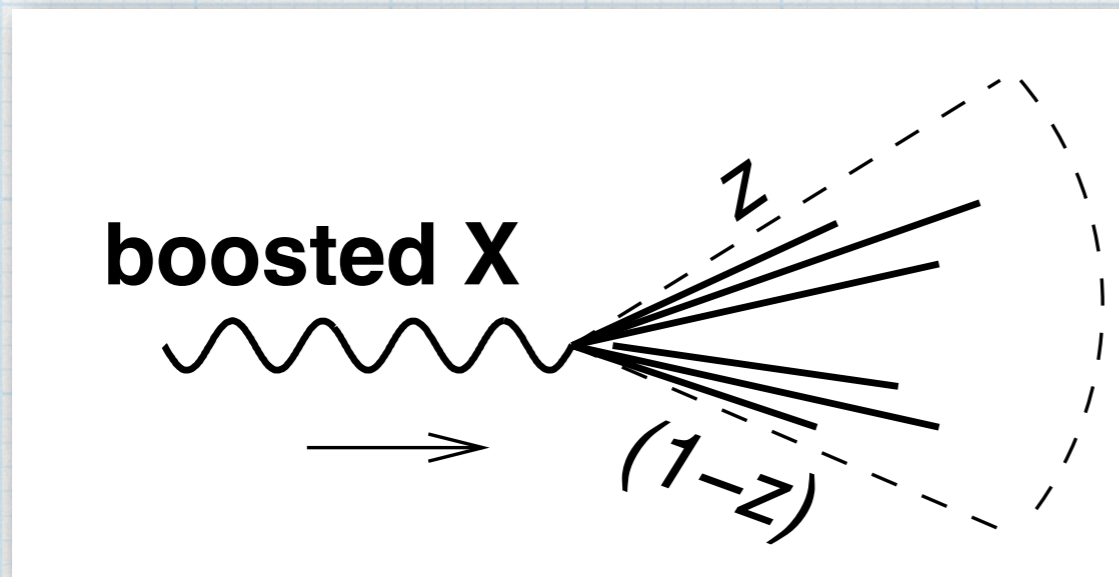
- \* Look for bumps in the dijet invariant mass distribution

- \* What about EW-scale particles at the LHC?



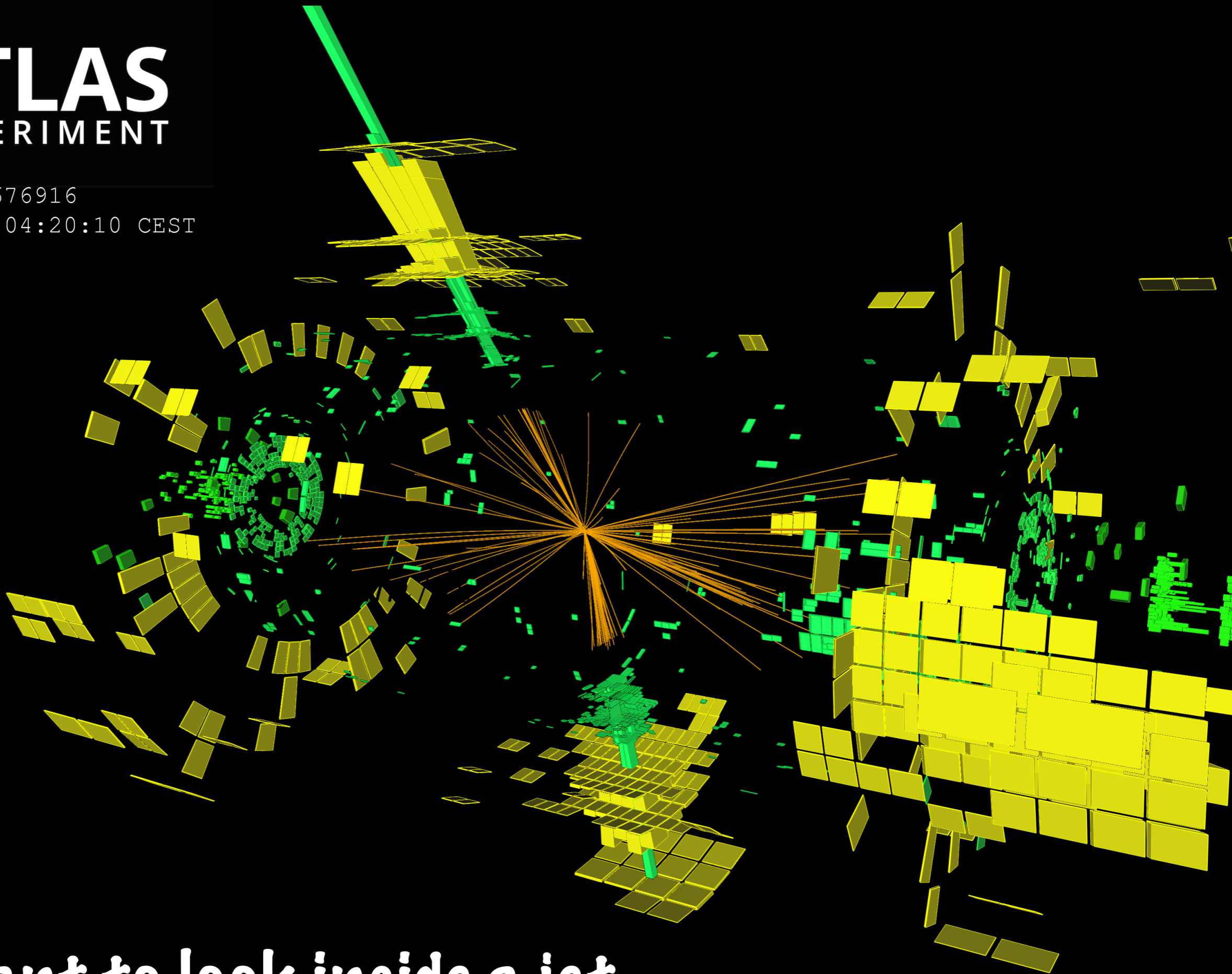
# searching for new particles (II)

- \* LHC energy ( $10^4$  GeV)  $\gg$  electro-weak scale ( $10^2$  GeV)
- \* EW-scale particles (new physics, Z/W/H/top) are abundantly produced with a large boost



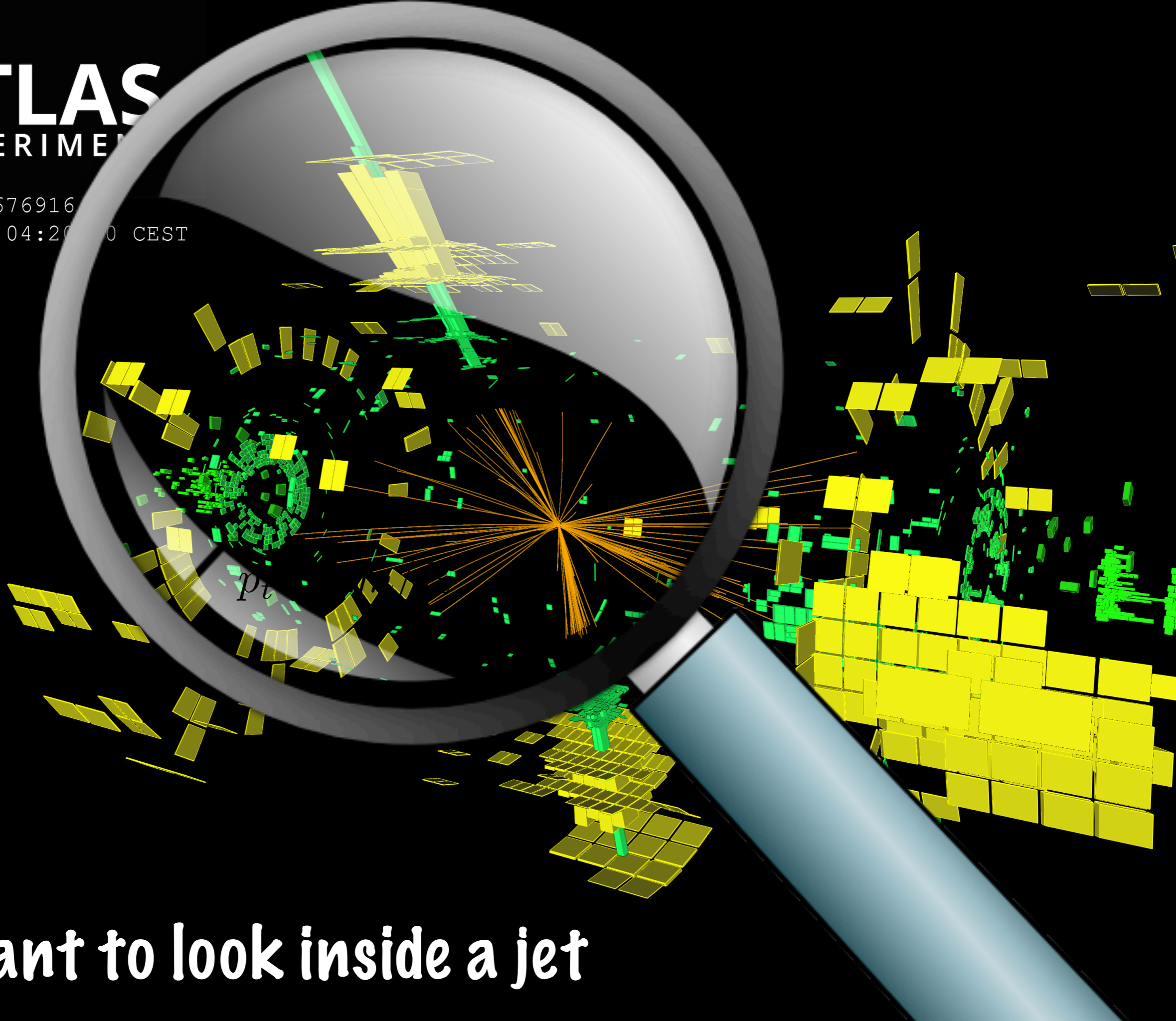
- \* their decay-products are then collimated
- \* if they decay into hadrons, we end up with localised deposition of energy in the hadronic calorimeter: **a jet**

Event: 531676916  
2015-08-22 04:20:10 CEST



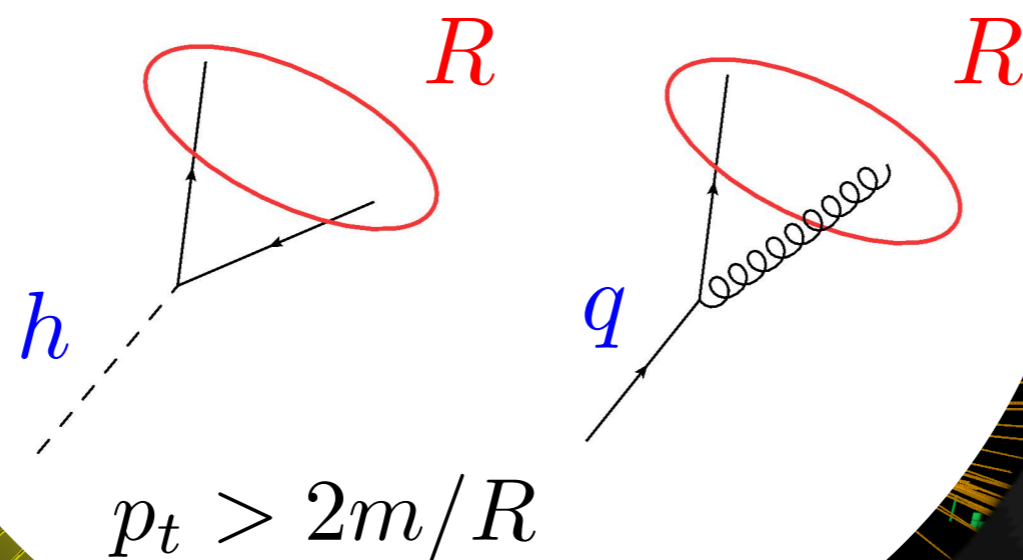
**we want to look inside a jet**

Event: 531676916  
2015-08-22 04:20:00 CEST



we want to look inside a jet

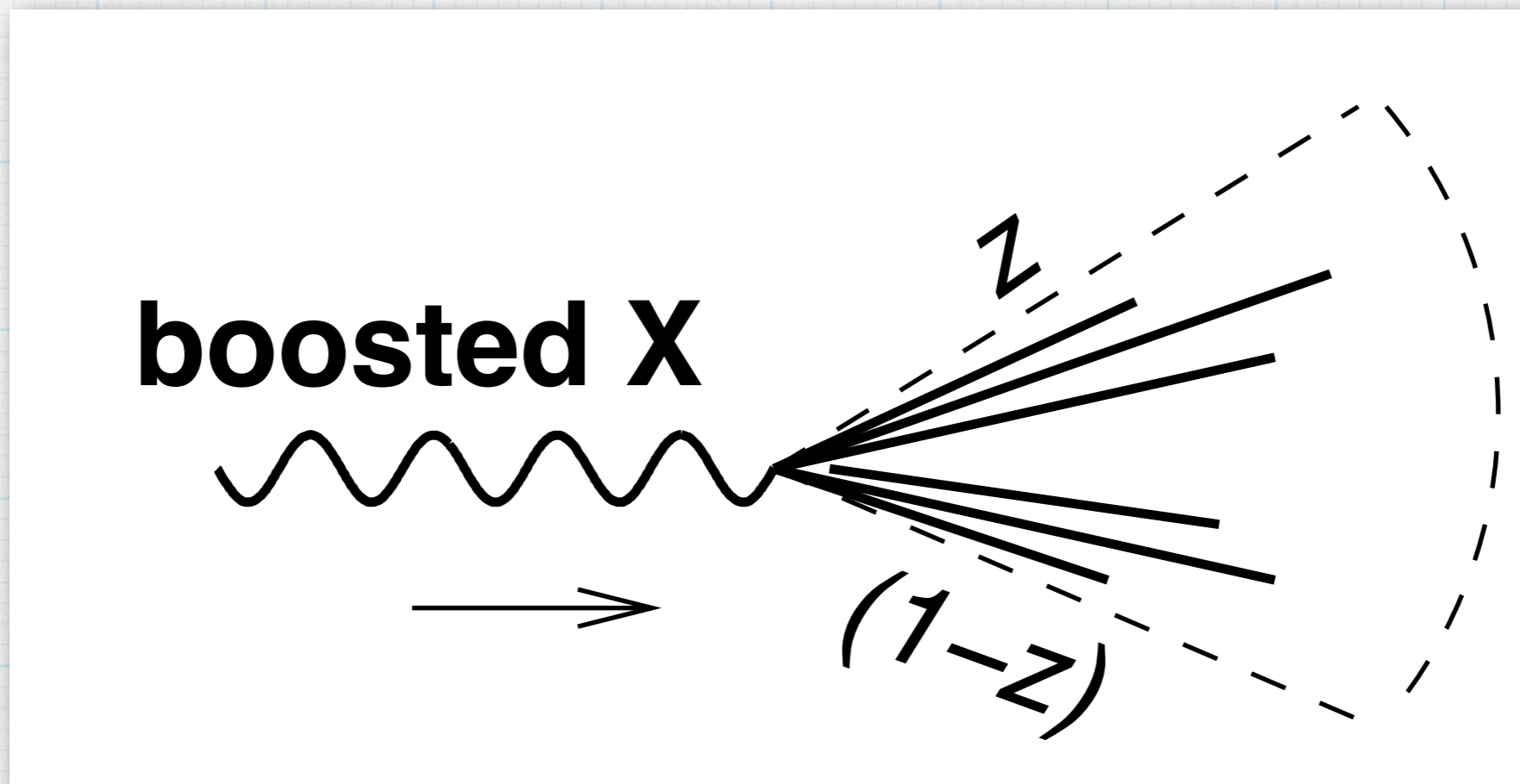
exploit jets' properties  
to distinguish signal  
jets from bkgd jets



we want to look inside a jet

# signal-jet mass

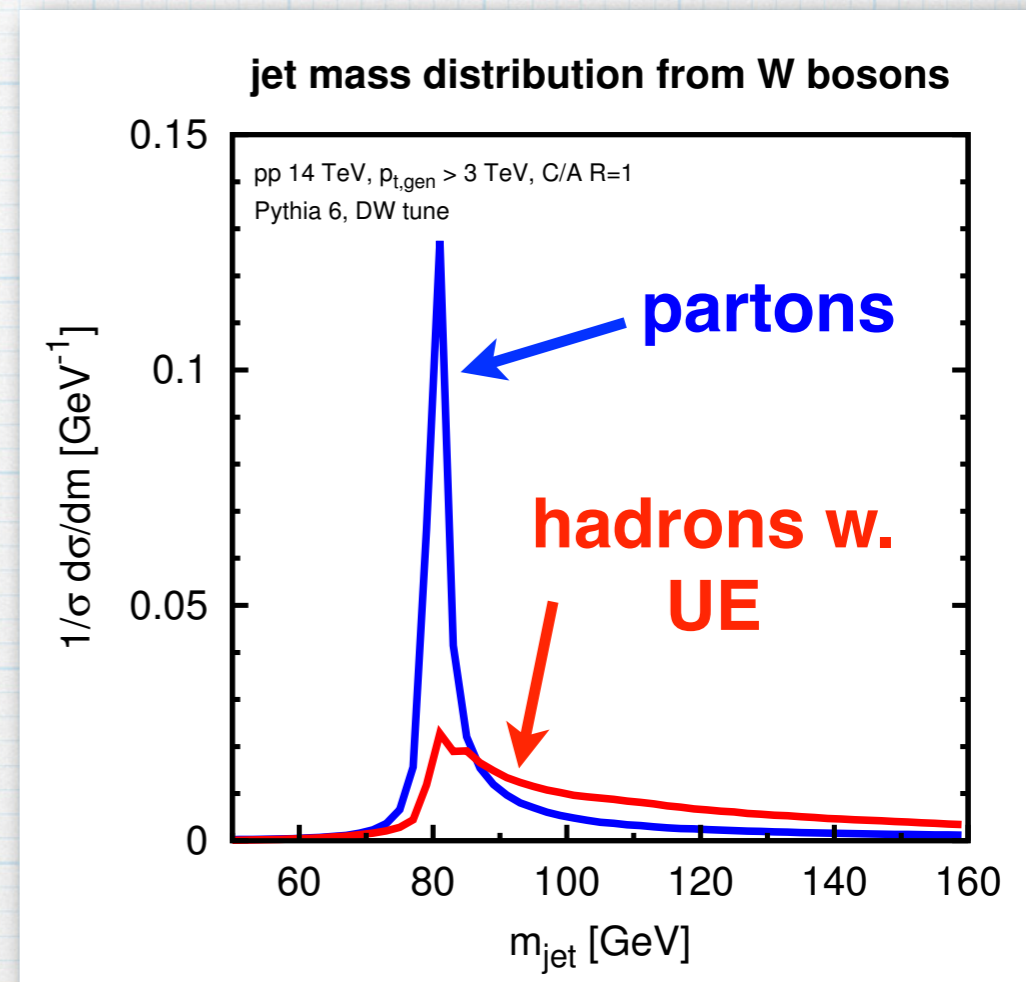
- \* first jet-observable that comes to mind
- \* signal jets should have a mass distribution peaked near the resonance





