Jets and their structure

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summer school

Lecture 1: jets

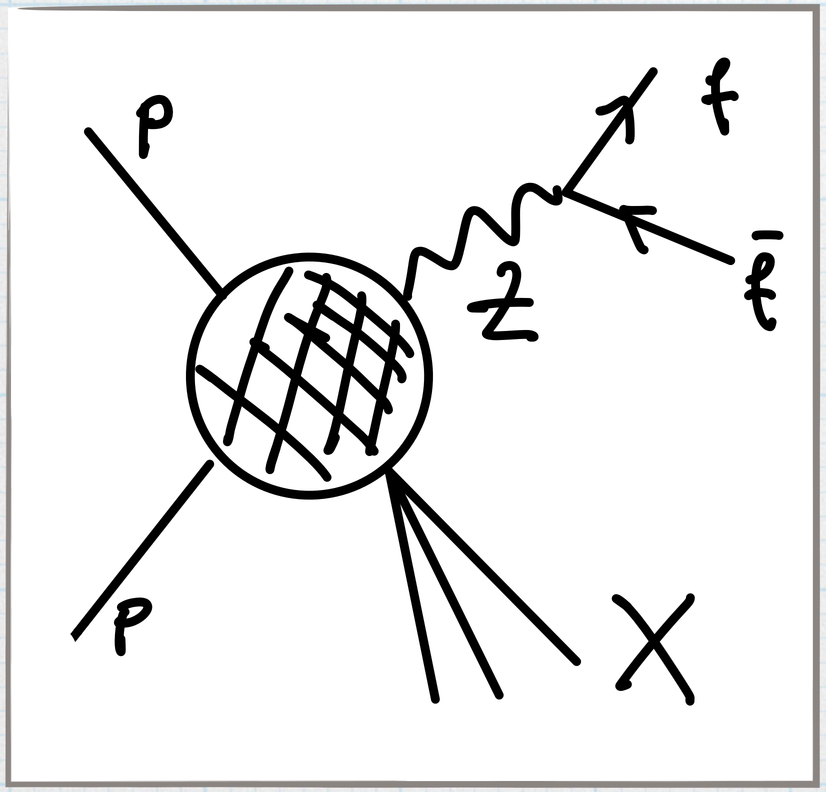
* in these two lectures we study hadronic final states in terms of so-called jets
* lecture1: we'll discuss jet definitions: the focus will be on the theory and experimental motivations behind certain choices
* lecture2: basics concepts of jet substructure and our first principle understanding
* I have only 2 hours, so I had to compromise. Two big topics are missing: energy-correlators and machine-learning approaches. Ask me during recitation if you're interested!

resources

* G. Salam: "Towards jetography"
* G. Soyez: "Pileup mitigation at the LHC: a theorist's view"
* SM, M. Spannowsky, G. Soyez, "Looking inside jets: an introduction to jet substructure and boosted-object phenomenology"

Lecture 1: jets

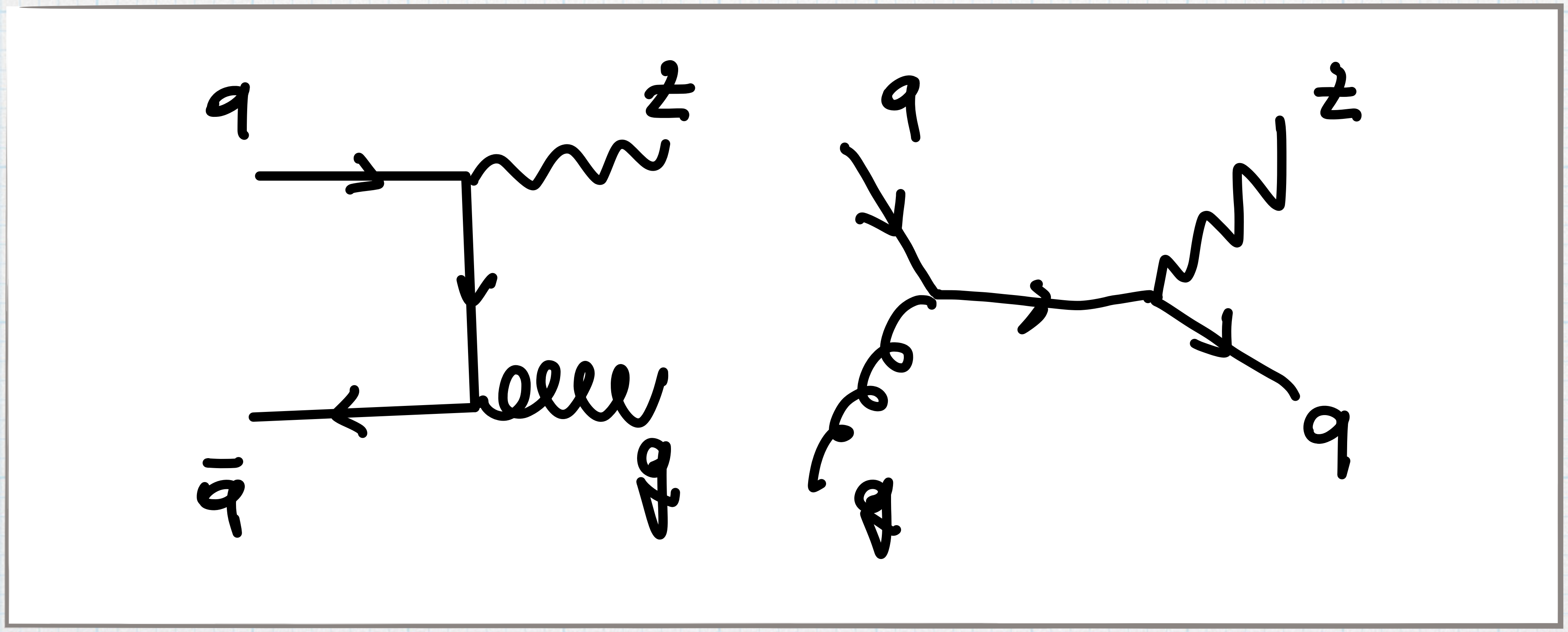
* inspire by Dave's lectures, let us start from perturbative QCD



* $pp \to Z(\to f\bar{f}) + X$
* key-process at the LHC: SM tests and background (e.g. monojets)
* Can we characterise X?

X at lowest order

* we can employ perturbation theory: at $O(\alpha_s)$, X is just a quark or a gluon

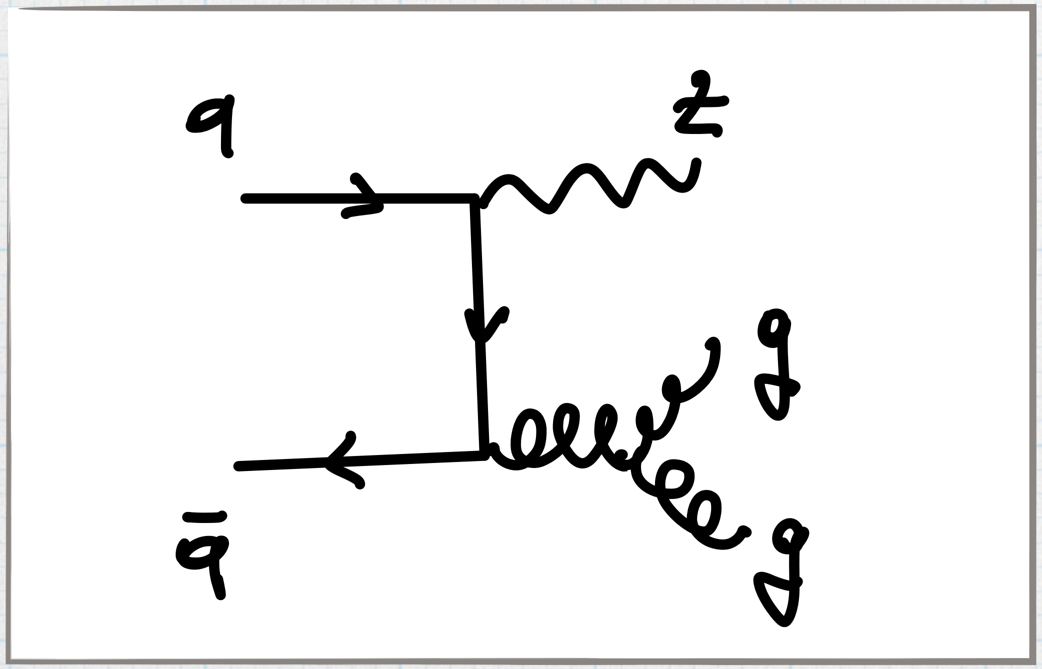


* momentum conservation relates the kinematics of X to the Z one

X beyond LO

* at $O(\alpha_s^2)$, we have
  - real emission (2 partons)
  - virtual correction (1 parton)

* can we compute the cross section for Z+2 partons?



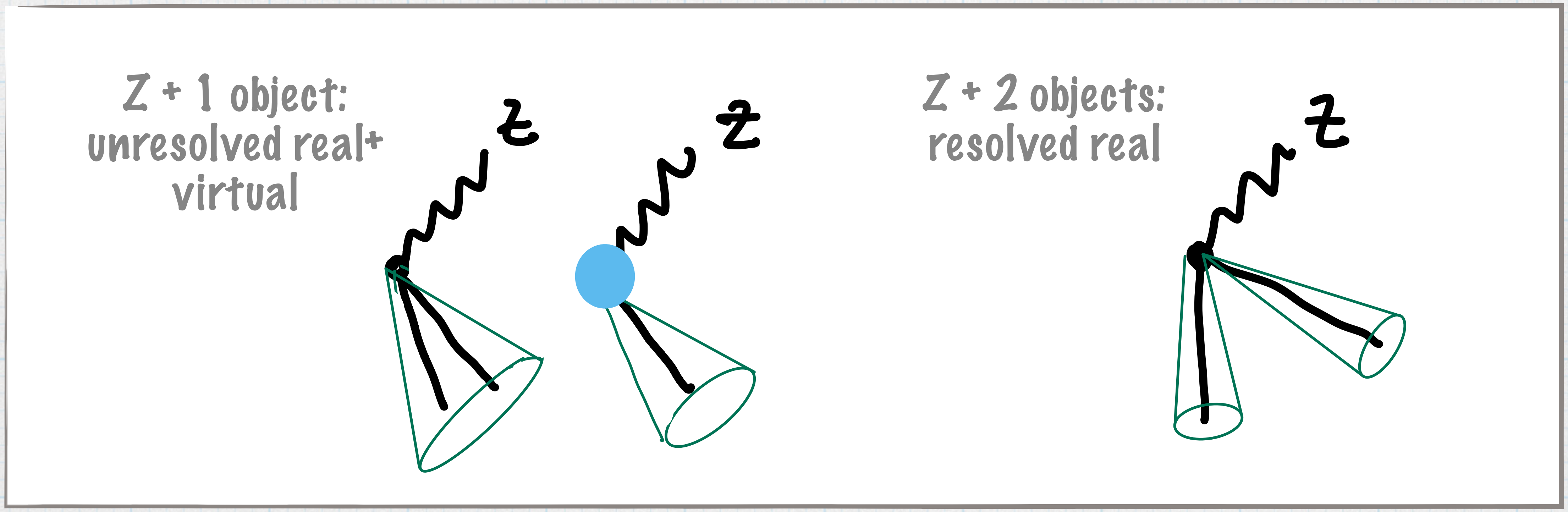
* ISR collinear singularities absorbed by PDFs
* FSR singularities should cancel against virtual corrections... but we don't have them!

* similarly, we cannot compute the cross section for Z+1 parton

Jets come to rescue us

* pert. theory gives us a divergent result for Z+fixed number (n) of partons!

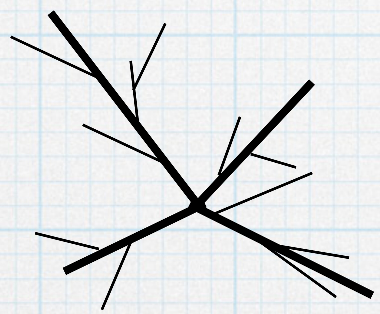
* we need to be more inclusive: Z+ n "objects"



* these objects are called jets

jets for theorists

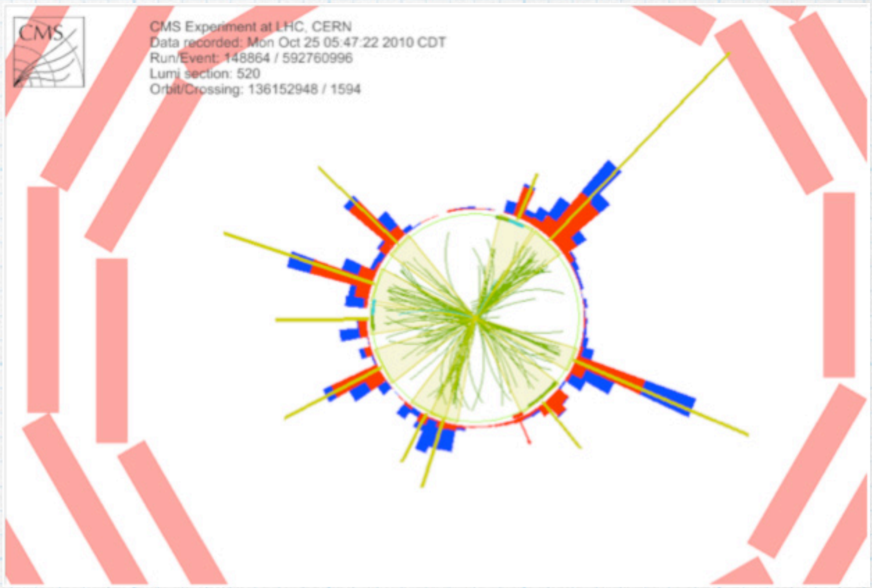
* jets are extremely useful for theorists
* powerful way of turning calculations into predictions



theory-land:
(quarks & gluons)

??

