Electroweak and Higgs Physics (Theory)

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Outline of lectures

- EW symmetry breaking within the SM
- Extension to 2 Higgs Doublet Models (2HDM)
- Kinematics of LHC events (rapidity, pT, R separation, invariant mass, phase space)
- Higgs production and decay channels at the LHC

- Effective field theory (EFT) parameterization of BSM effects
- Beyond Higgs production: vector boson scattering (VBS)
- Extension of EFT for VBS: dimension 8 operators
- UV complete model(s) with fermions or scalars and their EFT
- Conclusions

SM of particle physics: basic structure

Interactions are described by gauge theory with gauge group

$$SU(3) \quad \times \quad SU(2) \times U(1)$$

Strong interactions: **QCD**

$SU(3)$ — 8 massless gluons

Electroweak interactions:

$SU(2) \times U(1)$ — γ massless

$W^{\pm}, Z$ massive

EW gauge-boson sector of the SM

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any mass terms for $W^\pm$ and Z, i.e. forbidden are terms like

$$\mathcal{L}_{Mass} = \frac{1}{2} m_W^2 W^a_\mu W_a^\mu$$

As in QCD, the nonabelian component of the field strength tensor, $g\,\epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu}$ gives rise to WWZ and WWphoton triple gauge couplings (TGC) as well as quartic gauge couplings (QGC), already at tree level

Spontaneous symmetry breaking

Experimentally, the weak bosons are massive. We give mass to the gauge bosons through the Higgs mechanism: generate mass terms from the kinetic energy term of a scalar doublet field Φ that undergoes spontaneous symmetry breaking.

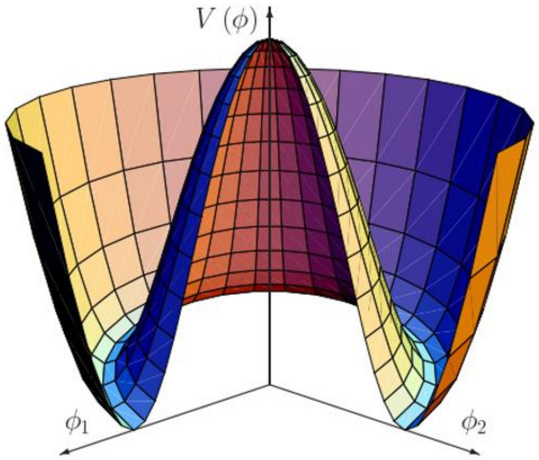
Postulate existence of a complex scalar doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} + \text{Goldstone terms},$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V\left(\Phi^\dagger\Phi\right)$$

$$D^\mu = \partial^\mu - igW_i^\mu\frac{\sigma^i}{2} - ig'\frac{Y_\Phi}{2}B^\mu$$

$$V\left(\Phi^\dagger\Phi\right) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2$$



$V\left(\Phi^\dagger\Phi\right)$ is $SU(2)_L \times U(1)_Y$ symmetric.

Goldstone modes and unitary gauge

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Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i\theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an $SU(2)_L$ gauge transformation

$$\Phi(x) \to \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i\theta^i(x)}{v}\right]$.

This gauge choice, called unitary gauge, is equivalent to absorbing the Goldstone modes $\theta^i(x)$.

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Notice that only a scalar field can have a vacuum expectation value.The VEV of a fermion or vector field would break Lorentz invariance.

Consequences for the scalar field H

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The scalar potential

$$V\left(\Phi^\dagger\Phi\right) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

becomes

$$V = \frac{\lambda}{4}\left(2vH + H^2\right)^2 = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

Consequences:

- the scalar field H gets a mass which is given by the quartic coupling λ

  $$m_H^2 = 2\lambda v^2 \quad \Longrightarrow \quad \lambda \approx 0.13 \quad \text{since} \quad m_H \approx 125\,\text{GeV} \quad \text{and} \quad v = 246.22\,\text{GeV}$$

- there is a term of cubic and quartic self-coupling.
- The coupling $\lambda \approx 0.13$ is small, i.e. perturbation theory is warranted.

Higds kinetic term and couplings to W,Z

$$D^\mu\Phi = \left(\partial^\mu - igW_i^\mu\frac{\sigma^i}{2} - ig'\frac{1}{2}B^\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H(x)\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H\end{pmatrix} - \frac{i}{2\sqrt{2}}\left[g\begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu\end{pmatrix} + g'B^\mu\right]\begin{pmatrix} 0 \\ v+H\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H\end{pmatrix} - \frac{i}{2}(v+H)\begin{pmatrix} g(W_1^\mu - iW_2^\mu) \\ -gW_3^\mu + g'B^\mu\end{pmatrix}\right]$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H\end{pmatrix} - \frac{i}{2}\left(1+\frac{H}{v}\right)\begin{pmatrix} vg\,W^{\mu+} \\ -v\sqrt{(g^2+g'^2)/2}\;Z^\mu\end{pmatrix}$$

$$(D^\mu\Phi)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu H\partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu+}W_\mu^- + \frac{1}{2}\frac{(g^2+g'^2)\,v^2}{4}Z^\mu Z_\mu\right]\left(1+\frac{H}{v}\right)^2$$

(Identification of properly normalized Z field: explained later)

Consequences

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad m_Z^2 = \frac{(g^2 + g'^2)\, v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22\ \mathrm{GeV}$$

- the photon stays massless
- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W_\mu^+ W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv g m_W W_\mu^+ W^{-\mu} H + \frac{1}{2} \frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Higgs coupling proportional to mass

- tree-level HVV (V = vector boson) coupling requires VEV! e.g. $g m_W = g^2 v/2$
Normal scalar couplings give $\Phi^\dagger \Phi V$ or $\Phi^\dagger \Phi V V$ couplings only.

