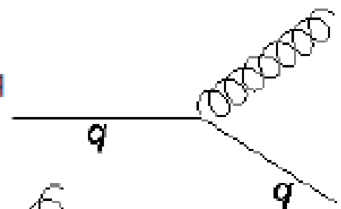
Gluons will evolve similarly DGLAP

$$\frac{d\,g(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{gq}\left(\frac{x}{y}\right)q(y,Q^2) + P_{gg}\left(\frac{x}{y}\right)g(y,Q^2)\right]$$

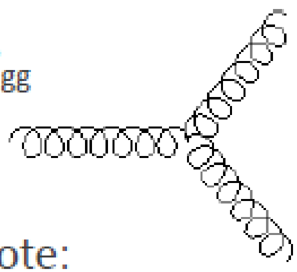
Summed over all quark parents

P_{gq}



Gluon of momentum ***xP*** from *quark* of momentum ***yP***

P_{gg}



Gluon of momentum ***xP*** from *gluon* of momentum ***yP***

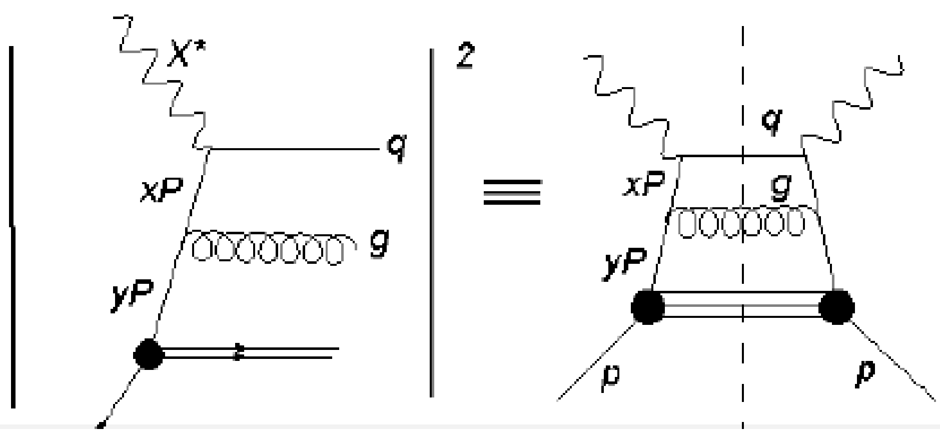
Note:

$\Delta q(x,Q^2) \sim \alpha_s \ln Q^2$ So far considered fixed α_s, but $\alpha_s \sim \dfrac{1}{\ln Q^2}$ RUNS, $\alpha_s \ln Q^2$ is $O(1)$

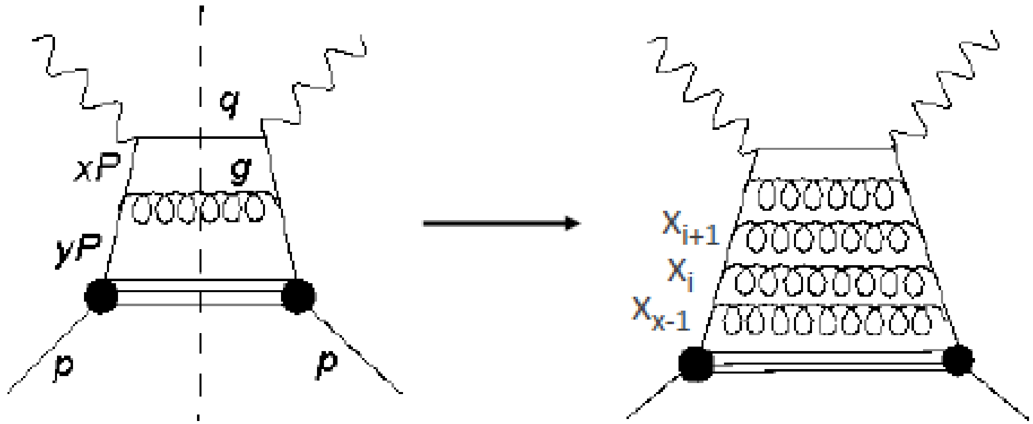
So must sum all terms $\alpha_s^{\,n}(\ln Q^2)^n$ "leading logs" LLA. Formally, $\alpha_s \to \alpha_s(Q^2)$

Consider the optical theorem equivalence

Total cross section = Imaginary part of forward elastic scattering amplitude



Extension to leading logs is,



A ladder of gluon rungs

where $x_{i-1} > x_i > x_{i+1}$

x decreasing from target to probe

$$p^2_{t_{i-1}} < p^2_{t_i} < p^2_{t_{i+1}}$$

$\boldsymbol{p_t}^2$ increasing from target to probe

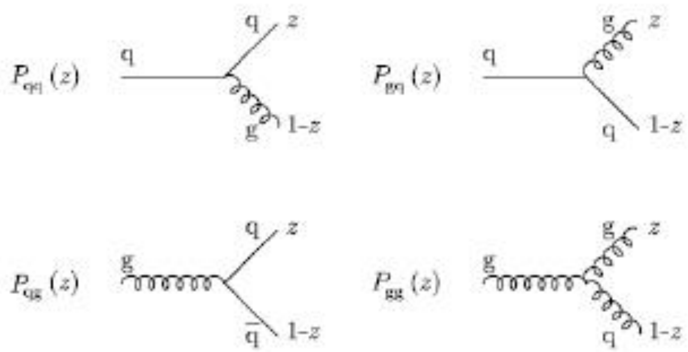
The dominant diagrams are STRONGLY ordered in $\boldsymbol{p_t}^2$

BUT NOTE: These are NOT the only diagrams which may need to be summed. They are only the dominant ones in the kinematic region considered $\left(Q^2 \gtrsim 4\,\mathrm{GeV}^2 \quad x \gtrsim 0.01\right)$

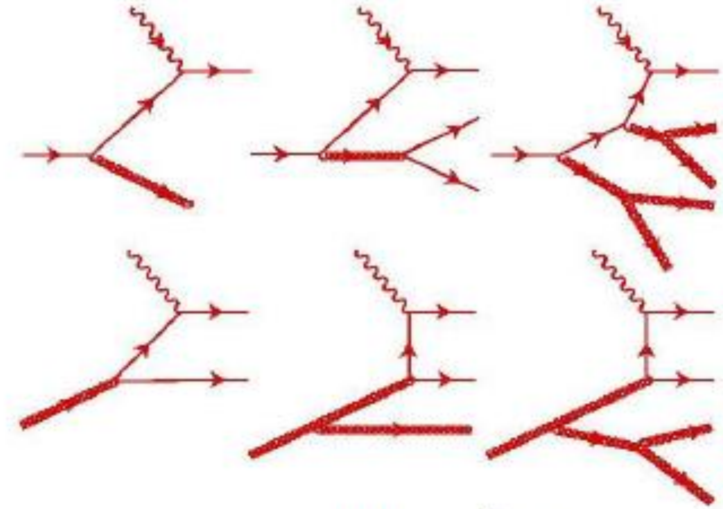
The DGLAP equations are a coupled set of equations for the evolution of quark and gluon densities

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q_i(x,Q^2)\\ g(x,Q^2)\end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\sum_j\int_x^1\frac{d\xi}{\xi}\begin{pmatrix}P_{q_iq_j}(\frac{x}{\xi},\alpha_s(Q^2)) & P_{q_ig}(\frac{x}{\xi},\alpha_s(Q^2))\\ P_{gq_j}(\frac{x}{\xi},\alpha_s(Q^2)) & P_{gg}(\frac{x}{\xi},\alpha_s(Q^2))\end{pmatrix}\begin{pmatrix}q_j(\xi,Q^2)\\ g(\xi,Q^2)\end{pmatrix}$$

Where at LO



But we may want to go beyond LO



$$P_{q_iq_j}(z,\alpha_s) = \delta_{ij}P^{(0)}_{qq}(z) + \frac{\alpha_s}{2\pi}P^{(1)}_{q_iq_j}(z) + \ldots$$
$$P_{qg}(z,\alpha_s) = P^{(0)}_{qg}(z) + \frac{\alpha_s}{2\pi}P^{(1)}_{qg}(z) + \ldots$$
$$P_{gq}(z,\alpha_s) = P^{(0)}_{gq}(z) + \frac{\alpha_s}{2\pi}P^{(1)}_{gq}(z) + \ldots$$
$$P_{gg}(z,\alpha_s) = P^{(0)}_{gg}(z) + \frac{\alpha_s}{2\pi}P^{(1)}_{gg}(z) + \ldots$$

And F2 is no longer so neatly expressed in terms of parton distributions at NLO

And FL is no longer zero

It has a strong gluon dependence

$$\frac{F_2(x,Q^2)}{x} = \int_0^1\frac{dy}{y}\left[\Sigma_i C_2(z,\alpha_s)q_i(x,Q^2) + C_g(z,\alpha_s)g(y,Q^2)\right]$$

$$C_2(z,\alpha_s) = e_i^2\left[\delta(1-z) + \alpha_s f_2(z)\right]$$

$$C_g(z,\alpha_s) = \alpha_s f_g(z)$$

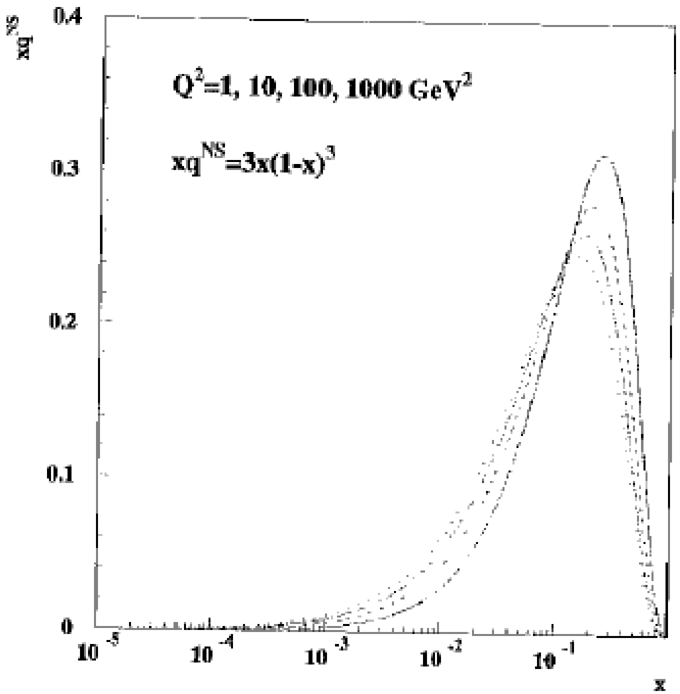
$$F_L(x,Q^2) = \frac{\alpha_s}{x}\left[\frac{4}{3}\int_0^1\frac{dy}{y}z^2F_2(y,Q^2) + 2\Sigma_i e_i^2\int_0^1\frac{dy}{y}z^2(1-z)yg(y,Q^2)\right]$$

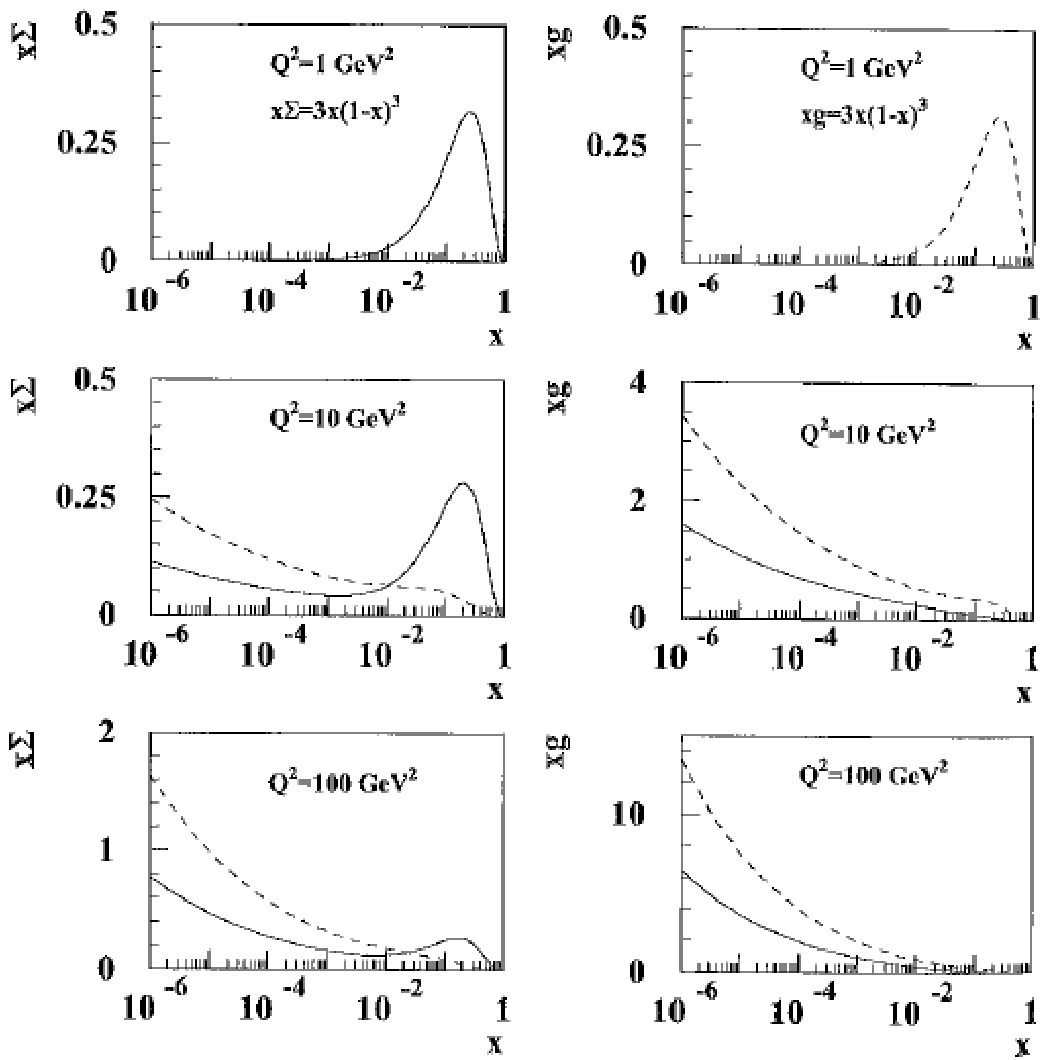
But everything is still perturbatively QCD calculable apart from the parton starting distributions