jets

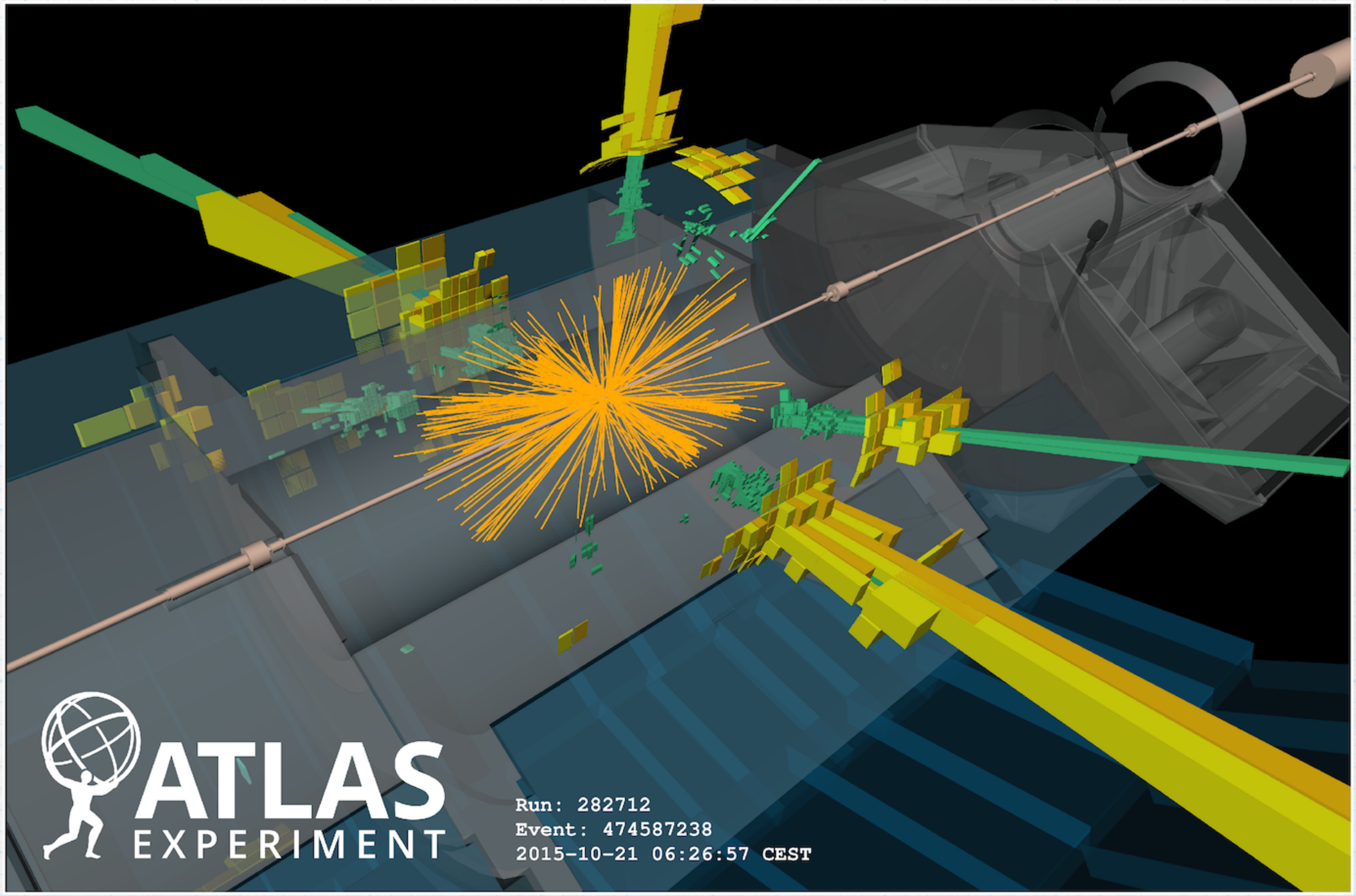


real data
(π,K,p,e,μ)

thanks to jets we can
reduce the complexity of the final state, simplifying many hadrons to simpler objects that one can hope to calculate

jets for experimentalists

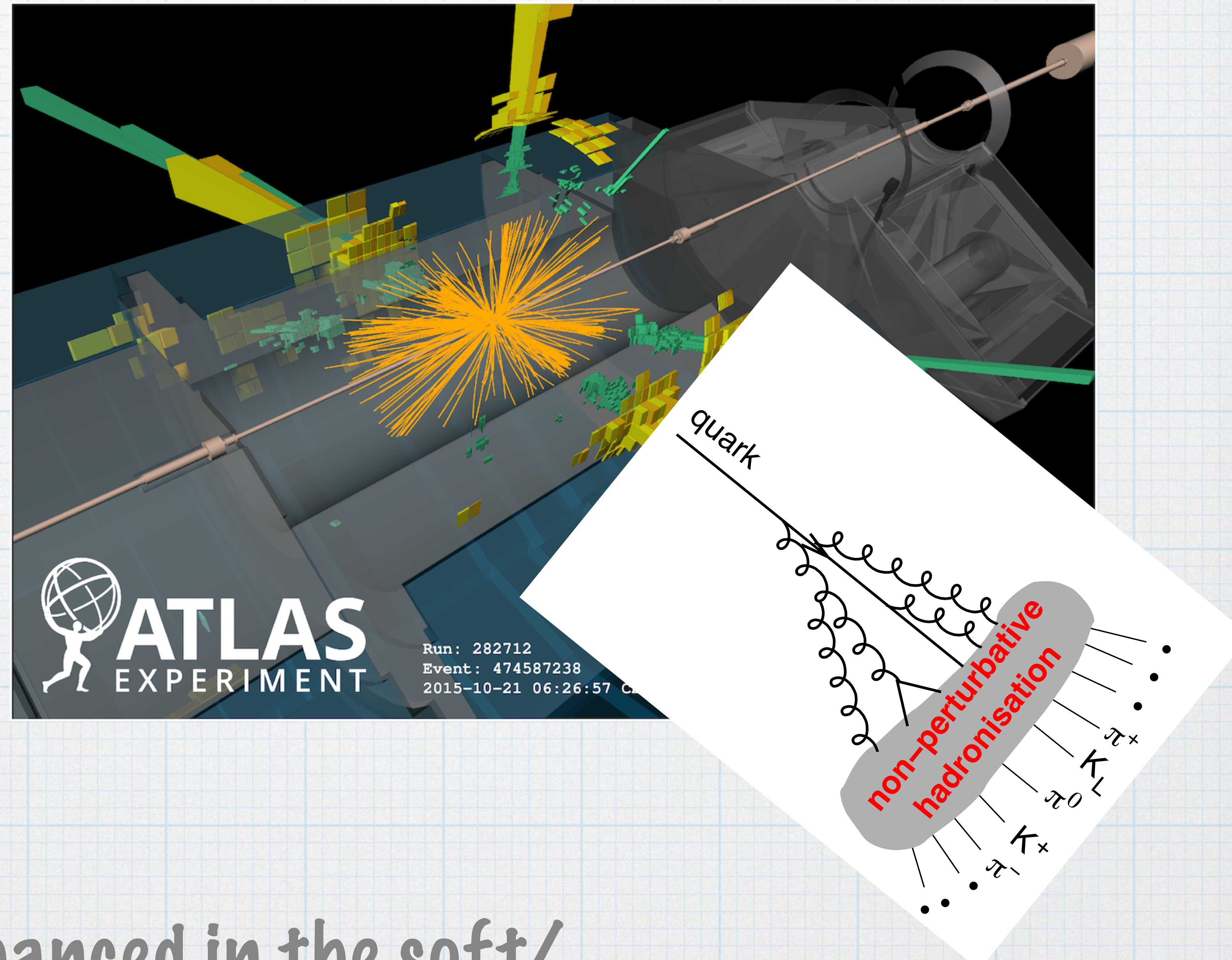
* high-energy collisions often results into collimated sprays of particles



* why?

jets for experimentalists

- high-energy collisions offer results into collimated sprays of particles



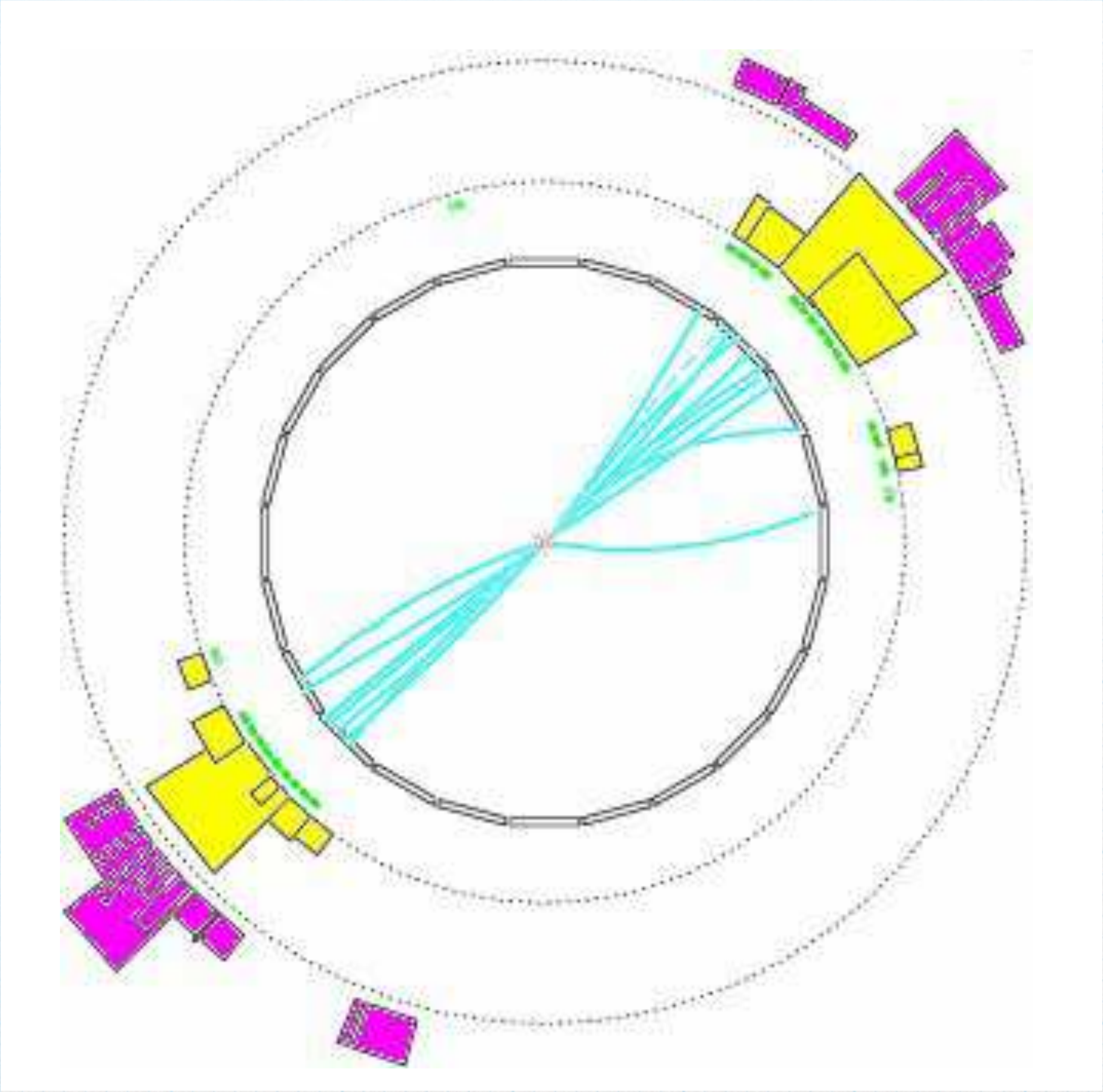
- why?

gluon emission enhanced in the soft/collinear limit

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s \gg 1$$

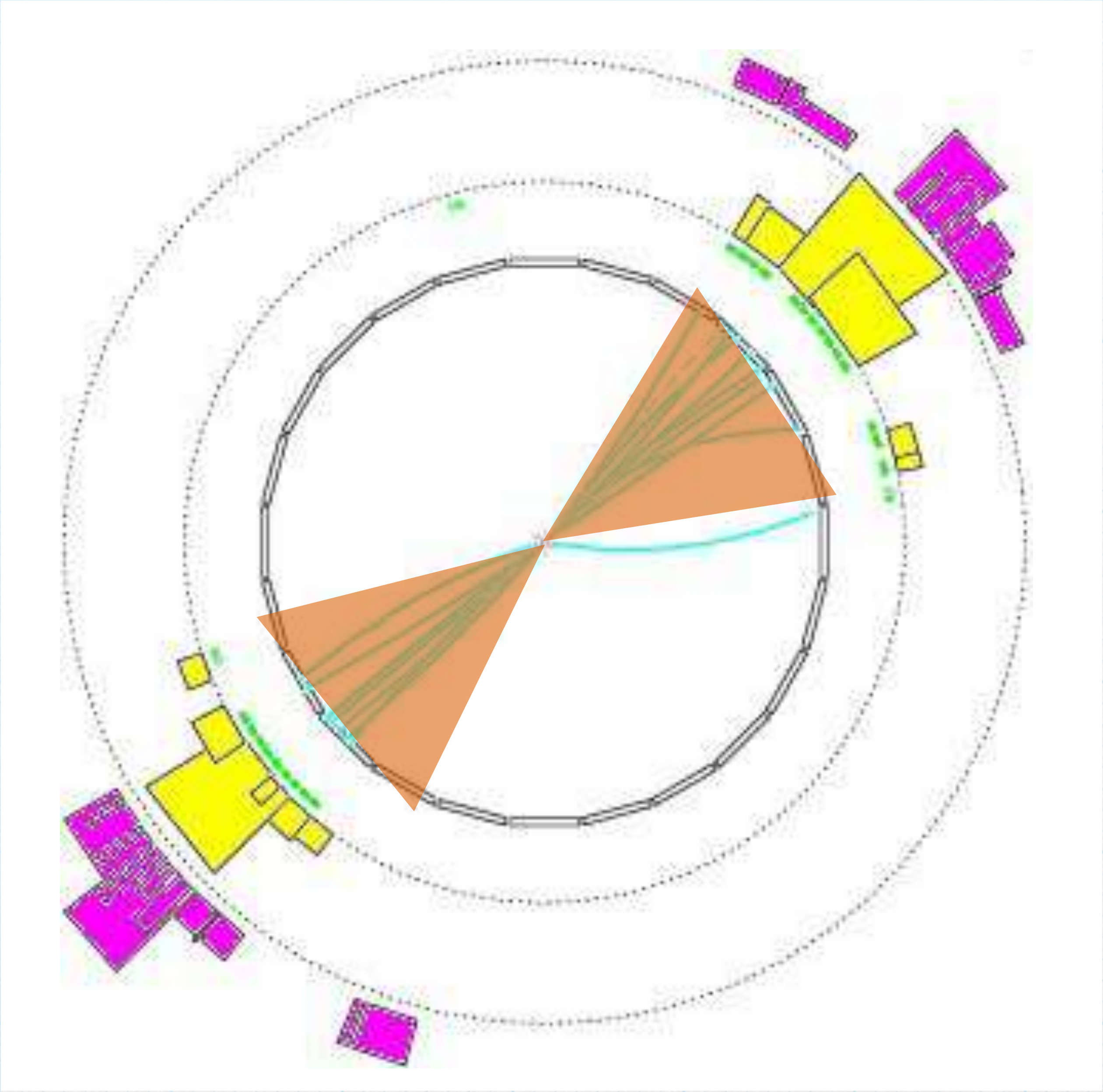
what is a jet?