SM fermions and their gauge representations

				$SU(3)$	$SU(2)$	$U(1)_Y$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0

Could add more, e.g. vector-like fermions, if discovered by experiments

Fermion Lagrangian fixed by renormalizability and gauge quantum numbers

Following the gauge recipe (for one generation of leptons, quarks work the same way)

$$\mathcal{L}_\psi = i\,\bar{L}_L\,\not{D}\,L_L + i\,\bar{\nu}_{eR}\,\not{D}\,\nu_{eR} + i\,\bar{e}_R\,\not{D}\,e_R$$

where

$$D^\mu = \partial^\mu - igW_i^\mu\,T^i - ig'Y_\psi\,B^\mu \qquad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0, \qquad i = 1,2,3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\begin{aligned}
\mathcal{L}_{kin} &= i\,\bar{L}_L\,\not{\partial}\,L_L + i\,\bar{\nu}_{eR}\,\not{\partial}\,\nu_{eR} + i\,\bar{e}_R\,\not{\partial}\,e_R \\
\mathcal{L}_{CC} &= g\,W_\mu^1\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_1}{2}\,L_L + g\,W_\mu^2\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_2}{2}\,L_L = \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_L\,\gamma^\mu\,e_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \\
\mathcal{L}_{NC} &= \frac{g}{2}\,W_\mu^3\,[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L] + g'\,B_\mu\Big[Y_L\,(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L) \\
&\quad + Y_{\nu_{eR}}\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y_{e_R}\,\bar{e}_R\,\gamma^\mu\,e_R\Big]
\end{aligned}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

Weak mixing angle

W_μ^3 and B_μ mix to produce two orthogonal mass eigenstates

$$\begin{aligned}
\text{massive partner :} \quad g\,W_\mu^3 - g'\,B_\mu &= \sqrt{g^2+g'^2}\,Z_\mu = \sqrt{g^2+g'^2}\left(W_\mu^3\cos\theta_W - B_\mu\sin\theta_W\right) \\
\text{orthogonal, massless :} \quad g'\,W_\mu^3 + g\,B_\mu &= \sqrt{g^2+g'^2}\,A_\mu = \sqrt{g^2+g'^2}\left(W_\mu^3\sin\theta_W + B_\mu\cos\theta_W\right)
\end{aligned}$$

with mixing angle fixed by $\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$ $\qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$

Write the NC Lagrangian in terms of these mass eigenstates

$$\mathcal{L}_{NC} = \bar\psi\gamma_\mu\left(gT_3W_3^\mu + g'YB^\mu\right)\psi = \bar\psi\gamma_\mu\left(\frac{1}{\sqrt{g^2+g'^2}}(g^2T_3 - g'^2Y)Z^\mu + \frac{gg'}{\sqrt{g^2+g'^2}}(T_3+Y)A^\mu\right)\psi$$

Must identify electron charge, e, as

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$$

and the charge of a particle, as a multiple of the positron charge, is given by the
Gell-Mann–Nishijima formula: $Q = T_3 + Y$

The neutral current

It is customary to write the Z coupling to fermions in terms of the electric charge Q and the third component of isospin ($T_3 = \pm 1/2$ for left-chiral fermions, 0 for right-chiral fermions)

$$\mathcal{L}_{NC} = \bar{\psi}\gamma_\mu \left(\frac{1}{\sqrt{g^2+g'^2}}(g^2 T_3 - g'^2 Y) Z^\mu + \frac{g g'}{\sqrt{g^2+g'^2}}(T_3+Y)A^\mu \right)\psi = e\bar{\psi}\gamma_\mu Q \psi A^\mu + \bar{\psi}\gamma_\mu Q_Z \psi Z^\mu$$

Q_Z is given by

$$Q_Z = \frac{1}{\sqrt{g^2+g'^2}}(g^2 T_3 - g'^2(Q-T_3)) = \frac{e}{\cos\theta_W \sin\theta_W}\left(T_3 - Q\sin^2\theta_W\right)$$

This procedure works for leptons and also for the quarks (see more later)

$$Q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \begin{array}{l} u_R^i = u_R, c_R, t_R \\ d_R^i = d_R, s_R, b_R \end{array}$$

Weak mixing angle from W/Z mass ratio and from Zff couplings receive different loop corrections, especially from heavy degrees of freedom/fields in the loops (top quark, Higgs boson, sparticles…)

- distinguish source of mixing angle determination
- gain sensitivity to BSM physics
- → perform precision measurement of $\sin\theta_W$ in all possible ways, and compare….

Fermion mass generation

A direct mass term is not invariant under $\mathrm{SU}(2)_L$ or $\mathrm{U}(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

since left- and righthanded fields have different gauge quantum numbers

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d \bar{Q}_L \Phi d_R - \Gamma_d^* \bar{d}_R \Phi^\dagger Q_L \\
&\quad -\Gamma_u \bar{Q}_L \Phi_c u_R + \text{h.c.} \\
&\quad -\Gamma_e \bar{L}_L \Phi e_R + \text{h.c.} \\
&\quad -\Gamma_\nu \bar{L}_L \Phi_c \nu_R + \text{h.c.}
\end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q, L are left-handed doublet fields and d_R, u_R, e_R, ν_R are right-handed $SU(2)$ -singlet fields.

Notice: neutrino masses can be implemented via Γ_ν term. Since $m_\nu \approx 0$ we neglect it in the following.

Fermion masses for three generations

A direct mass term is not invariant under $SU(2)_L$ or $U(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned} \mathcal{L}_{\rm Yukawa} &= -\Gamma_d^{ij} \bar{Q}_L^{\prime i} \Phi d_R^{\prime j} - \Gamma_d^{ij*} \bar{d}_R^{\prime i} \Phi^\dagger Q_L^{\prime j} \\ &\quad -\Gamma_u^{ij} \bar{Q}_L^{\prime i} \Phi_c u_R^{\prime j} + \text{h.c.} \\ &\quad -\Gamma_e^{ij} \bar{L}_L^i \Phi e_R^j + \text{h.c.} \end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$\mathcal{L}_{Yukawa}$ is gauge invariant and renormalizable and thus it can (and should) be added to the Lagrangian