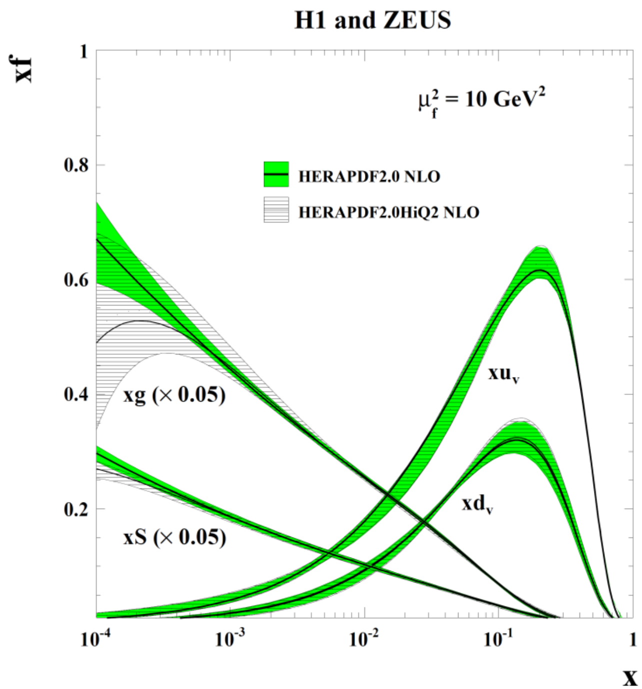
The evolution of valence quarks does not involve the gluon since the gluon splits to q-qbar flavour blind and thus makes sea quarks, hence valence distributions evolve slowly

whereas the evolution of the singlet combination and the gluon are coupled

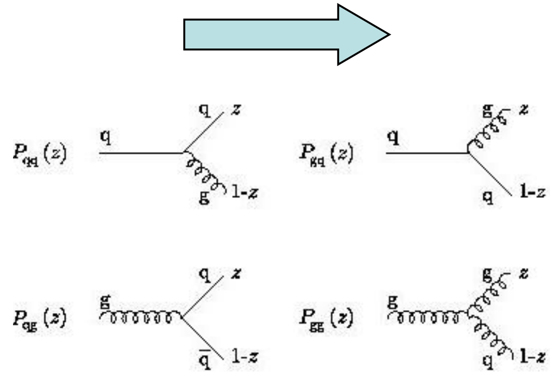
Take sea=xΣ valence-like and gluon zero at Q^2=1	Take sea=xΣ zero and gluon valence-like at Q^2=1

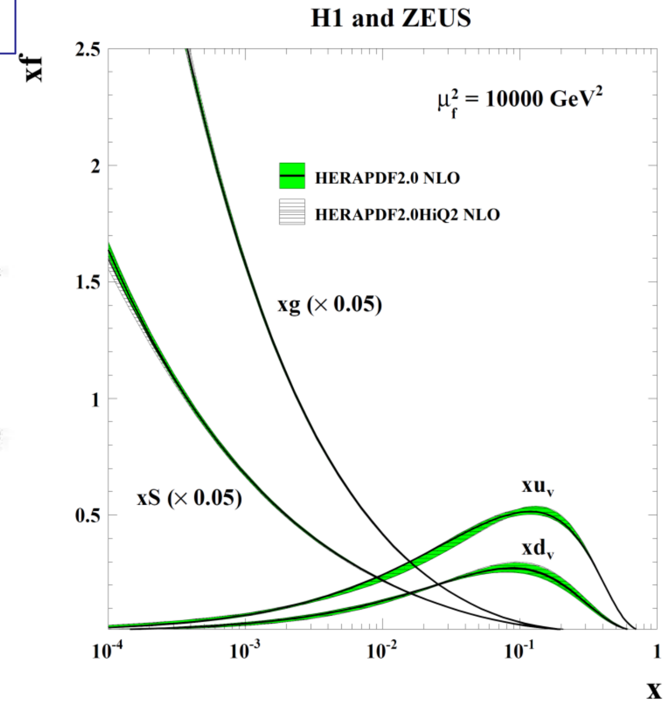






So at high scale…..





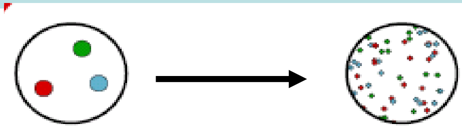
The valence quarks are radiating gluons and the gluons are splitting into quark-antiquark pairs

Parton momentum distributions change with the scale of the probe: $Q^2=p^2-E^2 \sim 10\ \mathrm{GeV}^2$ is typical scale for low energy experiments

Whereas $Q^2 = \sim 10^{4-6}\ \mathrm{GeV}^2$ are the scales that we are now probing at the Large Hadron Collider

And at this scale the proton is pretty well all glue

And the harder we hit the more of this activity we see- rather than seeing further sub-structure



So how do we determine parton distributions?- they are not perturbatively calculable— lattice gauge theory not yet able to tell us

We parametrise the parton distribution functions (PDFs) at some low starting scale:

Q^2_0 (~1-7 GeV^2)

$xu_v(x) = A_{uv}x^{Buv}(1-x)^{Cu\,v}(1+ D_{uv}\,x + E_{uv}\,x^2)$

$xd_v(x) = A_{dv}x^{Bdv}(1-x)^{Cdv}(1+ D_{dv}\,x + E_{dv}\,x^2)$

$xubar(x) = A_u x^{Bu}(1-x)^{Cu}(1+ D_u\,x + E_u\,x^2)$

$xdbar(x) = A_d x^{Bd}(1-x)^{Cd}(1+ D_d\,x + E_d\,x^2)$

$xg(x) = A_g x^{Bg}1-x)^{Cg}(1+ D_g\,x + E_g\,x^2)$

$- A'_g\,x^{Bg'}(1-x)^{Cg'}$

Alternative forms

$xf(x) = A_0 x^{A1}(1-x)^{A2}\,e^{A3x}\,(1+e^{A4}x)^{A5}$

Chebyshev polynomials

Bernstein polynomials

Or don't use a starting parametrization at all let neural nets learn the shape of the data- NNPDF)

Not all parameters are independent,

- A_g is determined from the momentum sum-rule

$$\int_0^1 \sum_{q,\bar{q}} (x\,q(x) + x\,\bar{q}(x)) + x\,g(x)\,dx = 1$$

- A_u, A_d from the number sum-rules : $\int_0^1 u_v(x)\,dx = 2 \qquad \int_0^1 d_v(x)\,dx = 1$

Various other restrictions have been imposed --and then dropped ---historically

Then measurable quantities like, $F_2,\ xF_3$ for $\nu,\ \bar{\nu},\ e^{\pm},\ \mu^{\pm} \to p, D$

Depend on a finite number of parameters (~ 15-20)

These structure functions are measured over a very wide, (x,Q^2) range
 ~2500 data points

So you evolved the partons –using the DGLAP equations--to a Q^2 value at which you have data and then you predict the measured structure functions from them:

Simply at LO

And by convolution with QCD calculable coefficient functions at NLO and NNLO

Then you fit the data to determine the parameters of the PDFs

The fact that so few parameters allows us to fit so many data points established QCD as the THEORY OF THE STRONG INTERACTION and provided the first measurements of α_s (as one of the fit parameters)

Recap how measurable structure functions depend on parton distributions?

Fixed target e/μ p/D data from NMC, BCDMS, E665, SLAC

$$F_2(lp) = x(\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s}) + \frac{4}{9}(c+\bar{c})$$

$$F_2(lN) = \frac{5}{18}\, x\left[u+\bar{u}+d+\bar{d}+\frac{2}{5}(s+\bar{s})+\frac{8}{5}(c+\bar{c})\right]$$

ν, νbar fixed Fe target data from CCFR, NuTeV, Chorus

$$F_2(\nu,\bar{\nu}N) = x(u+\bar{u}+d+\bar{d}+s+\bar{s}+c+\bar{c})$$

$$xF_3(\nu,\bar{\nu}N) = x(u-\bar{u}+d-\bar{d}) = x(u_v+d_v)$$

Valence information for 0< x < 1

These data are shot on nuclear targets like Fe which suffer from heavy target corrections- even deuterium is not safe

Can get ~4 distributions from this: e.g. u, d, ubar, dbar

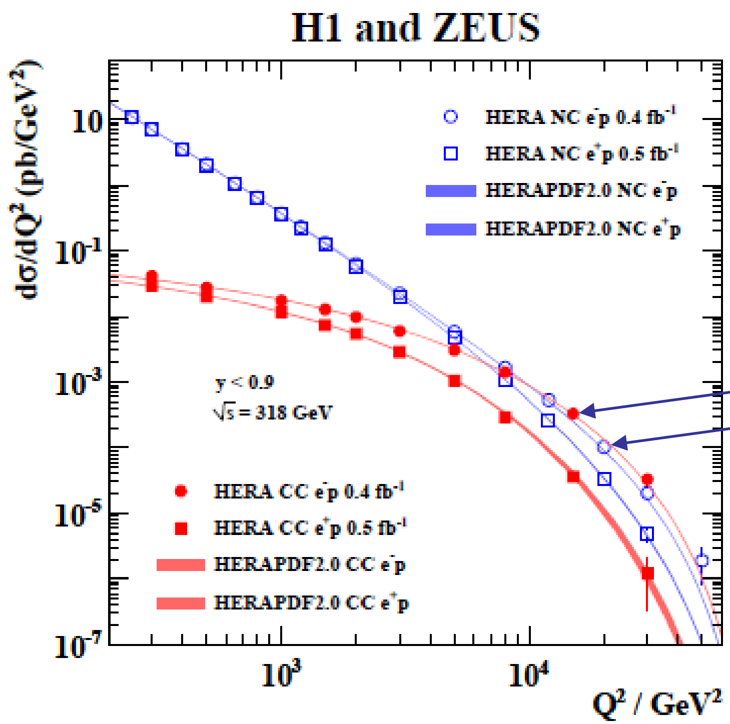
– but note we have already assumed

- u in proton= d in neutron and q=qbar in the sea (in practice violations are very small)
- And we need further assumptions like sbar = 1/4 (ubar+dbar) and a heavy quark treatment

→the assumption on sbar is questionable

→ but the heavy quarks contributions can be calculated from pQCD

Note gluon enters indirectly via DGLAP equations for Q^2 evolution AND directly in the longitudinal structure function F_L at NLO and higher orders



More information from High Q^2 ep scattering data at HERA

HERA data have also provided information at high $Q^2 \rightarrow Z^0$ and $W^{+/-}$ become as important as γ exchange → NC and CC cross-sections comparable

For NC processes

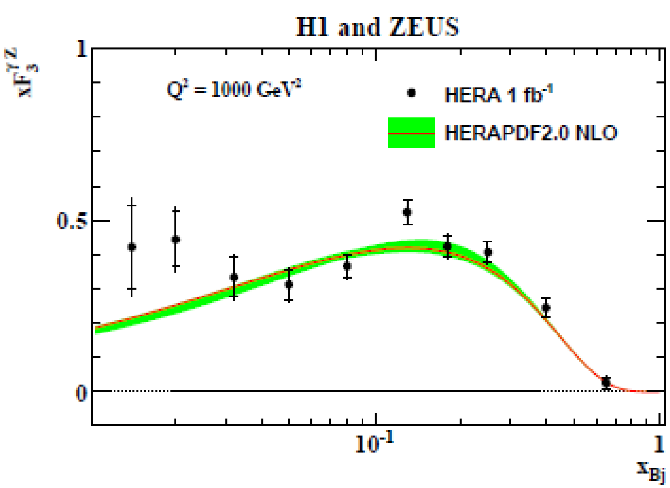
$$F_2 = \Sigma_i\, A_i(Q^2)\, [xq_i(x,Q^2) + x\bar{q}_i(x,Q^2)] -$$

$$xF_3 = \Sigma_i\, B_i(Q^2)\, [xq_i(x,Q^2) - xq_i(x,Q^2)]$$

$$A_i(Q^2) = e_i^2 - 2\, e_i\, v_i\, v_e\, P_Z + (v_e^2+a_e^2)(v_i^2+a_i^2)\, P_Z^2$$

$$B_i(Q^2) = \quad -2\, e_i\, a_i\, a_e\, P_Z + \quad 4a_i\, a_e\, v_i\, v_e \quad P_Z^2$$