* how many jets do you see?



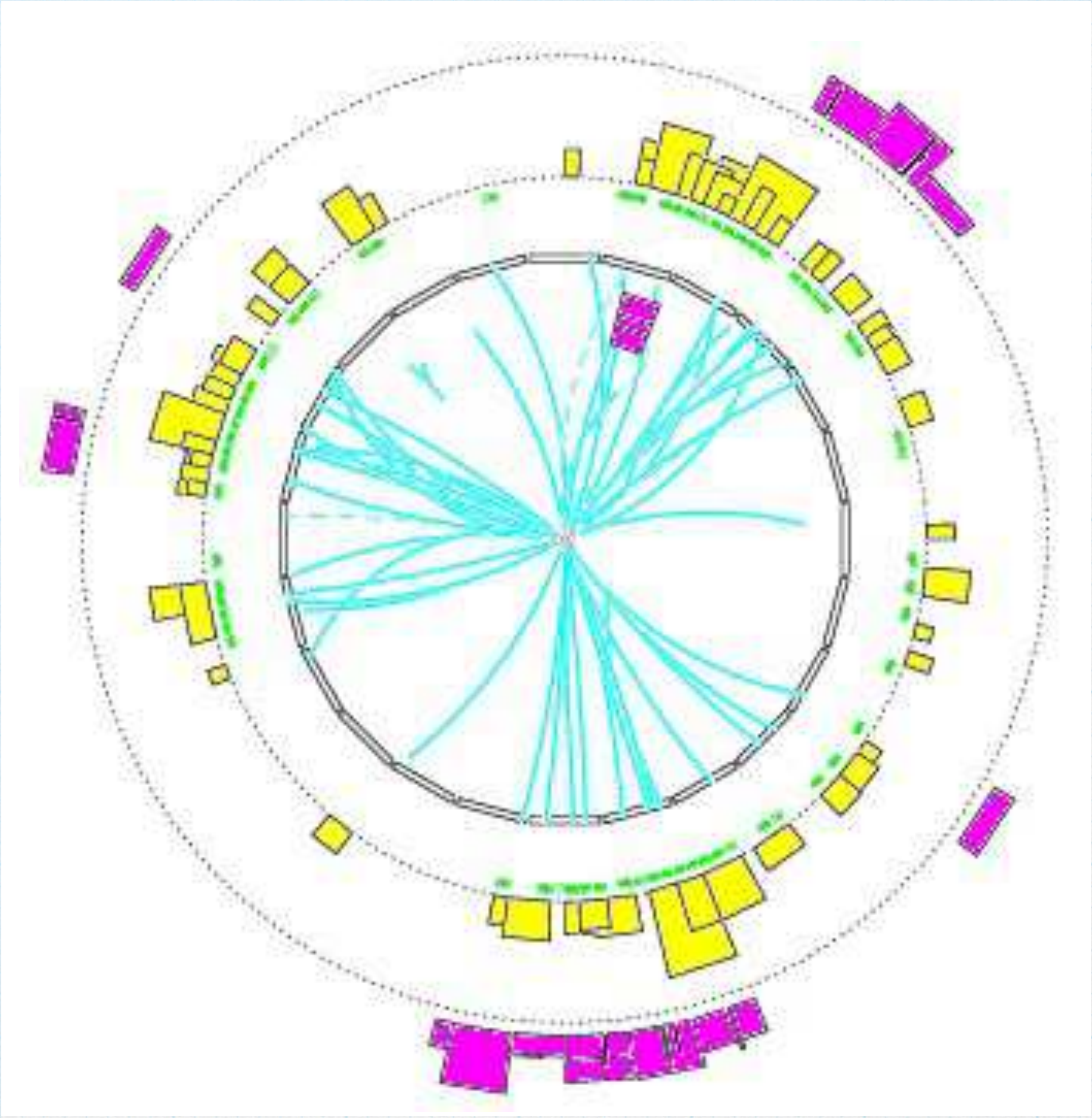
what is a jet?

* how many jets do you see?
* two is probably a good guess
* eyeballing not good enough!



what is a jet?

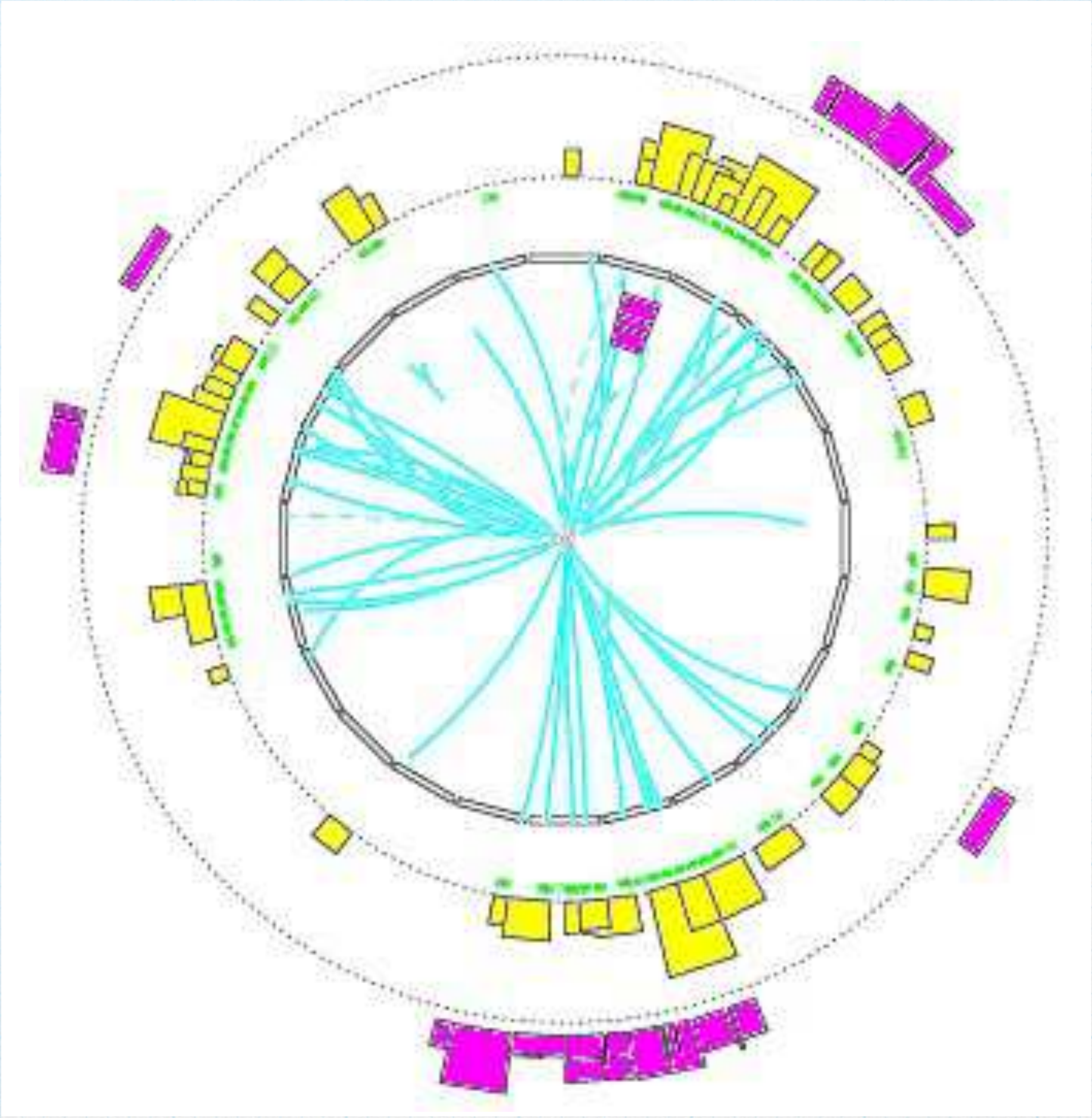
* what about now ?



what is a jet?

* what about now ?

* messy events are more ambiguous

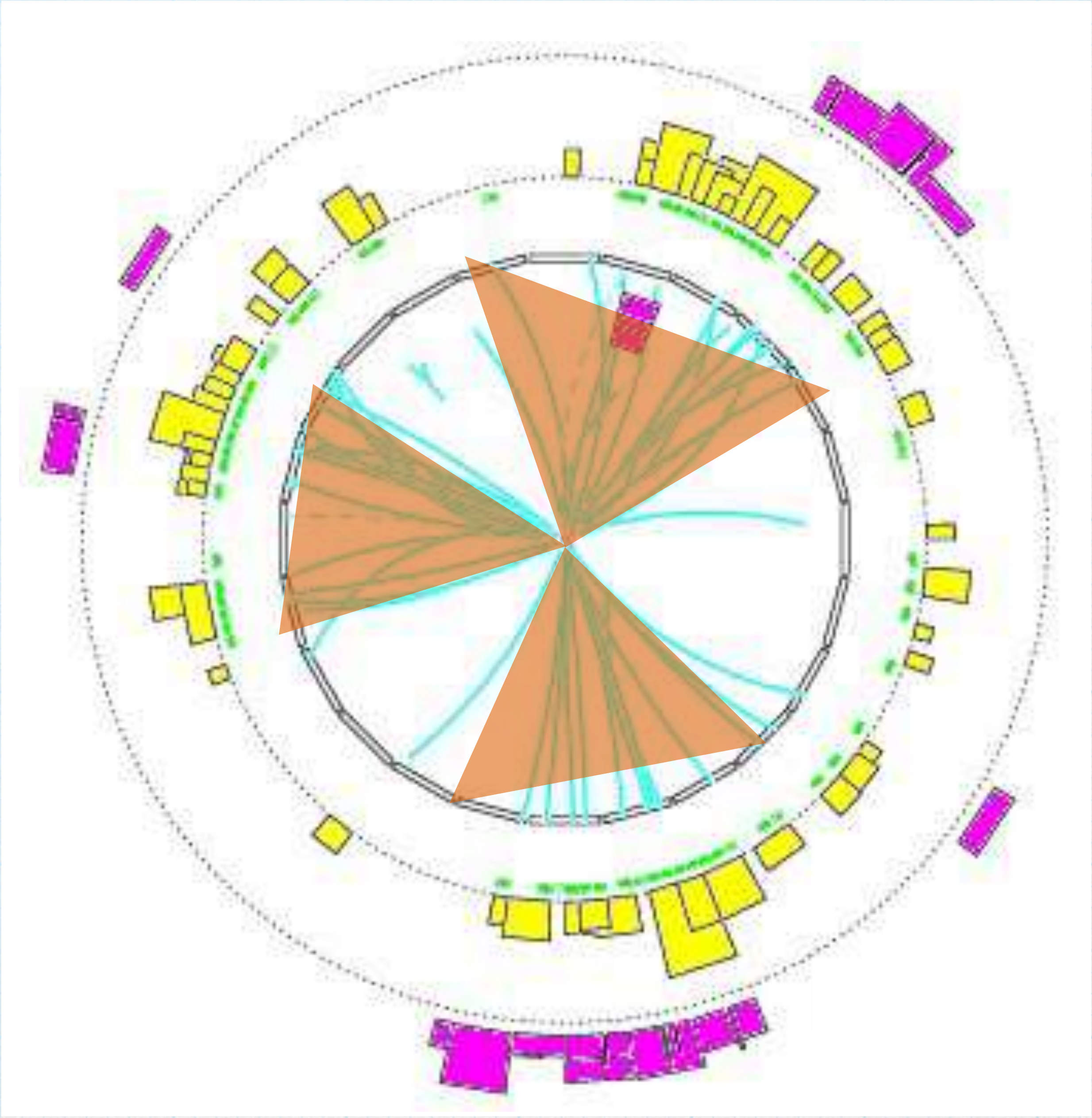


what is a jet?

* what about now ?

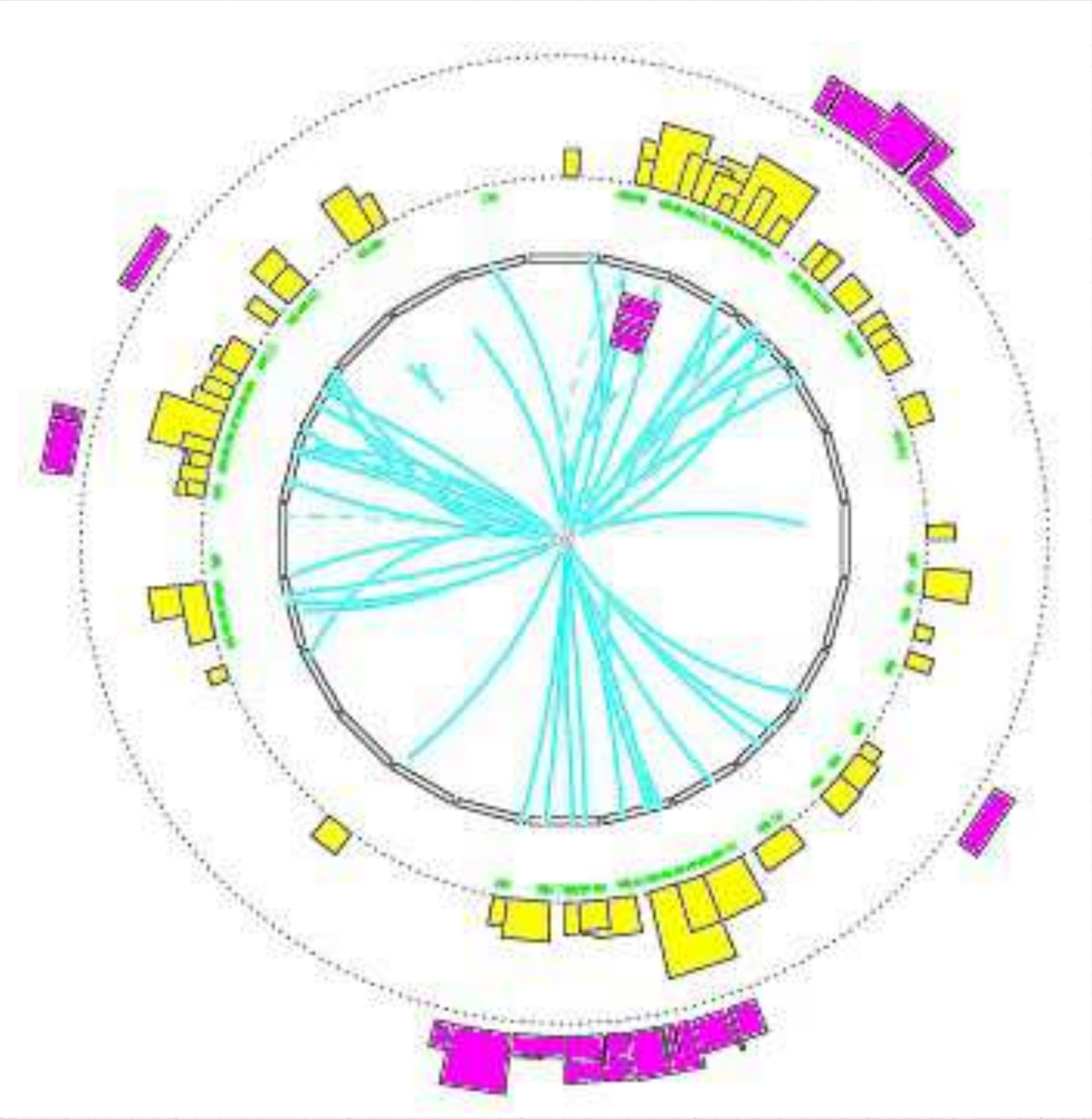
* messy events are more ambiguous

* 3 jet event?