3 x 3 mass matrix

In the unitary gauge we have

$$\bar{Q}_L^{\prime i}\,\Phi\, d_R^{\prime j} = \left(\bar{u}_L^{\prime i}\ \ \bar{d}_L^{\prime i}\right)\begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} d_R^{\prime j} = \frac{v+H}{\sqrt{2}}\,\bar{d}_L^{\prime i}\, d_R^{\prime j}$$

$$\bar{Q}_L^{\prime i}\,\Phi_c\, u_R^{\prime j} = \left(\bar{u}_L^{\prime i}\ \ \bar{d}_L^{\prime i}\right)\begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix} u_R^{\prime j} = \frac{v+H}{\sqrt{2}}\,\bar{u}_L^{\prime i}\, u_R^{\prime j}$$

and we obtain

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{d}_L^{\prime i}\, d_R^{\prime j} - \Gamma_u^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{u}_L^{\prime i}\, u_R^{\prime j} - \Gamma_e^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{e}_L^{i}\, e_R^{j} + \text{h.c.} \\ &= -\left[M_u^{ij}\,\bar{u}_L^{\prime i}\, u_R^{\prime j} + M_d^{ij}\,\bar{d}_L^{\prime i}\, d_R^{\prime j} + M_e^{ij}\,\bar{e}_L^{i}\, e_R^{j} + \text{h.c.}\right]\left(1+\frac{H}{v}\right)\end{aligned}$$

with mass matrices $M^{ij} = \Gamma^{ij}\,\frac{v}{\sqrt{2}}$

Mass matrix diagonalization

It is always possible to diagonalize M_f^{ij} $(f = u, d, e)$ with a bi-unitary transformation ($U^f_{L/R}$ must be unitary in order to preserve the form of the kinetic terms in the Lagrangian)

$$\begin{aligned} f'_{Li} &= \left(U^f_L\right)_{ij} f_{Lj} \\ f'_{Ri} &= \left(U^f_R\right)_{ij} f_{Rj} \end{aligned}$$

with U^f_L and U^f_R chosen such that

$$\left(U^f_L\right)^\dagger M_f U^f_R = \text{diagonal}$$

For example:

$$(U^u_L)^\dagger M_u U^u_R = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad \left(U^d_L\right)^\dagger M_d U^d_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

Mass terms

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\sum_{f',i,j} M_f^{ij}\, \bar{f}_L'^{\,i}\, f_R'^{\,j} \left(1+\frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_{f,i,j} \bar{f}_L^i \left[\left(U_L^f\right)^\dagger M_f U_R^f\right]_{ij} f_R^j \left(1+\frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_f m_f \left(\bar{f}_L f_R + \bar{f}_R f_L\right)\left(1+\frac{H}{v}\right)
\end{aligned}
$$

We succeed in producing fermion masses and we got a fermion-antifermion-Higgs coupling proportional to the fermion mass.

The Higgs Yukawa couplings are flavor diagonal: no flavor changing Higgs interactions.

Mass diagonalization and CKM matrix

The charged current interaction is given by

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}'^{i}_{L}\, \not{W}^{+}\, d'^{i}_{L} + \text{h.c.}$$

After the mass diagonalization described previously, this term becomes

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}^{i}_{L}\left[(U^{u}_{L})^{\dagger}\, U^{d}_{L}\right]_{ij} \not{W}^{+} d^{j}_{L} + \text{h.c.}$$

and we define the Cabibbo-Kobayashi-Maskawa matrix V_{CKM}

$$V_{CKM} = (U^{u}_{L})^{\dagger}\, U^{d}_{L}$$

- V_{CKM} is not diagonal and then it mixes the flavors of the different quarks.

- It is a unitary matrix and the values of its entries must be determined from experiments.

By contrast, the neutral current remains flavor diagonal, no FCNC,
since the unitary matrices of mass diagonalization cancel (GIM mechanism)

Summary of Higgs-boson Couplings

We have identified the relevant terms in the SM Lagrangian for Higgs boson couplings
to gauge bosons:

$$\mathcal{L}_{\text{kin}}^{\Phi} = (D^\mu\Phi)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu H\partial_\mu H + \left[m_W^2 W^{\mu+}W_\mu^- + \frac{1}{2}m_Z^2 Z^\mu Z_\mu\right]\left(1+\frac{H}{v}\right)^2$$

which produces the HVV coupling term

$$\frac{2m_V^2}{v}V_\mu V^\mu H = \frac{2m_V^2}{v}g^{\mu\nu}V_\mu V_\nu H$$

to fermions:

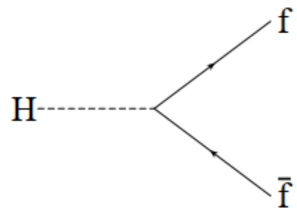
$$\mathcal{L}_{\text{Yukawa}} = -\sum_f m_f \bar{f}f\left(1+\frac{H}{v}\right) = -\sum_f m_f\bar{f}f - \sum_f \frac{m_f}{v}H\bar{f}f$$

and the Higgs self-couplings

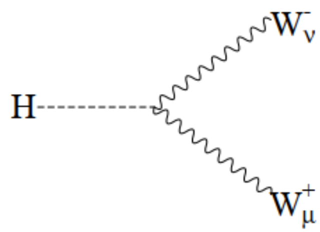
$$\mathcal{L}_V = -\frac{1}{2}(2\lambda v^2)H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4 = -\frac{1}{2}m_H^2 H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4$$

Note that the Higgs couplings increase with the mass of particles the Higgs boson couples to.

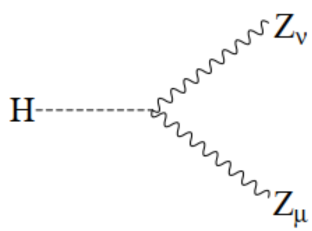
Feynman rules for SM Higgs vertices



$$-i\frac{m_f}{v}$$



$$i g\, m_W\, g_{\mu\nu}$$



$$i g\, \frac{1}{\cos\theta_W}\, m_Z\, g_{\mu\nu}$$

Within the Standard Model, the Higgs couplings are completely constrained since the masses of all SM particles[a] have been measured.

[a]except neutrinos

2 Higgs Doublet Models (2HDM): the MSSM case

The SM uses the conjugate field $\Phi_c = i\sigma_2\Phi^*$ to generate down quark and lepton masses. In supersymmetric models this must be an independent field

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d \bar{Q}_L \Phi_1 d_R - \Gamma_e \bar{L}_L \Phi_1 e_R + \text{h.c.} \\ &\quad -\Gamma_u \bar{Q}_L \Phi_2 u_R + \text{h.c.}\end{aligned}$$

Two complex Higgs doublet fields Φ_1 and Φ_2 receive mass and VEVs v_1, v_2 from generalized Higgs potential. Mass eigenstates constructed out of these 8 real fields are

Neutral sector:
2 CP even Higgs bosons: h and H
1 CP odd Higgs boson: A
1 Goldstone boson: χ_0

Charged sector:
charged Higgs bosons: $H^\pm$
charged Goldstone boson: $\chi^\pm$