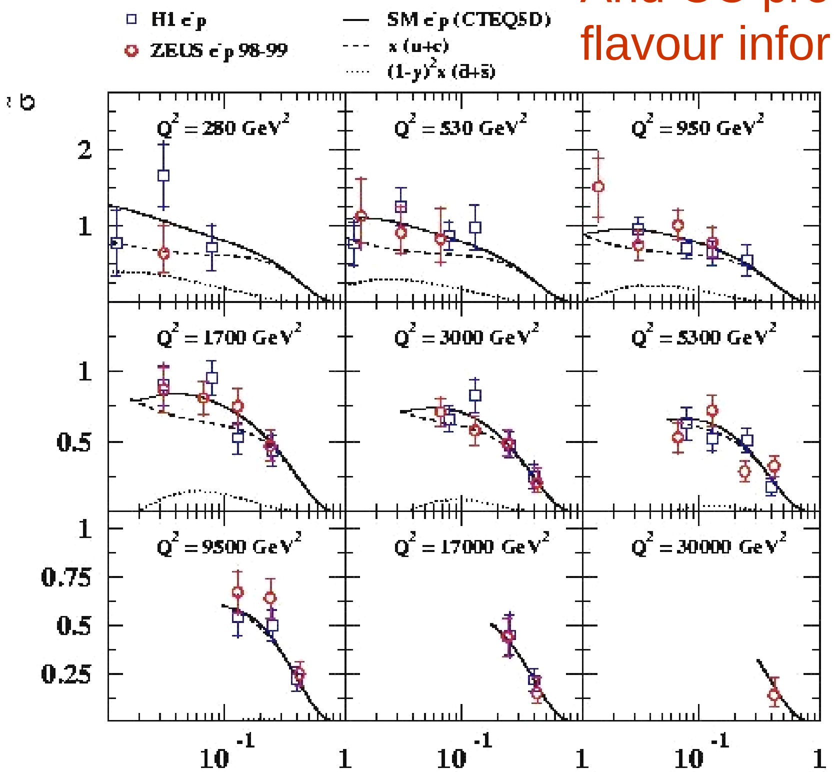
$$P_Z^2 = Q^2/(Q^2 + M^2_Z)\; 1/\sin^2\theta_W$$

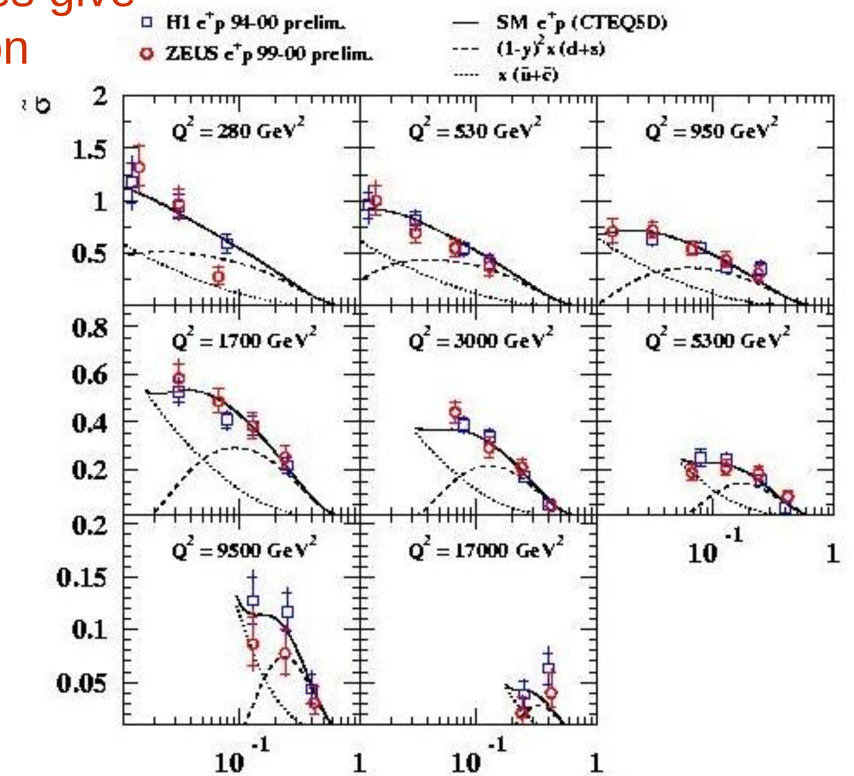


→F2 gives the usual information on the Sea but we also have a new valence structure function xF_3 due to Z exchange

This is measurable from low to high x- on a pure proton target → no heavy target corrections- no assumptions about strong isospin

And CC processes give flavour information





$$\frac{d^2\sigma(e^-p)}{dxdy} = \frac{G_F^2 M^4_W}{2\pi x(Q^2+M^2_W)^2}[x\,(u+c) + (1-y)^2 x\,(\bar{d}+\bar{s})]$$

$$\frac{d^2\sigma(e^+p)}{dxdy} = \frac{G_F^2 M^4_W}{2\pi x(Q^2+M^2_W)^2}[x\,(\bar{u}+\bar{c}) + (1-y)^2 x\,(d+s)]$$

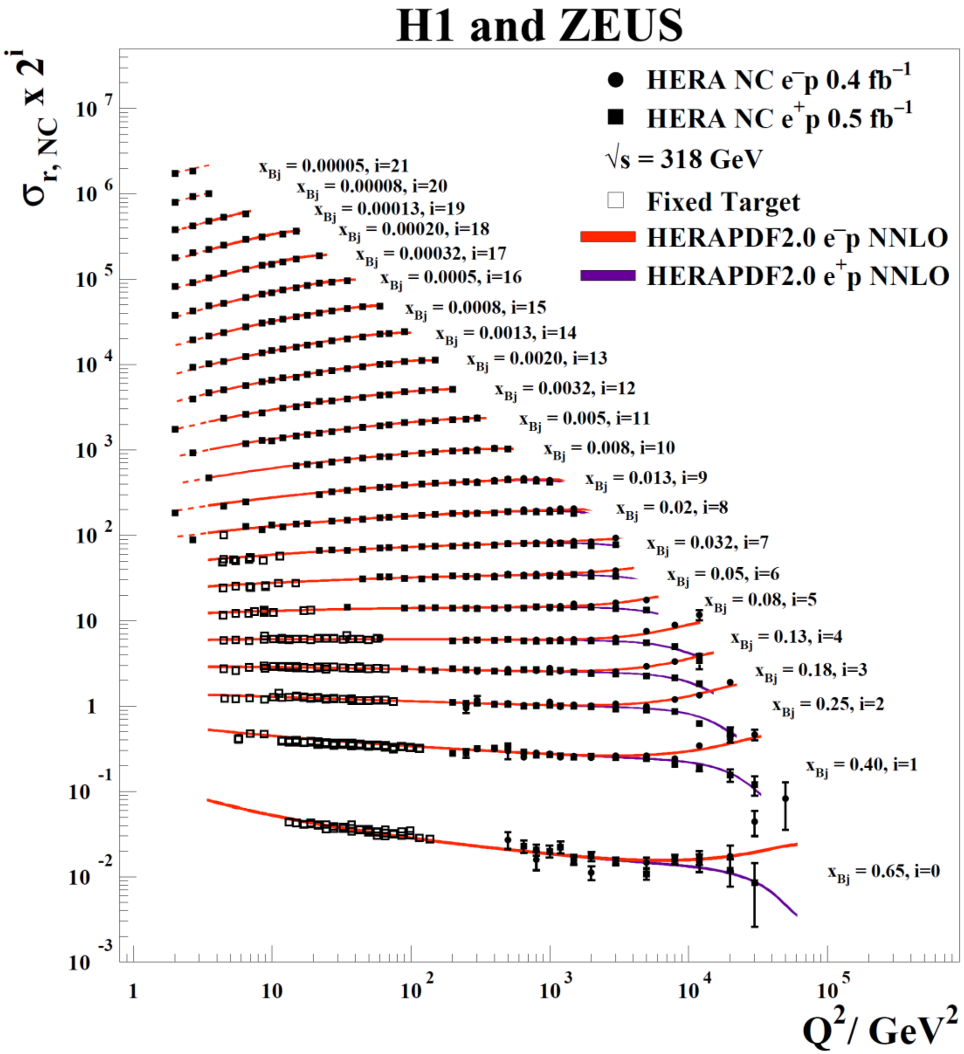
M_W information

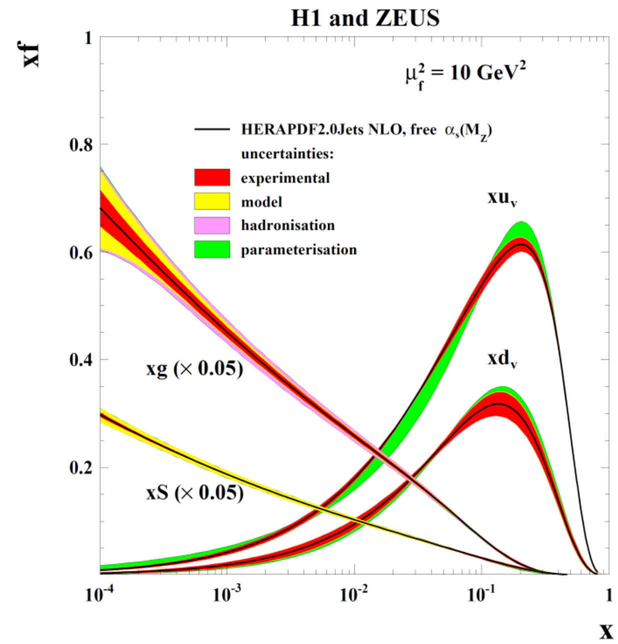
u_v at high x

d_v at high x

Measurement of high-x d_v on a pure proton target (one caveat data only up to x~0.65)

d is not well known because u couples more strongly to the photon. Historically information has come from deuterium targets –but even Deuterium needs binding corrections. And you have to assume d in proton = u in neutron





The PDFs extracted from a fit to the HERA charged and neutral current scattering data

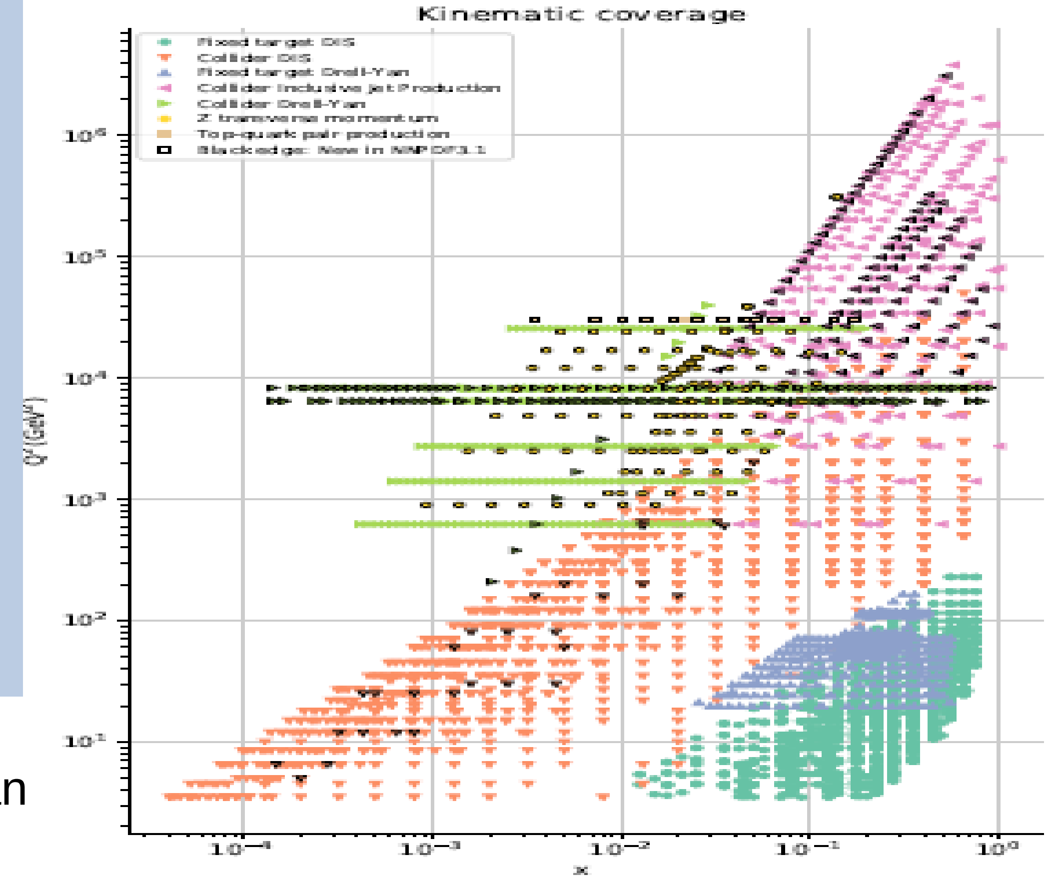
NOTE uncertainties are NOT just from those of experimental data but also from model assumptions and parametrisation variations

Thus there is enough information to make a PDF using only HERA data, with a consistent set of systematic uncertainties

AND PDFs are extracted by several different groups …..