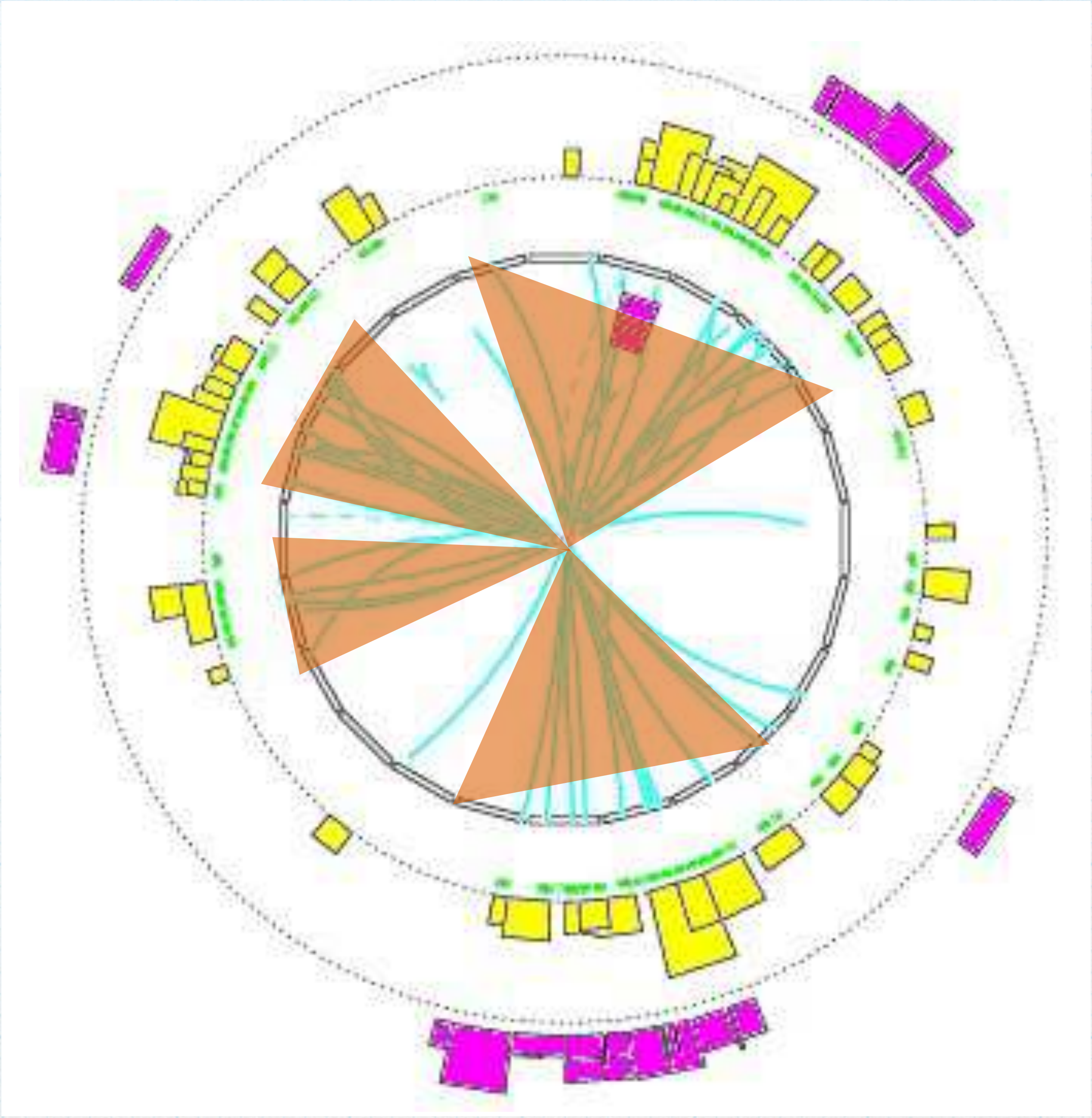
what is a jet?

* what about now ?



what is a jet?

* what about now ?
* messy events are more ambiguous
* or 4 jet event?



* we need a way to define jets in a given event

jet definition

a jet algorithm
+
its parameters (e.g. R)
+
a recombination scheme
=
a jet definition

* examples of recombination schemes:
  * E-scheme: sum all the four momenta
  * winner-take-all

jet clustering algorithm

* an algorithm that maps the momenta of the final state particles into the momenta of a certain number of jets

jet algorithm

$$\{p_i\} \longrightarrow \{j_k\}$$

often comes with resolution parameters e.g. R

particles,
4-momenta,
calorimeter towers, ...

jets

* jet definitions must make sense for both theorists and experimentalists!

what do theorists want?

* Infra-Red and Collinear Safety!

* An observable is IRC safe if, in the limit of a collinear splitting, or the emission of an infinitely soft particle, the observable remains unchanged:

we need IRC safety if we want to compute things beyond LO!

$$O(X; p_1, \ldots, p_n, p_{n+1} \to 0) \to O(X; p_1, \ldots, p_n)$$

$$O(X; p_1, \ldots, p_n \parallel p_{n+1}) \to O(X; p_1, \ldots, p_n + p_{n+1})$$

what do experimentalists want?

* jet algorithms must be usable on real events

* fast and easy to calibrate

the Snowmass accord

* simple to implement in an experimental analysis;
* simple to implement in theoretical calculations;
* defined at any order of perturbation theory;
* yields finite cross-sections at any order of perturbation theory;
* yields cross-sections and distributions that are relatively insensitive to hadronisation

types of algorithms

* cone algorithms
  * top-down approach: find coarse regions of energy flow.
  * how? Find stable cones (i.e. their axis coincides with sum of momenta of particles in it)
  * can be programmed to be fairly fast, at the price of being complex and IRC unsafe
  * Examples: JetClu, MidPoint, ATLAS cone, CMS cone, SISCone ...

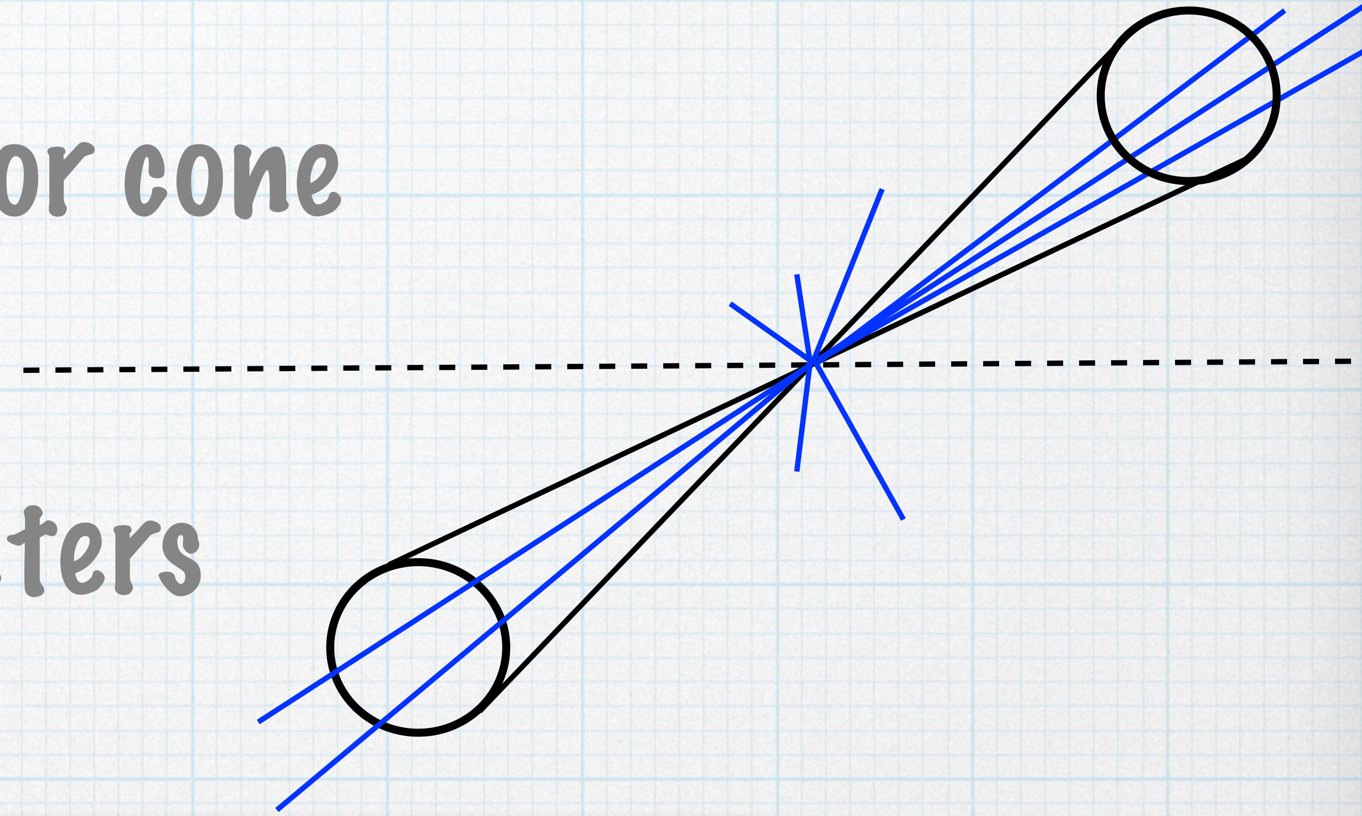
* sequential recombination algorithms
  * bottom-up approach: combine particles starting from closest ones
  * how? Choose a distance measure, iterate recombination until few objects left, call them jets
  * usually trivially made IRC safe, but their algorithmically complex (unless you're clever)
  * Examples: Jade, k_t, Cambridge/Aachen, anti-k_t ...

for a complete review see G. Salam, Towards jetography (2009)

a bit of history

* first calculation done for cone algorithm

* two resolution parameters



To study jets, we consider the partial cross section $\sigma(E,\theta,\Omega,\epsilon,\delta)$ for e^+e^- hadron production events, in which all but a fraction $\epsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measur-

Sterman and Weinberg,
Phys. Rev. Lett. 39, 1436 (1977):

Jet cross section at NLO

* let's start with the NLO 2-jet cross-section for a generic algorithm

* and separate out the divergent (IRC) from the finite (hard)

* QCD tells us that the virtual and real in the IRC limits are equal and opposite. We need IRC safety of the jet algorithm to ensure a finite result

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$$\sigma_{2\,\mathrm{jets}} = \int d\Phi_2(k_1,k_2)\,|\,\mathcal{M}_0 + \mathcal{M}_{1-\mathrm{loop}}\,|^2\, J_r(k_1,k_2)$$

$$+\int d\Phi_3(k_1,k_2,k_3)\,|\,\mathcal{M}_{\mathrm{real}}\,|^2\, J_r(k_1,k_2,k_3)$$

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jet definition for 2 and 3 particles

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$$+ \int d\Phi_2(k_1,k_2)\left[2\text{Re}\mathcal{M}^*_0\mathcal{M}^{\text{IRC}}_{1-\text{loop}} J_r(k_1,k_2) + \int d\Phi_1(k_3)\,|\,\mathcal{M}^{\text{IRC}}_{\text{real}}\,|^2 J_r(k_1,k_2,k_3)\right]$$

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IRC safety of Sterman-Weinberg jets

* let's go back to cone jets, at NLO we have

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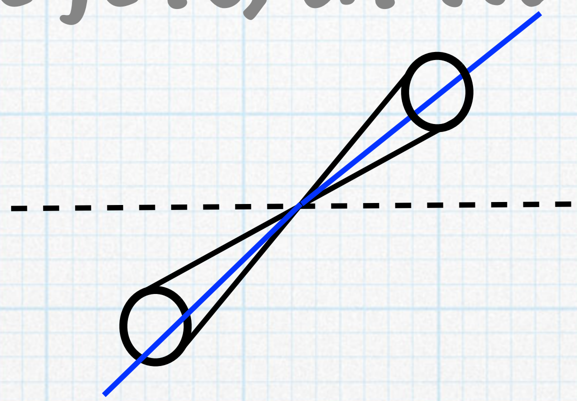
$$J_{\varepsilon,\delta}(k_1, k_2) = 1$$



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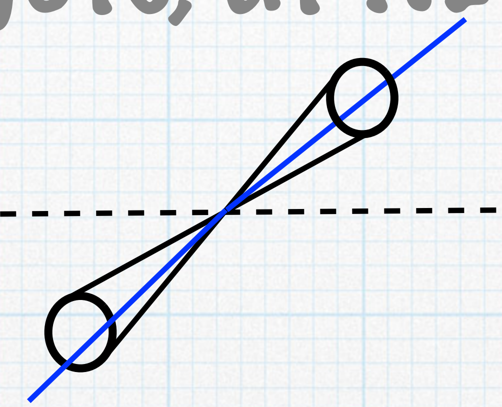
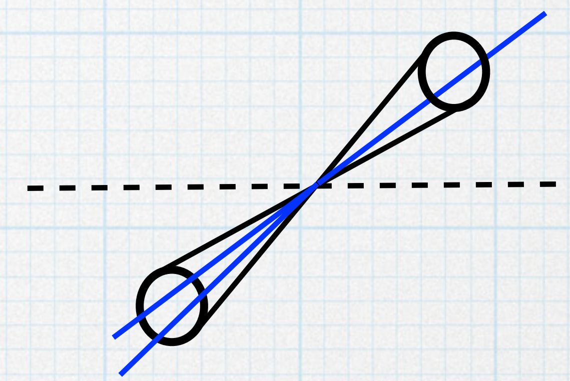


$$J_{\varepsilon,\delta}(k_1, k_2, k_3) = \Theta\left(\min(\theta_{12}, \theta_{13}, \theta_{23}) < \delta\right)$$