The Yukawa Lagrangian above makes the MSSM a 2HDM of type II
Type I: Φ_2 and its charge conjugate generate all SM fermion masses

Higgs mixing and Higgs mass eigenstates

The Higgs potential leads to general mixing of the 2 doublet fields

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}[H^+ \sin\beta - \chi^+ \cos\beta] \\ v_1 + [H \cos\alpha - h \sin\alpha] + i[A \sin\beta + \chi_0 \cos\beta] \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \sin\beta \\ v_1 + \varphi_1 + iA \sin\beta \end{pmatrix}$$

$$\Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 + [H \sin\alpha + h \cos\alpha] + i[A \cos\beta - \chi_0 \sin\beta] \\ \sqrt{2}[H^- \cos\beta + \chi^- \sin\beta] \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 + \varphi_2 + iA \cos\beta \\ \sqrt{2} H^- \cos\beta \end{pmatrix}$$

The angle β is determined by the VEVs:

$$v_1 = v \cos\beta\,, \qquad v_2 = v \sin\beta\,, \qquad \Longrightarrow \qquad \frac{v_2}{v_1} = \tan\beta$$

The mixing angle α between the 2 CP even scalars and the masses are determined by

$$\tan\beta\,, \qquad m_A\,, \qquad v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV}$$

Decoupling behavior for large m_A

Higgs potential in the MSSM produces distinct mass relations at tree level

$$
\begin{aligned}
m_h^2, m_H^2 &= \frac{1}{2}\left[m_A^2 + m_Z^2 \pm \sqrt{\left(m_A^2 + m_Z^2\right)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}\right] \\
m_{H^\pm} &= \sqrt{m_A^2 + m_W^2} > m_W
\end{aligned}
$$

Mixing angle α is also fixed by masses and $\tan\beta$

$$
\cos(\beta - \alpha) = \frac{m_h^2 (m_Z^2 - m_h^2)}{m_A^2 (m_H^2 - m_h^2)}
$$

Behaviour for $m_A \gg m_Z$:

$$
\begin{aligned}
m_H^\pm &\approx m_A \approx m_H\,, \\
\cos(\beta - \alpha) &\approx \frac{m_Z^4 \sin^2 4\beta}{4 m_A^4} \rightarrow 0 \quad \text{for} \quad m_A \rightarrow \infty \qquad \text{(decoupling limit)}
\end{aligned}
$$

- Loop corrections substantially change mass relations, e.g. raise light Higgs mass
- Qualitative features of decoupling limit are preserved

Coupling to gauge bosons

$$\begin{aligned}\mathcal{L} &= (D^\mu\Phi_1)^\dagger D_\mu\Phi_1 + (D^\mu\Phi_2)^\dagger D_\mu\Phi_2 \\ &= \frac{1}{2}|\partial_\mu\varphi_1|^2 + \frac{1}{2}|\partial_\mu\varphi_2|^2 + \left(\frac{g_Z^2}{8}Z_\mu Z^\mu + \frac{g^2}{4}W_\mu^+W^{-\mu}\right)\left[(v_1+\varphi_1)^2 + (v_2+\varphi_2)^2\right] + \ldots\end{aligned}$$

The $v_1^2 + v_2^2 = v^2$ term gives same masses to W, Z as in the SM

$$m_W^2 = \frac{g^2v^2}{4} \qquad m_Z^2 = \frac{(g^2+g'^2)\,v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

The couplings to the gauge bosons arise from

$$\begin{aligned}2v_1\varphi_1 + 2v_2\varphi_2 &= 2v\cos\beta[H\ \cos\alpha - h\ \sin\alpha] + 2v\sin\beta[H\ \sin\alpha + h\ \cos\alpha] \\ &= 2v\ [\ H\cos(\beta-\alpha) + h\sin(\beta-\alpha)\ ]\end{aligned}$$

$\Longrightarrow$ extra coupling factors for hVV and HVV couplings as compared to SM

$$hVV \sim \sin(\beta-\alpha) \qquad HVV \sim \cos(\beta-\alpha)$$

Note: $\cos(\beta-\alpha)\to 0$ for $m_A\to\infty \Longrightarrow$ H decouples from WW and ZZ, h has SM coupling

Coupling to quarks and leptons

$$\mathcal{L}_{\text{Yuk.}} = -\Gamma_b \bar{b}_L \Phi_1^0 b_R - \Gamma_t \bar{t}_L \Phi_2^0 u_R + \text{h.c.}$$

$$= -\Gamma_b \bar{b}_L \frac{v_1 + H\cos\alpha - h\sin\alpha + iA\sin\beta}{\sqrt{2}} b_R - \Gamma_t \bar{t}_L \frac{v_2 + H\sin\alpha + h\cos\alpha + iA\cos\beta}{\sqrt{2}} t_R + \text{h.c.}$$

The v_1, v_2 terms are the fermion masses

$$m_b = \frac{\Gamma_b v_1}{\sqrt{2}} \qquad m_t = \frac{\Gamma_t v_2}{\sqrt{2}} \qquad \Longrightarrow \qquad \frac{\Gamma_b}{\sqrt{2}} = \frac{m_b}{v\cos\beta} \qquad \frac{\Gamma_t}{\sqrt{2}} = \frac{m_t}{v\sin\beta}$$

Expressed in terms of masses the Yukawa Lagrangian is

$$\mathcal{L}_{\text{Yuk.}} = -\frac{m_b}{v}\bar{b}\left(v + H\frac{\cos\alpha}{\cos\beta} - h\frac{\sin\alpha}{\cos\beta} - i\gamma_5 A\tan\beta\right)b - \frac{m_t}{v}\bar{t}\left(v + H\frac{\sin\alpha}{\sin\beta} + h\frac{\cos\alpha}{\sin\beta} - i\gamma_5 A\cot\beta\right)t$$

$\Longrightarrow$ coupling factors compared to SM hff coupling $-i\, m_f/v$

Behavior in decoupling limit

Consider limit $\sin(\beta-\alpha)\to 1, \quad \cos(\beta-\alpha)\to 0$

- $hbb, h\tau\tau$:

$$-\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha) \to 1$$

- htt:

$$\frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \frac{\cos(\beta-\alpha)}{\tan\beta} \to 1$$

- $Hbb, H\tau\tau$:

$$\frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha) \to \tan\beta$$

- Htt:

$$\frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \frac{\sin(\beta-\alpha)}{\tan\beta} \to \frac{-1}{\tan\beta}$$