Why do PDF sets differ?

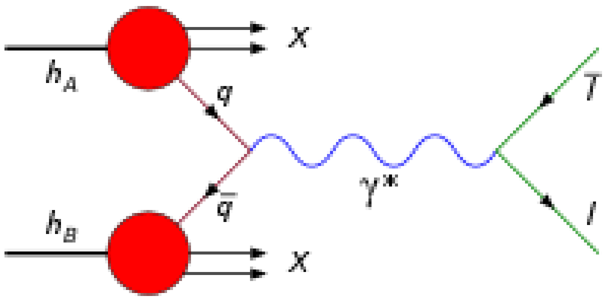
- Data sets included
- Cuts applied to remain in kinematic region of DGLAP evolution: Q^2 cut, W^2 cut, x cuts?
- Form of parametrization at Q^2_0, value of Q^2_0
- Assumptions on flavour structure of sea and valence
- heavy flavour scheme, heavy quark masses
- the value of $\alpha_S(M_Z)$ assumed, or fitted



We now use many other processes than deep-inelastic scattering:

- Drell-Yan data from fixed targets and the Tevatron and LHC
- W,Z rapidity spectra from Tevatron and LHC
- Jet pT spectra from Tevatron and LHC
- Top-anti-top differential cross-sections
- W and Z +jet spectra, or W,Z pt spectra
- W and Z +heavy flavours

PDFs also differ in how they evaluate their uncertainties some use inflated χ2 tolerances --closer to the hypothesis testing criterion– but this is a whole lecture series in itself



Drell-Yan is one of the simplest processes

$$\sigma(pp \to cdX) = \sum \iint dx_a dx_b f_a(x_a, Q^2) f_b(x_b, Q^2) \hat{\sigma}(ab \to cd)$$

- **Parton kinematics**

$$\vec{p}_A = x_A\left(\sqrt{s}/2\right); \vec{p}_B = -x_B\left(\sqrt{s}/2\right)$$

$$\hat{s} = (E_A + E_B)^2 - (\vec{p}_A - \vec{p}_B)^2$$

$$\hat{s} = 4 p_A p_B = x_A x_B s$$

Q^2: scale of parton –parton reaction, e.g. M^2

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- **$\mu^+\mu^-$ via virtual photon: parton-parton cross section.**

$$\hat{\sigma}(q_a\bar{q}_a \to \mu^+\mu^-) = \frac{e_a^2}{3}\frac{4\pi\alpha^2}{3\hat{s}}$$

- **Master formula:**

$$\frac{d^2\sigma(\mu^+\mu^-)}{dx_a dx_b} = \left[f_A(x_a) f_B(x_b) + \leftrightarrow\right]\frac{e_a^2}{3}\frac{4\pi\alpha^2}{3\hat{s}}$$

$$\tau = M^2/s = x_a x_b$$

$$2y = \ln\left(\frac{x_a}{x_b}\right)$$

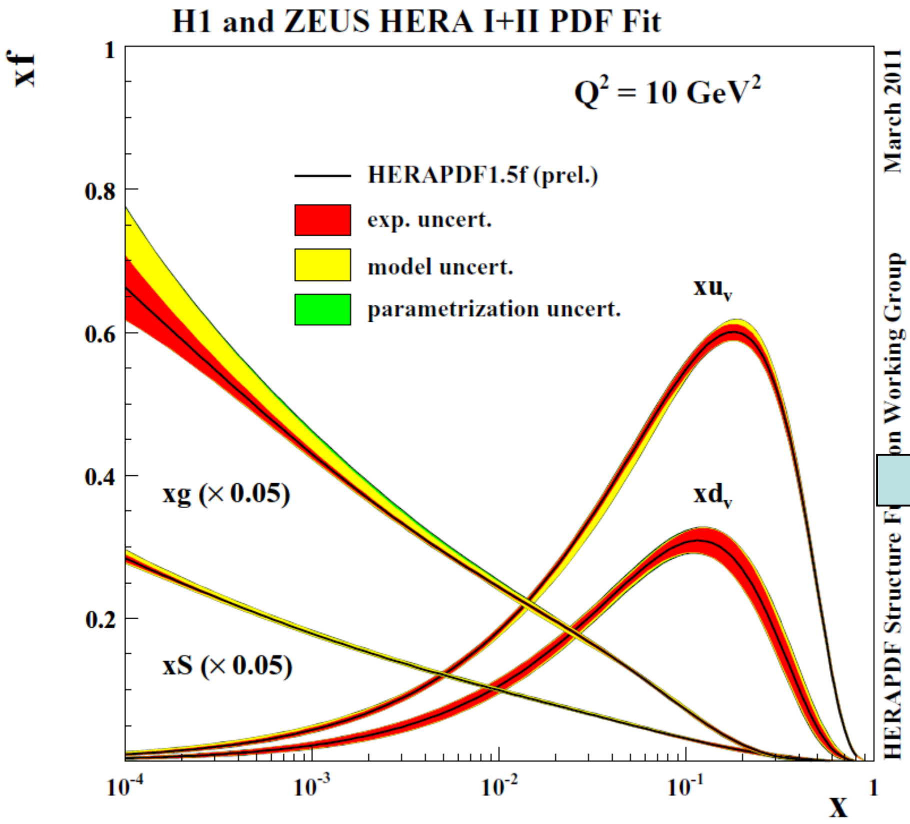
- **Change variables (appendix)** $y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right); \tau = \hat{s}/s = M^2/s$
- **Scaling cross section:**

$$\frac{d^2\sigma(\mu^+\mu^-)}{d\tau dy} = \left[f_A(x_a) f_B(x_b) + \leftrightarrow\right]\frac{e_a^2}{3}\frac{4\pi\alpha^2}{3\hat{s}}$$

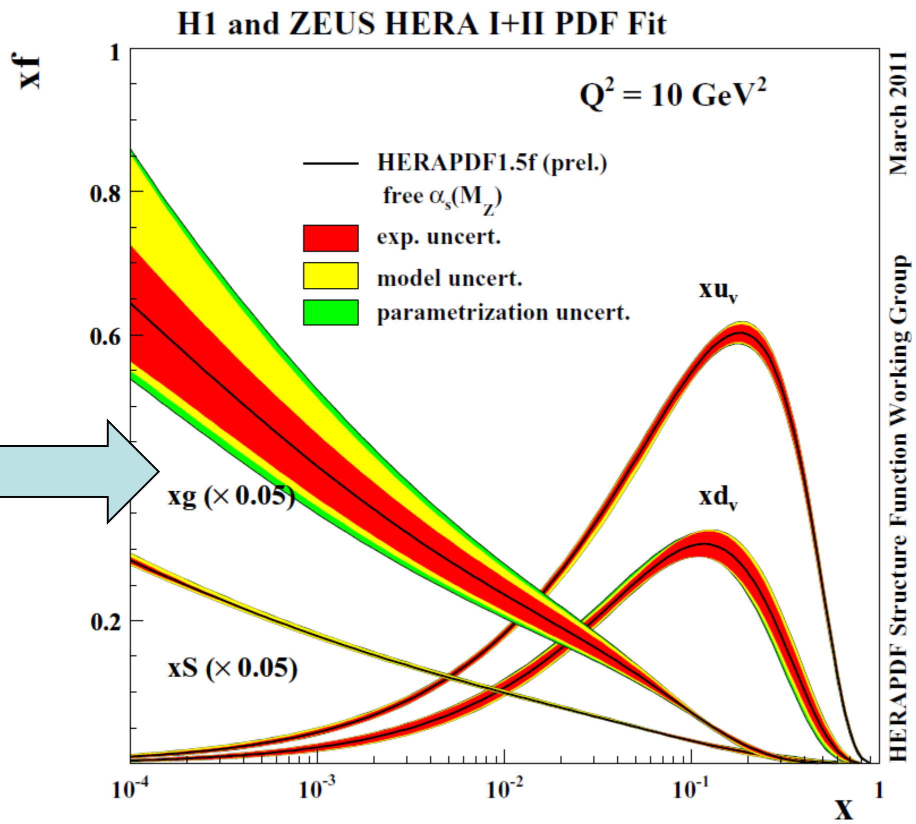
Can get x_a and x_b and Q^2 from the kinematics of the process

What are the consequences of using a free value of $\alpha_S(M_Z)$ in a PDF fit?

Fixed $\alpha_S(M_Z)$



Free $\alpha_S(M_Z)$

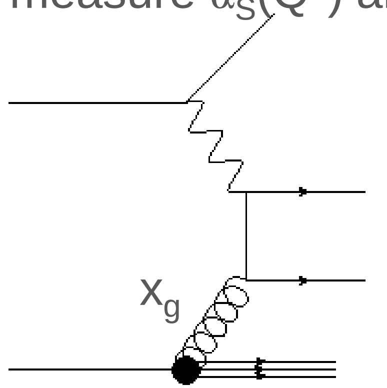


Many PDFs use a fixed value of $\alpha_S(M_Z)$
CT (EQ) , NNPDF, HERAPDF
And supply PDFs for various different fixed values

Look what happens when you free $\alpha_S(M_Z)$
… and you ONLY have Deep Inelastic Scattering data.
The gluon density and $\alpha_S(M_Z)$ are coupled by the DGLAP equations

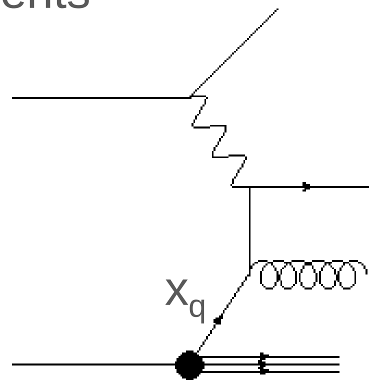
BEYOND inclusive DIS: Jet studies in the Hadron Final state gives us more information

- You can measure $\alpha_S(Q^2)$ and $xg(x,Q^2)$ from 2+1 jet events



BGF

+1 means proton remnant



QCDC

$$\sigma_{2+1} \sim \alpha_s\left[A\, x_g\, g\left(x_g, Q^2\right) + B\, x_q\, q\left(x_q, Q^2\right)\right]$$

(glue) + (quark)

This helps to break the $\alpha_S(Q^2)$ / gluon PDF correlation
Use more information that depends directly on the gluon -- jet cross-sections

To get x $g(x,Q^2)$

- Assume α_S is known
- Choose kinematic region BGF > QCDC (i.e. low x, Q^2)

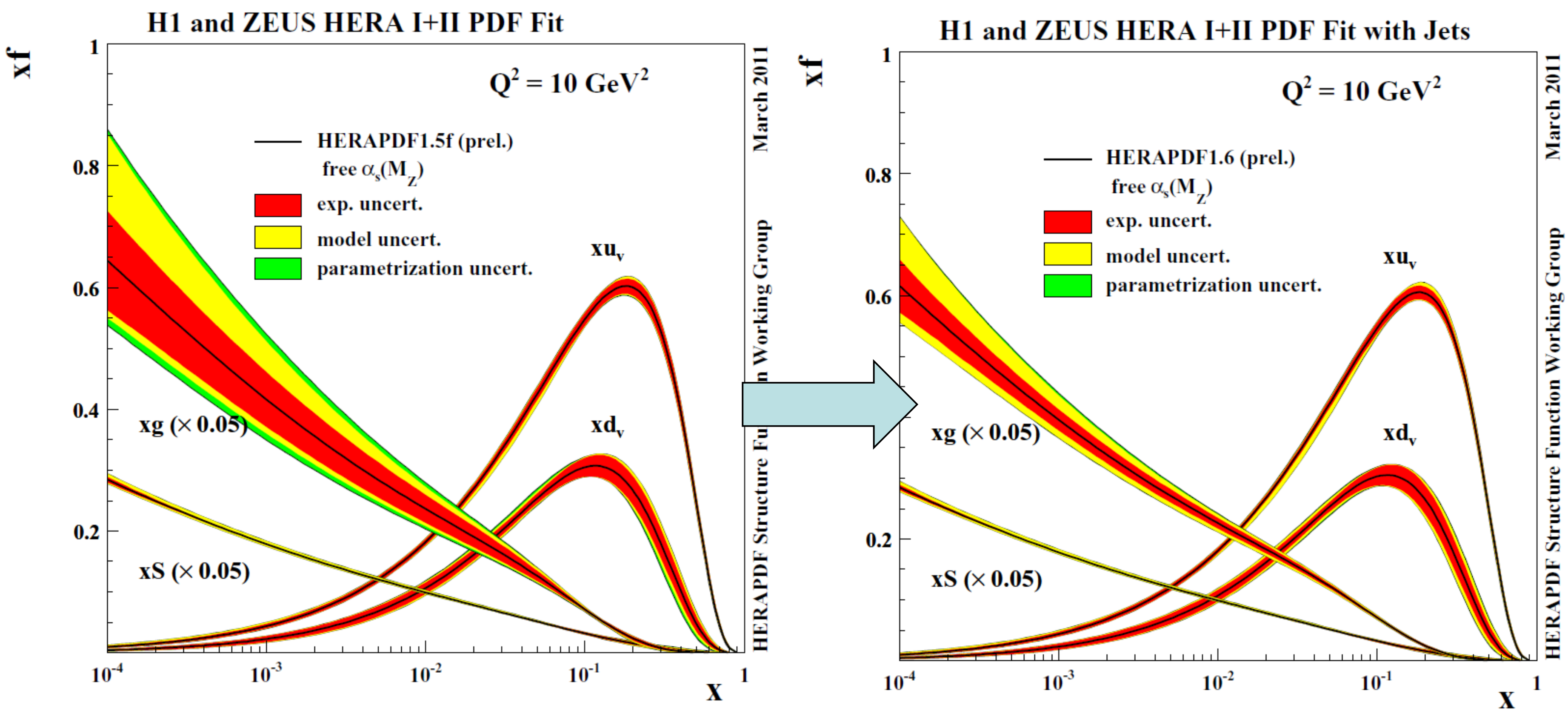
To get $\alpha_S(Q^2)$

- Choose kinematic region where PDFs $xq(x)$, $x\,g(x)$ are well known. (i.e. $x_g > 10^{-2}$, $x_q > 10^{-3} – 10^{-2}$ and $\sigma_{BGF} \sim \sigma_{QCDC}$