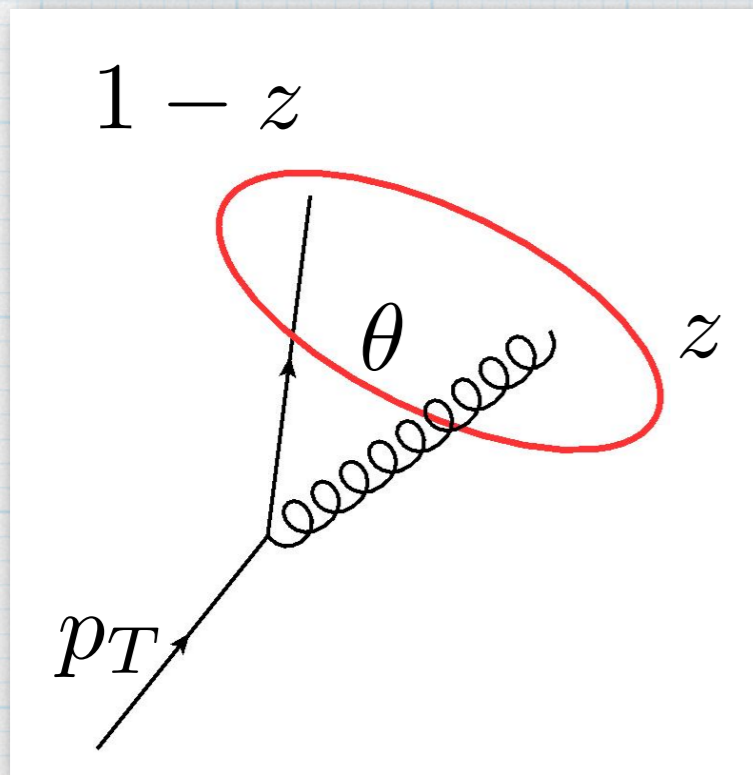
# signal-jet mass

- \* first jet-observable that comes to mind
- \* signal jets should have a mass distribution peaked near the resonance
- \* however, that's a simple partonic picture
- \* perturbative and non-pert. emissions from the qq̄ pair broadens and shift the peak
- \* underlying event and pile-up typically enhance the jet mass



# QCD-jet mass

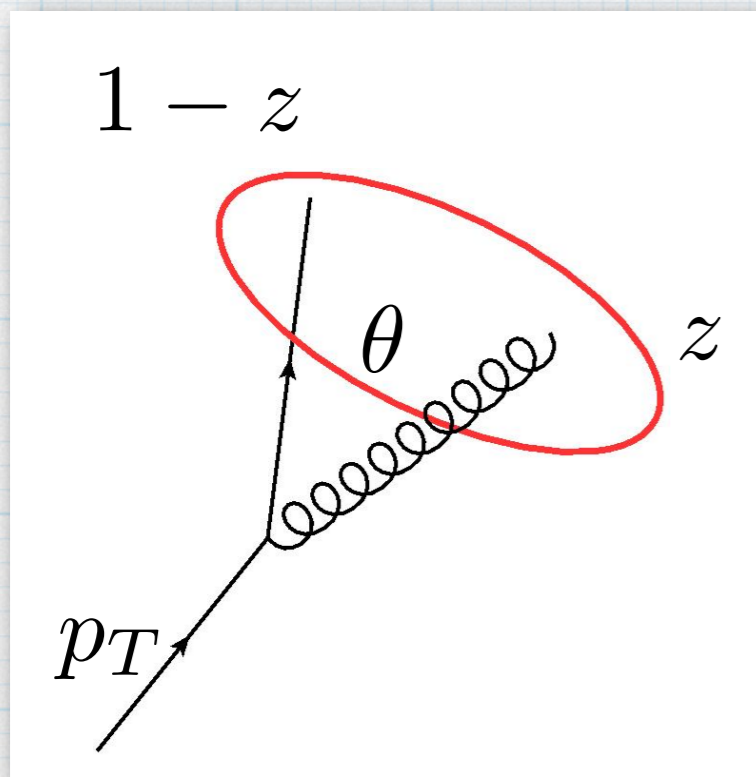
- \* first jet-observable that comes to mind
- \* background (QCD) jets acquire mass through showering



$$m^2 = 2p_q \cdot p_g \simeq z(1 - z)\theta^2 p_T^2$$

# QCD-jet mass

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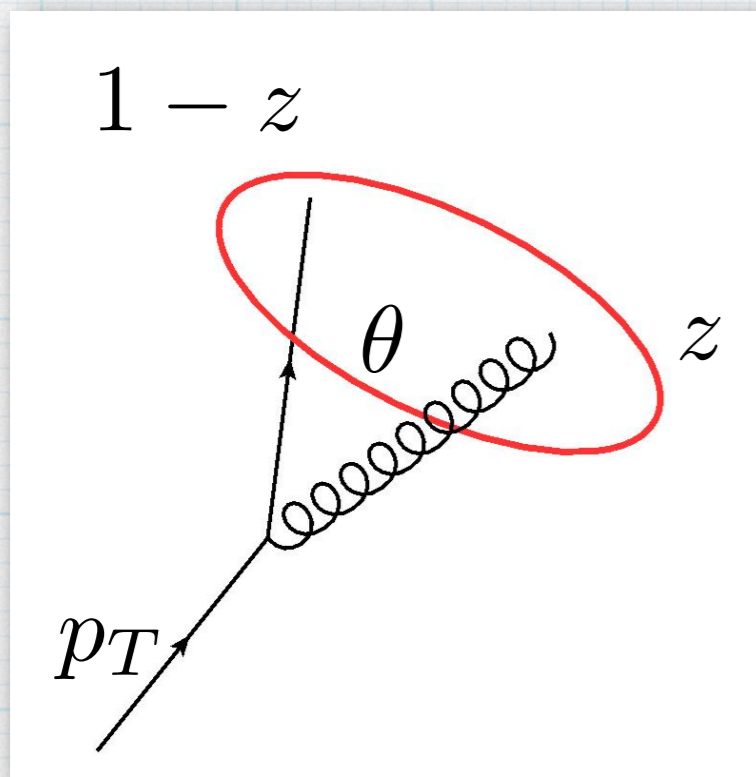


$$m^2 = 2p_q \cdot p_g \simeq z(1-z)\theta^2 p_T^2$$

$$\langle m^2 \rangle \simeq \frac{\alpha_s}{2\pi} p_T^2 \int_0^{R^2} \frac{d\theta^2}{\theta^2} \int_0^1 dz z(1-z)\theta^2 P_{gq}(z)$$

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$$= \frac{\alpha_s C_F}{\pi} p_T^2 R^2 \int_0^1 dz z(1-z) \frac{2-2z+z^2}{2z}$$

mass grows with  $p_T$

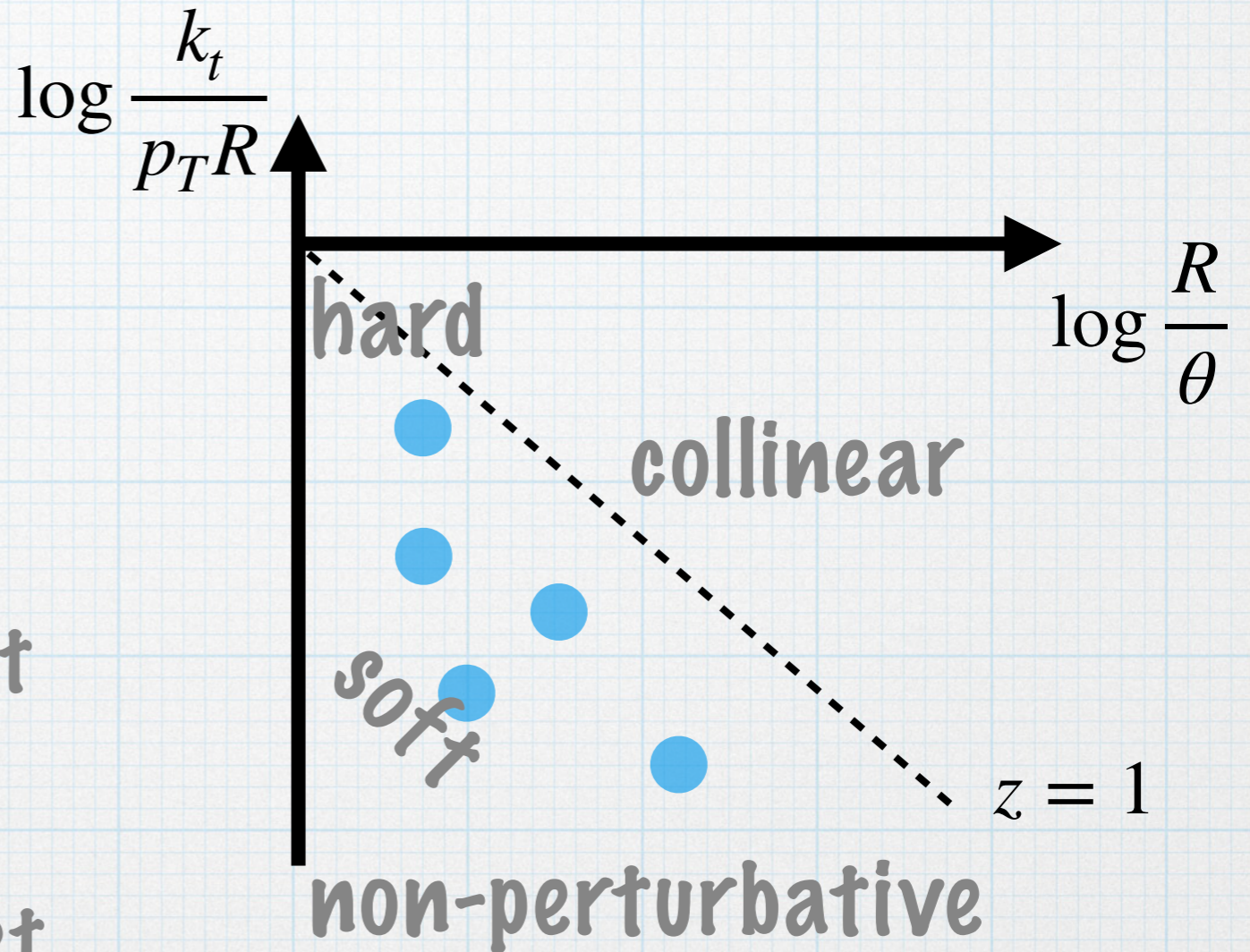
$$= 3/8$$

# jet properties

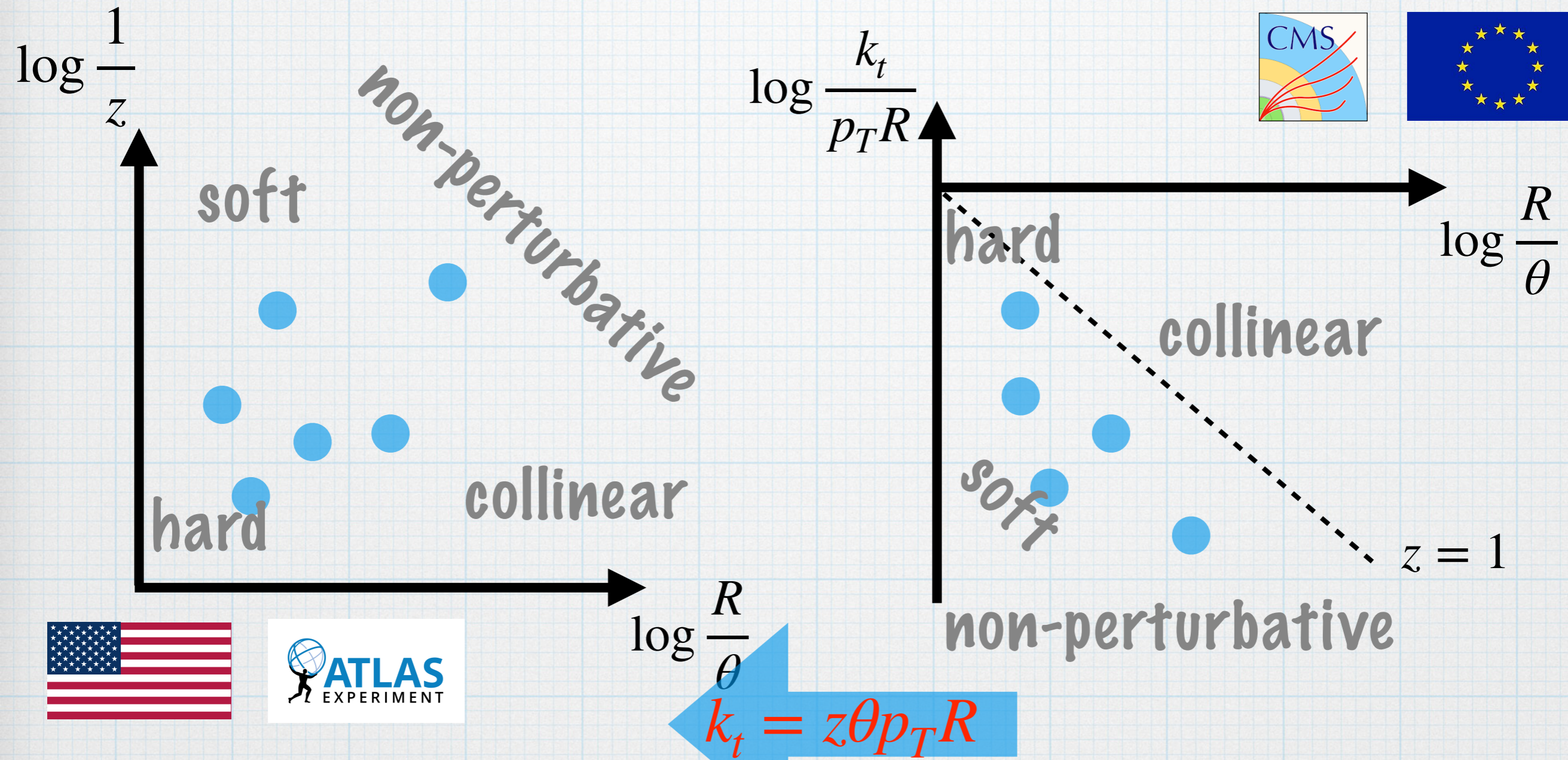
- \* we want to study the properties of jets
- \* hence, we resolve a (high  $p_t$ ) jet down to a smaller scale, e.g. its mass
- \* large logarithms appear invalidating the fixed-order expansion
- \* we need to reorganise the calculation so that we can consider any number of soft/collinear partons: **resummation**
- \* vast field with many approaches: dQCD, SCET, etc.

# aside: the Lund plane

- \* the Lund plane is a powerful representation of soft-collinear emissions kinematics
- \* as the name suggests it was first developed in the context of Monte Carlo studies
- \* useful representation of a jet (also for ML!)
- \* soft-collinear emissions populate the Lund plane uniformly: equal area = equal probability
- \* now also a powerful observables (measured at the LHC)



# aside: the Lund plane



- \* soft-collinear emissions populate the Lund plane uniformly: equal area = equal probability
- \* now also a powerful observables (measured at the LHC)

# how do we model it ?

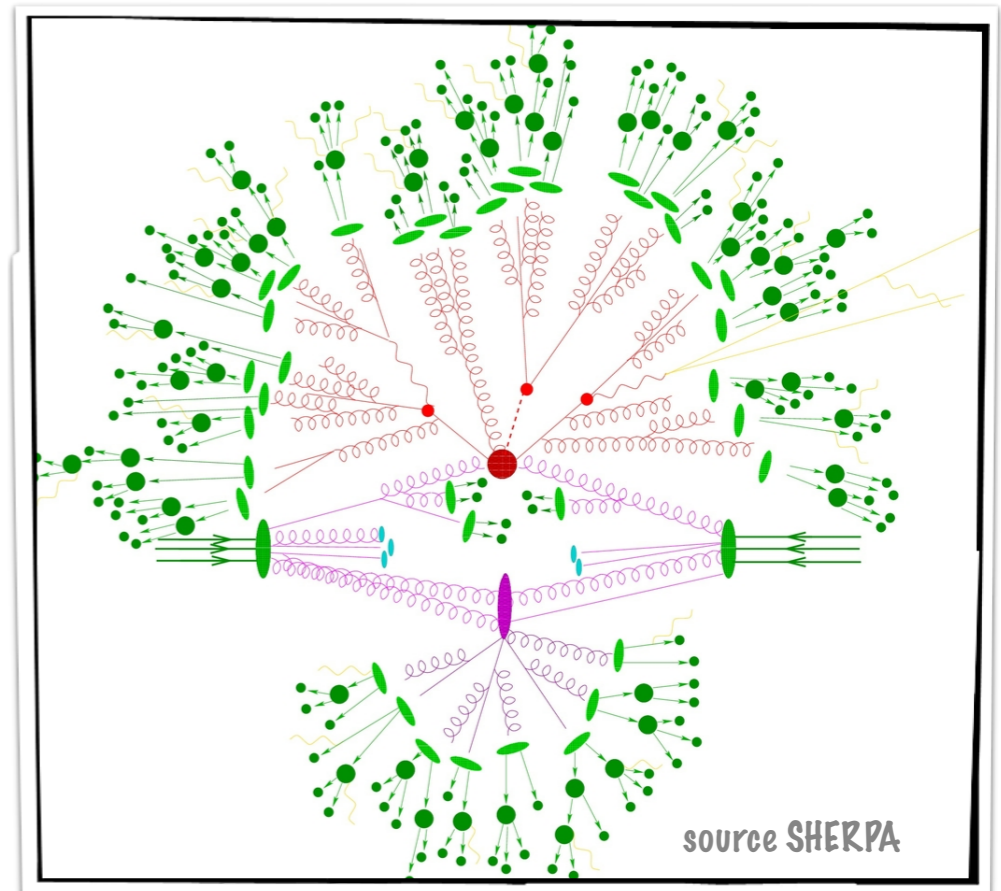
- \* jet properties: we want to compute x-sections and distributions with many particles in the final state
- \* fixed-order perturbation theory seems inadequate
- \* interesting physics happens at small angular separation and small energies
- \* all-order (resummed) calculations are possible and necessary !