In the large m_A regime

- light h couplings to fermions approach SM values
- $H\bar{b}b$ (and $A\bar{b}b$, $H/A\tau\tau$) couplings are enhanced $\sim \tan\beta$ $\Longrightarrow$ potentially large cross sections at LHC

Higgs properties and collider signatures

Importance of decoupling limit in MSSM (large m_A) $\Longrightarrow$ Concentrate on SM case

Higgs couples to fermions and gauge bosons proportional to their mass $\Longrightarrow$
Heavy SM particles are involved in both production and decay processes

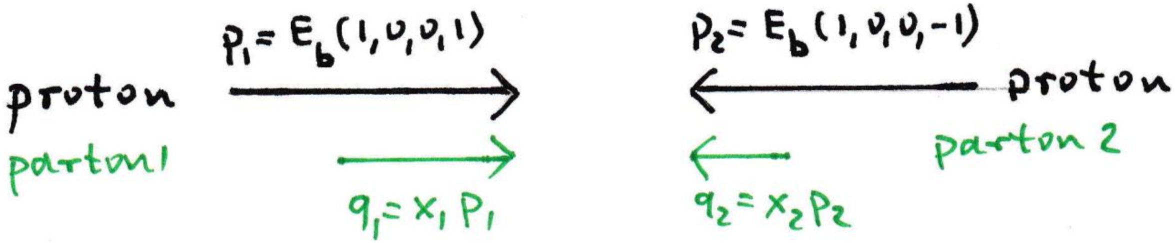
$$W, Z, t, b, \tau$$

Consider

- Higgs decay: partial widths, total width and decay branching fractions
- Production cross sections at LHC
- Signatures

Hadron Collider Kinematics

- Collisions are between partons not between incident protons



- Lab frame and center of mass frame are not the same
  → a boost along the beam axis (taken as z-axis) connects the two

$$\vec{\beta} = \frac{\vec{Q}}{Q^0} = \frac{x_1 - x_2}{x_1 + x_2}\hat{z} \quad \text{with c.m. momentum} \quad Q = q_1 + q_2 = E_b(x_1 + x_2, 0, 0, x_1 - x_2)$$

Described by Lorentz transformation with rapidity y_{cm}

$$\Lambda = \Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} = \begin{pmatrix} \cosh y_{cm} & 0 & 0 & \sinh y_{cm} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh y_{cm} & 0 & 0 & \cosh y_{cm} \end{pmatrix}$$

- Relation to Feynman x is $x_1 = \sqrt{\frac{\hat{s}}{s}}\, e^{y_{cm}}$ and $x_2 = \sqrt{\frac{\hat{s}}{s}}\, e^{-y_{cm}}$ $\left(\hat{s} = Q^2 = (2E_b)^2 x_1 x_2\right)$

Consecutive boosts along beam axis

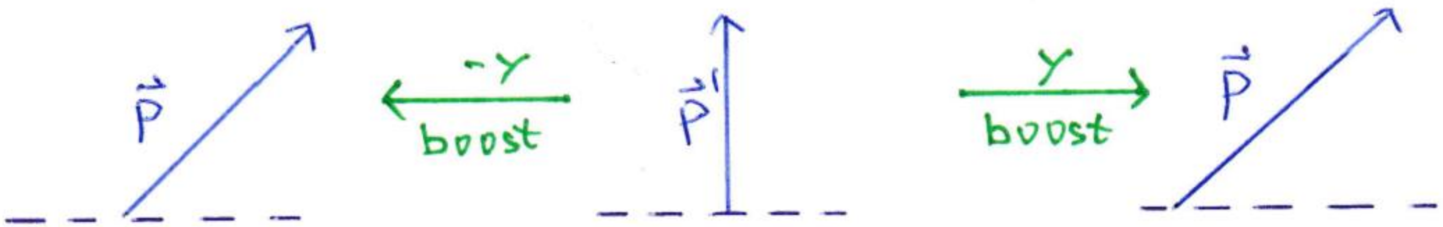
- The generator for boosts along the beam = z-axis is

$$K = \frac{1}{i}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$ i.e. boost matrix is $\Lambda = e^{iyK}$ with y the rapidity of the boost

- As a result, rapidities are additive for boosts along a common direction

$$\Lambda^{\mu}{}_{\nu}(y_2)\,\Lambda^{\nu}{}_{\rho}(y_1) = \Lambda^{\mu}{}_{\rho}(y_1+y_2)$$

Now consider the momentum of a particle of mass m in the lab frame



- Boost with e^{-iyK} to frame in which momentum is perpendicular to beam axis

$p'^{\mu} = (m_T, p_T\cos\varphi, p_T\sin\varphi, 0)$ with transverse energy $m_T = \sqrt{m^2+p_T^2}$

- Boost back with rapidity +y to lab frame

$$p^{\mu} = \Lambda^{\mu}{}_{\nu}(y)\,p'^{\nu} = (m_T\cosh y, p_T\cos\varphi, p_T\sin\varphi, m_T\sinh y)$$

Rapidity and transverse momentum of a single particle

- This procedure gives momentum parameterization

$$p^\mu = \Lambda^\mu{}_\nu(y)\, p'^\nu = (m_T \cosh y,\ p_T \cos\varphi,\ p_T \sin\varphi,\ m_T \sinh y)$$

  in terms of
  - particle mass, m
  - transverse momentum, p_T or transverse energy, $m_T = \sqrt{m^2 + p_T^2}$ (also called transverse mass)
  - the particle's rapidity, y
  - its azimuthal angle, around the beam axis, φ
- Since $\frac{p_z}{p^0} = \tanh y = \frac{p_z}{E}$ the rapidity in the lab frame is $y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$