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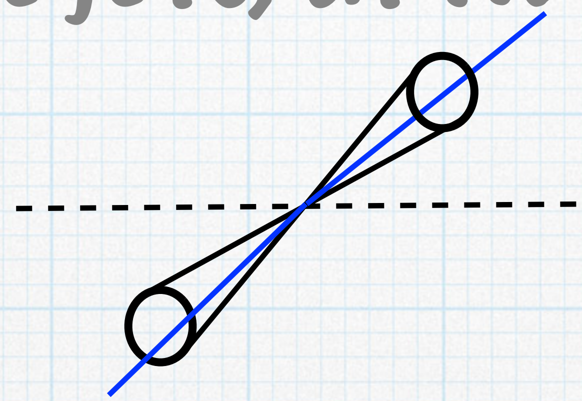

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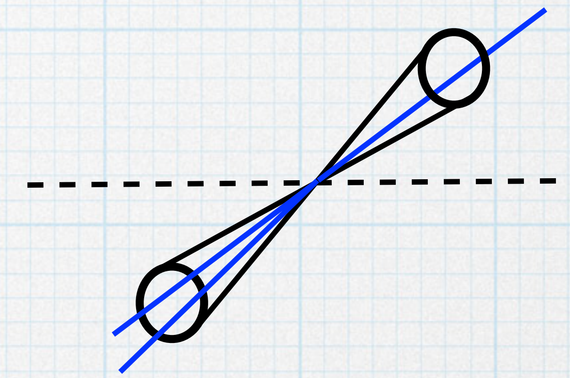
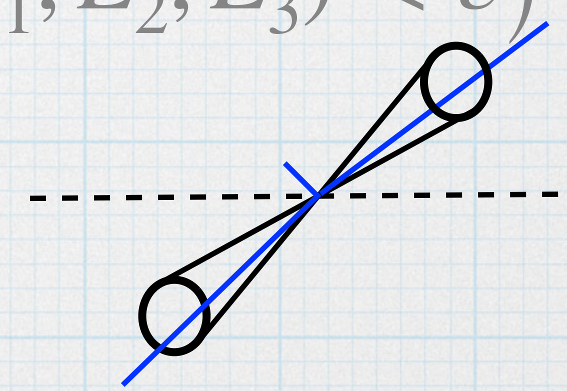


$$+\Theta\left(\min(\theta_{12}, \theta_{13}, \theta_{23}) > \delta\right)\Theta\left(\min(E_1, E_2, E_3) < \varepsilon\right)$$



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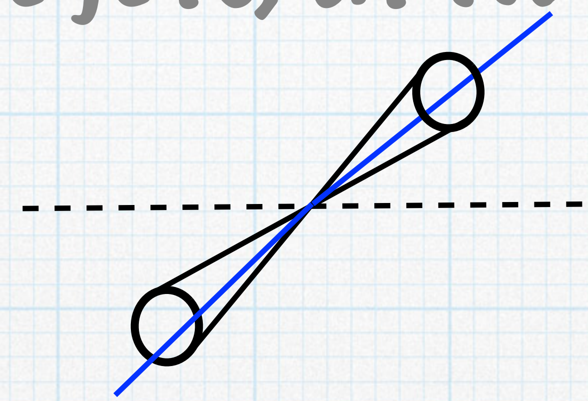

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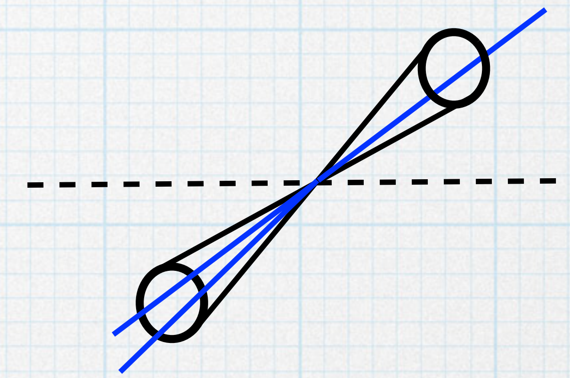

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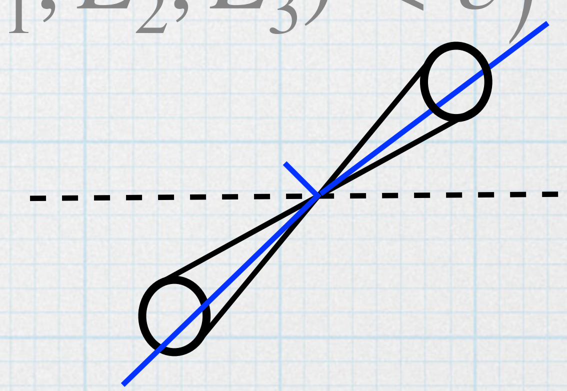
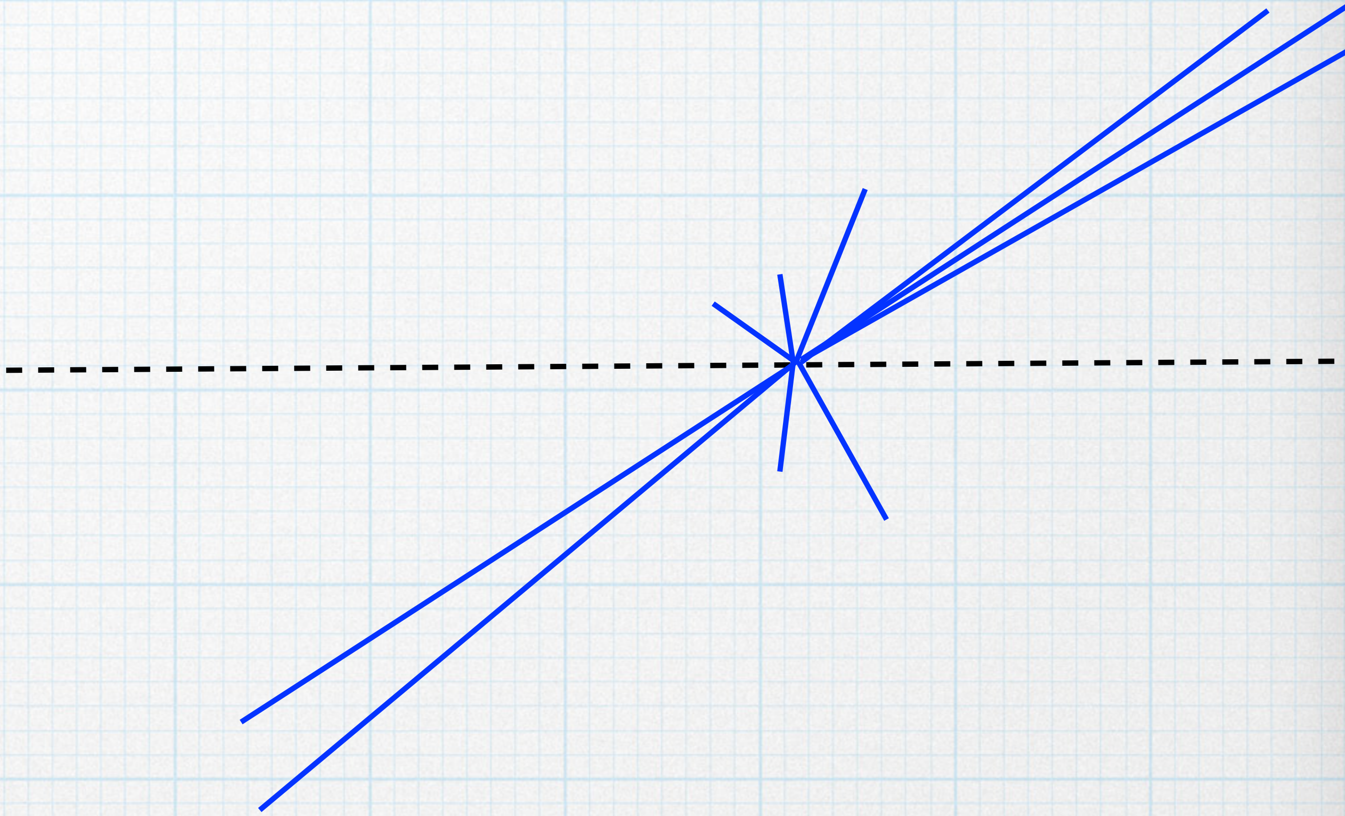


* it is straightforward to check that in any soft and/or collinear limit:

$$J_{\varepsilon,\delta}(k_1, k_2, k_3) \to 1$$

sequential recombination

* start with a list of particles,
* compute all distances d_{ij} and d_{iB}
* find the minimum of all d_{ij} and d_{iB}



d_{ij} (weighted) distance between i j
d_{iB} external parameter or distance from the beam ...

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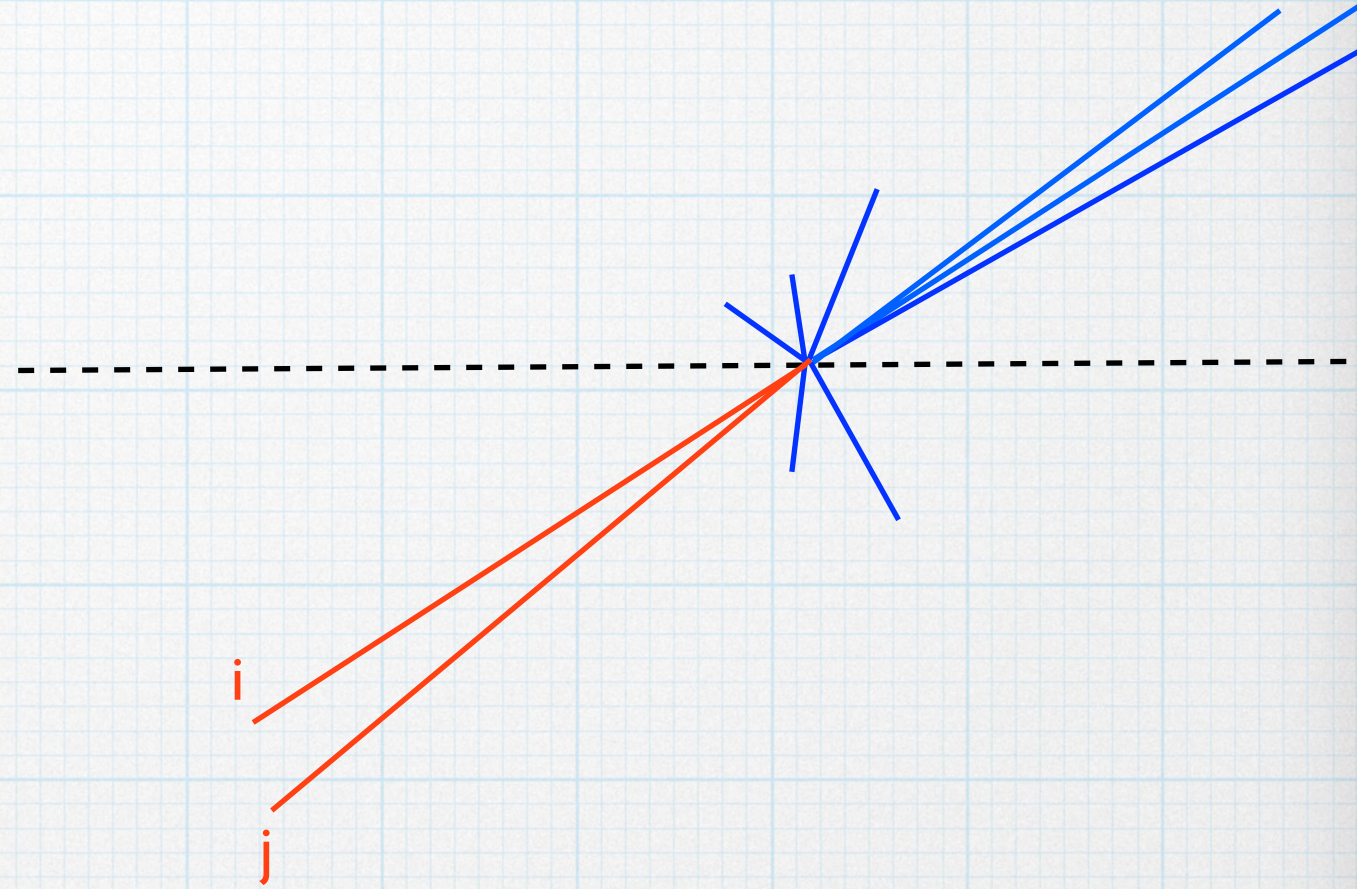