## Monte Carlo Parton Showers

emissions at small angles factorize

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz d\phi P(z, \phi)$$

we can write a computer program that simulates these classical branchings





# how do we model it ?

- \* jet properties: we want to compute x-sections and distributions with many particles in the final state
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## Analytic Resummation

emissions at small angles factorize

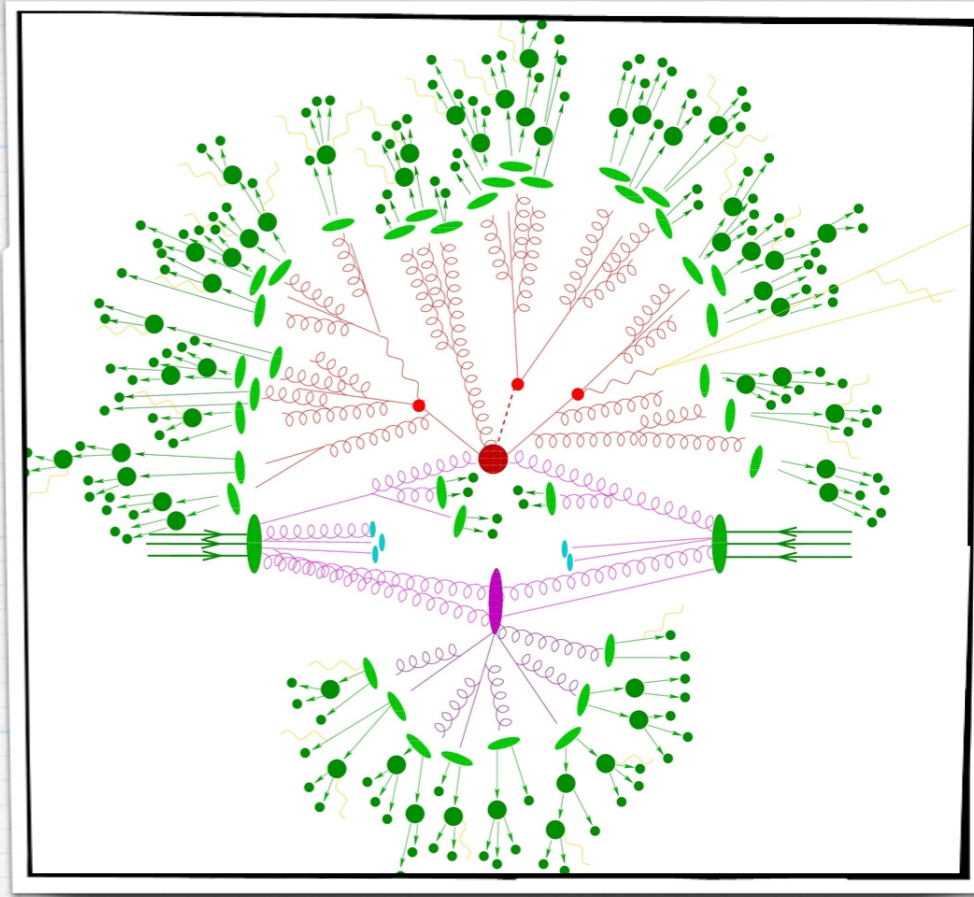
$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz d\phi P(z, \phi)$$

soft emissions factorise in a subtle way

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s}{2\pi} d\theta dz d\phi \sum_{i < j} C_{ij} D_{ij}(z, \theta, \phi)$$

$$\sigma_{\text{res}} = g_0 \exp \left[ g_1(\alpha_s L) / \alpha_s + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots \right]$$

- \* powerful general-purpose tools
- \* provide fully differential events on which any observable can be measured
- \* interfaced with non-perturbative models to give a realistic description
- \* theoretical accuracy difficult to assess (often low)



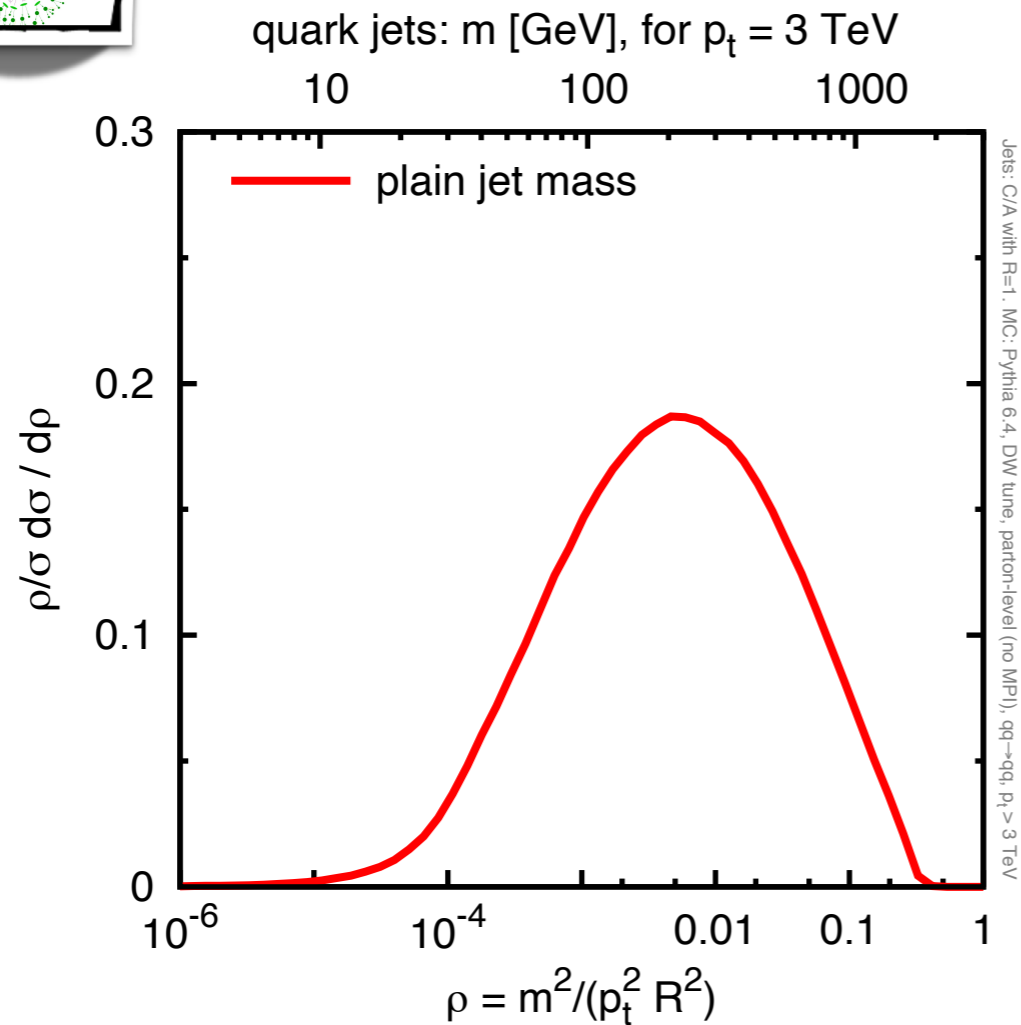
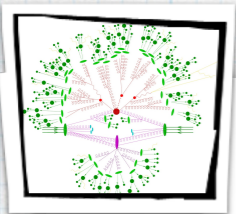
VS

$$\sigma_{res} = g_0 \exp \left[ \frac{g_1(\alpha_s L)}{\alpha_s + g_2(\alpha_s L) + g_3(\alpha_s L) + \dots} \right]$$

- \* feasible for a limited number of observables
- \* well defined and improvable accuracy
- \* state-of-the art (resummation + fixed order)
- \* provide insights and understanding

# the jet mass

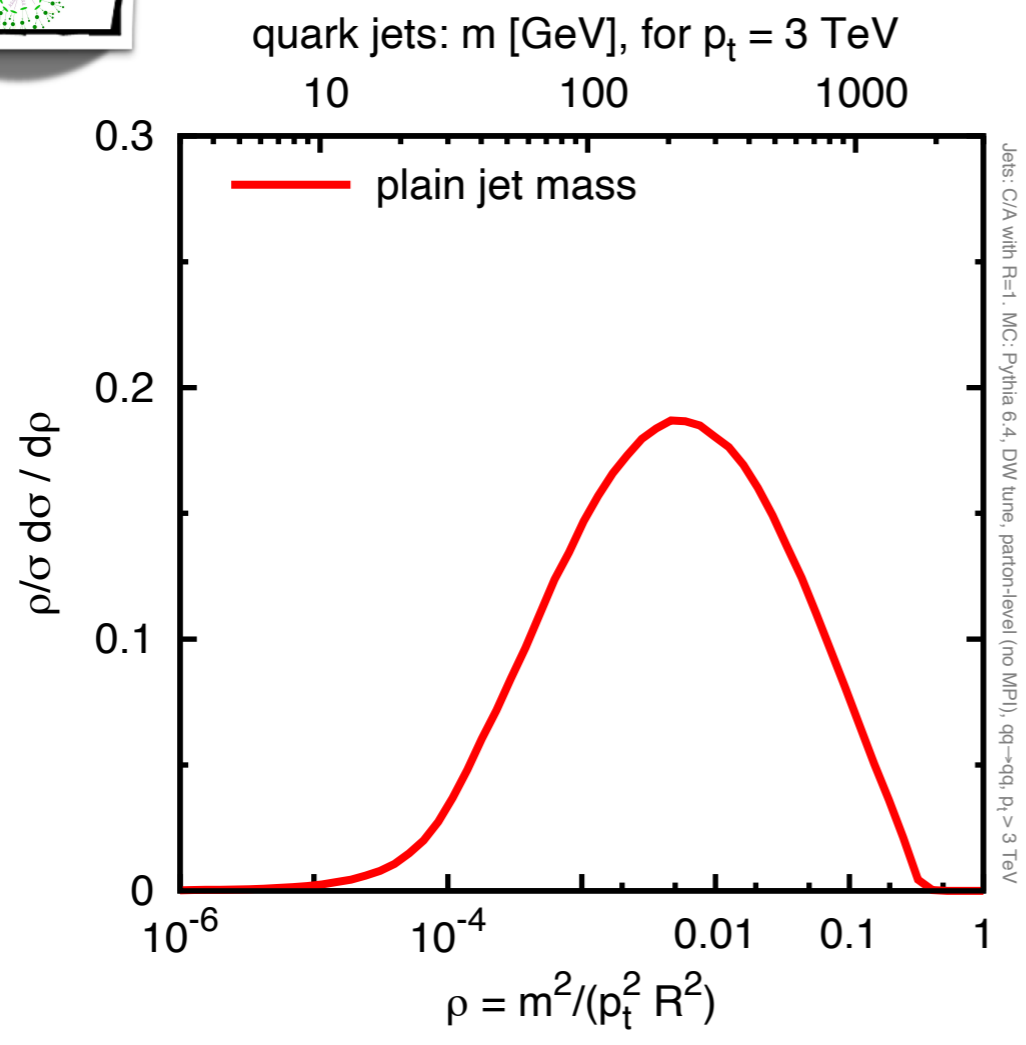
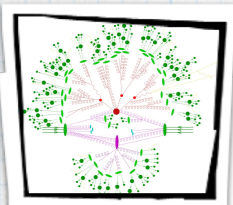
$$\sigma_{\text{res}} = g_0 \exp[g_1(a_s L) / a_s + g_2(a_s L) + a_s g_3(a_s L) + \dots]$$



- \* plain jet mass: Sudakov peak, where does it come from ?
- \* let's do an easy calculation: one gluon emission in the collinear limit

# the jet mass

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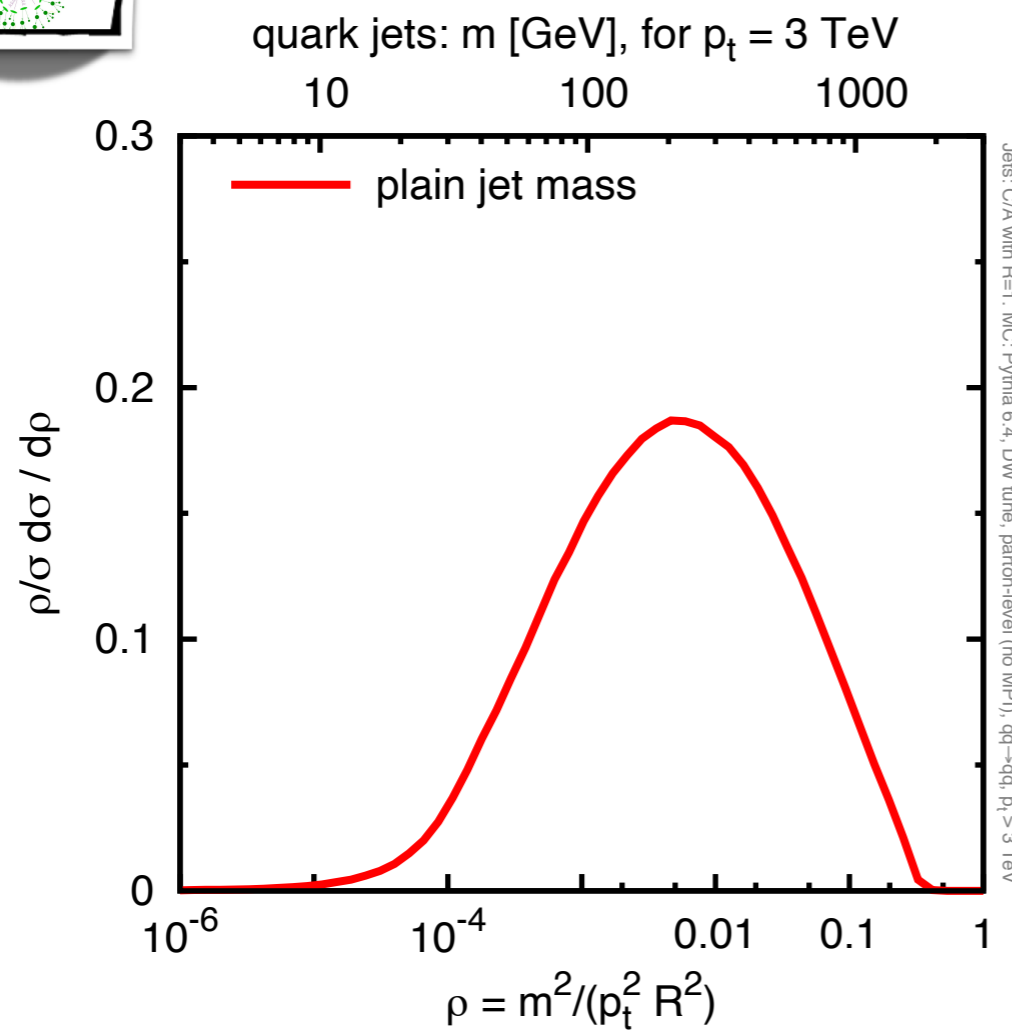
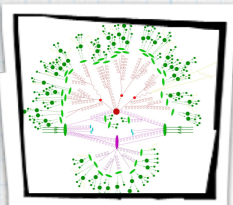
$$\frac{1}{\sigma} \frac{d\sigma^{LO}}{dm^2} = \frac{\alpha_s}{2\pi} \int_0^{R^2} \frac{d\theta^2}{\theta^2} \int_0^1 dz P_{gq}(z) \times \delta(m^2 - z(1-z)\theta^2 p_T^2)$$

**we demand a mass  $m$**

- \* plain jet mass: Sudakov peak, where does it come from ?
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# the jet mass

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$$\frac{1}{\sigma} \frac{d\sigma^{LO}}{dm^2}$$

$$= \frac{\alpha_s}{2\pi} \int_0^{R^2} \frac{d\theta^2}{\theta^2} \int_0^1 dz P_{gq}(z)$$

$$\times \delta(m^2 - z(1-z)\theta^2 p_T^2)$$

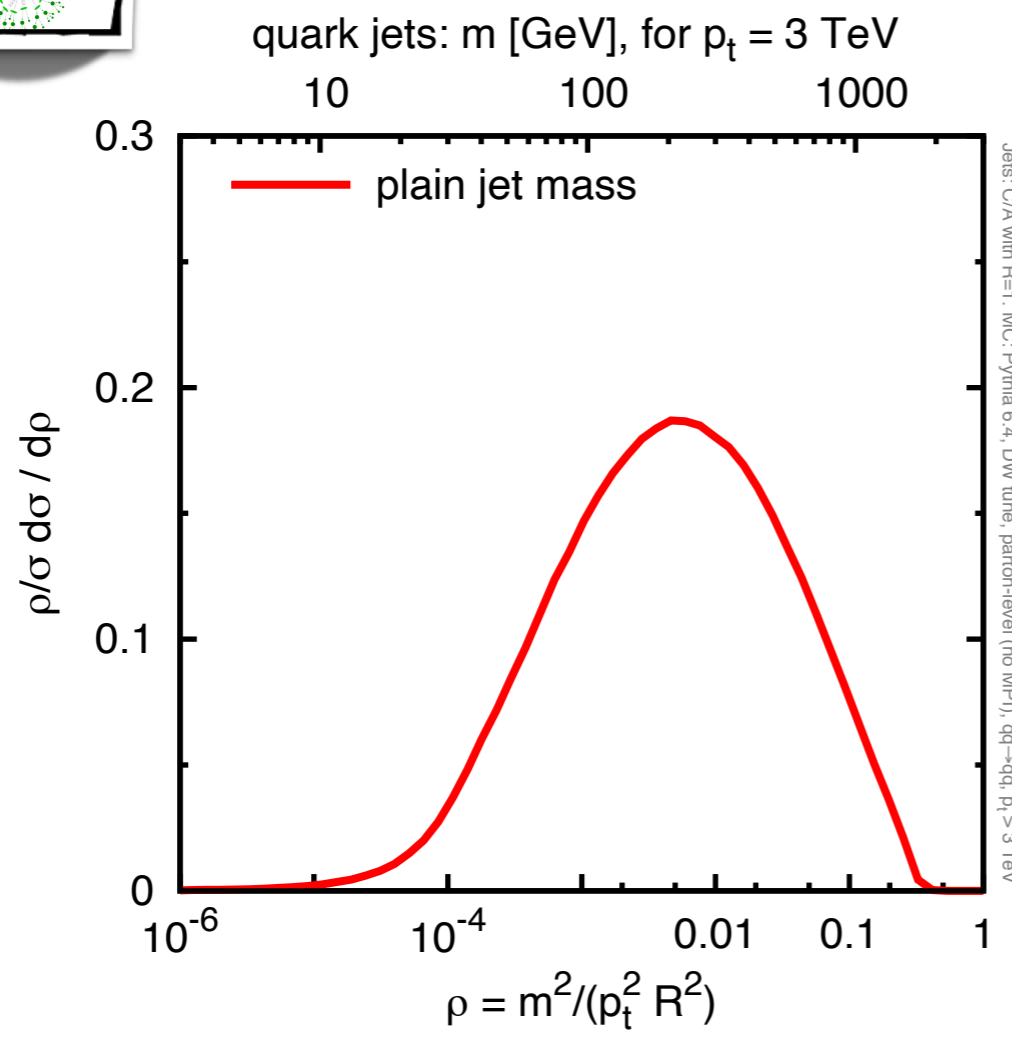
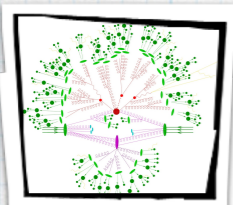
$$= \frac{\alpha_s C_F}{2\pi} m^{-2} \int_{m^2 / (p_T^2 R^2)}^1 dz P_{gq}(z)$$