In practice we fit jets in all kinematic regions and hope to determine xg(x,Q2) and $\alpha_S(Q^2)$ simultaneously

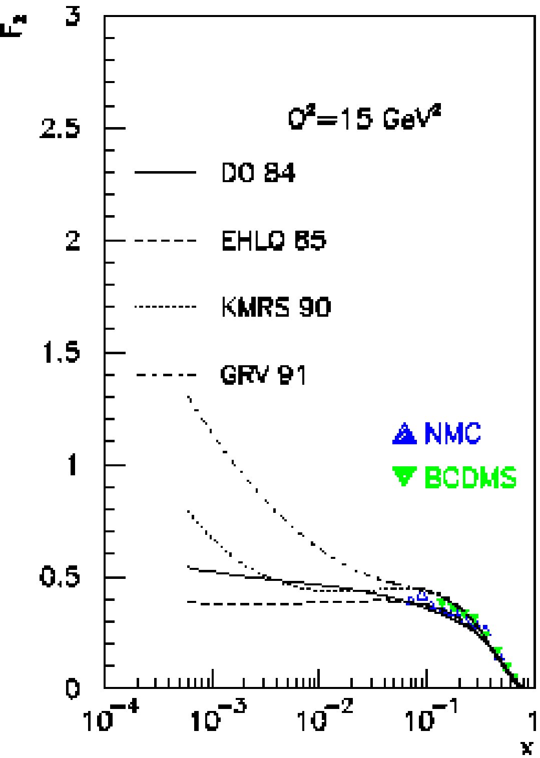
PDF fit with free $\boldsymbol{\alpha_S(M_Z)}$ **w**ithout jets

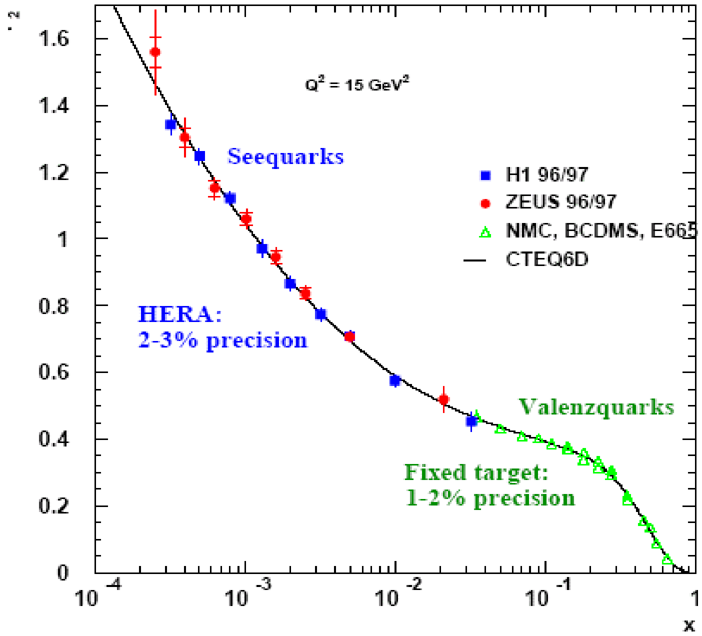
PDF fit with free $\boldsymbol{\alpha_S(M_Z)}$ **w**ith jets

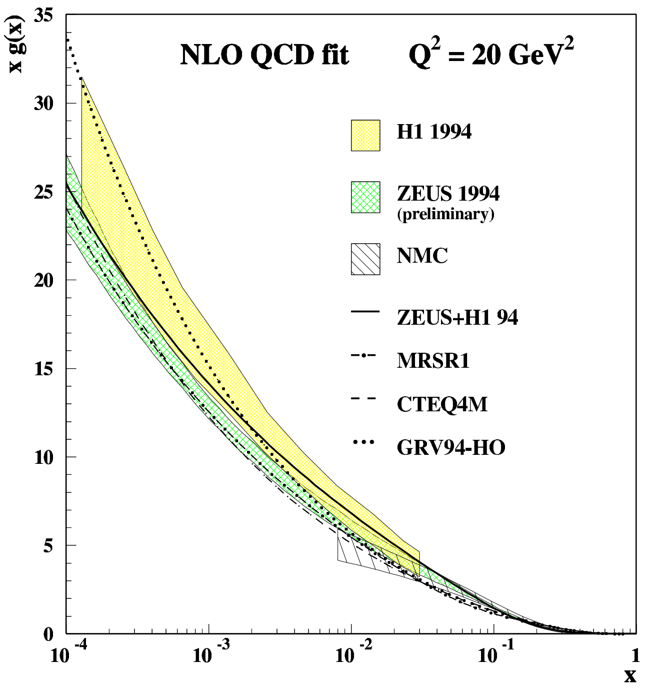


Look what happens when you keep $\alpha_S(M_Z)$ free but add jets---

Beyond DGLAP? QCD at low-x







Before the HERA measurements most of the predictions for low-x behaviour of the structure functions and the gluon PDF were wrong

It was expected that F2- and the gluon that we deduce from its scaling violations- would flatten – WHY?

Now it seems that the conventional NLO DGLAP formalism works TOO WELL !

$$\frac{d\,g(x,Q^2)}{d\ln Q^2}=\frac{\alpha_s}{2\pi}\int_x^1\frac{dy}{y}\left[P_{gq}\left(\frac{x}{y}\right)q(y,Q^2)+P_{gg}\left(\frac{x}{y}\right)g(y,Q^2)\right]$$

At low x, $\frac{x}{y}=z\rightarrow 0 \qquad P_{gq}\rightarrow\frac{C_F}{z} \qquad P_{gg}\rightarrow\frac{2C_A}{z}$

The gluon splitting functions are singular *Pgg* dominates so the equation becomes

$$\frac{d\,g(x,Q^2)}{d\ln Q^2}=\frac{\alpha_s}{2\pi}\int_x^1\frac{dy}{y}\,\frac{\sigma}{z}\,g(y,Q^2)$$

Which gives, $x\,g(x,Q^2)\sim x^{-\lambda g}$

$$\lambda_g=\left[\frac{12\ln(t/t_0)}{\beta_0\ln(1/x)}\right]^{\frac{1}{2}} \qquad t=\ln\left(\frac{Q^2}{\Lambda^2}\right) \qquad (\Lambda \text{ relates to } \alpha_S)$$

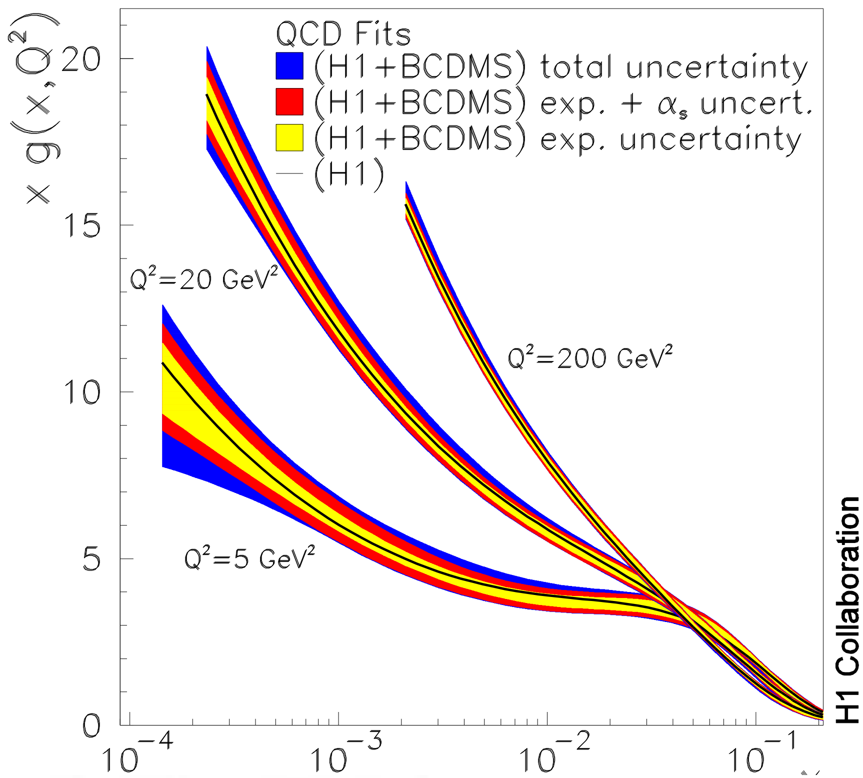
At low-x the evolution of F_2 becomes gluon dominated

$$\frac{\partial F}{\partial\ln Q^2}=\frac{\alpha_s}{2\pi}\int_x^1\frac{dy}{y}\left[P_{qg}\left(\frac{x}{y}\right)2\sum_i e_i^2\,x\,g(y,Q^2)\right]$$

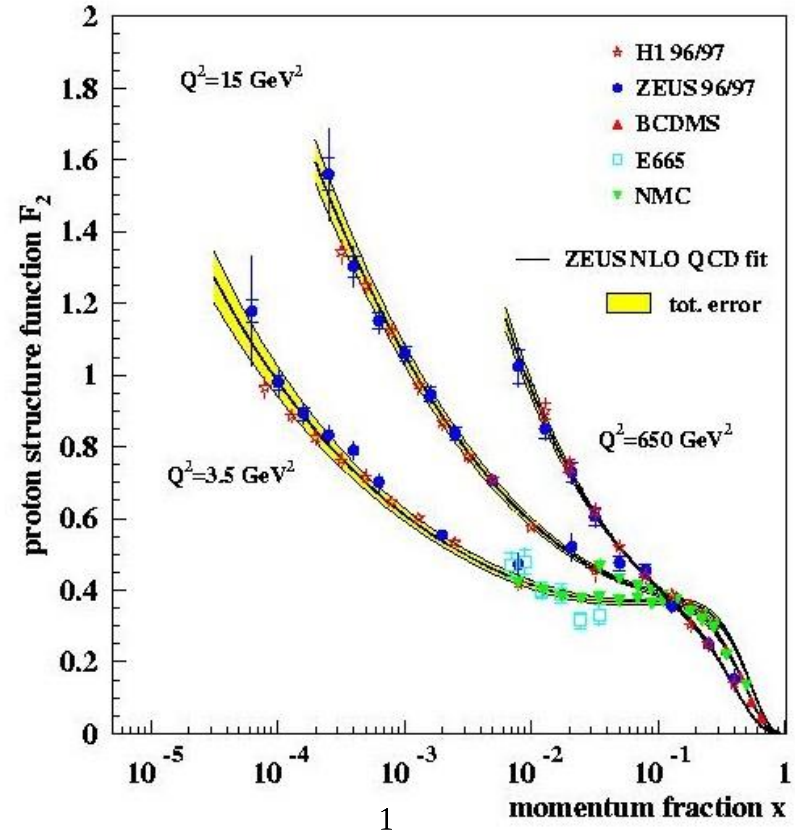
$$\rightarrow F_2(x,Q^2)\sim x^{-\lambda s} \quad \text{where } \lambda_s=\lambda_g-\epsilon$$

So slope of low x gluon gets steeper as Q^2 increases.

→ Slope of F_2 at low x gets steeper as Q^2 increases.