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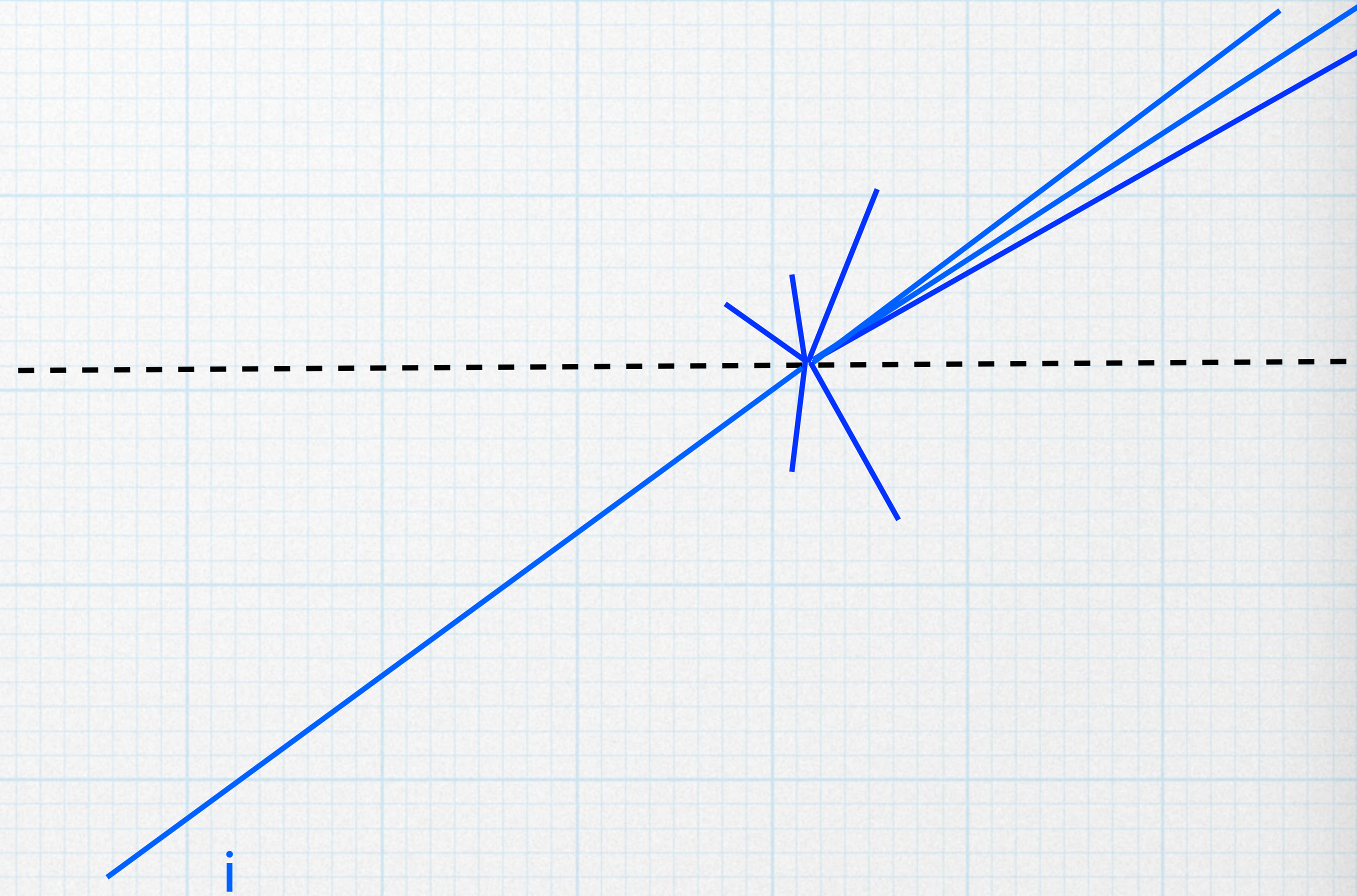

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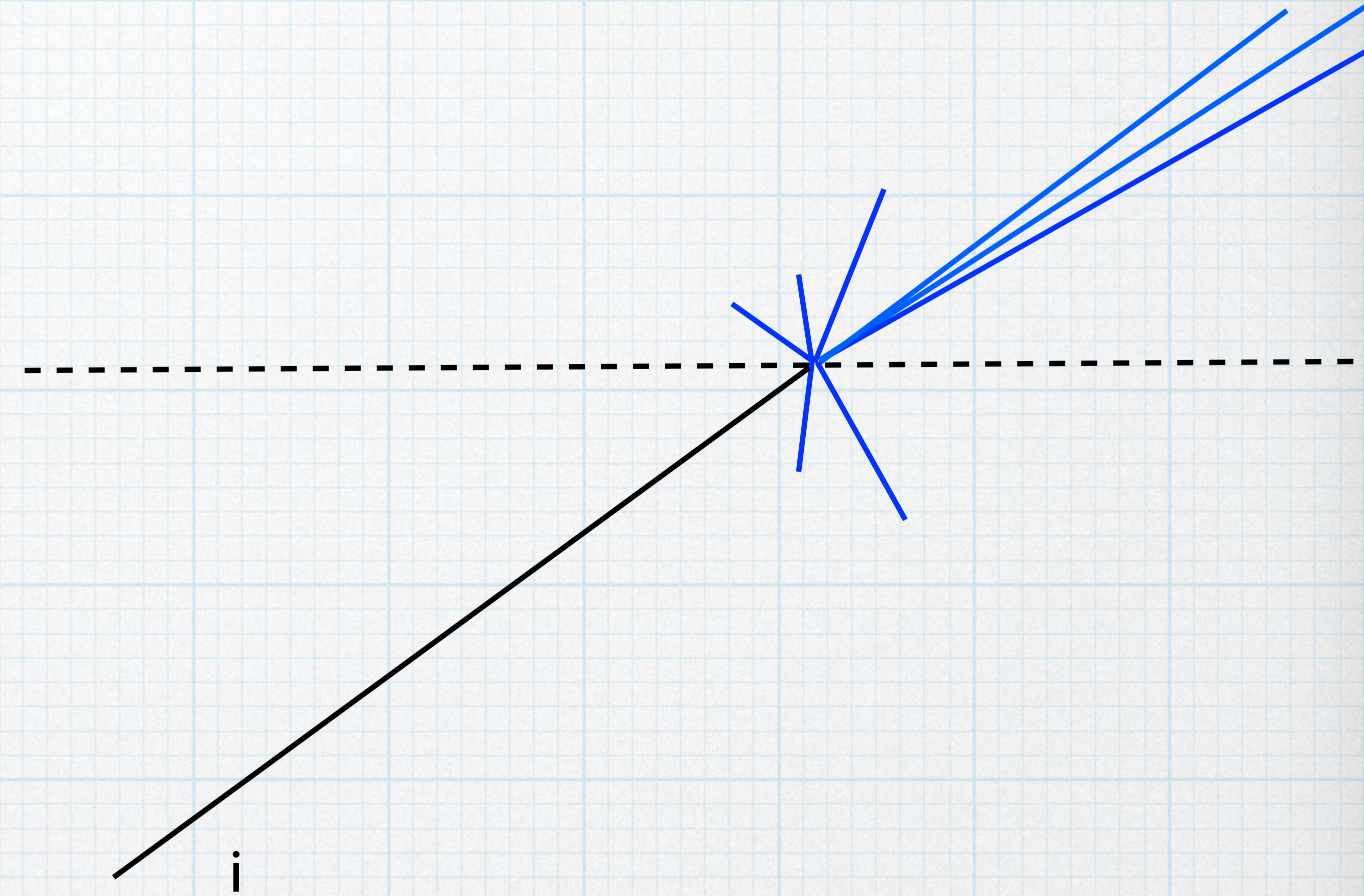
* find the minimum of all d_{ij} and d_{iB}
* if the minimum is a d_{ij}, recombine i and j and iterate
* otherwise call i a final-state jet, remove it from the list and iterate



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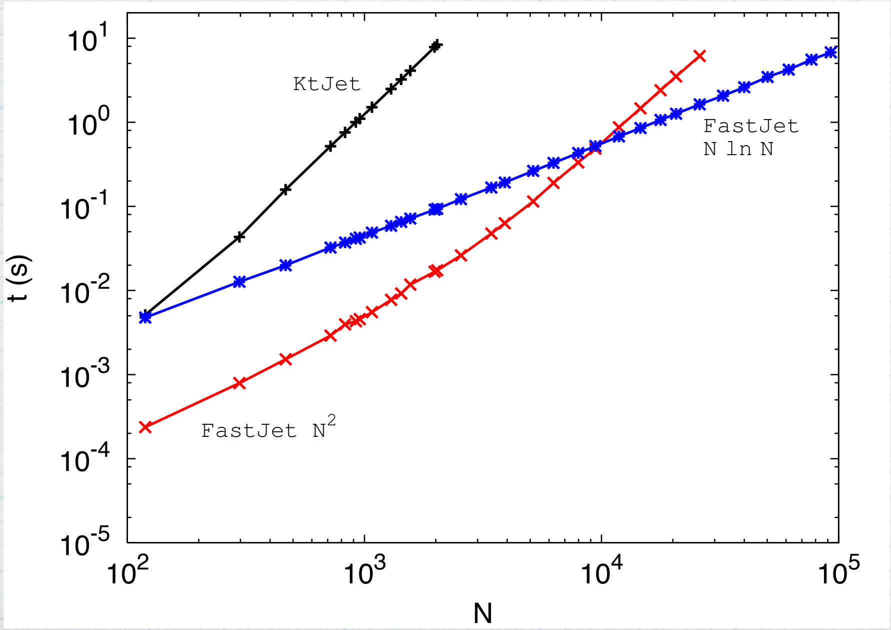
speeding-up the algorithms

- from combinatorics sequential recombination should scale like N^3
- an approach based on geometry (Voronoi diagrams) leads to notable improvements
- Sequential recombination algorithms could be implemented with $O(N^2)$ or even $O(N\ln N)$ complexity rather than $O(N^3)$

Cacciari, Salam, 2006

- Cone algorithms could be implemented exactly (and therefore made IRC safe) with $O(N^2 \ln N)$ rather than $O(N\,2^N)$ complexity

Salam, Soyez, 2007



method implemented in FastJet

JADE and k_t algorithm

* actual choice of d_{ij} determines the algorithm

JADE

$$d_{ij} = (p_i + p_j)^2 = 2E_iE_j(1 - \cos\theta_{ij})$$

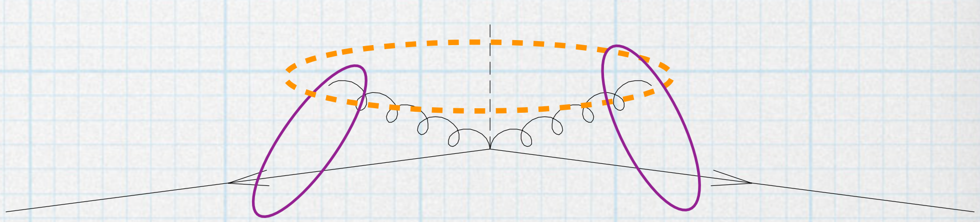
$$d_{iB} = y_{\mathrm{cut}}$$

k_t

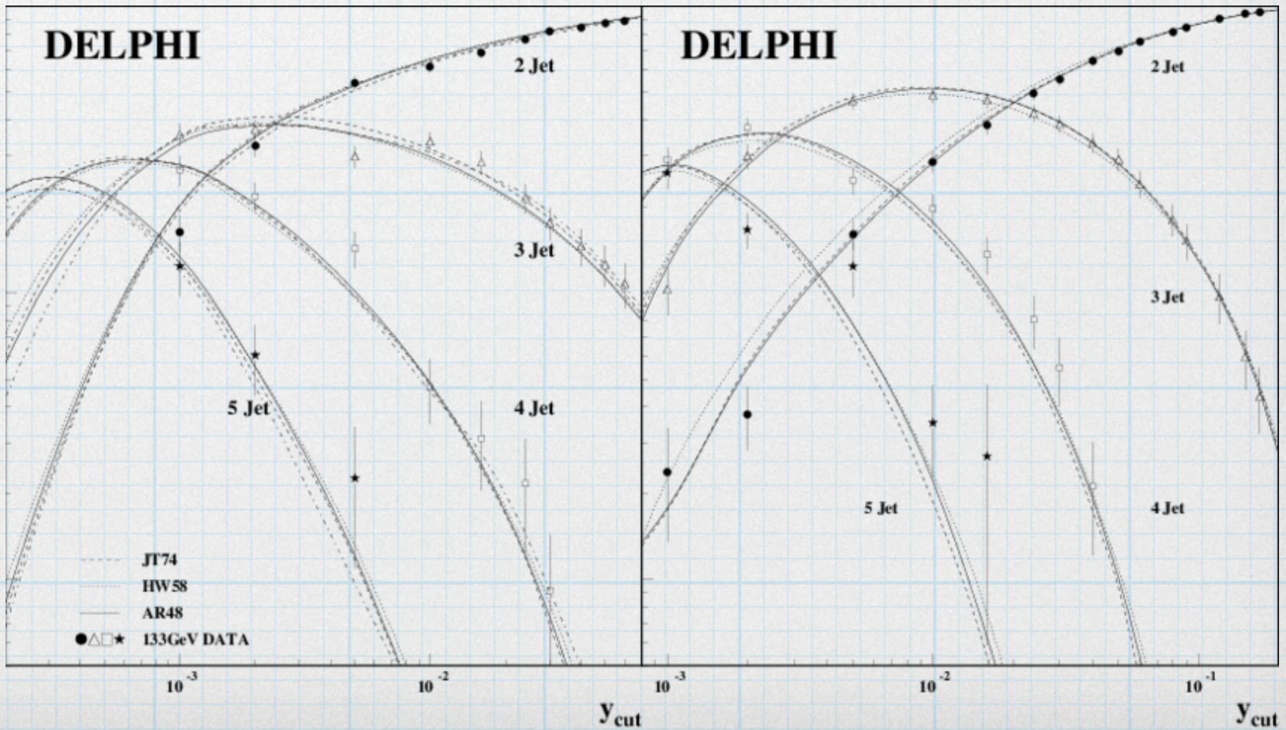
$$d_{ij} = \min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$$

$$d_{iB} = y_{\mathrm{cut}}$$

- both algorithms for e+e- collisions
- k_t algorithm theory friendly



- generic issue: problems when the resolution parameter becomes smaller as real radiation is constrained to a small (Born-like) region of phase space
- singularities still avoided but finite parts can become large (typically large logs of y_{cut})
- All-order calculations in QCD are necessary to resum these large contributions: active area of research, many theses available!
- See A. Larkoski: An unorthodox introduction to QCD



the k_t algorithm

* the k_t distance is the inverse of the QCD splitting probability

$$\frac{dP_{k\to ij}}{dE_i d\theta_{ij}} \sim \frac{\alpha_s}{\min(E_i, E_j)\theta_{ij}}$$

* the algorithm roughly inverts the QCD shower, bringing us back to the hard scattering
* the clustering history has physical meaning
* jets grow around soft particles, which is a problem in a noisy environment as the LHC

the generalised k_t family

* actual choice of d_{ij} determines the algorithm

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta y^2 + \Delta\phi^2}{R^2}$$