**we do the angular integral with the delta function**

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# the jet mass

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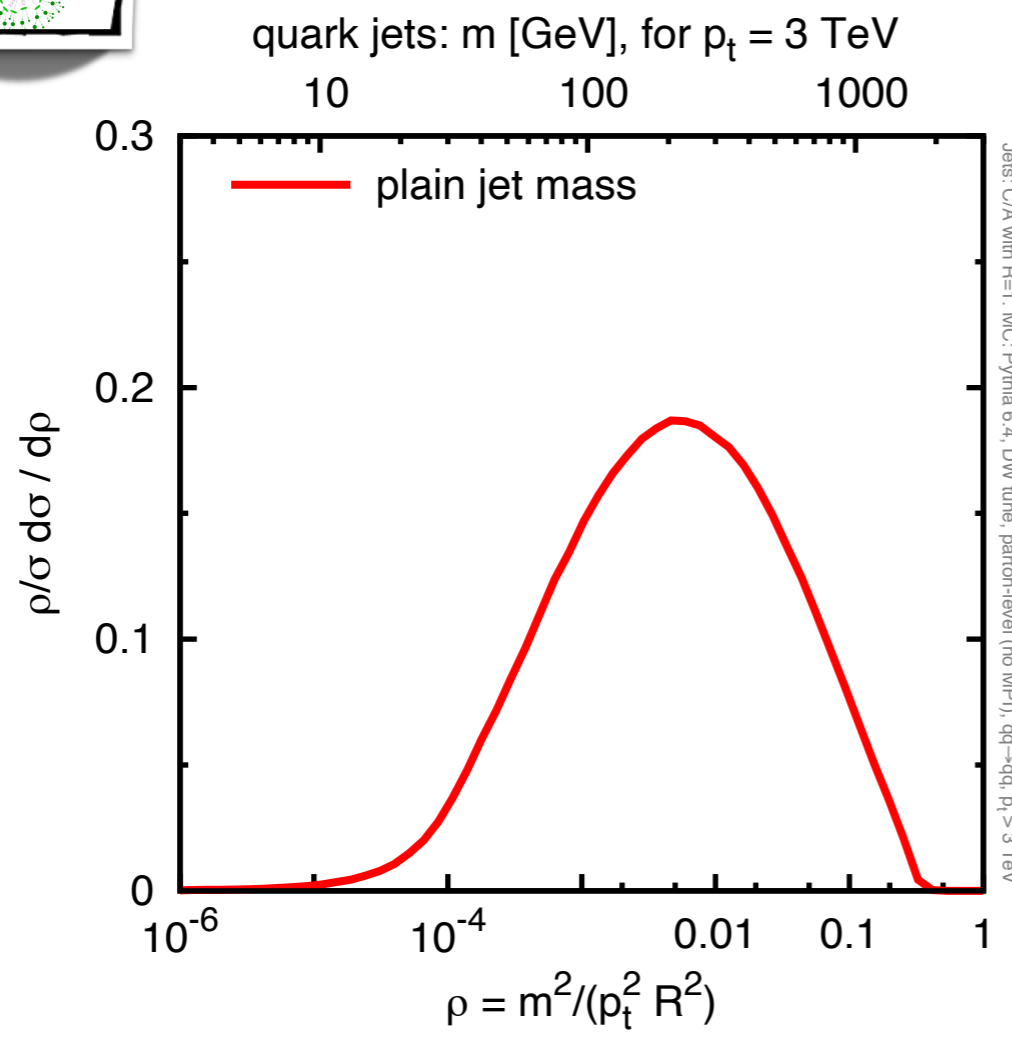
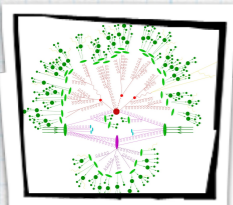
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$$\simeq \frac{\alpha_s C_F}{\pi} m^{-2} \left[ \ln \frac{p_T^2 R^2}{m^2} - \frac{3}{4} \right]$$

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# the jet mass

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$$= \frac{\alpha_s}{2\pi} \int_0^{R^2} \frac{d\theta^2}{\theta^2} \int_0^1 dz P_{gq}(z)$$

$$\times \delta(m^2 - z(1-z)\theta^2 p_T^2)$$

$$= \frac{\alpha_s C_F}{2\pi} m^{-2} \int_{m^2/(p_T^2 R^2)}^1 dz P_{gq}(z)$$

$$\simeq \frac{\alpha_s C_F}{\pi} m^{-2} \left[ \ln \frac{p_T^2 R^2}{m^2} - \frac{3}{4} \right]$$

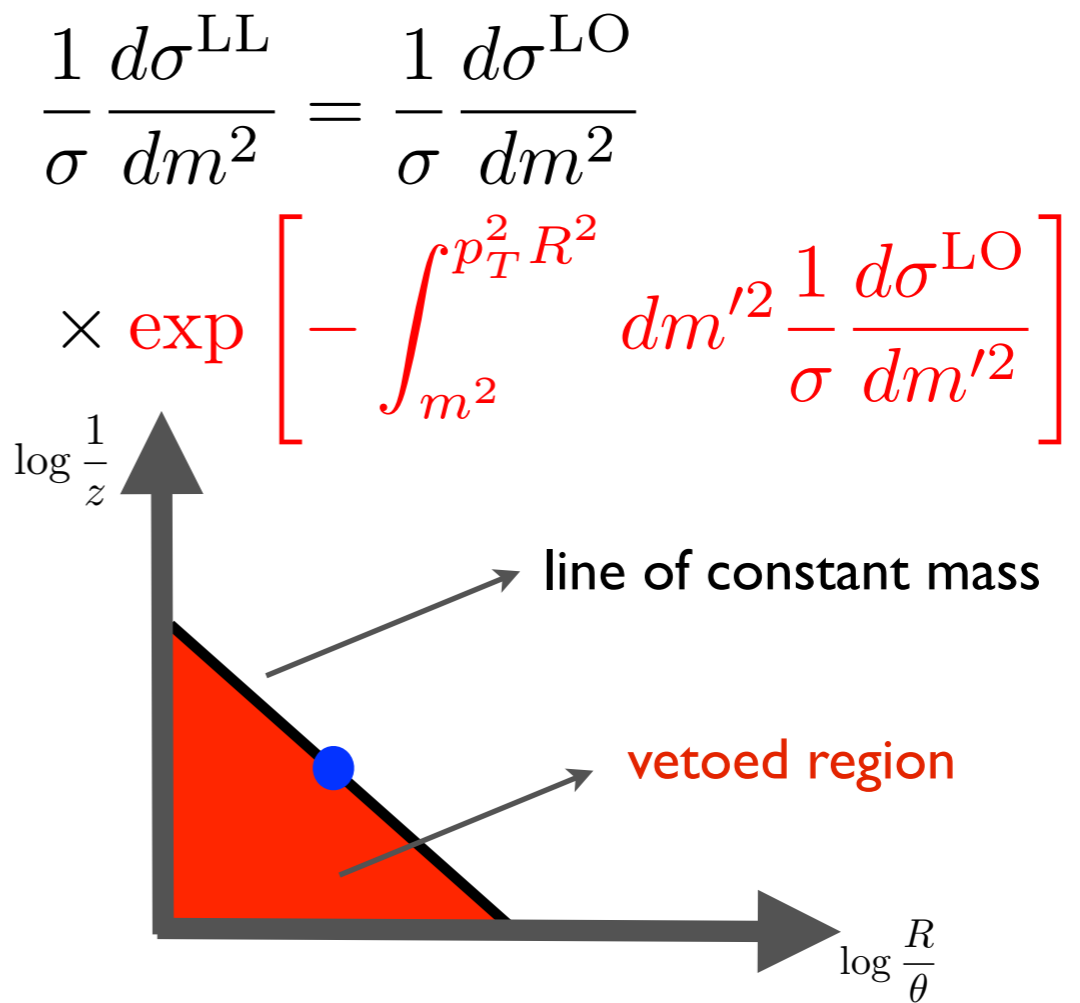
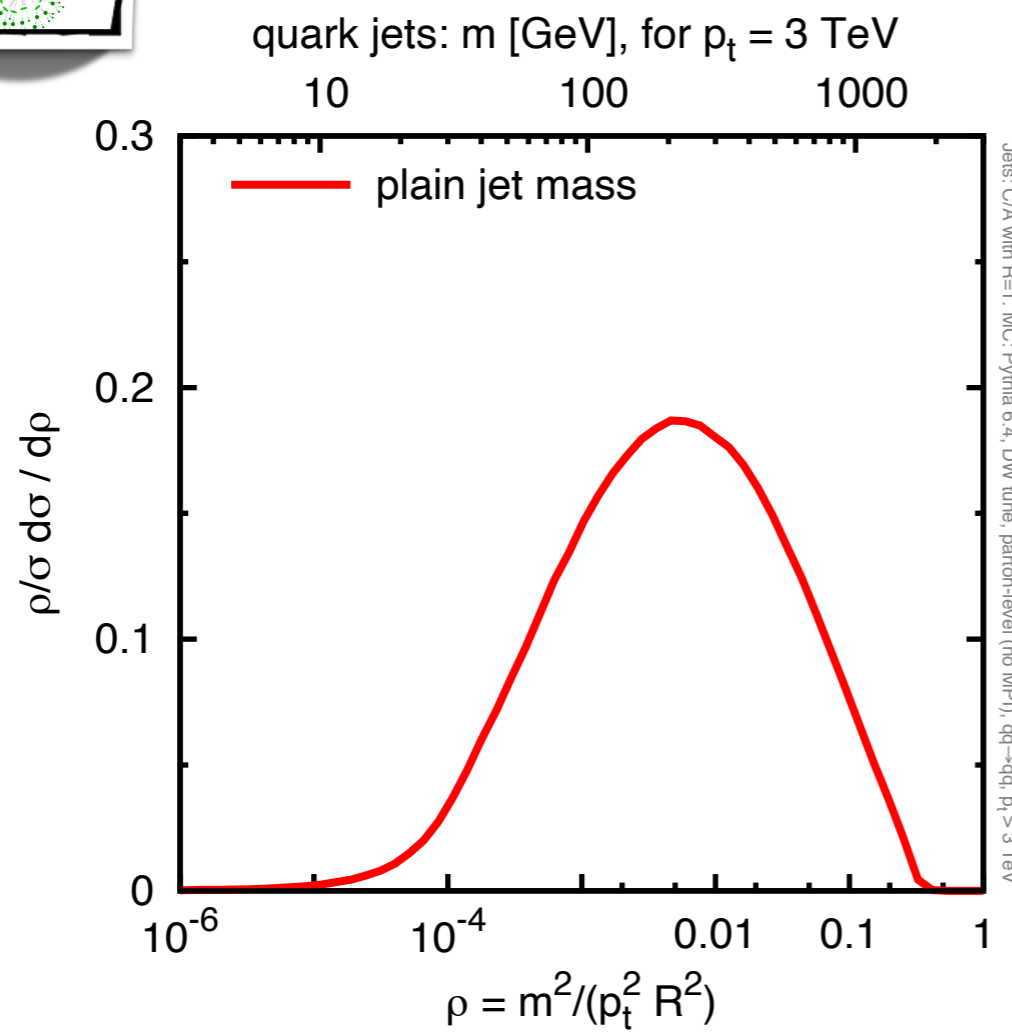
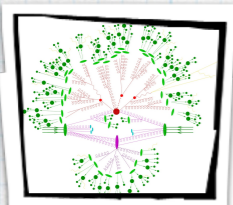
double log: soft & coll.

single log: hard coll.

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# the jet mass

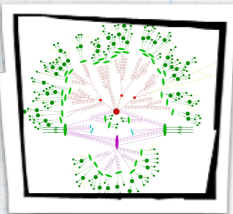
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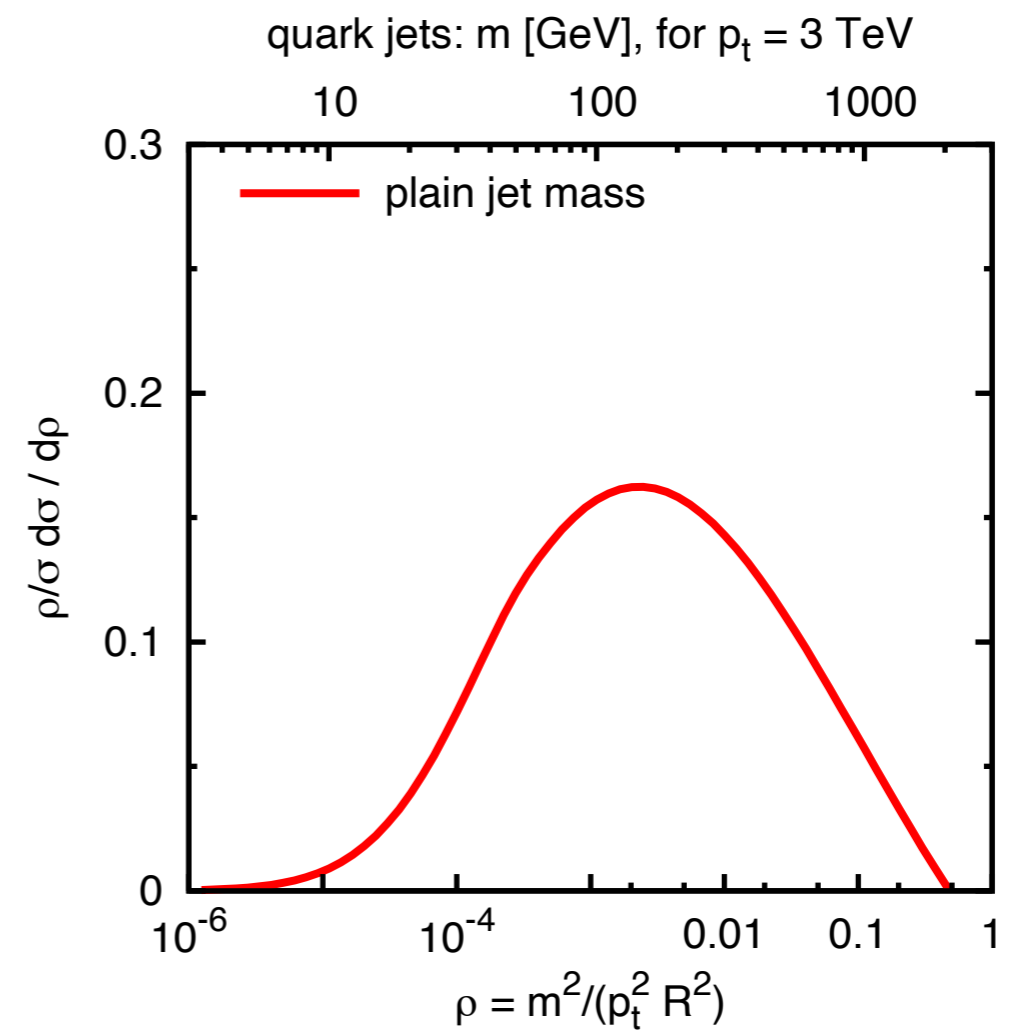
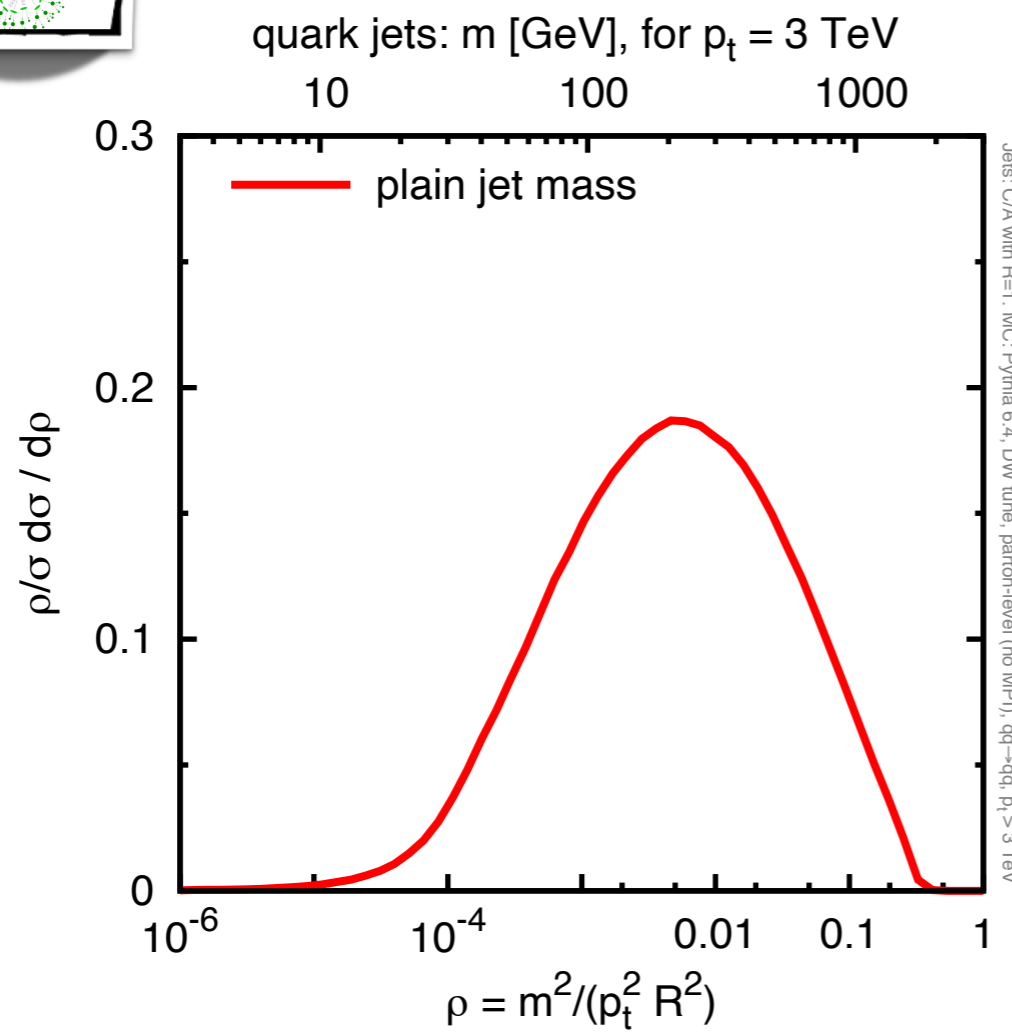
- \* all-order leading logs: veto emissions which would give too big a mass
- \* exponential that gives the no-emission probability