Low-x



$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[\Sigma_q P_{gq}(z)\, q(y,Q^2) + P_{gg}(z)\, g(y,Q^2)\right]$$

At small x,

small z=x/y

$$P_{gq} \to \frac{C_F}{z}, \quad P_{gg} \to \frac{2C_A}{z}$$

Gluon splitting functions become singular

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\frac{6}{z}\, g(y,Q^2)$$

$xg(x,Q^2) \sim x^{-\lambda_g}$

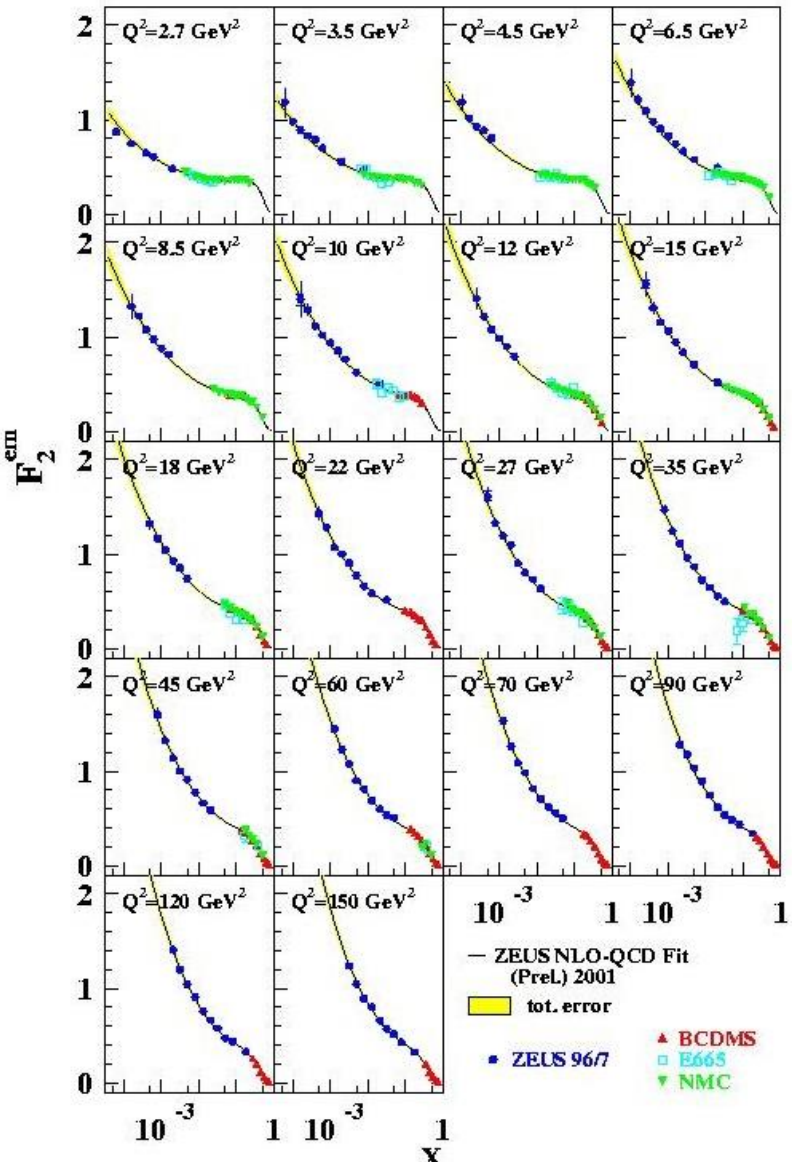
$$\lambda_g = \left(\frac{12\ln(t/t_0)}{\beta_0 \ln(1/x)}\right)^{\frac{1}{2}}, \qquad t = \ln Q^2/\Lambda^2, \qquad \alpha_s \sim 1/\ln Q^2/\Lambda^2$$

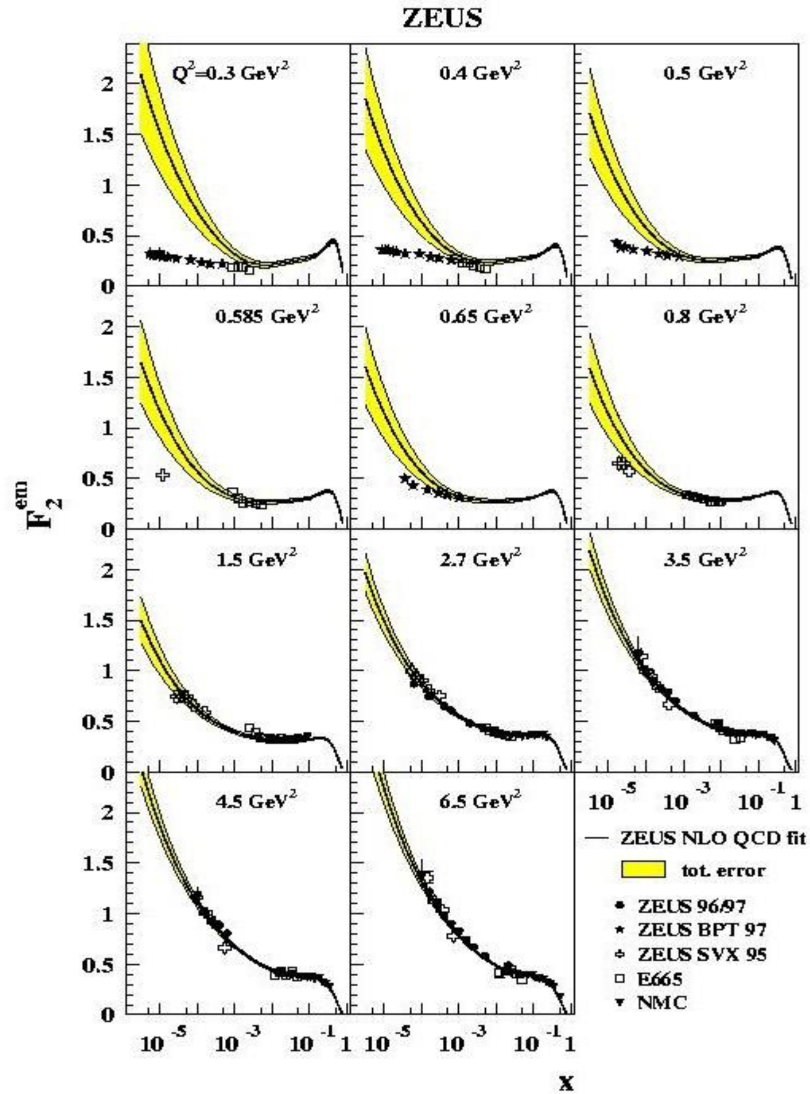
We expect that a flat gluon at low Q^2 becomes very steep **AFTER** **Q^2 evolution** AND F_2 becomes gluon dominated

$F_2(x,Q^2) \sim x^{-\lambda_s}$, $\lambda_s = \lambda_g - \varepsilon$

So what's the problem?– this expected steepness of F_2 is happening TOO EARLY

ie too low in Q^2 with no lever arm for Q^2 evolution





So it was a surprise to see F_2 steep at small x - for low Q^2, Q^2 ~ 1 GeV^2

Should perturbative QCD work? α_s is becoming large - α_s at Q^2 ~ 1 GeV^2 is ~ 0.4

There is another reason why the application of conventional DGLAP at low x is questionable:

The splitting functions, $P(x) = P^0(x) + P^1(x)\,\alpha_s\left(Q^2\right) + P^2(x)\,\alpha_s^2\left(Q^2\right)$

have contributions,

$$P^n(x) = \frac{1}{x}\left[a_n \ln^n\left(\frac{1}{x}\right) + b_n \ln^{n-1}\left(\frac{1}{x}\right)\right]$$

dominant at small x

Their contribution to the PDF comes from,

$$\frac{\partial q\left(x, Q^2\right)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P(x)\, q\left(y, Q^2\right)$$

→ and thus give rise to contributions to the PDF of the form,

$$\alpha_s^P\left(Q^2\right)\left(\ln Q^2\right)^q\left(\ln\frac{1}{x}\right)^r$$

Leading log(Q^2):

conventionally in LO DGLAP: p = q ≥ r ≥ 0 — LL(Q^2)

NLO: p = q +1 ≥ r ≥ 0 — NLL(Q^2)

But if ln($^1/_x$) is large, we should also consider,

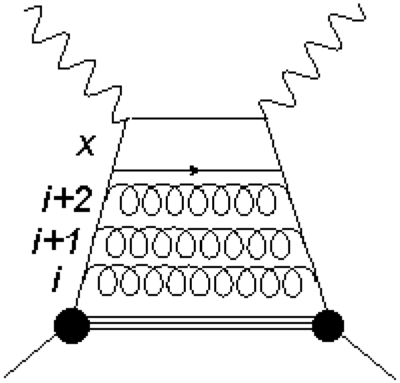
Leading log(1/x):

p = r ≥ q ≥ 1 — LL(1/x)

p = r+1 ≥ q ≥ 1 — NLL(1/x)

This is what is meant by BFKL summation.

Diagrammatically, Leading $\log Q^2 \rightarrow$ *strong* p_t ordering



$$Q^2 >> p_{t_i}^2 >> p_{t_{i-1}}^2 \ldots >> p_{t_1}^2$$

and at small *x* we also have strong ordering in *x*

$$x << x_i << x_{i-1} \ldots << p_{t_1}^2$$

$$\Rightarrow \text{leading } \ln(1/x)$$

$\rightarrow$ double leading logs $\alpha_s \ln Q^2 \ln(1/x)$ at small x

But why not sum up $\alpha_s \ln(1/x)$ independent of Q^2?

$\rightarrow$ Diagrams ordered in *x*, but *not* in p_t

BFKL formalism

$\rightarrow$ $$x\, g\left(x, Q^2\right) \sim x^{-\lambda}$$

$$\lambda = \frac{\alpha_s}{\pi} C_A \ln 2 \simeq 0.5$$ for $\alpha_s \sim 0.25$ (low Q^2)

$\rightarrow$ A singular gluon behaviour even at moderate Q^2

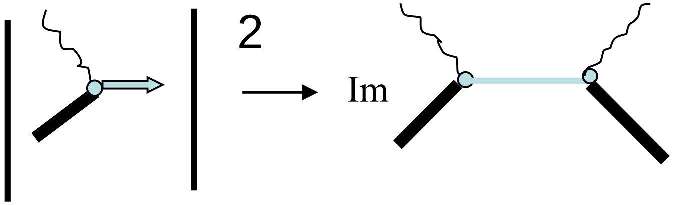
$\rightarrow$ Is this the reason for the steep behaviour of F2 at low-x ?

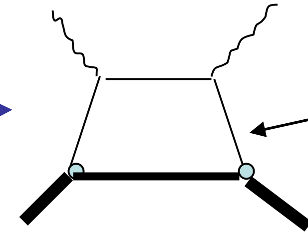
IS there a "BFKL Pomeron".

Pomerons are another story: basically they control the rise with energy of total hadron-hadron cross section, related by the optical theorem to the hadron-hadron elastic amplitude. The flavourless exchange in this could be mediated by a 'Pomeron' now believed to be multi-gluon exchange

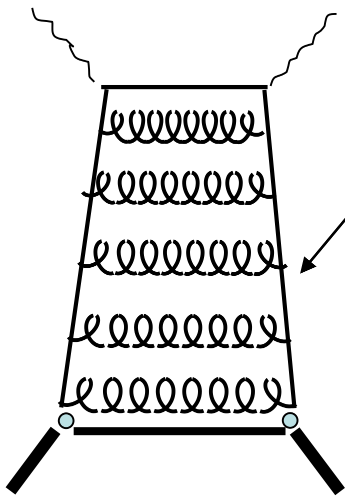
Need to extend the formalism?

Optical theorem





The handbag diagram- QPM



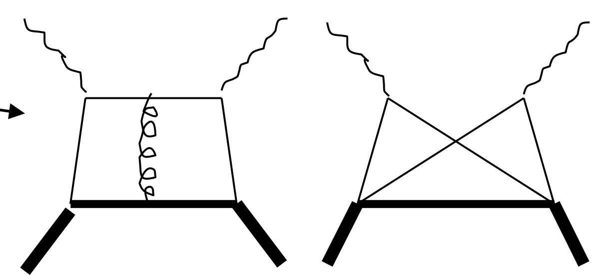
QCD at LL(Q^2)

Ordered gluon ladders ($\alpha_s^n \ln Q^{2\,n}$)

NLL(Q^2) one rung disordered $\alpha_s^n \ln Q^{2\,n-1}$

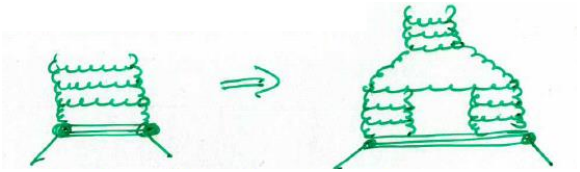
BUT what about disordered (in p_t) Ladders?
at small x there may be a need for BFKL ln(1/x) resummation?

And what about Higher twist diagrams ?
Are they always subdominant?
Important at high x, low Q^2



AND there are further theoretical problems from non-linear effects.