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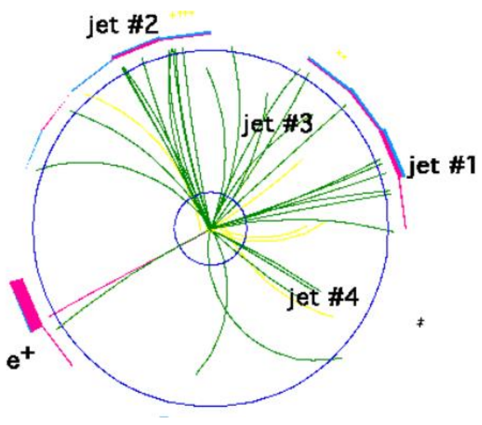
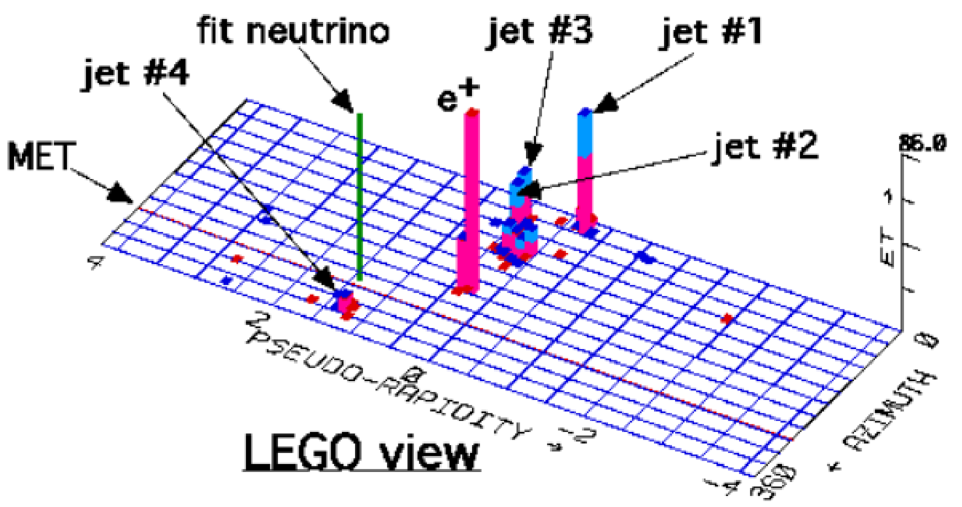
- These variables are also called "legoplot variables"





Example: CDF top-pair candidate event

Massless momenta as special case

- Special case: m=0 (or negligible particle mass), i.e. $E = |\vec{p}|, \quad p_z = |\vec{p}| \cos\theta$

$$y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z} = \frac{1}{2} \ln \frac{1+\cos\theta}{1-\cos\theta} \equiv \eta$$

- η is called pseudo-rapidity, also for massive momenta

  rapidity = pseudo-rapidity only for massless (i.e. light-like) momenta
- Using pseudo-rapidity for massive objects (e.g. jets) can lead to severe distortions in forward/backward region (i.e. high y)

Rapidity and the invariant mass of two massless objects

2 momenta $p_i^\mu = p_{Ti}(\cosh y_i, \cos\varphi_i, \sin\varphi_i, \sinh y_i)$ have invariant mass

$$m_{12}^2 = (p_1+p_2)^2 = 2\, p_1 \cdot p_2 = 4\, p_{T1} p_{T2} \left(\sinh^2 \frac{y_1-y_2}{2} + \sin^2 \frac{\varphi_1-\varphi_2}{2} \right)$$

- For modest angle between the momenta

$$m_{12}^2 \approx p_{T1} p_{T2} \left((y_1-y_2)^2 + (\varphi_1-\varphi_2)^2 \right) \qquad \text{or} \qquad m_{ij}^2 \approx p_{Ti} p_{Tj} R_{ij}^2$$

with separation $R_{ij} = \sqrt{(y_i-y_j)^2 + (\varphi_i-\varphi_j)^2}$ in the lego-plane

Rapidity and pT as phase space variables

- Compare momentum parameterizations via polar vs. legoplot coordinates

$$p^\mu = (E, p\sin\theta\cos\varphi, p\sin\theta\sin\varphi, p\cos\theta)$$
$$= (m_T\cosh y, p_T\cos\varphi, p_T\sin\varphi, m_T\sinh y)$$

with

$$y = \frac{1}{2}\ln\frac{E+p\cos\theta}{E-p\cos\theta}$$
$$p_T = p\sin\theta$$

- m and φ are common to both parameterizations,
- The transformation of (p_T, y) to $(p, \cos\theta)$ has a very simple Jacobian

$$dy\, dp_T = \frac{p^2}{E p_T}\, d\cos\theta\, dp$$

which leads to the Lorentz invariant 1-particle measure in legoplot variables

$$\frac{d^3\vec{p}}{(2\pi)^3 2E} = \frac{1}{16\pi^2}\, dp_T^2\, dy\, \frac{d\varphi}{2\pi}$$

- This is used to build the Lorentz-invariant phase space measure (Lips) in e.g. the VBFNLO Monte Carlo

$$d\,\mathrm{Lips} = (2\pi)^4\,\delta(q_1+q_2-\sum_i p_i)\prod_{i=1}^{n}\frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}$$

Some further comments

- The rapidity of a particle in the c.m. frame, y*, and in the lab frame, y, are connected by a boost with y_{cm} and hence y = y* + y_{cm}
- In rapidity differences, $y_i - y_j$, the dependence on this boost disappears. More generally, rapidity differences are invariant under boosts along the beam axis
- The physics is Lorentz invariant, i.e. we should remember that only rapidity differences matter in the theoretical description at the parton level
- Of course, low vs. high rapidities pose very different challenges at the detector level! But one should (and does!) strive for high, uniform detection efficiency over a wide rapidity range.
- … and in the theoretical description the c.m. rapidity enters via the parton distribution functions which depend on

$$x_1 = \sqrt{\frac{\hat{s}}{s}}\, e^{y_{cm}} \qquad \text{and} \qquad x_2 = \sqrt{\frac{\hat{s}}{s}}\, e^{-y_{cm}}$$

Higgs properties

Importance of decoupling limit in MSSM (large m_A) $\Longrightarrow$ Concentrate on SM case

Higgs couples to fermions and gauge bosons proportional to their mass $\Longrightarrow$
Heavy SM particles are involved in both production and decay processes