# the jet mass



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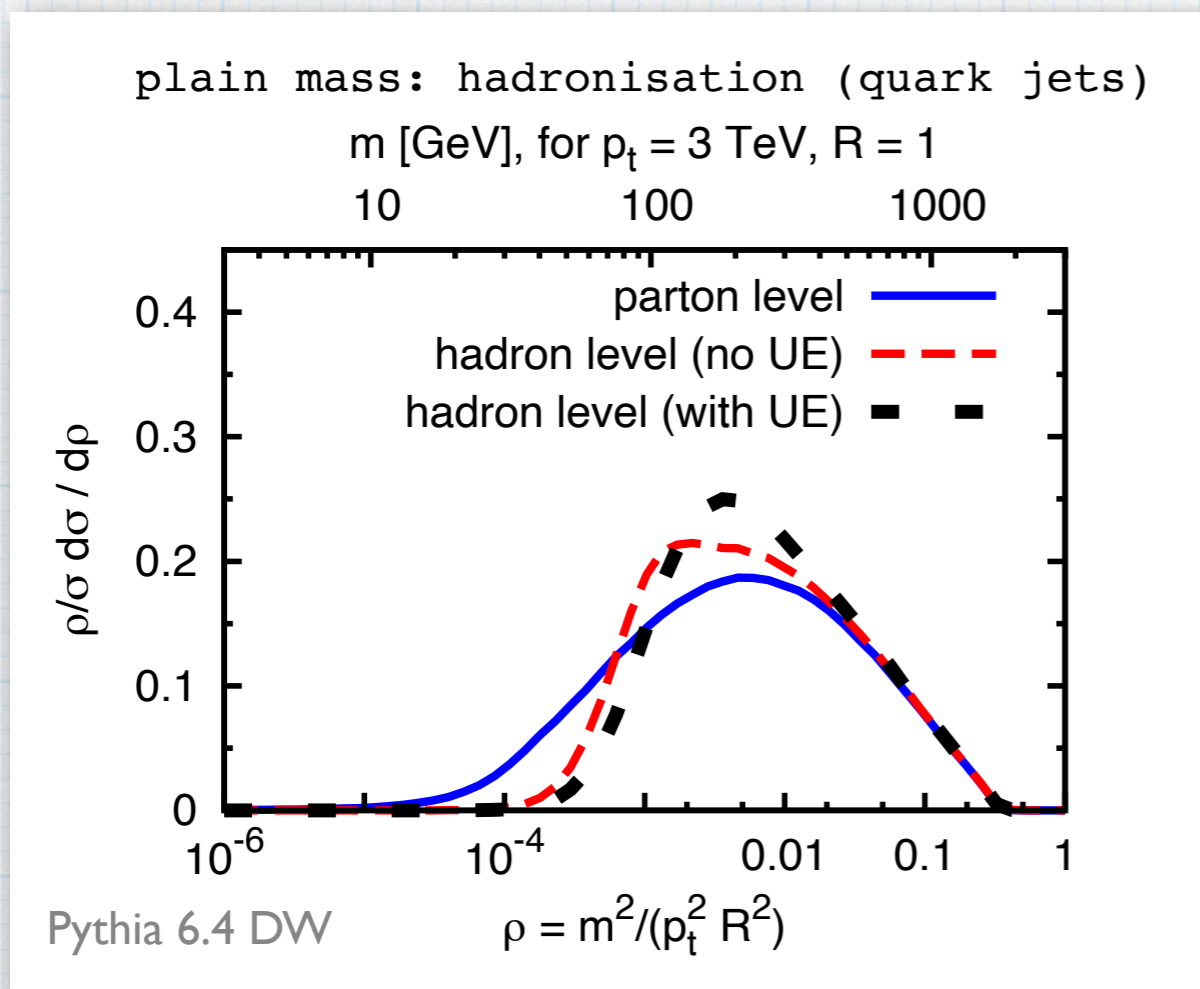


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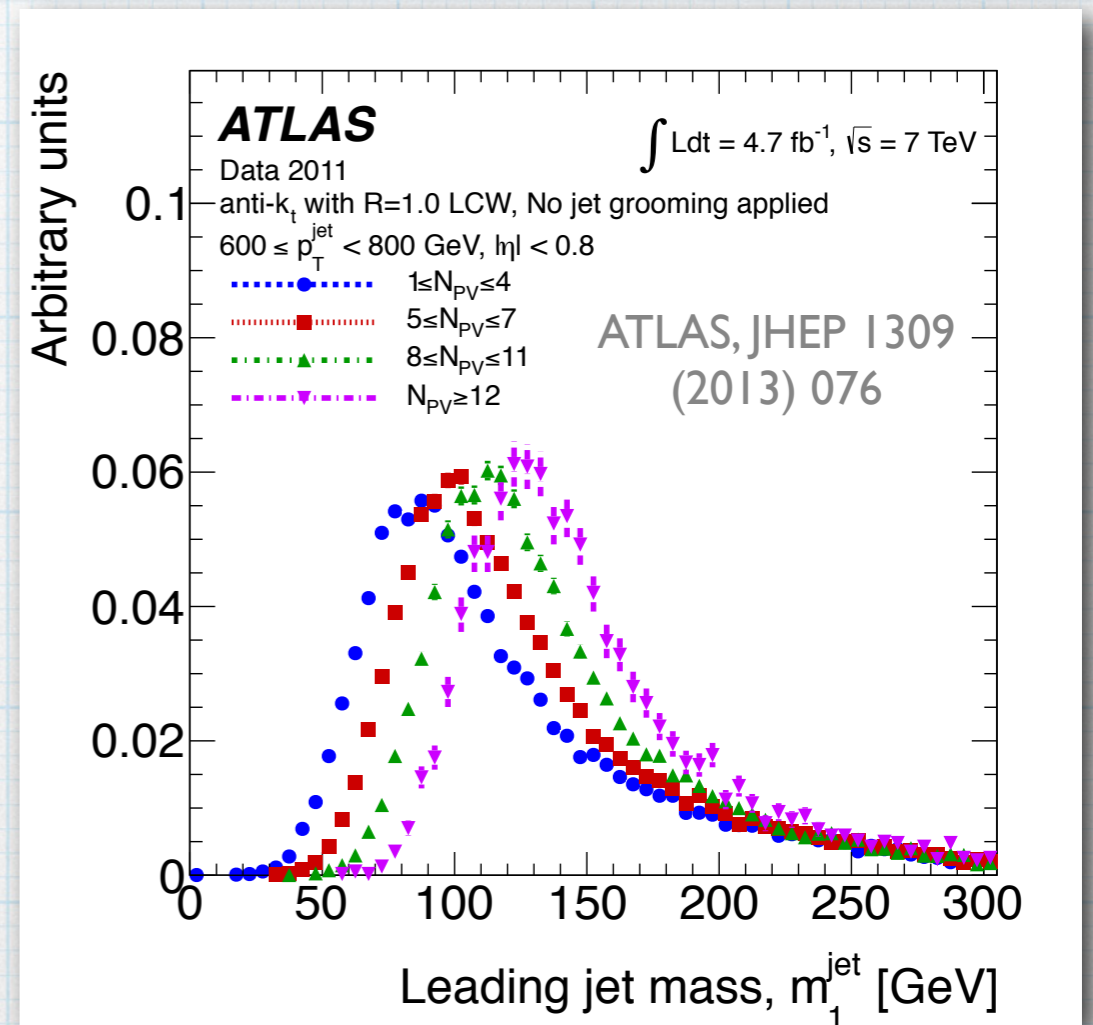
# QCD-jet mass: NP effects

\* first jet-observable that comes to mind

\* background (QCD) jets receive important non-pert contributions



hadronisation and UE



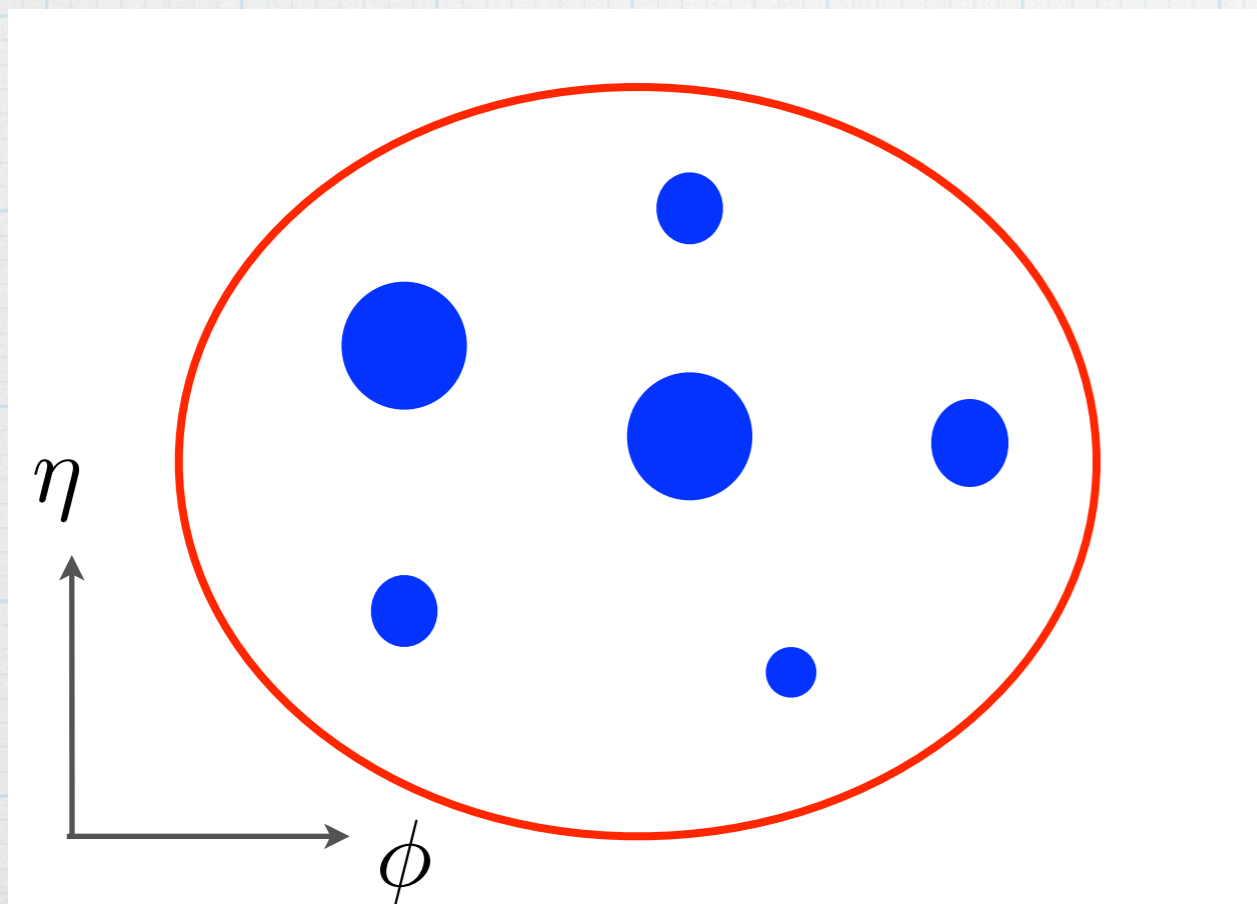
pile-up (data!)

# beyond the mass: substructure

- \* need to go beyond the mass and exploit jet substructure : **grooming** and **tagging**:
- \* clean the jets up by removing soft radiation
- \* identify the features of hard decays and cut on them

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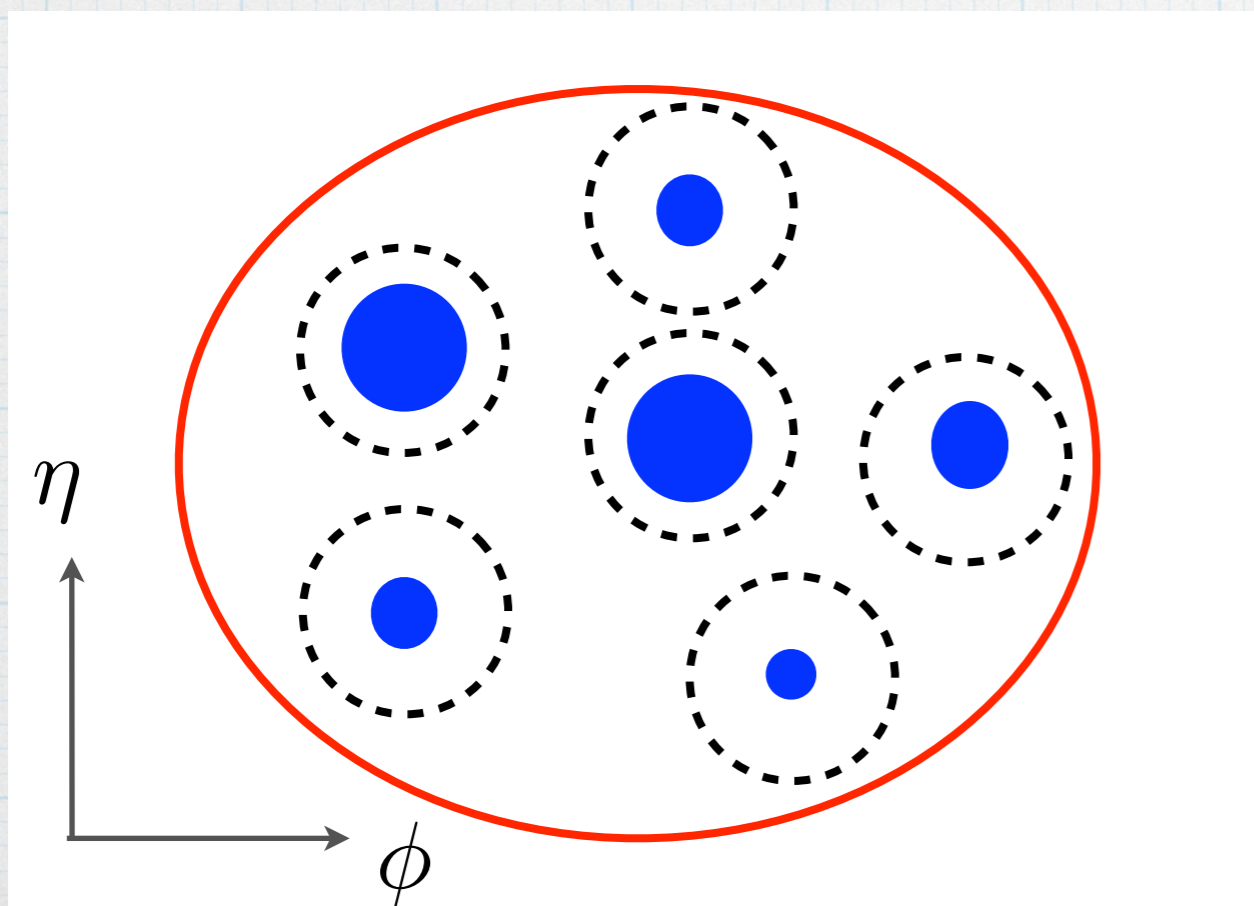
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core-idea for grooming:

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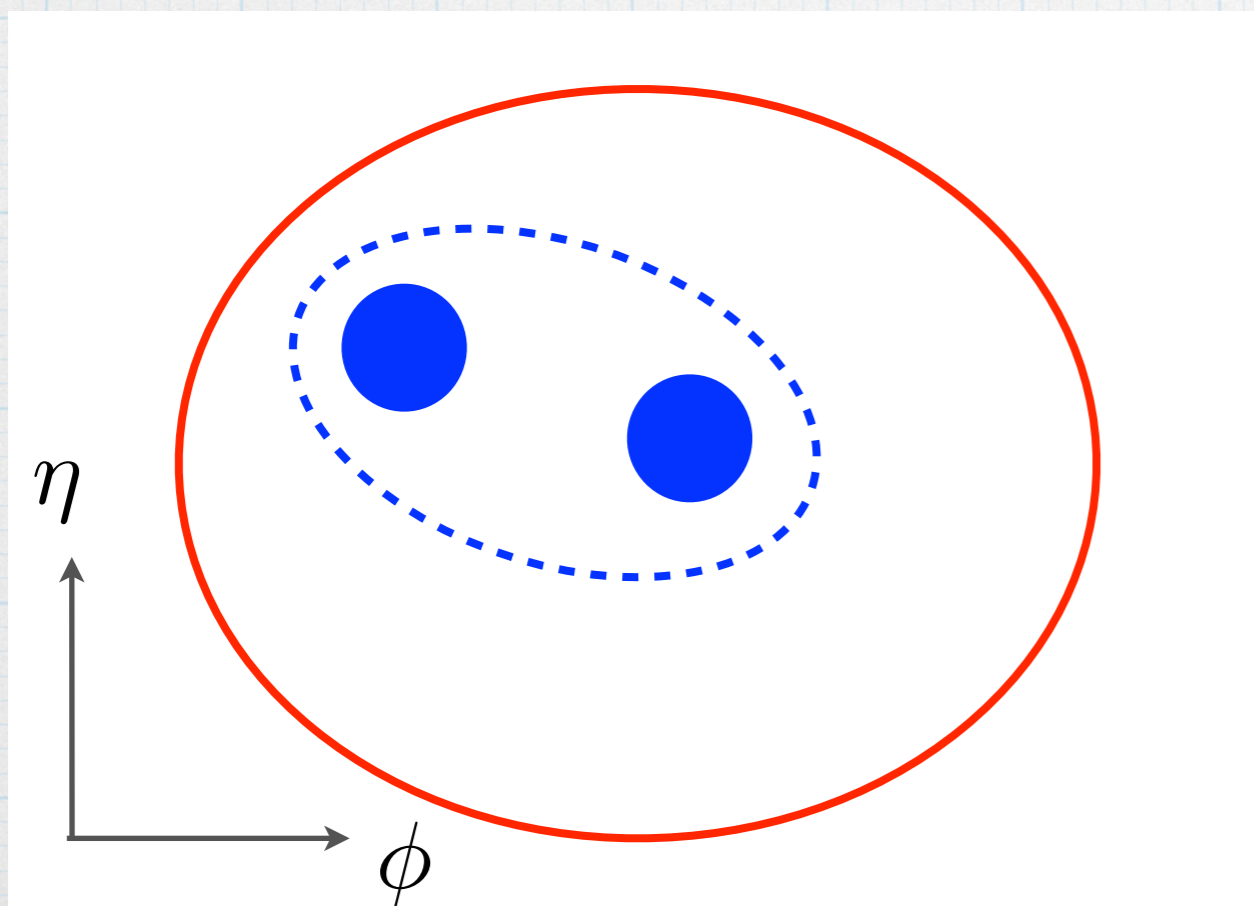


core-idea for grooming:

- \* identify the “right” angular scale

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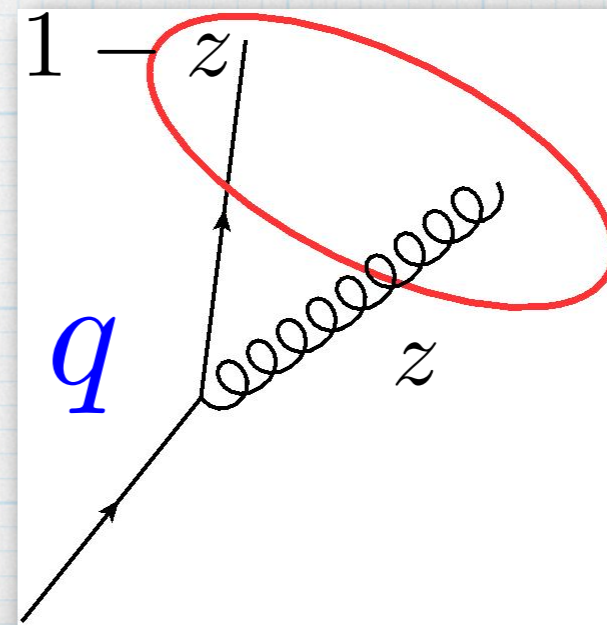
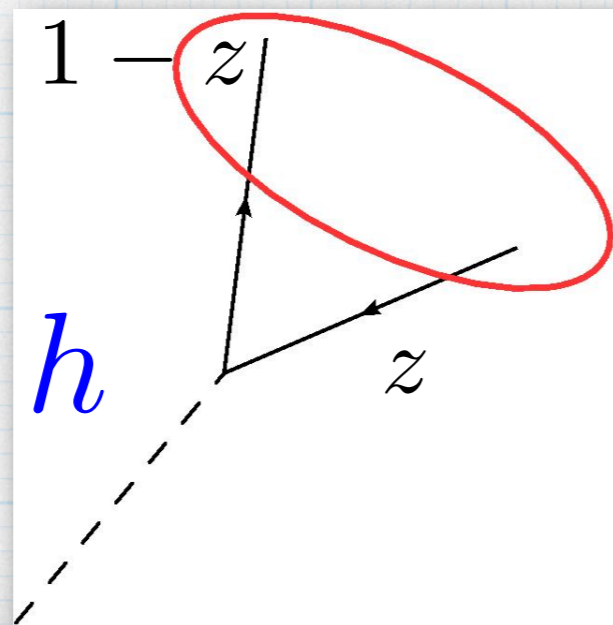
## core-idea for grooming:

- \* identify the "right" angular scale
- \* throw away what is soft & large angle
- \* left with a **groomed jet**

# beyond the mass: substructure

- \* need to go beyond the mass and exploit jet substructure : **grooming** and **tagging**:
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core-idea for 2-body tagging:



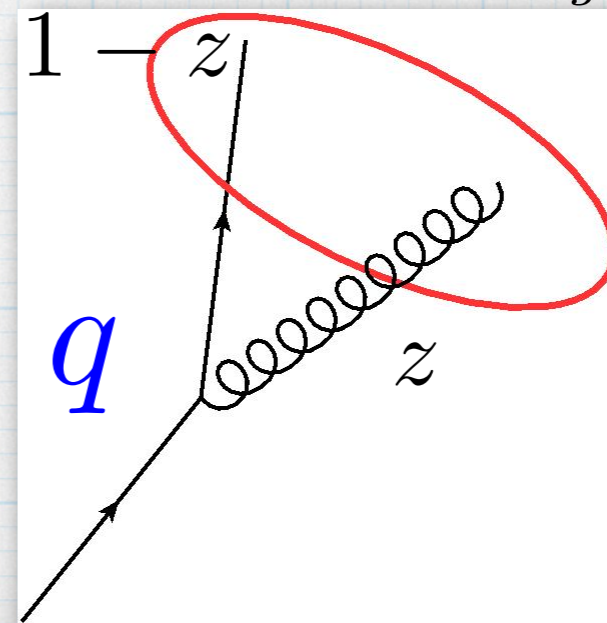
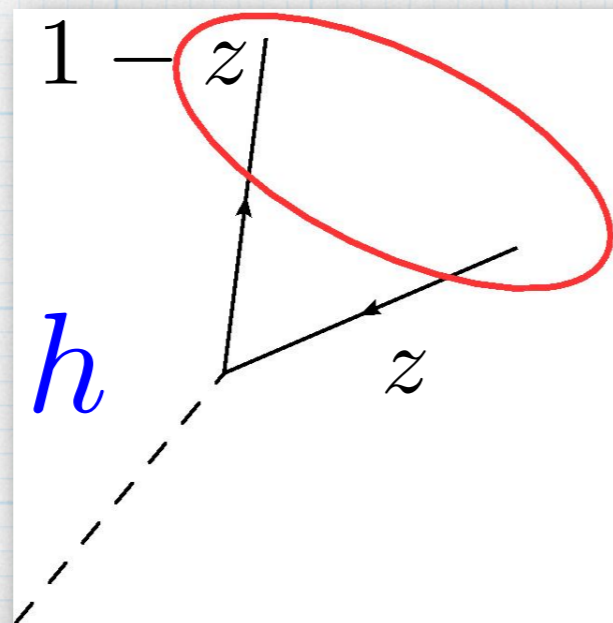
# beyond the mass: substructure

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core-idea for 2-body tagging:  $\min(z, 1 - z) > z_{\text{cut}}$

$$P_{gq} = C_F \frac{1 + (1 - z)^2}{z}$$

$P_{h \rightarrow q\bar{q}} = 1$   
symmetric  
sharing of  
the energy



asymmetric  
sharing of  
the energy



# analytic understanding at work: soft drop

Larkoski, SM, Soyez and Thaler (2014)

1. Undo the last stage of the C/A clustering. Label the two subjects  $j_1$  and  $j_2$ .

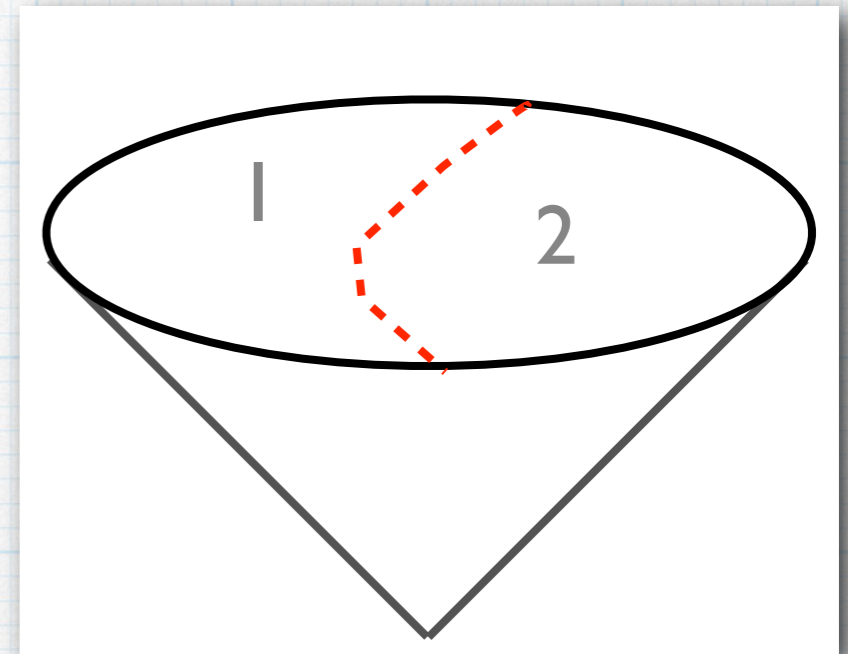
2. If 
$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left( \frac{\Delta R_{12}}{R_0} \right)^\beta$$

then deem  $j$  to be the soft-drop jet.

3. Otherwise redefine  $j$  to be the harder subject and iterate.

1-prong jets can be either kept (grooming mode) or discarded (tagging mode)

- \* generalisation of the (modified) Mass Drop procedure
- \* no mass drop condition (not so important)
- \* mMDT recovered for  $\beta=0$
- \* some inspiration from semi-classical jets



Butterworth, Davison, Rubin and Salam (2008)  
Dasgupta, Fregoso, SM and Salam (2013)

Tseng and Evans (2013)

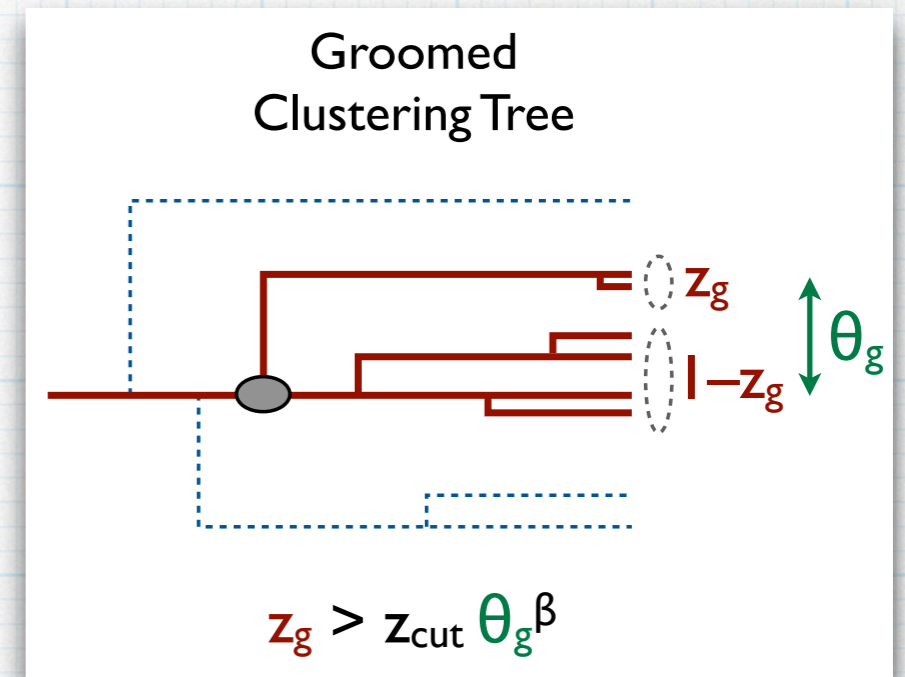
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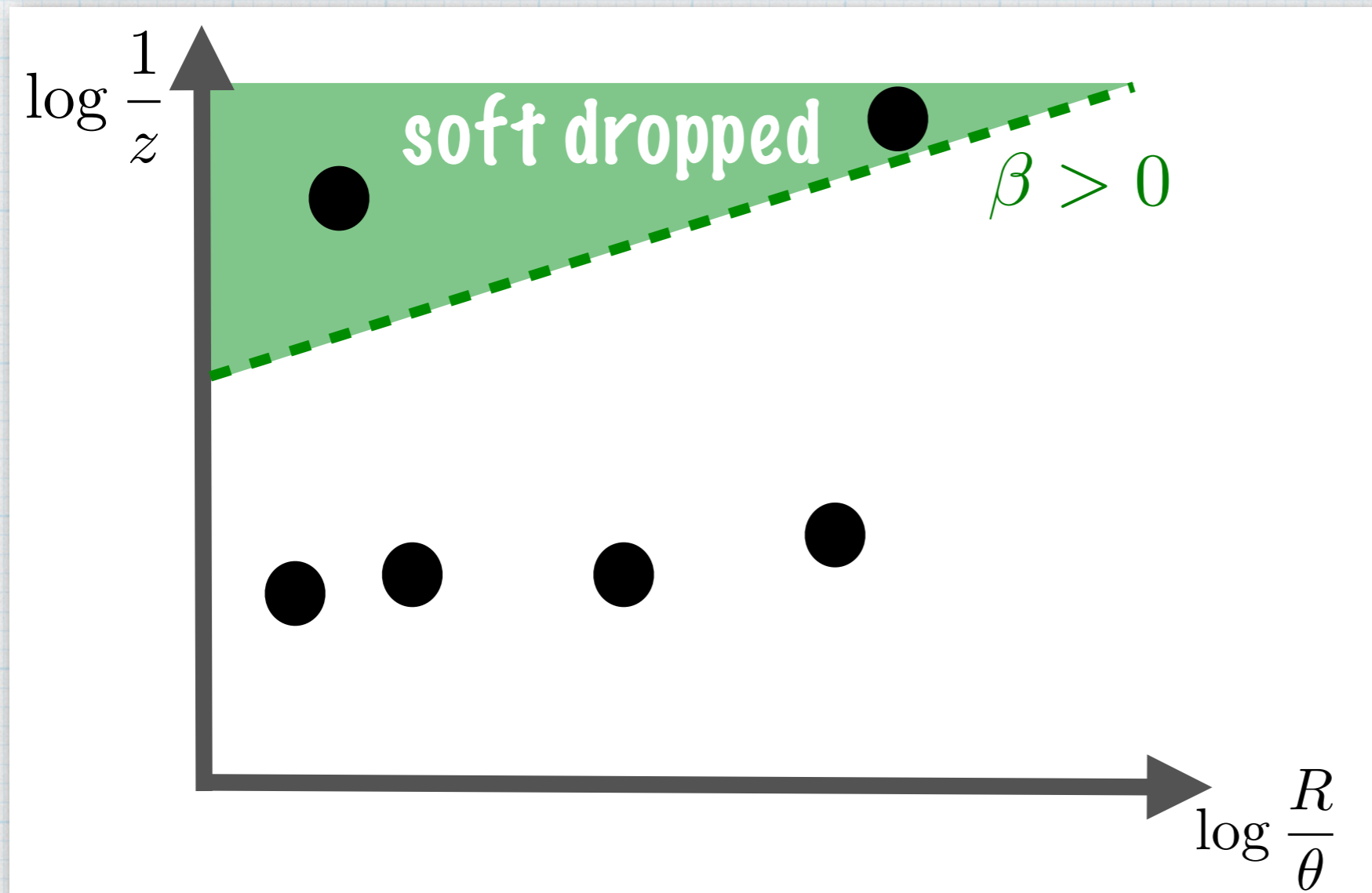
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Tseng and Evans (2013)

# soft drop as a groomer



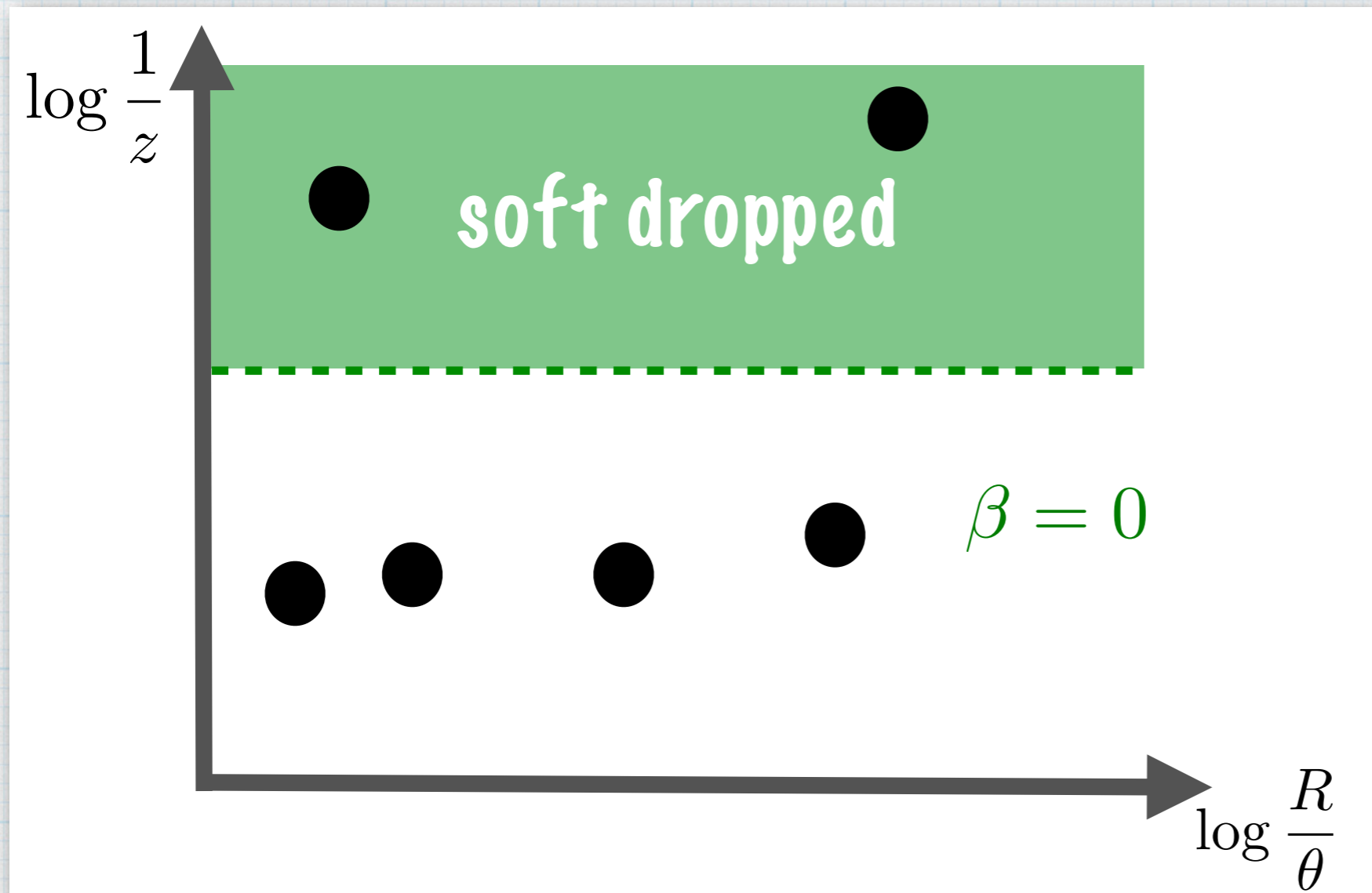
\* useful to consider the soft-gluon phase space

\* soft-drop condition becomes

$$z > z_{\text{cut}} \left( \frac{\theta}{R} \right)^{\beta}$$

- \* soft drop always removes soft radiation entirely (hence the name)
- \* for  $\beta > 0$  soft-collinear is partially removed