What if the steep rise of the gluon density at small x means that the gluon density becomes so large that gluon recombination becomes important?



gluon ladders recombine

and why stop at only 2? – this is a kind of low x “higher twist” effect

Furthermore if the **gluon density becomes large** there maybe **non-linear effects**

Gluon recombination $g\,g \to g$

$\sigma \sim \alpha_s^2 \rho^2/Q^2$

may compete with gluon evolution $g \to g\,g$

$\sigma \sim \alpha_s\, \rho$

where ρ is the gluon density

$\sim$

There is a lot of work in non-linear dynamics—Colour glass condensate, JIMWLK, BK equations..
It may be that if recombination competes with evolution then the gluon density will saturate and the saturation scale (the Q value below which saturation sets in)
$Q_s \sim x^{-\lambda}$
increases as x decreases

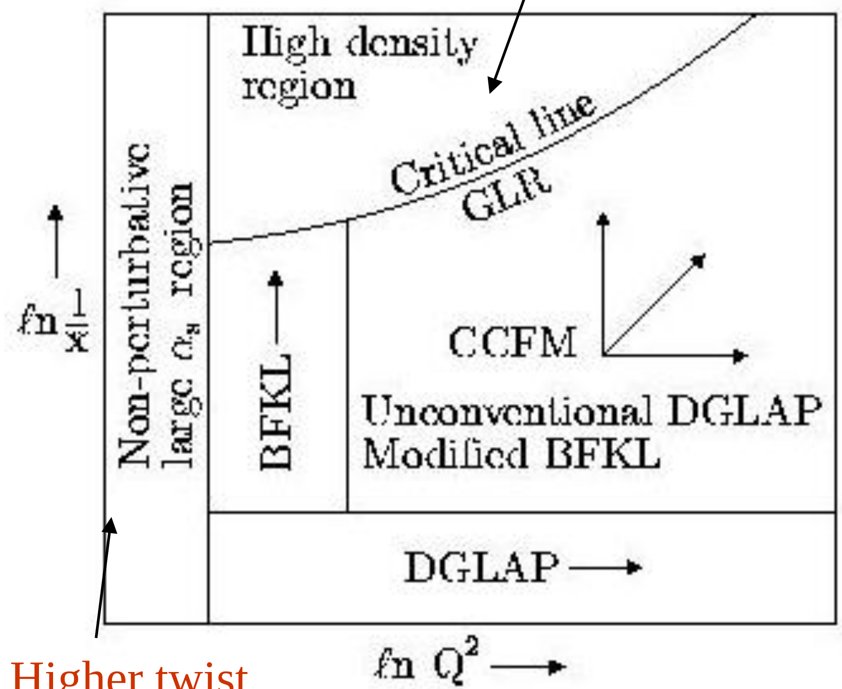
The gluon density also increases quicker in heavy nuclei as $Q_s \sim A^{1/3}\, x^{-\lambda}$

SO there are various reasons to worry that conventional LO, NLO, NNLO $\ln(Q^2)$ summations – as embodied in the DGLAP equations may be inadequate

It was a surprise to see F_2 steep at small x - even for very very low Q^2, $Q^2 \sim 1$ GeV^2

1. Should perturbative QCD work? α_s is becoming large - α_s at $Q^2 \sim 1$ GeV^2 is ~ 0.4
2. There hasn't been enough lever arm in Q^2 for evolution, but even the starting distribution is steep- **this HUGE rise at low-x makes us think**
3. there **should be** ln(1/x) resummation (BFKL) as well as the traditional $\ln(Q^2)$ DGLAP resummation- BFKL predicted $F_2(x,Q^2) \sim x^{-\lambda s}$, with $\lambda_s=0.5$, even at low Q^2
4. and/or there should be non-linear high density corrections for $x < 5\ 10^{-3}$

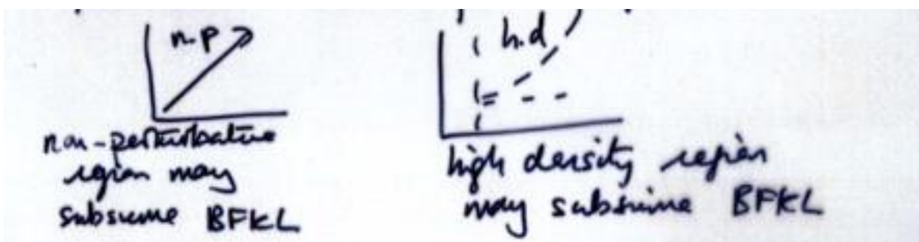
Colour Glass Condensate, JIMWLK, BK



Higher twist

Extending the conventional DGLAP equations across the x, Q2 plane

Plenty of debate about the positions of these lines!



Does the data *need* unconventional explanations?

- *ln(1/x)* terms in the splitting factors
- CCFM
- modified BFKL

Afficionados claim χ^2 improvements over conventional NLLA DGLAP..

But, one seems to be able to use DGLAP by absorbing unconventional behaviour in the boundary conditions i.e. the unknown shapes of the non-perturbative parton distributions at Q_0^2

We measure, $F_2 \sim x\,q$

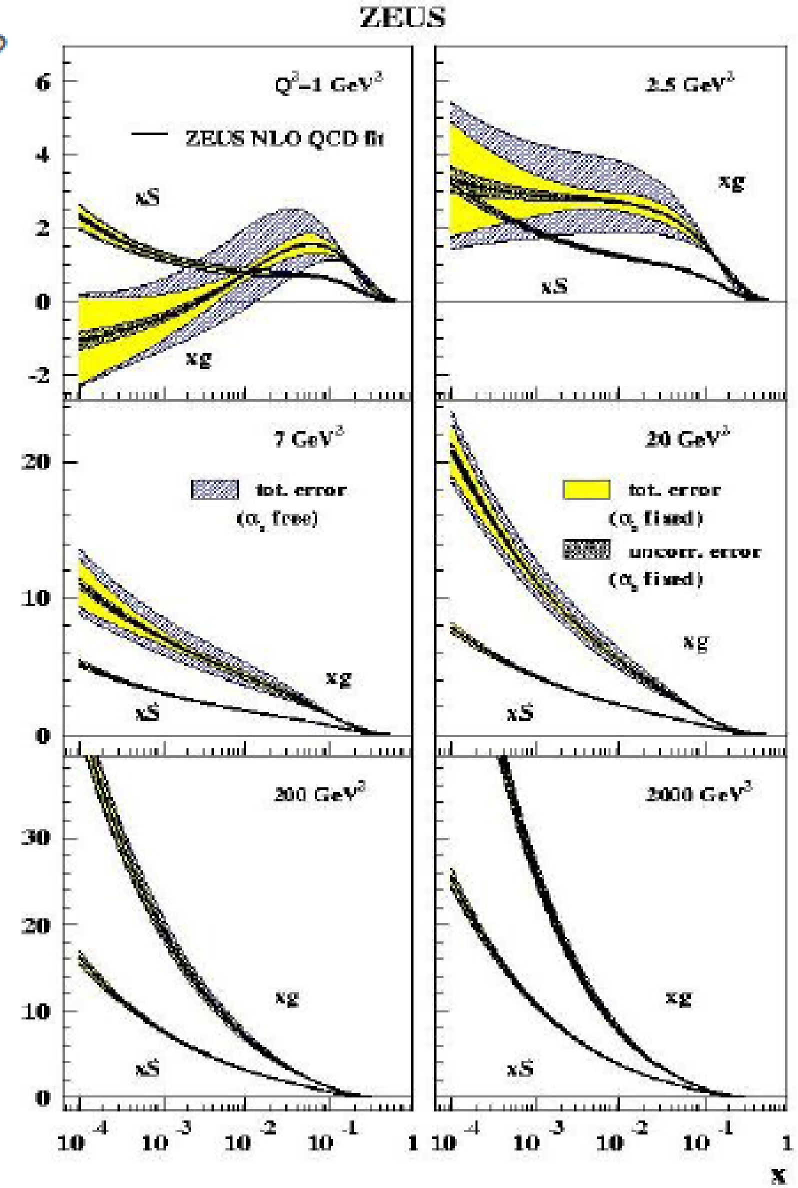
$$\frac{d\,F_2}{d\,\ln Q^2} \sim P_{qg} \cdot x\,g$$

we can explain unusually steep $\dfrac{d\,F_2}{d\,\ln Q^2}$ by:

unusual P_{qg} → eg *ln(1/x)*, BFKL

OR unusual $x\,g(x,Q_0^2)$ → "valence-like" gluon etc.

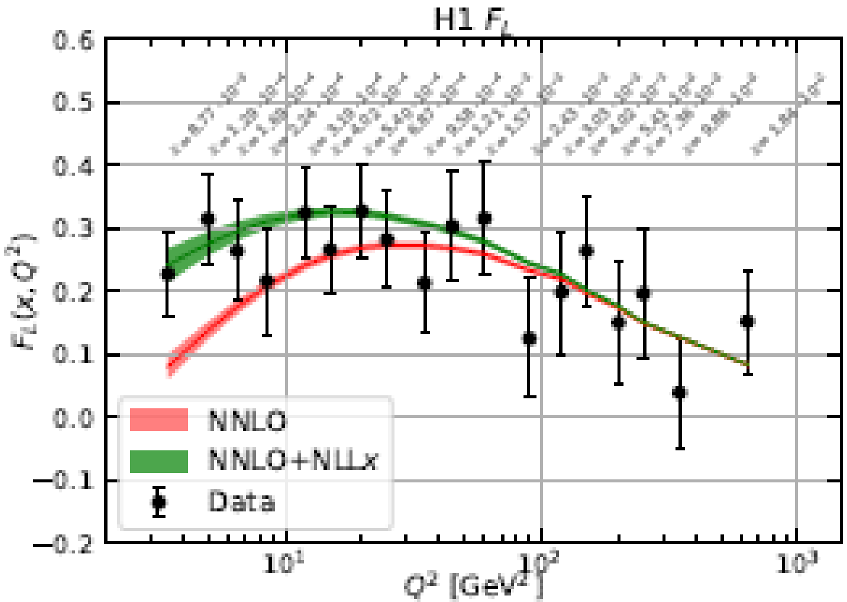
→ measure other gluon sensitive quantities at low x: F_L , $F^2_{c\bar{c}}$



Conventional DGLAP needs a valence-like gluon but a singular sea

But F^2_c gave us more information on the heavy quark scheme than on the gluon….

And F_L ? Well F_L was never very accurately measured at HERA (or anywhere else)



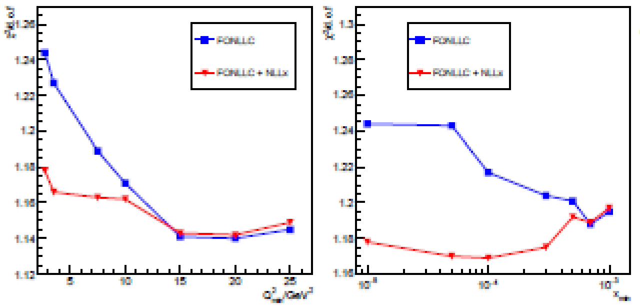
The red is the usual DGLAP

The green adds ln(1/x) resummation

BUT…
arXIV:1710.05935
A paper in which ln(1/x) BFKL resummation is worked out in detail and applied to the NNPDF fits giving NNPDF3.1sx PDF set.
WHY DID IT TAKE SO LONG?
This is partly because a) it's a very difficult calculation- the program is called HELL (High Energy Leading Log resummation)
b) Data were not accurate enough until the final HERA combination data arXIV: 1506.06042

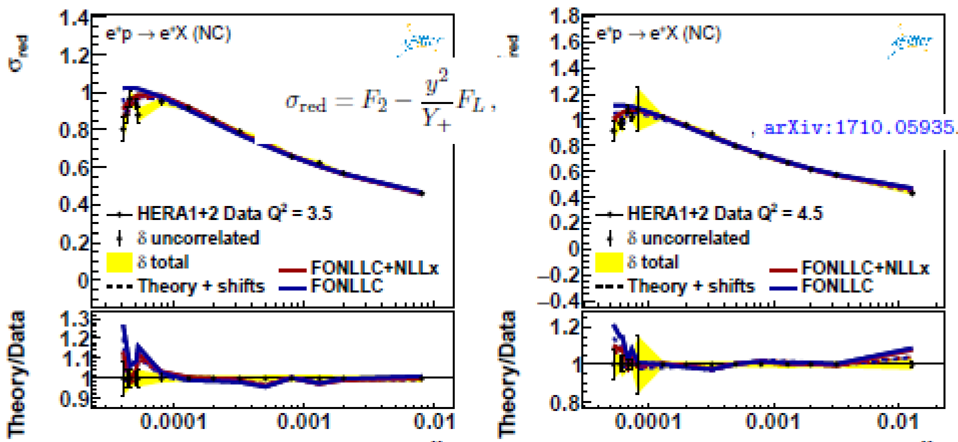
Consequences for the HERAPDF PDF fit (arXIV:1802.00064)