IRC behaviour

p = 1 k_t algorithm

S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187
S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

new soft particle ($p_t \to 0$) means that $d \to 0$ $\Rightarrow$ clustered first, no effect on jets

new collinear particle ($\Delta y^2 + \Delta\Phi^2 \to 0$) means that $d \to 0$ $\Rightarrow$ clustered first, no effect on jets

p = 0 Cambridge/Aachen algorithm

Y. Dokshitzer, G. Leder, S. Moretti and B. Webber, JHEP 08 (1997) 001
M. Wobisch and T. Wengler, hep-ph/9907280

new soft particle ($p_t \to 0$) can be new jet of zero momentum $\Rightarrow$ no effect on hard jets

new collinear particle ($\Delta y^2 + \Delta\Phi^2 \to 0$) means that $d \to 0$ $\Rightarrow$ clustered first, no effect on jets

p = -1 anti-k_t algorithm

M. Cacciari, G. Salam and G. Soyez, arXiv:0802.1189

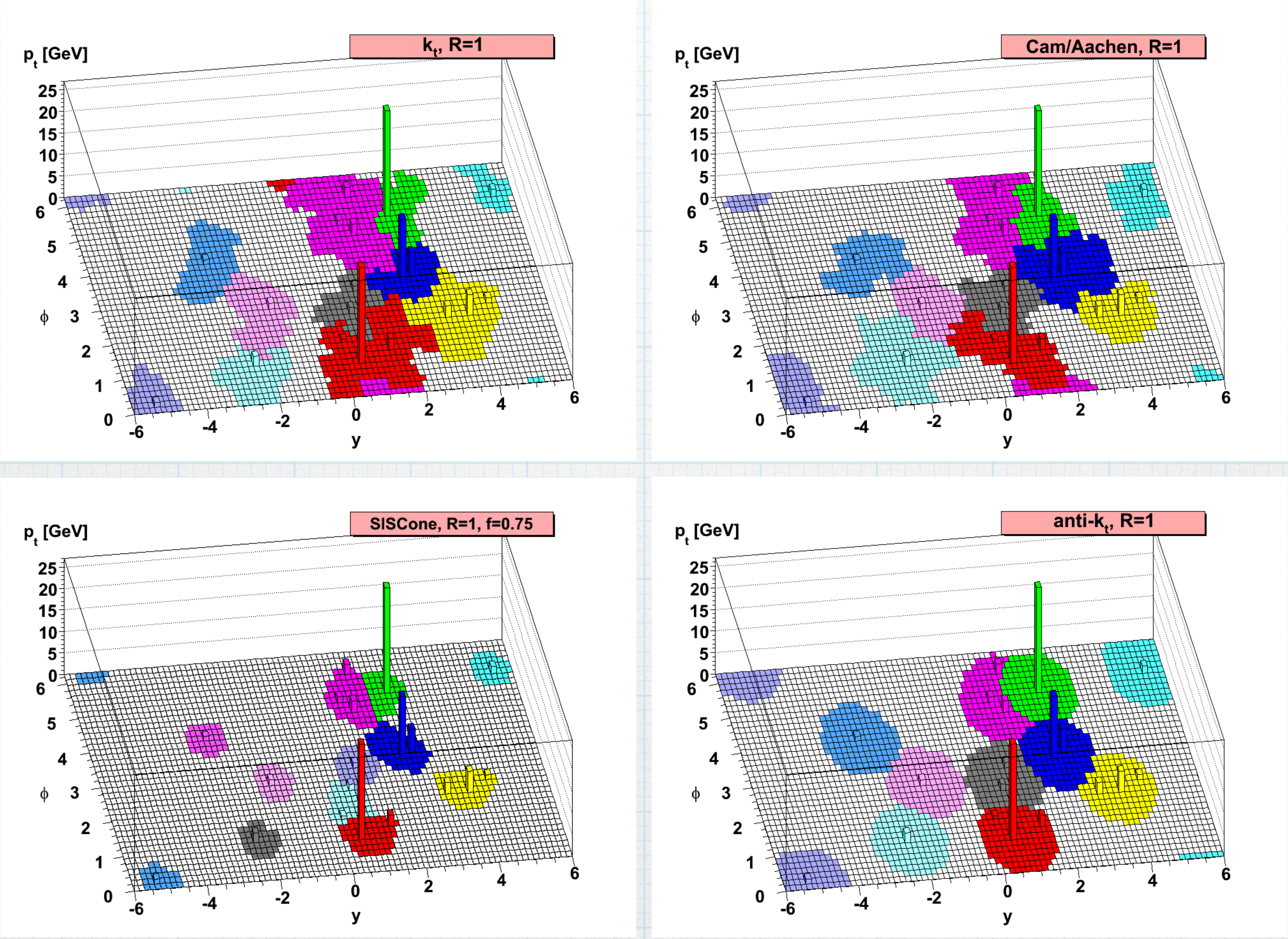
new soft particle ($p_t \to 0$) means $d \to \infty$ $\Rightarrow$ clustered last or new zero-jet, no effect on hard jets

new collinear particle ($\Delta y^2 + \Delta\Phi^2 \to 0$) means that $d \to 0$ $\Rightarrow$ clustered first, no effect on jets

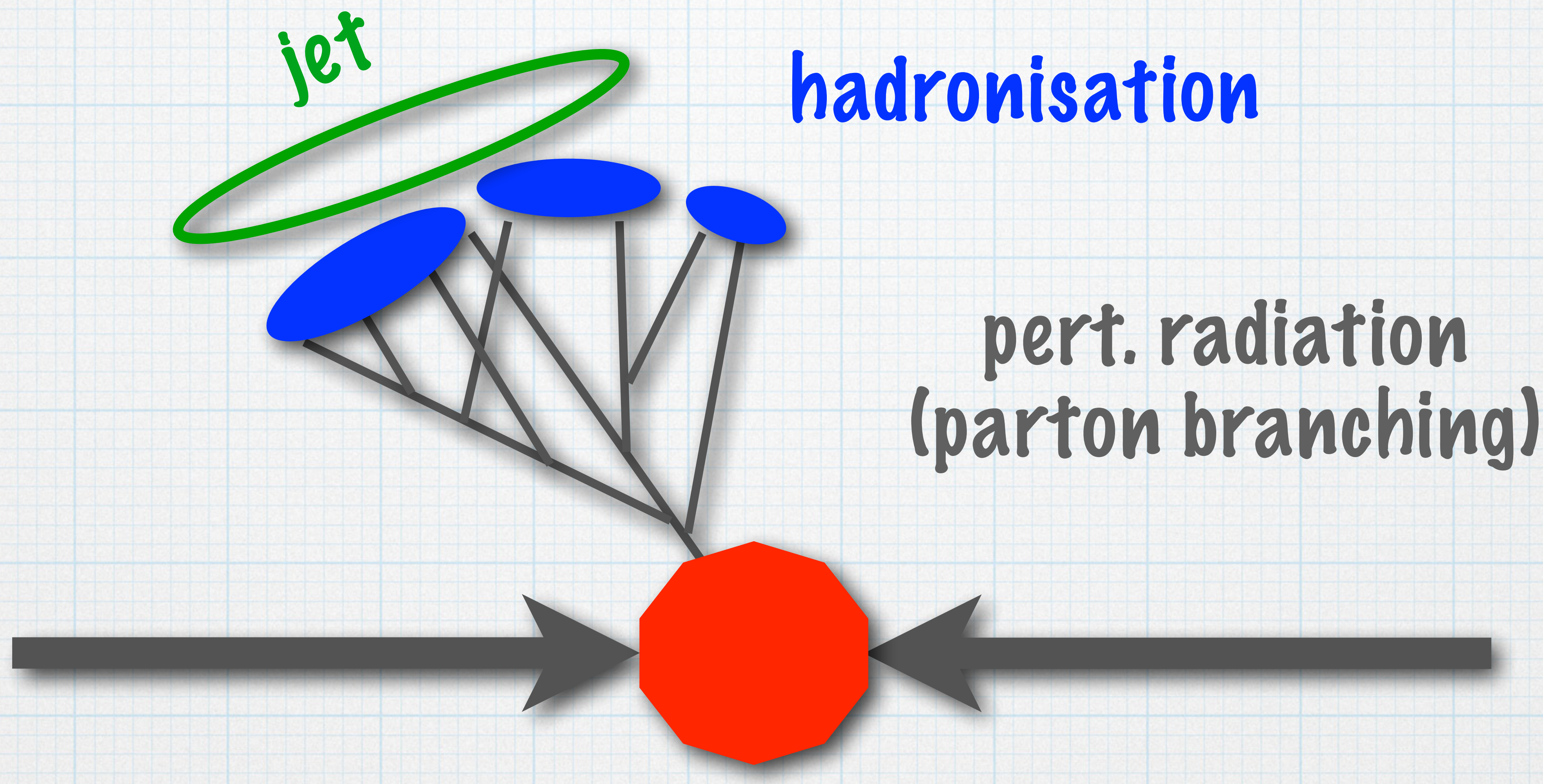
the anti-k_t algorithm

* with this measure soft particles are always far away
* jets grow around hard cores
* if no other hard particles are around the algorithm provides (ironically) perfect cones
* however, the clustering history carries little physics (re-clustering)

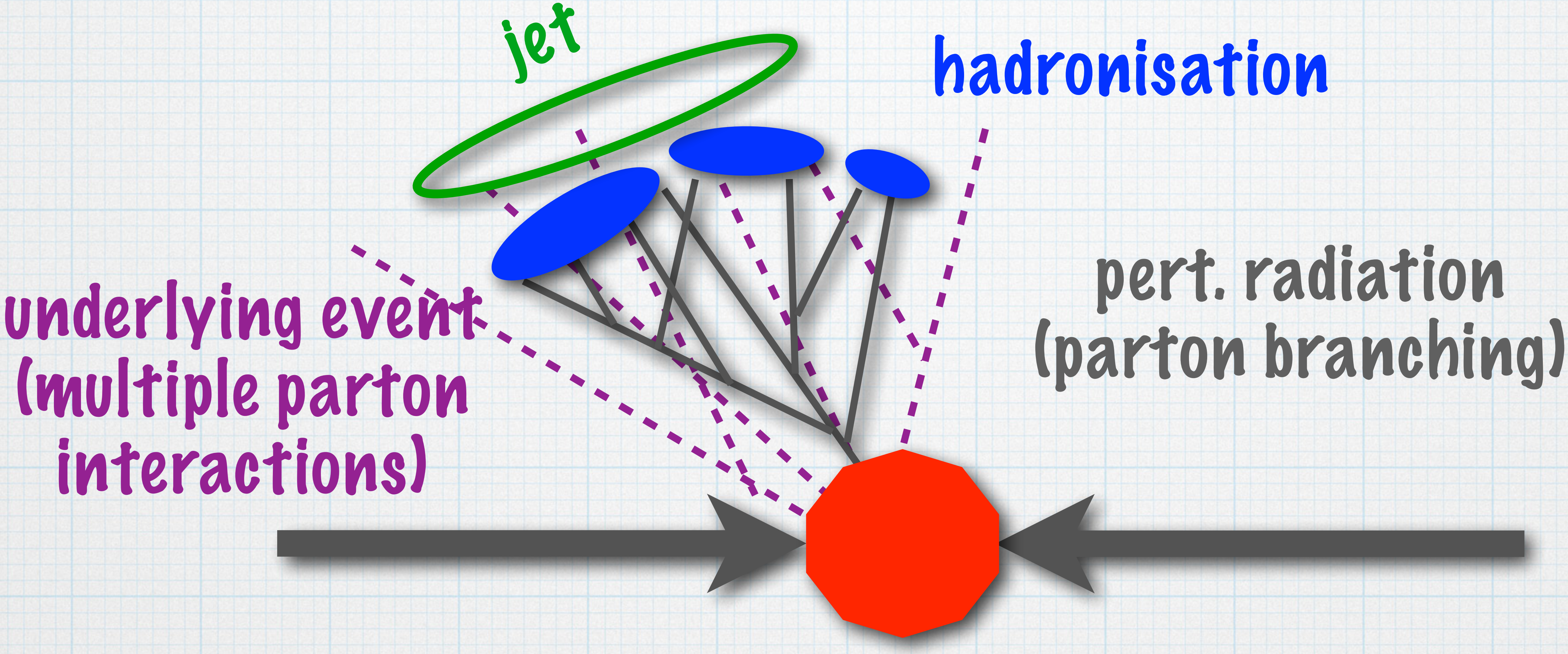
comparing them all



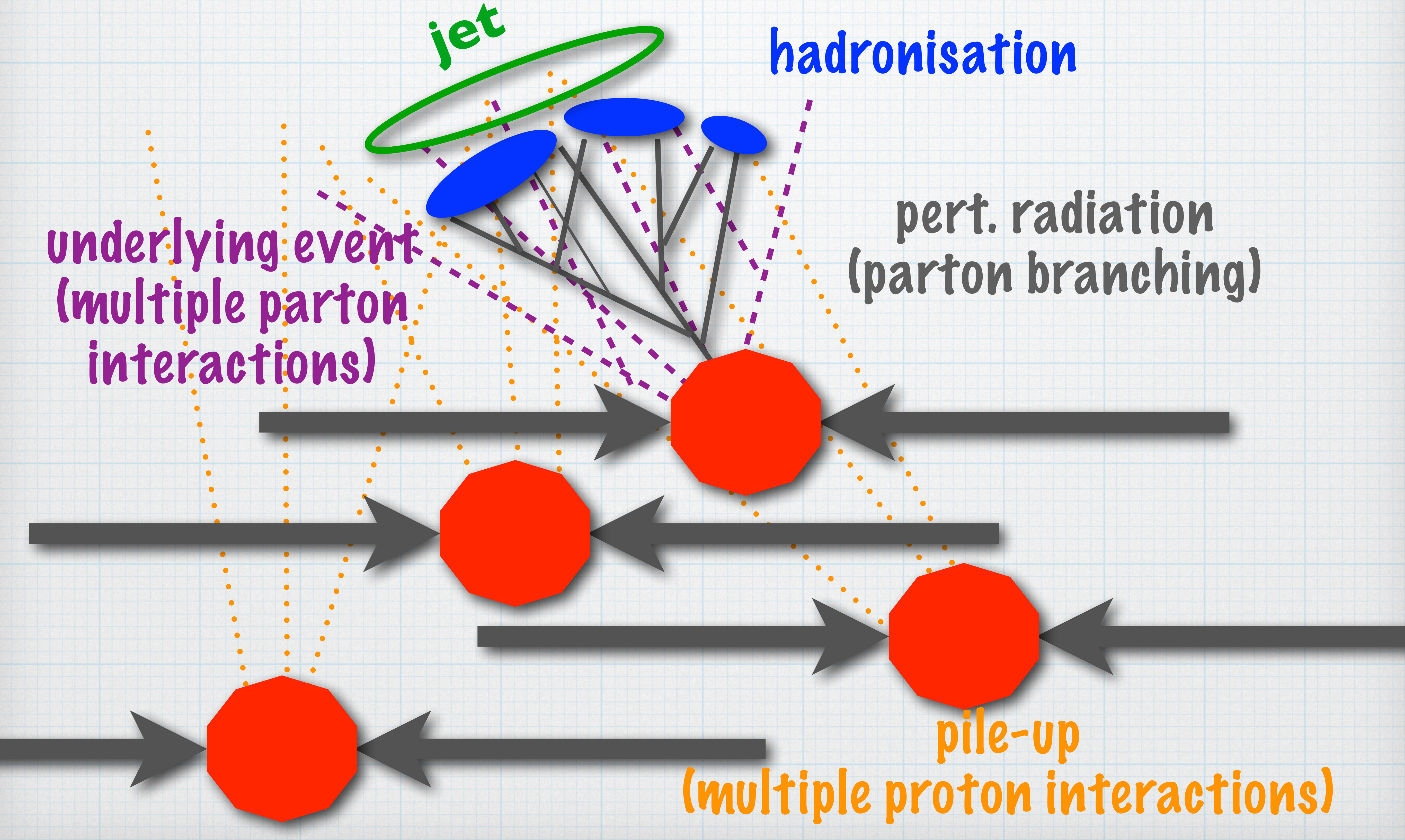
a useful cartoon



a useful cartoon



a useful cartoon



estimating p_t shifts

* we can use soft emission kinematics to estimate the changes in p_t from the hard parton to the measured quantities
* assume a finite coupling in the IR