# soft drop and mMDT

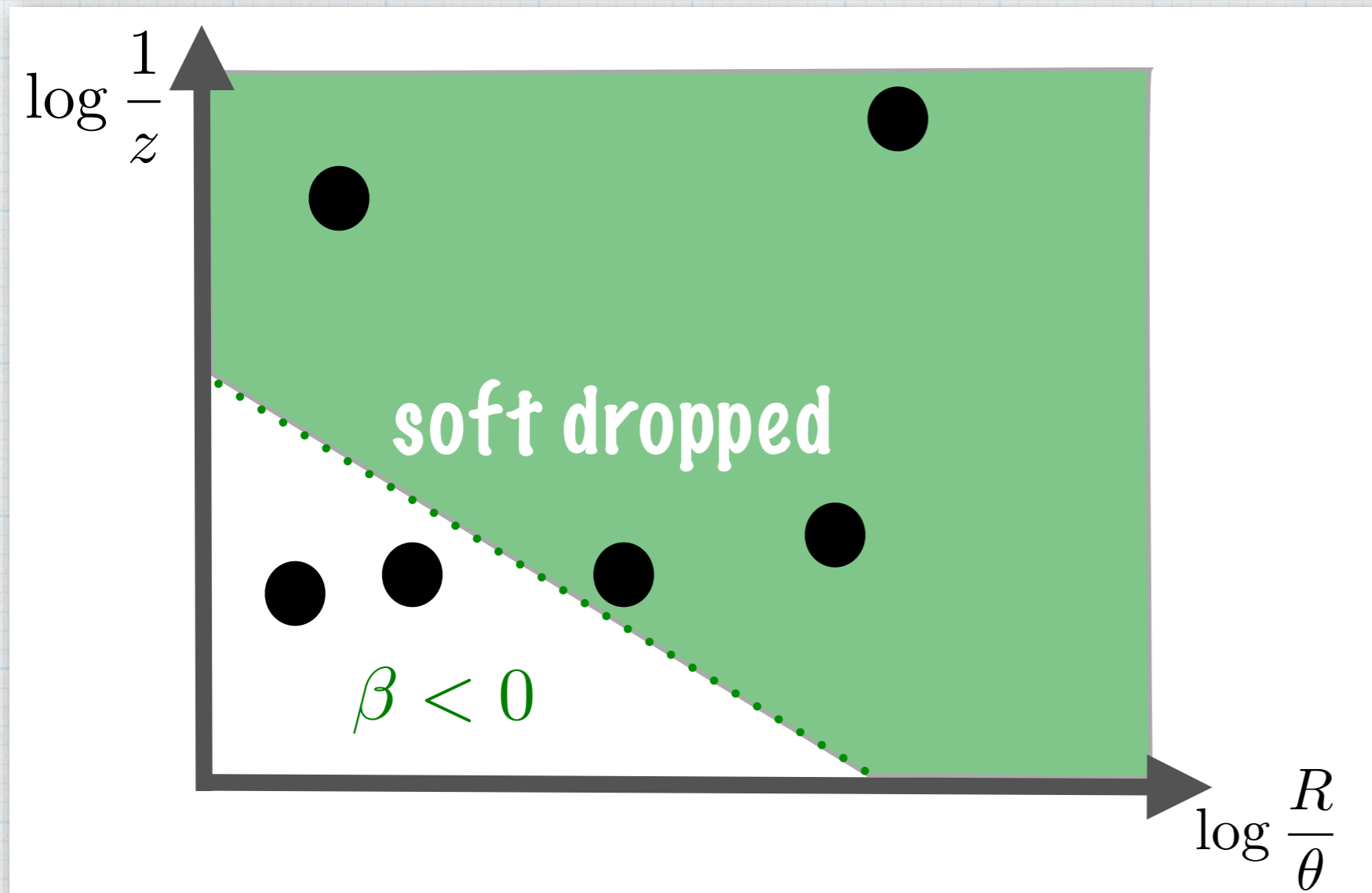


\* soft-drop condition becomes

$$z > z_{\text{cut}} \left( \frac{\theta}{R} \right)^{\beta}$$

\* for  $\beta=0$  soft-collinear is also entirely removed (mMDT limit)

# soft drop as a tagger

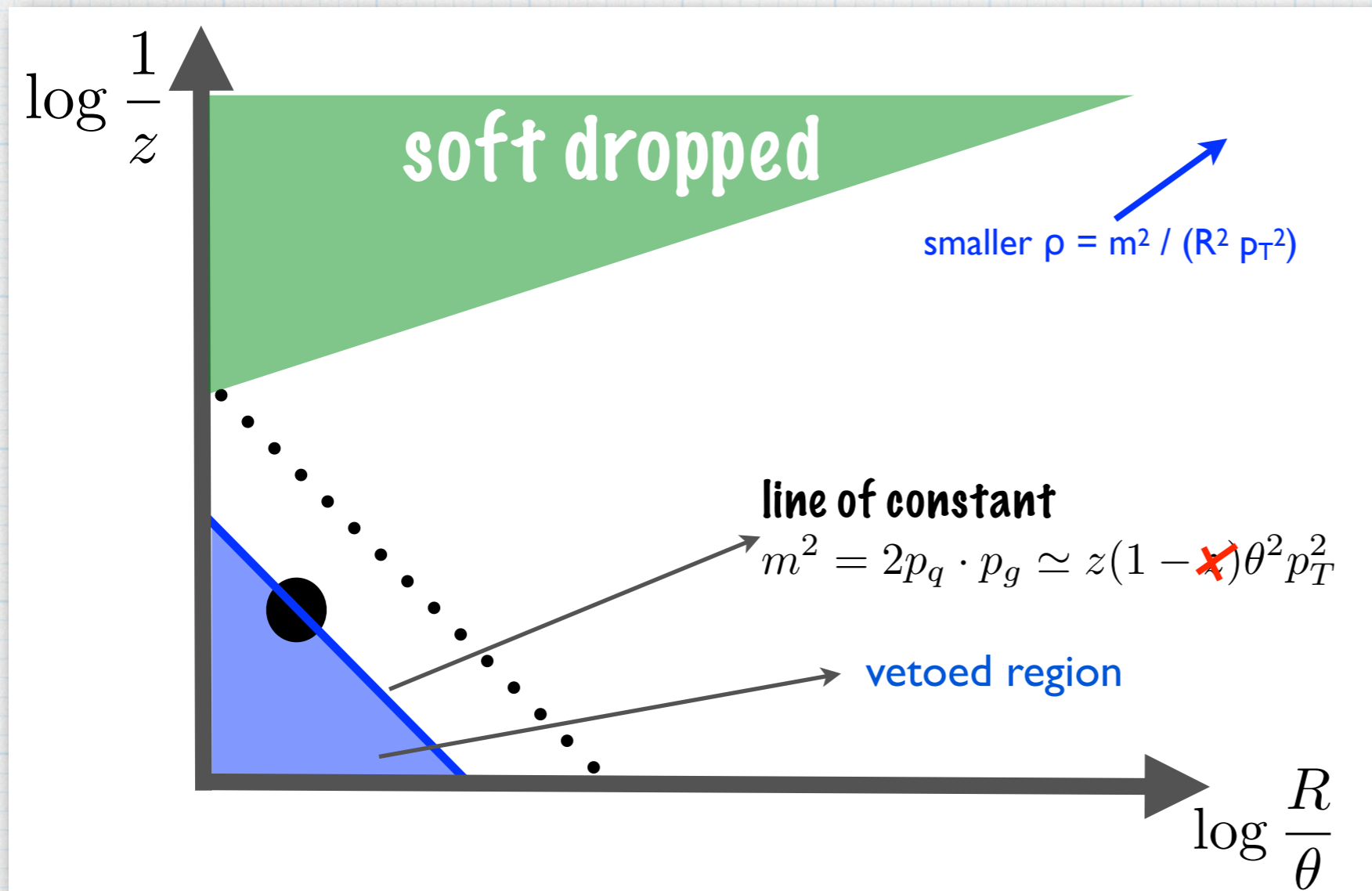


\* soft-drop condition becomes

$$z > z_{\text{cut}} \left( \frac{\theta}{R} \right)^{\beta}$$

\* for  $\beta < 0$  some hard-collinear is also partially removed

# soft-drop mass at LL

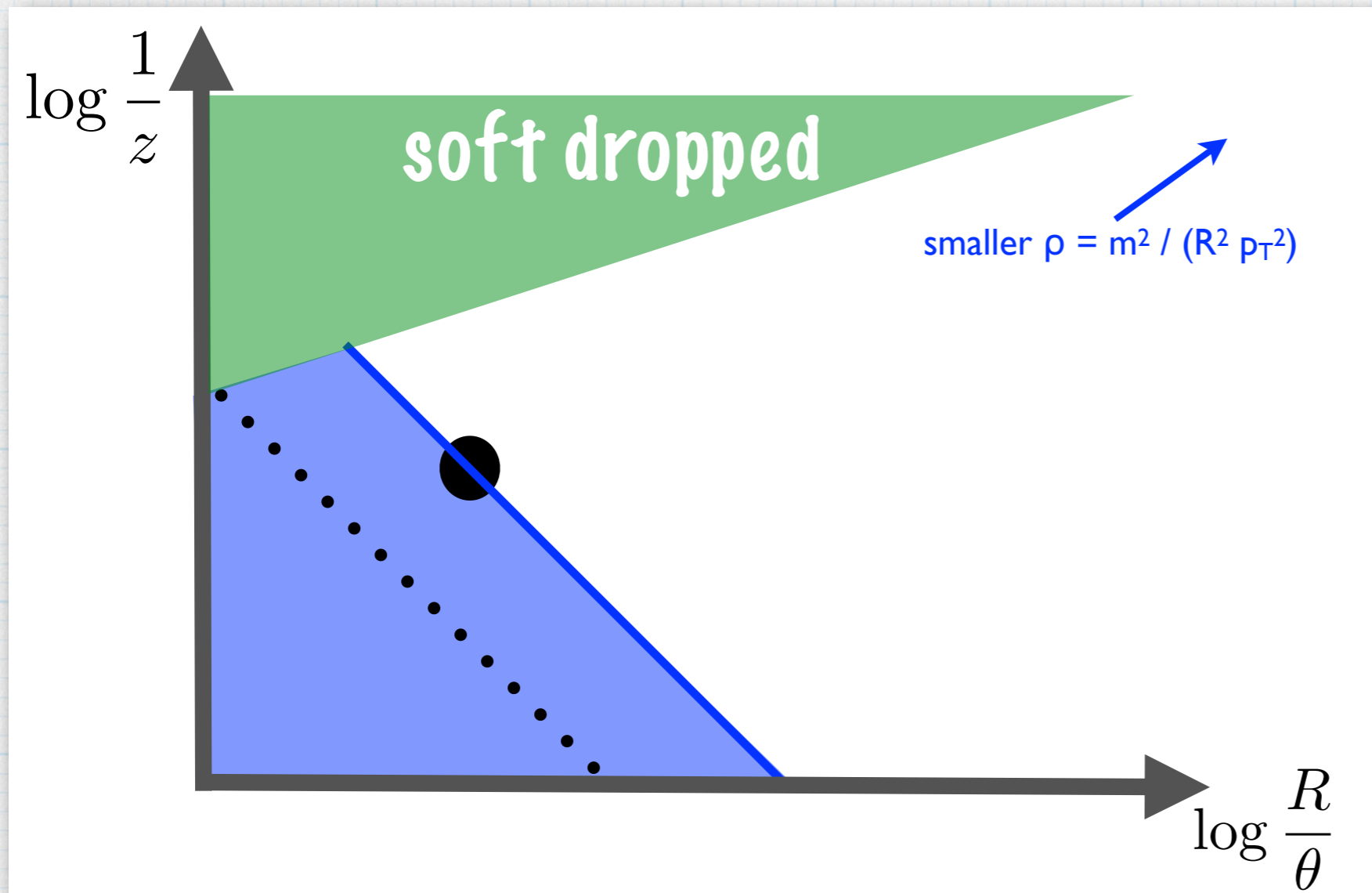


- \* one emission sets a **mass  $m$**
- \* veto emissions that would give too big a mass
- \* **soft drop here has no effect**

$$\Sigma(\rho) \equiv \int^{\rho} d\rho' \frac{1}{\sigma} \frac{d\sigma}{d\rho'}$$

$$\Sigma^{(s.d.)} = \exp \left[ -\frac{\alpha_s C_F}{2\pi} \left[ -\frac{3}{2} \ln \frac{1}{\rho} + \Theta(\rho > z_{\text{cut}}) \ln^2 \frac{1}{\rho} \right. \right. \\ \left. \left. + \Theta(z_{\text{cut}} > \rho) \left( \frac{\beta}{2 + \beta} \ln^2 \frac{1}{\rho} + \frac{2}{2 + \beta} \left( \ln^2 \frac{1}{z_{\text{cut}}} + 2 \ln \frac{z_{\text{cut}}}{\rho} \ln \frac{1}{z_{\text{cut}}} \right) \right) \right] \right]$$

# soft-drop mass at LL



\* only one transition point at  $\rho = z_{\text{cut}}$

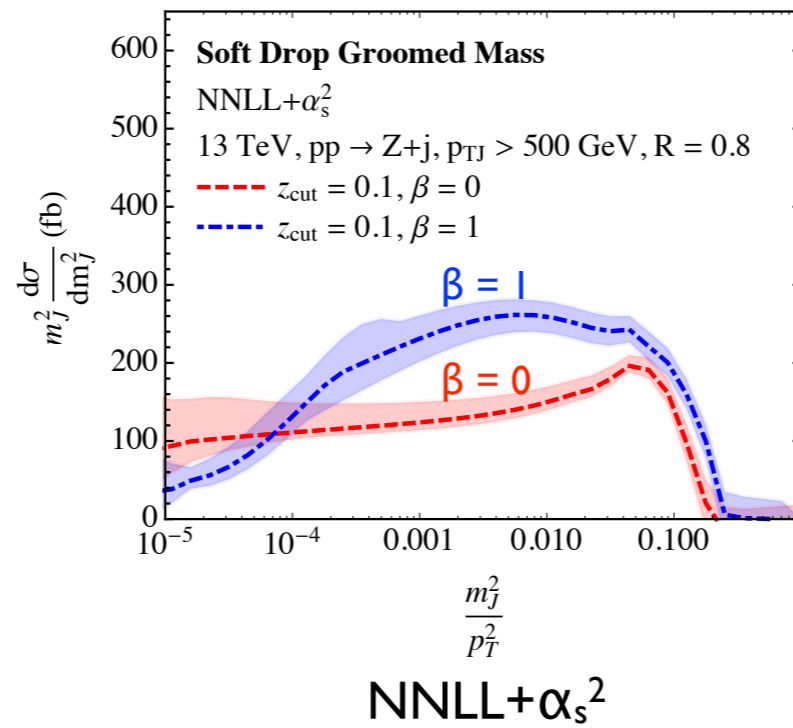
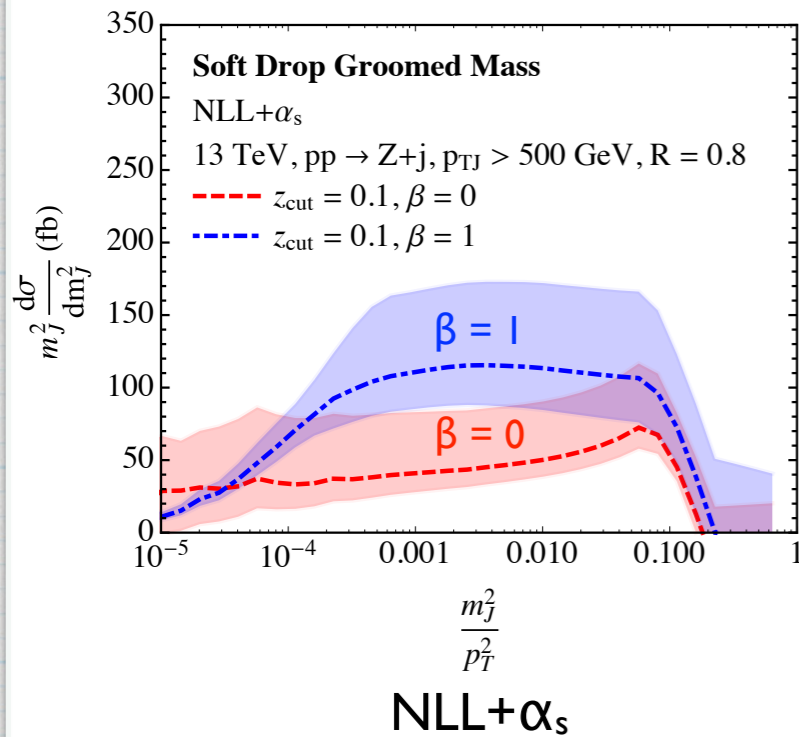
\* soft & soft collinear radiation is partially removed

\* only single logs for  $\beta=0!$

$$\Sigma^{(\text{s.d.})} = \exp \left[ - \frac{\alpha_s C_F}{2\pi} \left[ - \frac{3}{2} \ln \frac{1}{\rho} + \Theta(\rho > z_{\text{cut}}) \ln^2 \frac{1}{\rho} + \Theta(z_{\text{cut}} > \rho) \left( \frac{\beta}{2+\beta} \ln^2 \frac{1}{\rho} + \frac{2}{2+\beta} \left( \ln^2 \frac{1}{z_{\text{cut}}} + 2 \ln \frac{z_{\text{cut}}}{\rho} \ln \frac{1}{z_{\text{cut}}} \right) \right) \right] \right]$$