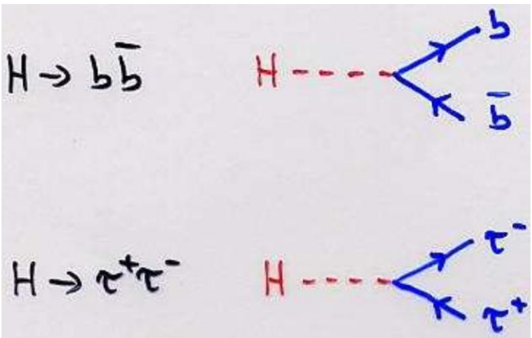
$$W, Z, t, b, \tau$$

Consider

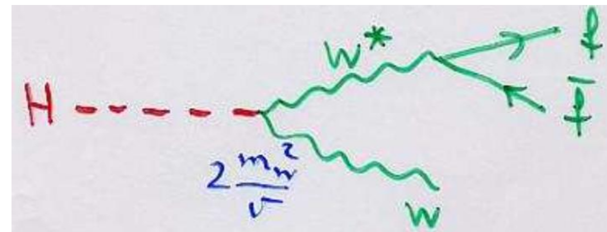
- Higgs decay: partial widths, total width and decay branching fractions
- Production cross sections at LHC
- Signatures

Higgs decay in the SM

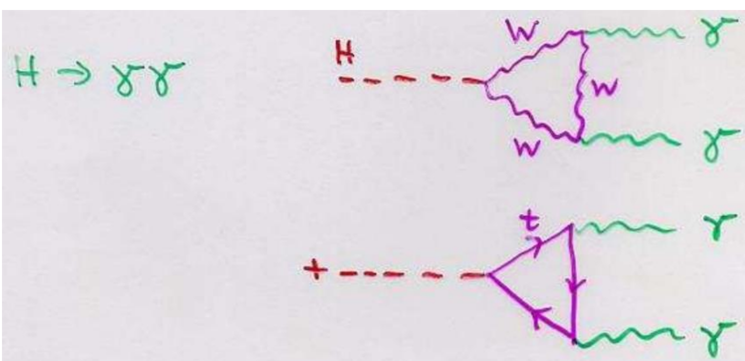
Decay to 3rd generation fermions

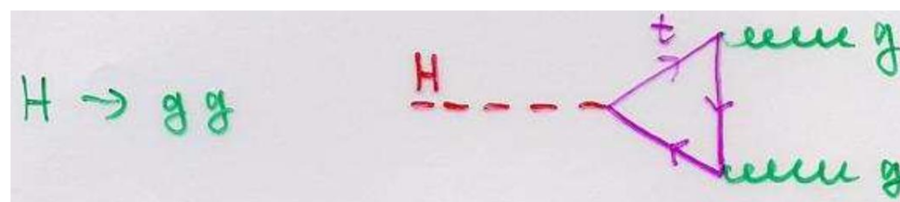


Due to low Higgs mass, H→WW or H→ZZ require off-shell weak boson



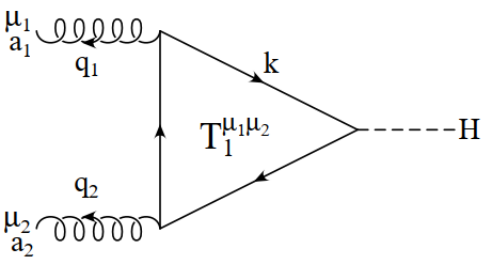
Even though there are no tree-level couplings to gluons or photons, decays into gamma-gamma or glue-glue proceed via top quark and W loops





Evaluating the top triangle

Two on-shell gluons. Graph is gauge-invariant i.e. $q_1^{\mu_1} T_{\mu_1\mu_2}(q_1,q_2) = q_2^{\mu_2} T_{\mu_1\mu_2}(q_1,q_2) = 0$



$$T^{\mu_1\mu_2} = -\frac{y_t\, g_s^2}{16\pi^2}\delta^{ab}\int \frac{d^4k}{i\pi^2}\mathrm{Tr}\left(\frac{1}{\not k - m_t}\gamma^{\mu_1}\frac{1}{\not k + \not q_1 - m_t}\gamma^{\mu_2}\frac{1}{\not k + \not q_1 + \not q_2 - m_t}\right)$$

$$= -y_t\, m_t \frac{\alpha_s}{4\pi}\delta^{ab}\left[A\left(q_1\cdot q_2\, g^{\mu_1\mu_2} - q_1^{\mu_2}\, q_2^{\mu_1}\right) + B\; q_1\cdot q_2\, q_1^{\mu_1} q_2^{\mu_2}\right]$$

- The B term does not contribute for light-like gauge bosons, while the SM value of the top Yukawa coupling, $y_t = m_t/v$ together with

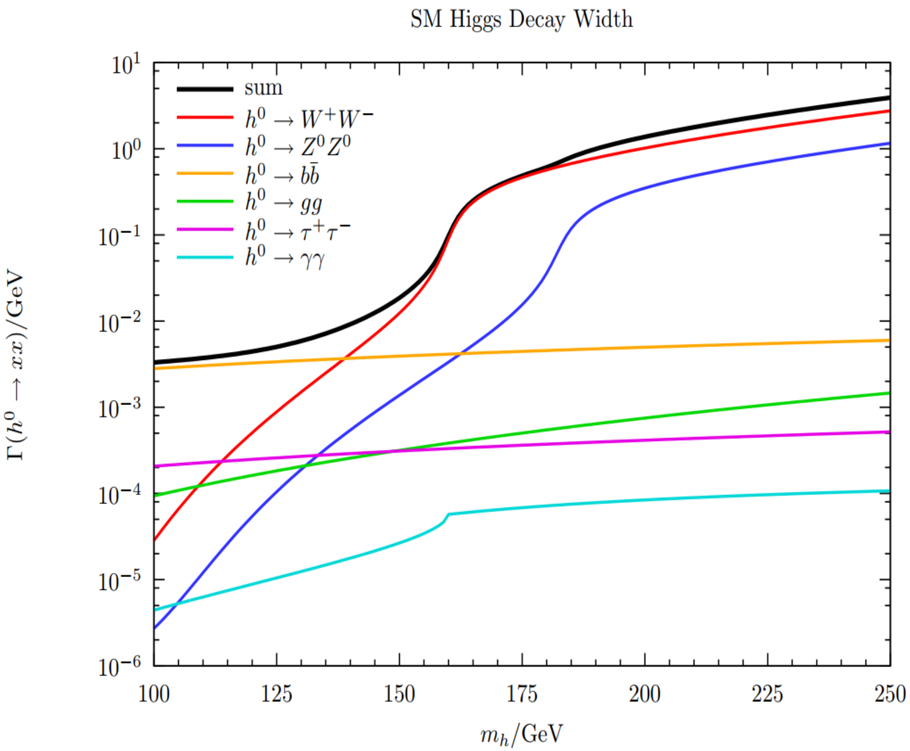
$$A = \frac{4}{3}\frac{1}{m_t^2}\left(1 + \mathcal{O}\left(\frac{m_H^2}{4m_t^2}\right)\right)$$