PT radiation:

$$q: \quad \langle \Delta p_t \rangle \simeq \frac{\alpha_s C_F}{\pi} p_t \ln R$$

Hadronisation:

$$q: \quad \langle \Delta p_t \rangle \simeq -\frac{C_F}{R} \cdot 0.4 \text{ GeV}$$

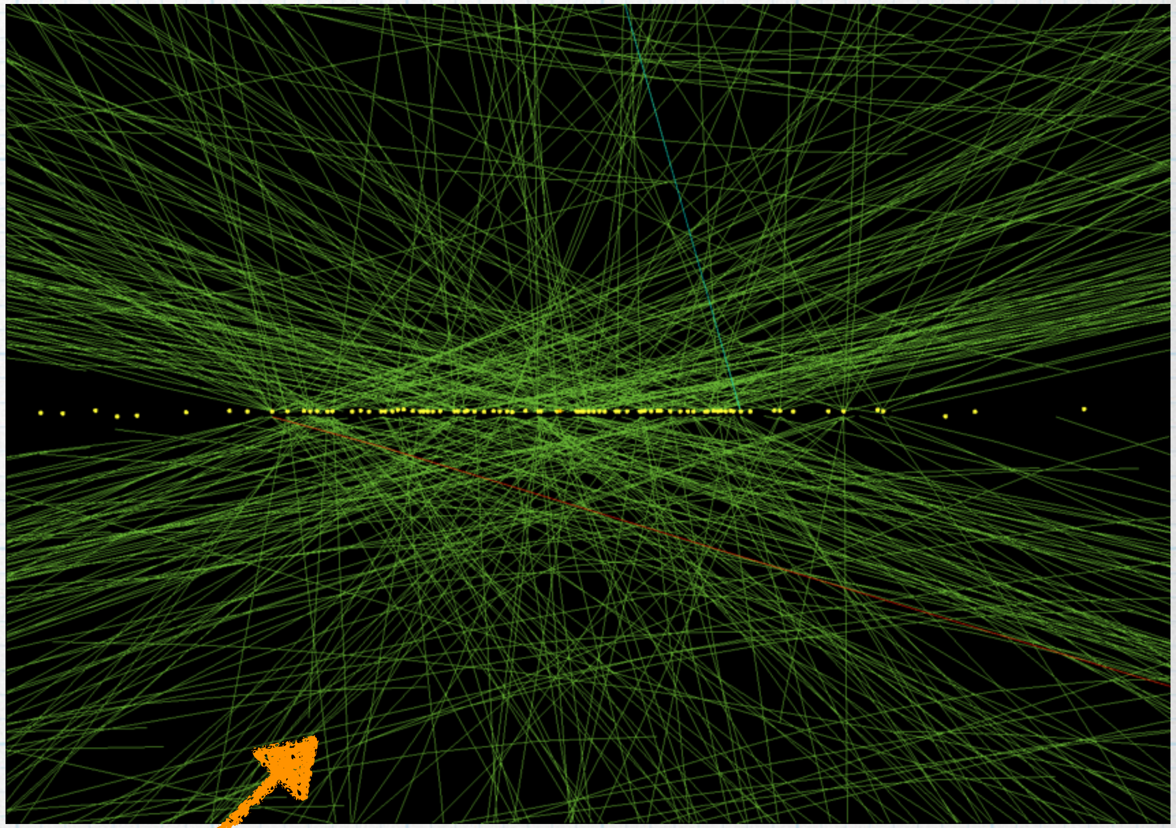
Underlying event:

$$q, g: \quad \langle \Delta p_t \rangle \simeq \frac{R^2}{2} \cdot 2.5 - 15 \text{ GeV}$$

Dasgupta, Magnea, Salam (2007)

pile-up

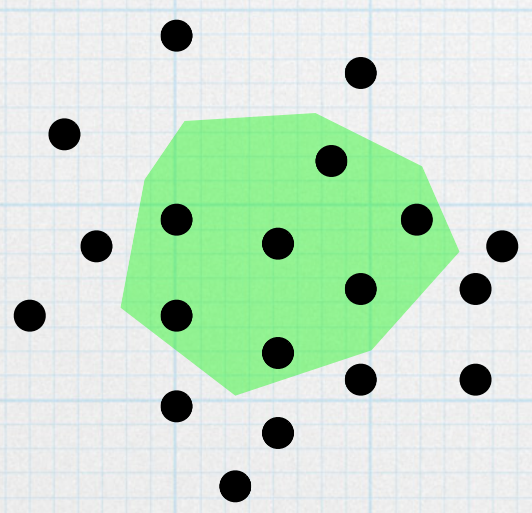
* pile-up can deposit several tens of GeV (or even hundreds, in a heavy ion collision) into a medium-sized jet
* it's a direct consequence of the desired high luminosity
* it hampers how ability of extracting useful information about the hard scatters



a 78-vertices event from CMS

hard jets and pile-up

* susceptibility measures how much background is picked up (jet area)
* resiliency measures how much the original jet is modified (backreaction)



hard jets and pile-up

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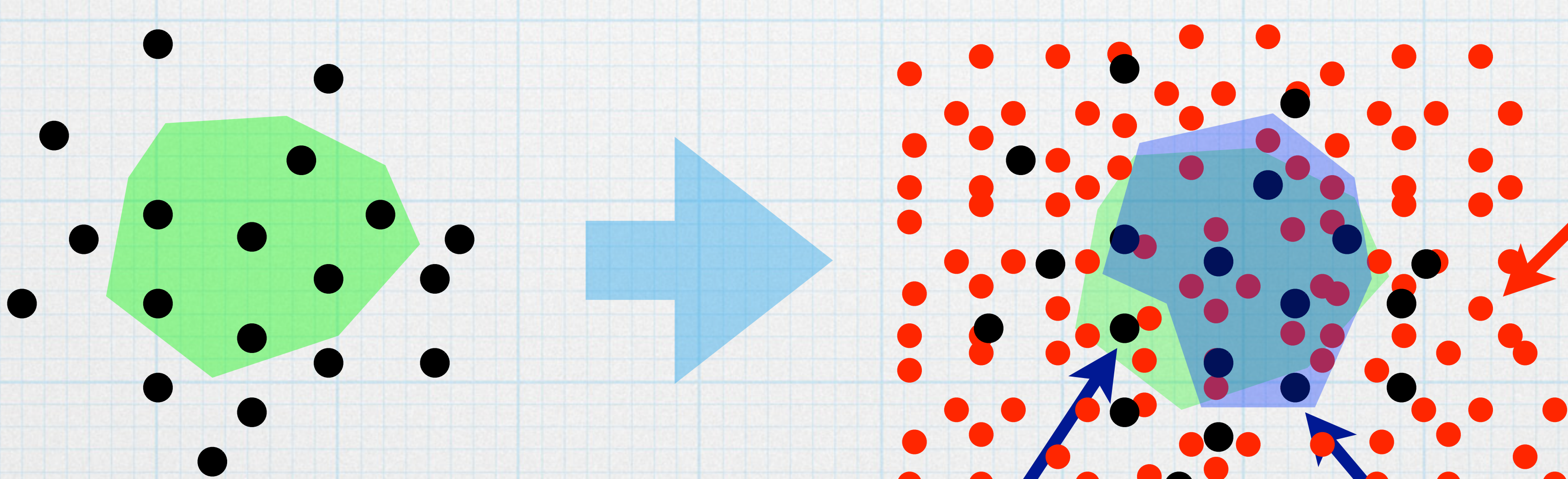


hard jets and pile-up

* **susceptibility** measures how much background is picked up (jet area)

* **resiliency** measures how much the original jet is modified (backreaction)

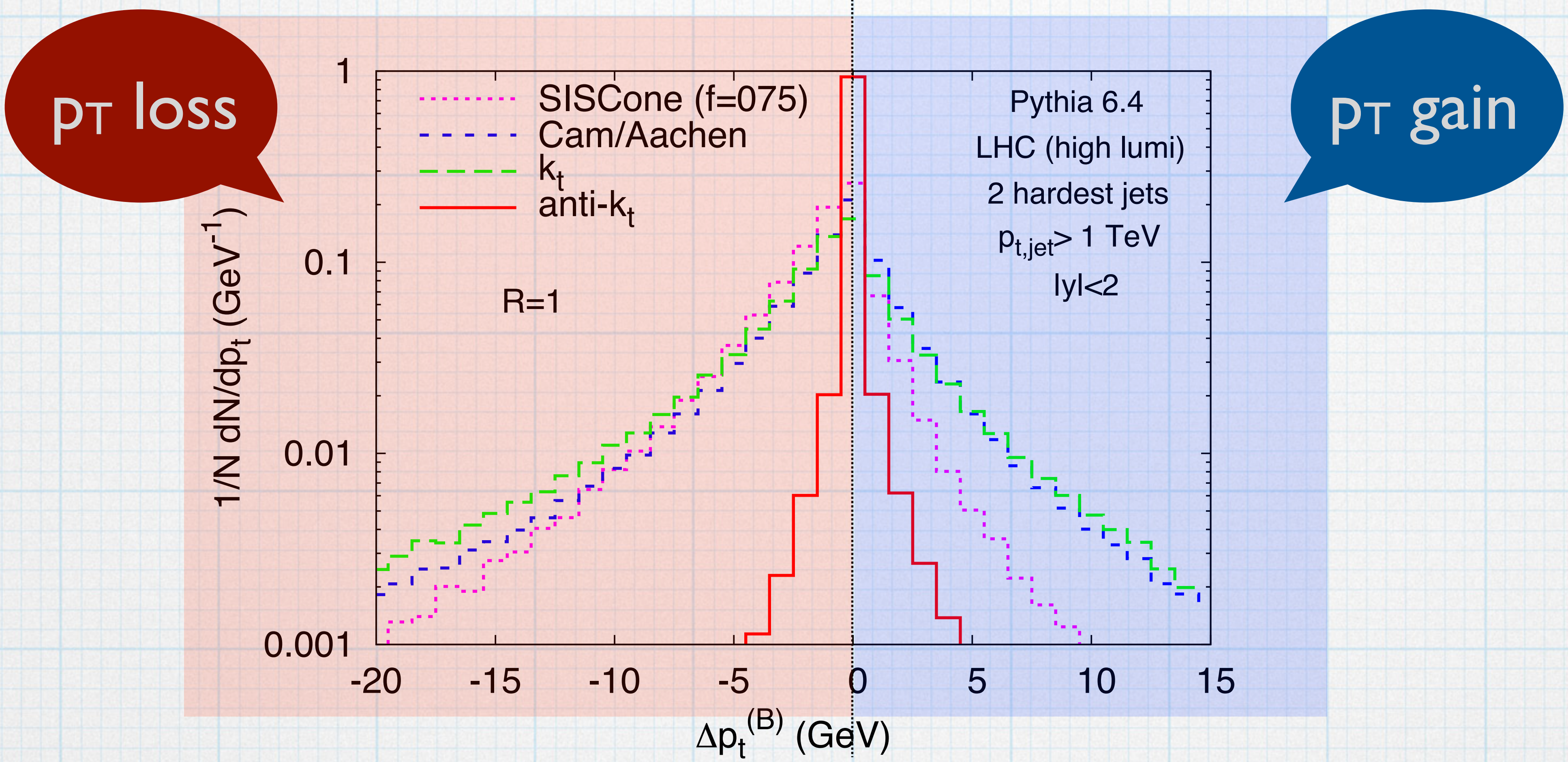
$$\Delta p_t = \rho A \pm (\sigma\sqrt{A} + \sigma_\rho A + \rho\sqrt{\langle A^2\rangle - \langle A\rangle^2}) + \Delta p_t^{BR}$$

background momentum density (per unit area)

background 'susceptibility'

backreaction 'resiliency'

resiliency



* anti-k_t jets are much more resilient to changes from background immersion

* their regular shape makes them easier to correct for detector effects

* default choice for LHC collaborations

mitigating pile-up

* Jet-based
  * Cluster the full event, determine the event-specific (ρ) and jet-specific (A) quantities, and subtract the relevant contamination from a given observable
  * Pros: largely unbiased subtraction
  * Cons: slow, potentially large(er) residual uncertainty
  * Examples: `jet area/median' in FastJet, GenericSubtractor for jet shapes, JetFFMoments for fragmentation functions, ...
* Particle-based
  * Produce a reduced event, by dropping some of the particles. Cluster this reduced event, and calculate from it the observables
  * Pros: fast, often small(er) residual uncertainty
  * Cons: not natively unbiased, can depend on choice of parameters
  * Examples: ConstituentSubtractor, SoftKiller, PUPPI, ...

for a complete review see G. Soyez, "Pile-up mitigation at the LHC: a theorist's view (2018)

summary of lecture 1

* jets to rescue perturbation theory
* jet definitions
* resilience against non-perturbative effects

homework 1

* which of the following observables are IRC safe (assuming the jet has been selected in an IRC safe fashion)?
    * the jet invariant mass
    * the invariant mass of tracks in a jet
    * generalised angularities (assume κ, β>0)

$$\lambda_{\kappa,\beta} = \sum_{i \in jet} \left(\frac{p_{Ti}}{p_T} \right)^{\kappa} \theta_i^{\beta}$$

homework 2

* show that for an event made up of two particles all gen. k_t algorithms recombine them is their azimuth-rapidity distance is less than R
* things dramatically changes with many particles!