# precision jet substructure

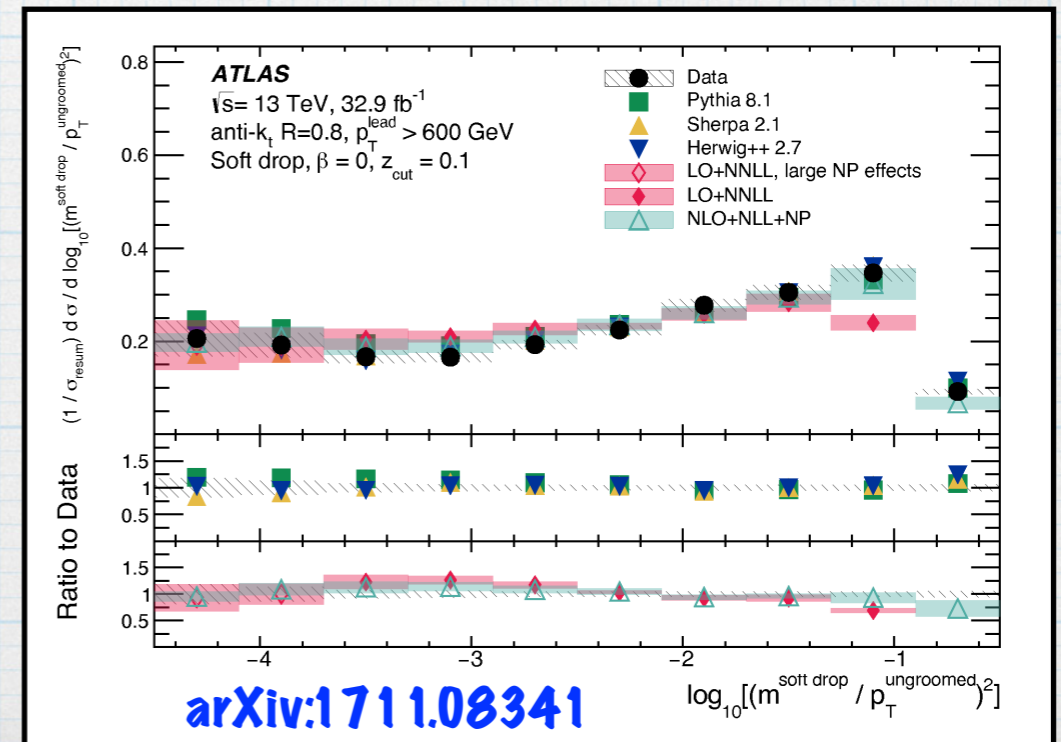
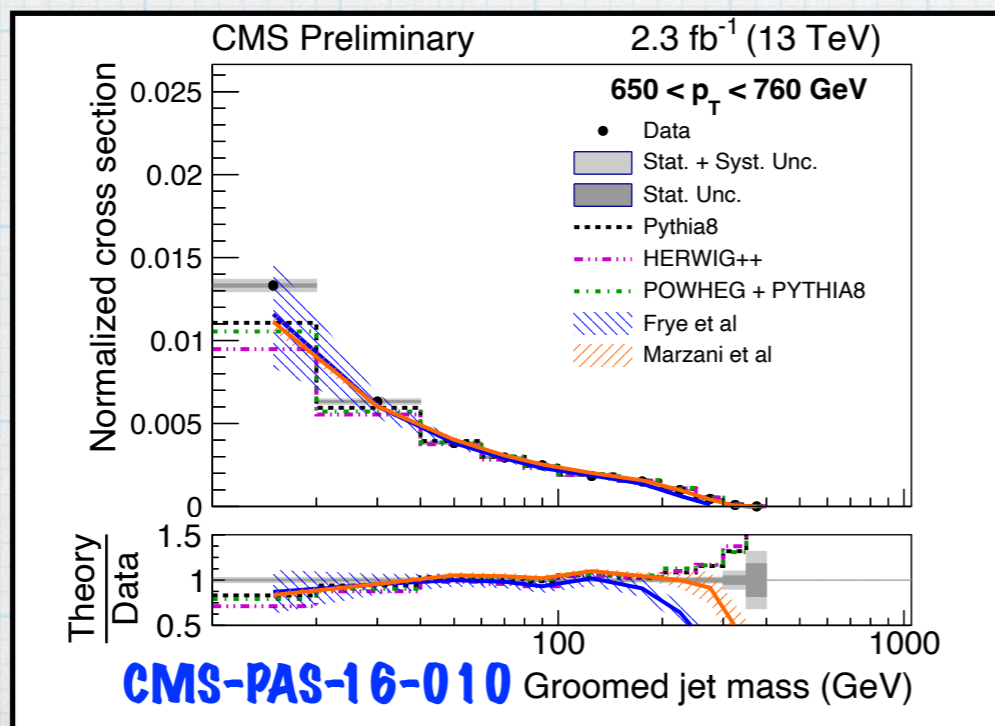
## Results: NNLL+ $\alpha_s^2$ Jet Substructure



- \* using SCET, precision pushed to NNLL
- \* no non-global logs
- \* no colour correlations

Frye, Larkoski, Schwartz, Yan (2016)

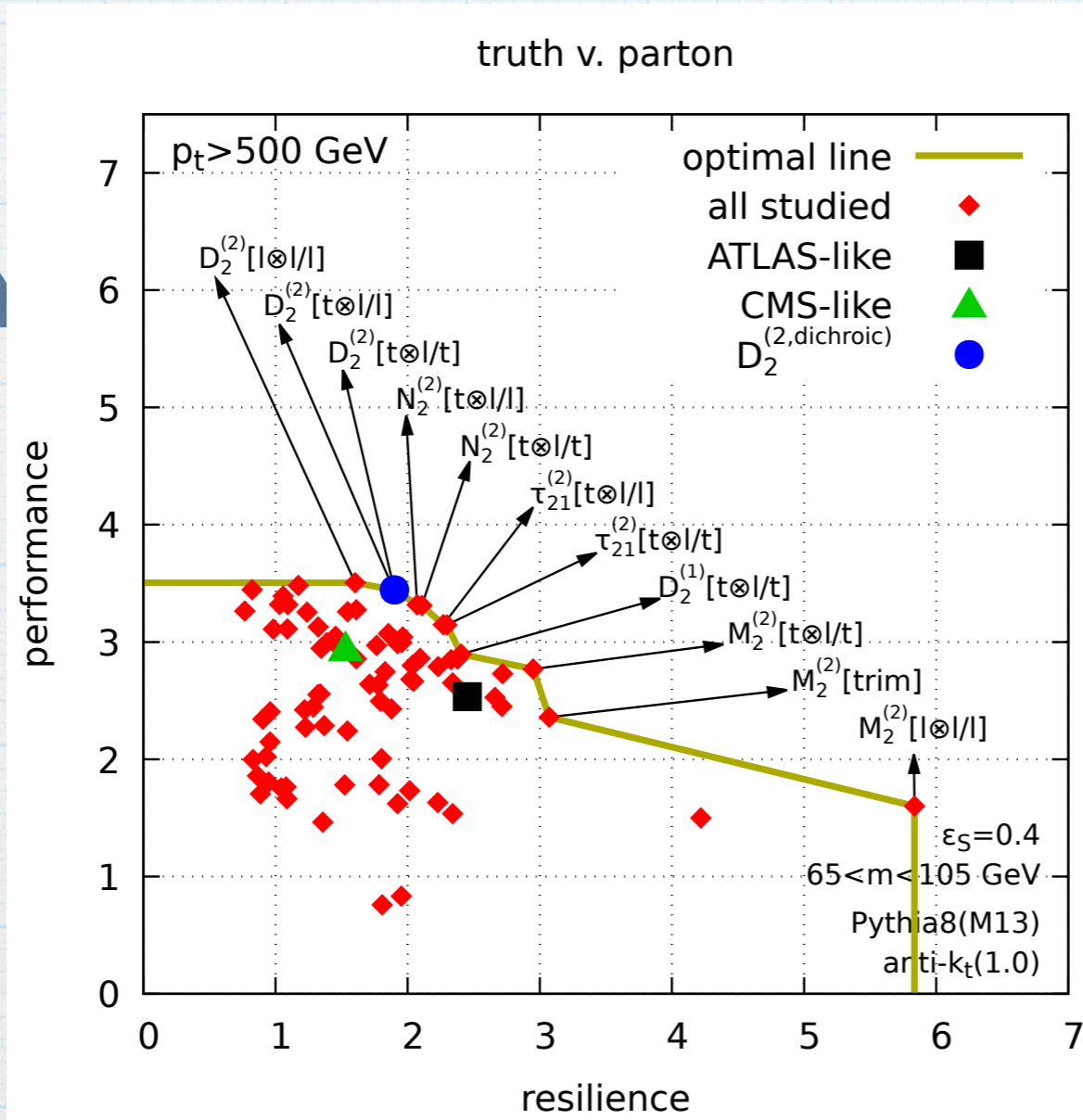
and data!





# performance & resilience

more efficient



$$\zeta = \left( \frac{\Delta\epsilon_S^2}{\langle\epsilon\rangle_S^2} + \frac{\Delta\epsilon_B^2}{\langle\epsilon\rangle_B^2} \right)^{-1/2}$$

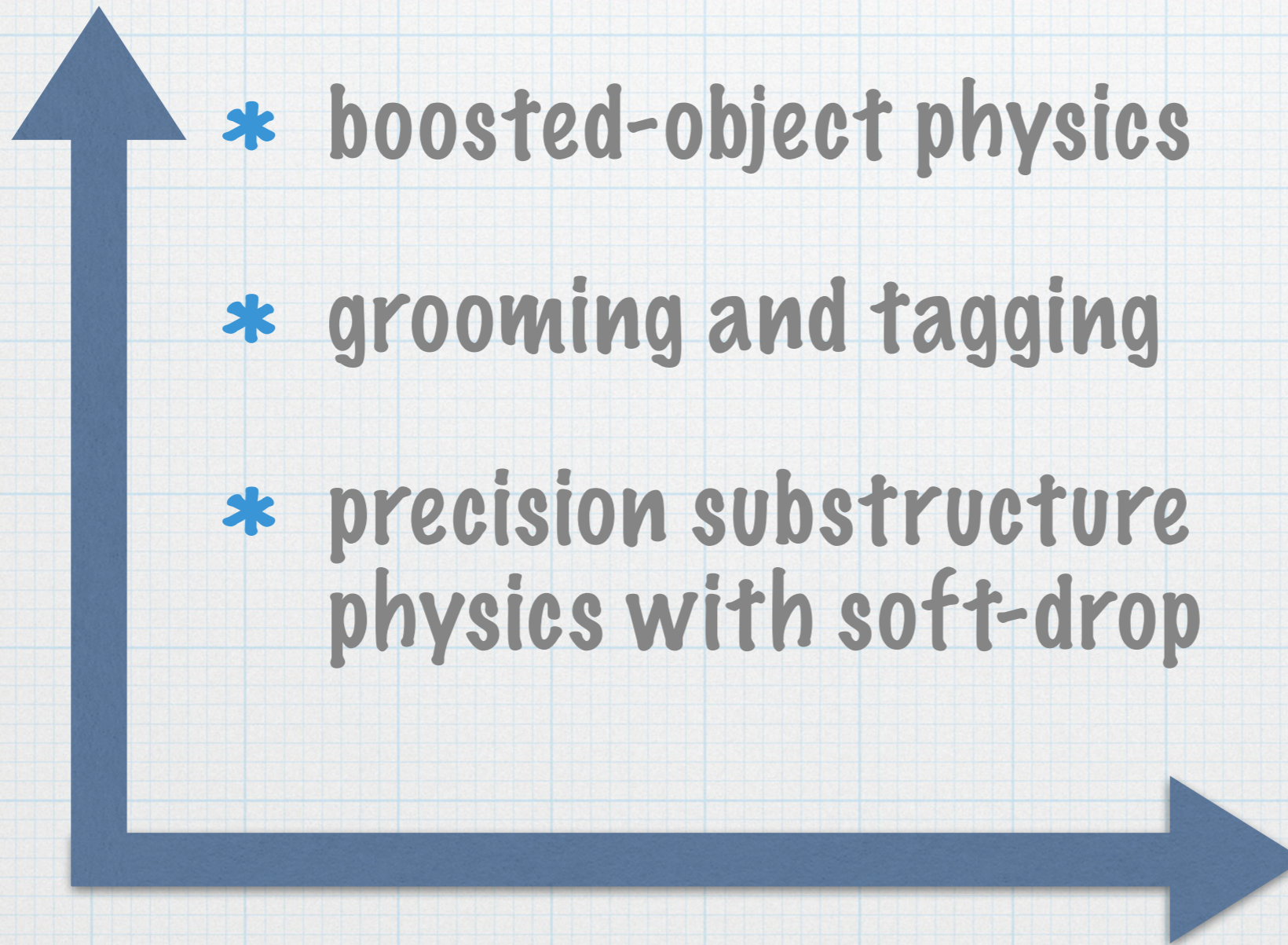
$$\Delta\epsilon_{S,B} = \epsilon_{S,B} - \epsilon'_{S,B},$$

$$\langle\epsilon\rangle_{S,B} = \frac{1}{2} (\epsilon_{S,B} + \epsilon'_{S,B})$$

more robust

# summary of lecture 2

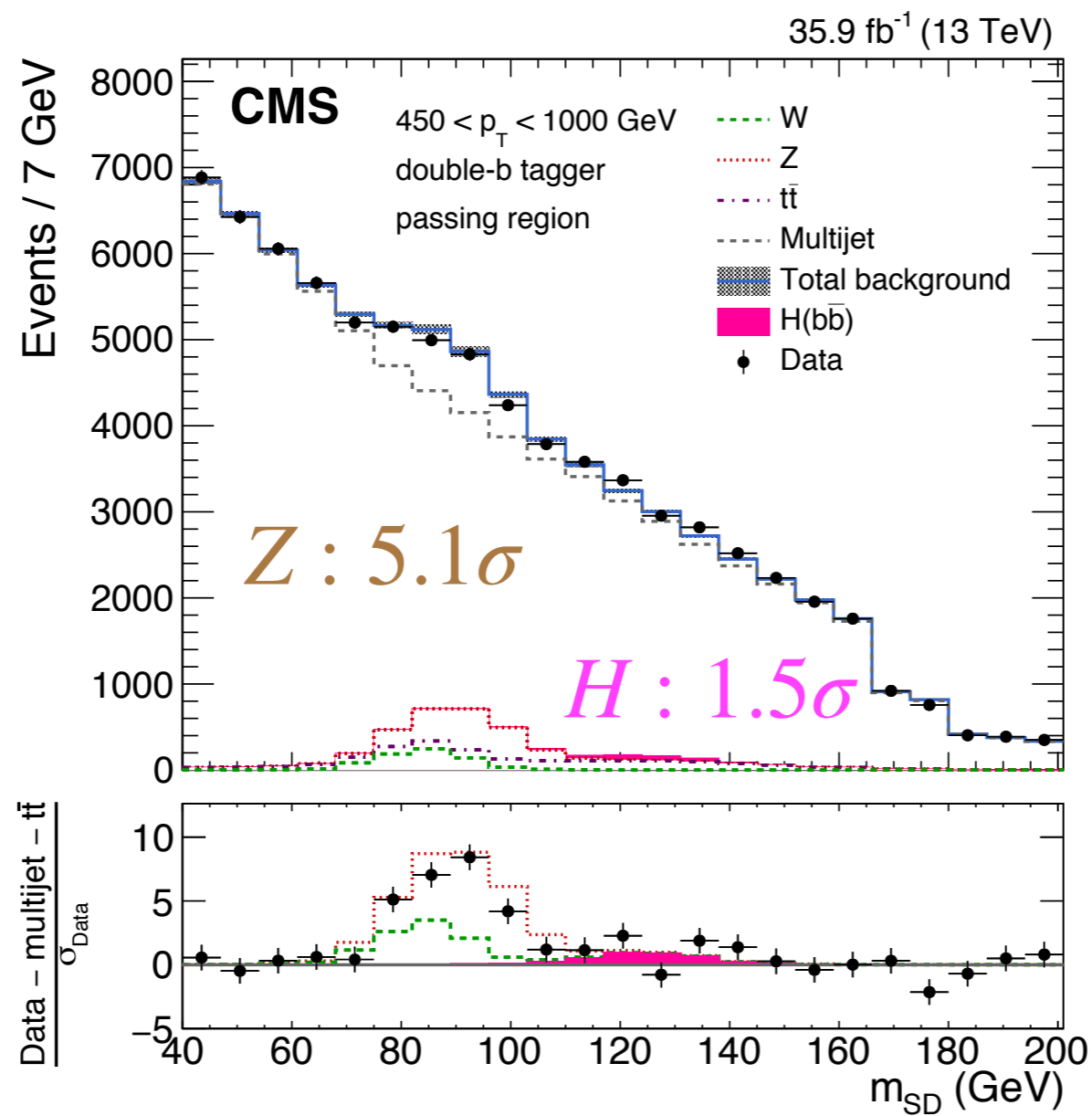
more efficient



more robust

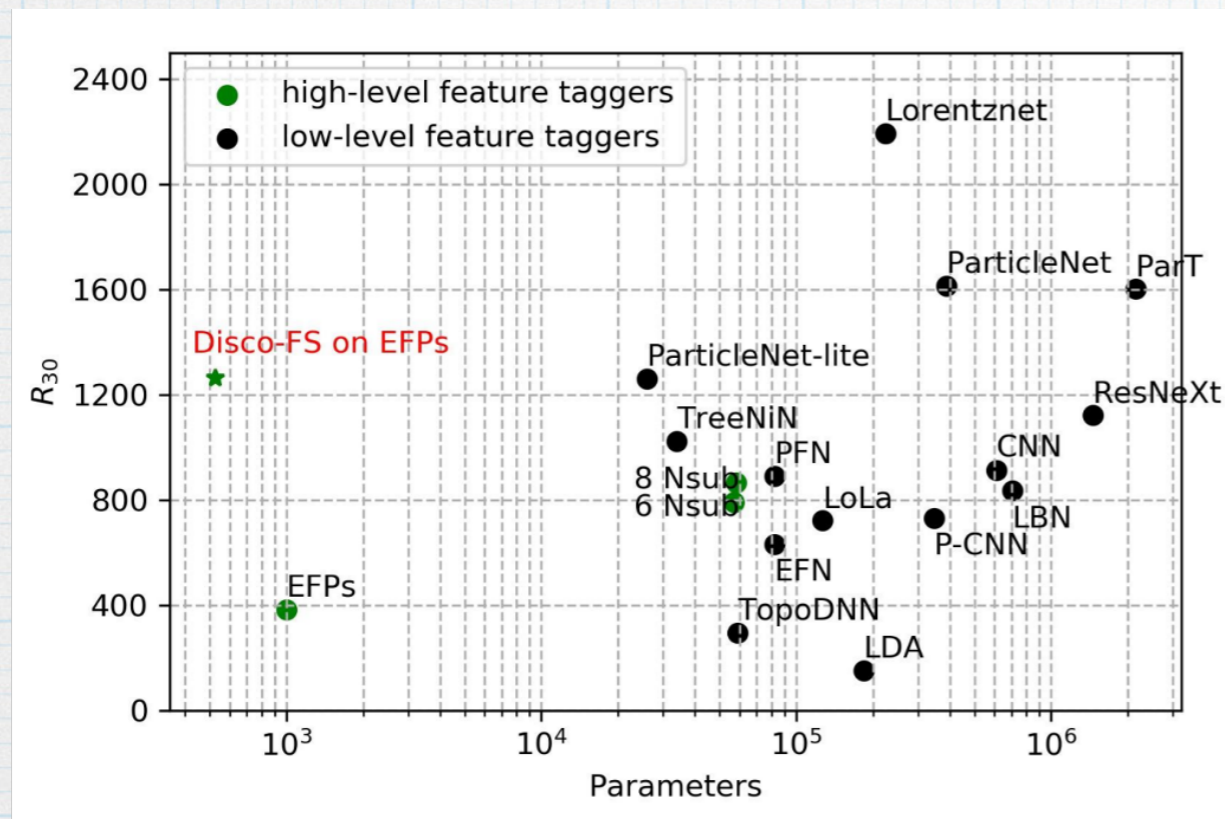
# summary of lecture 2

more efficient



more robust

# looking at ML



*adapted from Ranit Das talk at BOOST 2022*

**more efficient**

\* challenge: what is the best metric to measure robustness for ML algorithms?

**more robust ???**

# homework 3

- \* Gluon splitting into bottom quarks  $g \rightarrow bb$  is important for  $H \rightarrow bb$  studies. What's its average mass? (take  $m_b=0$ )