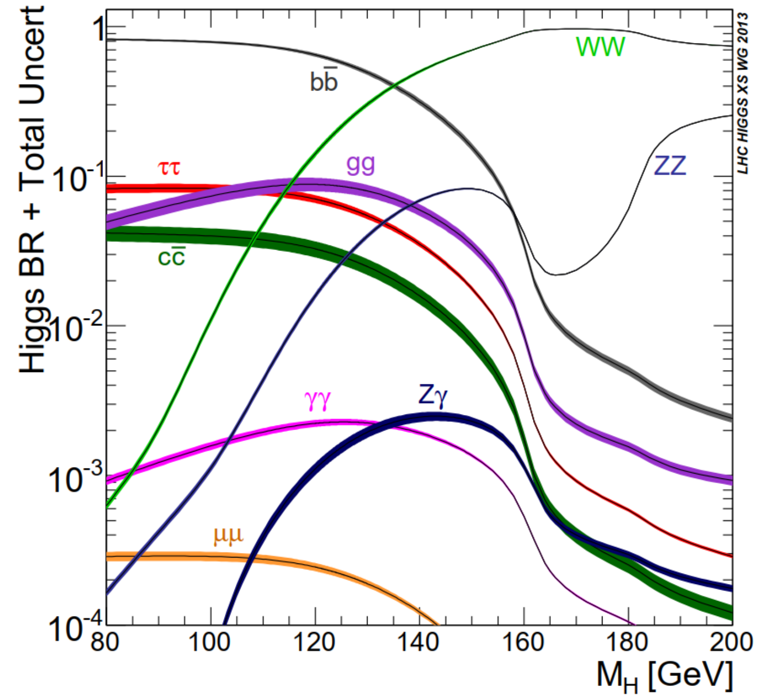
leads to

$$T^{\mu_1\mu_2} = \frac{\alpha_s}{3\pi v}\left[q_1\cdot q_2\, g^{\mu_1\mu_2} - q_1^{\mu_2} q_2^{\mu_1}\right]\delta^{ab}$$

in the large m_t limit

Decay width of the SM Higgs boson

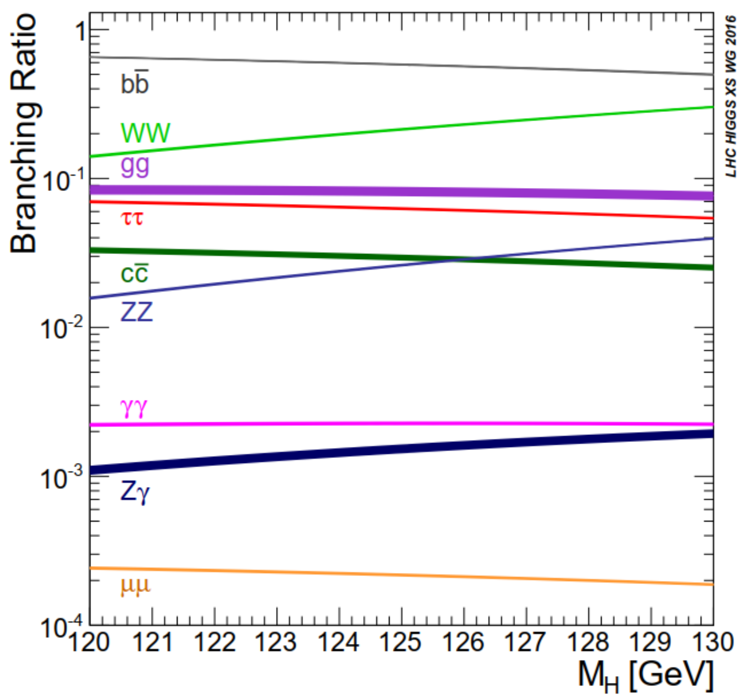


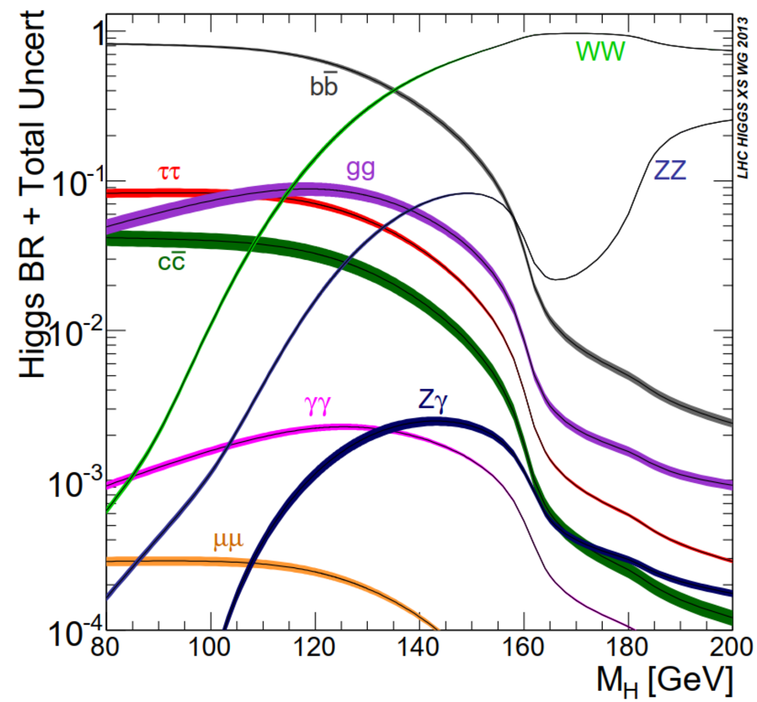


- Many accessible decay channels at mH=125 GeV

  (rich physics compared to a universe where the SM Higgs has a mass of e.g. 200 GeV)
- All partial decay widths calculated with loop corrections

Zeroing in on mH = 125 GeV





- Many accessible decay channels at mH=125 GeV
- For H→VV channels, one must in addition multiply by (small) leptonic branching ratios of W (11%) and Z (3.4%)

Tomorrow:

Higgs production and EFT