1.The χ2 is VASTLY improved– not just a bit



	NNLO fit with new settings	NNLO +NLLx fit with new settings
Total χ^2/d.o.f	1446/1178	1373/1178
subset NC 920 χ^2/n.d.p	446/377	413/377
subset NC 820 χ^2/n.d.p	70/70	65/70
subset charm χ^2/n.d.p	48/47	49/47
correlated shifts inclusive	102	77
correlated shifts charm	15	11
log term inclusive	20	−3
log term charm	-2	−1

2. The improvement comes at low-x and low-Q^2 the worst fitted region



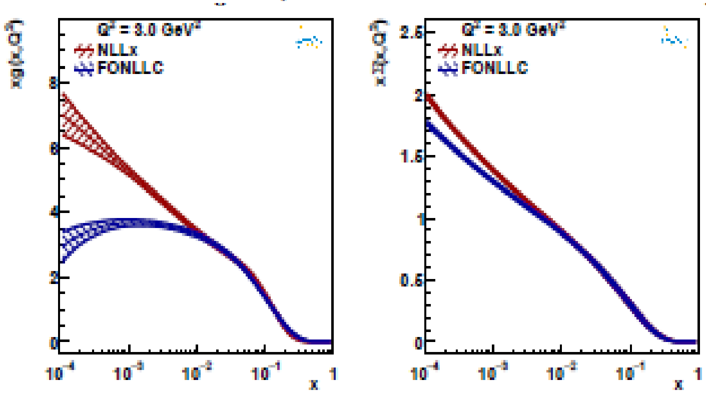
where the xF3 structure function is negligible and the reduced cross section is

$$\sigma_{red} = F_2 - \frac{y^2}{Y_+} F_L ,$$

3. And affects the high-y/low-x turn over of the cross-section $y=Q^2/(s.x)$, which fits much better because F_L is predicted to be larger

The blue is the usual DGLAP

The red adds ln(1/x) resummation



4. FL is gluon dominated and the gluon now has a much more reasonable shape…..and relationship to the sea

This is really all we have time for

But there are many other interesting topics

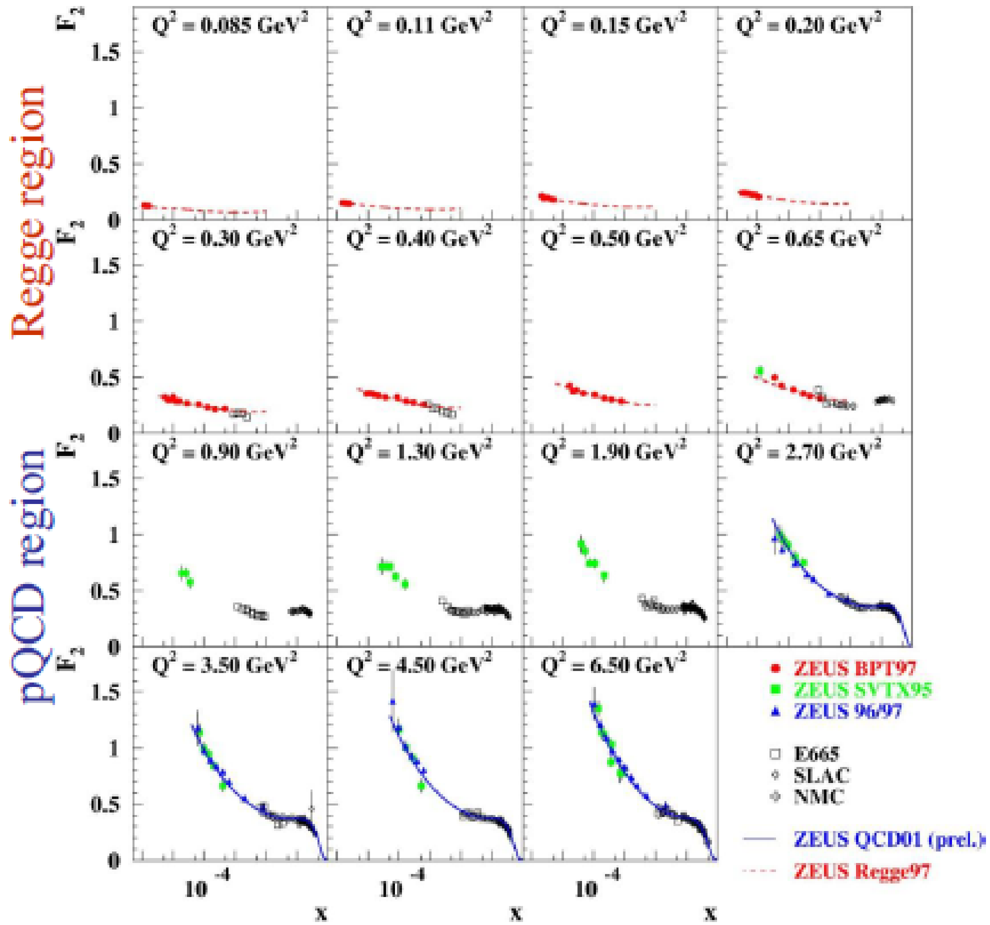
What happens at lower Q2, where the scattering is not so deep and perturbative QCD cannot be used?

What can semi-inclusive final states e,g, vector meson production tell us

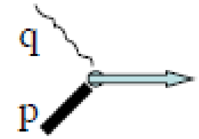
What can diffractive processes tell us

I encourage you to carry on studying

Linear DGLAP evolution doesn't work for $Q^2 < 1$ GeV2, WHAT does? – REGGE ideas?



Small x is high W^2, $x=Q^2/2p.q$ Q^2/W^2



$p_x^2 = W^2$

$\sigma(\gamma^* p) \sim (W^2)^{\alpha-1}$ – Regge prediction for high-energy hadron-hadron cross sections

α is the intercept of the Regge trajectory
α=1.08 for the SOFT POMERON

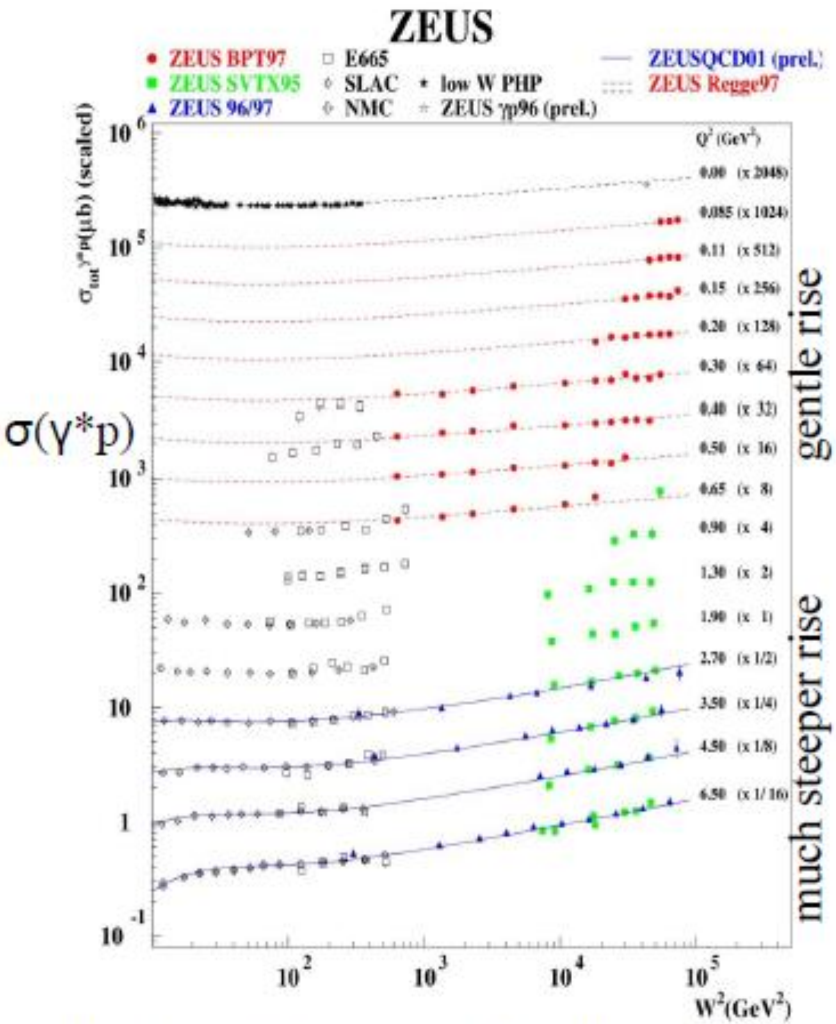
Such energy dependence is well established from the SLOW RISE of all hadron-hadron cross-sections - including
$\sigma(\gamma p) \sim (W^2)^{0.08}$
for real photon- proton scattering

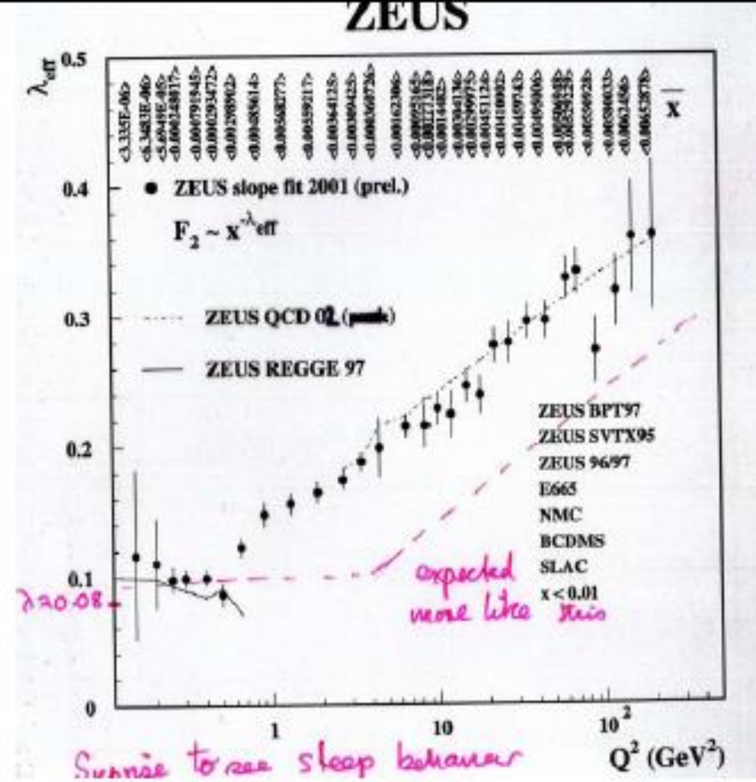
For virtual photons, at small x

$$\sigma(\gamma^* p) = \frac{4\pi^2\alpha}{Q^2} F_2$$

$\rightarrow$ $\sigma \sim (W^2)^{\alpha-1} \rightarrow F_2 \sim x^{1-\alpha} = x^{-\lambda}$
so a SOFT POMERON would imply
$\lambda = 0.08$ gives only a very gentle rise of F_2 at small x

For $Q^2 > 1$ GeV^2 we have observed a much stronger rise…..





$$F_2 \sim x^{-\lambda s}, \quad \lambda s = \frac{d \ln F_2}{d \ln 1/x}$$

The slope of F_2 at small x , $F_2 \sim x^{-\lambda}$, is equivalent to a rise of $\sigma(\gamma^* p) \sim (W^2)^{\lambda}$ which is only gentle for $Q^2 < 1$ GeV^2

So is there a HARD POMERON corresponding to this steep rise?

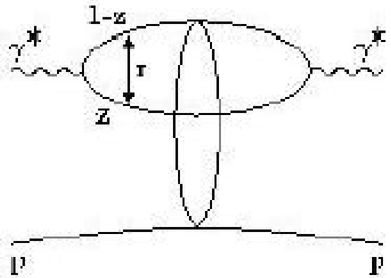
A QCD POMERON, $\alpha(Q^2) - 1 = \lambda(Q^2)$

A BFKL POMERON, $\alpha - 1 = \lambda \;\; = 0.5$

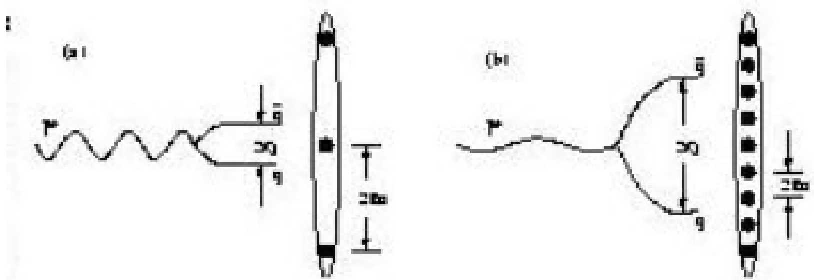
A mixture of HARD and SOFT Pomerons to explain the transition $Q^2 = 0$ to high Q^2?

Dipole models provide another way to model the transition $Q^2=0$ to high Q^2

At low x, γ^* to $q\bar{q}$ and the LONG LIVED ($q\bar{q}$) dipole scatters from the proton



The dipole-proton cross section depends on the relative size of the dipole $r \sim 1/Q$ to the separation of gluons in the target R_0



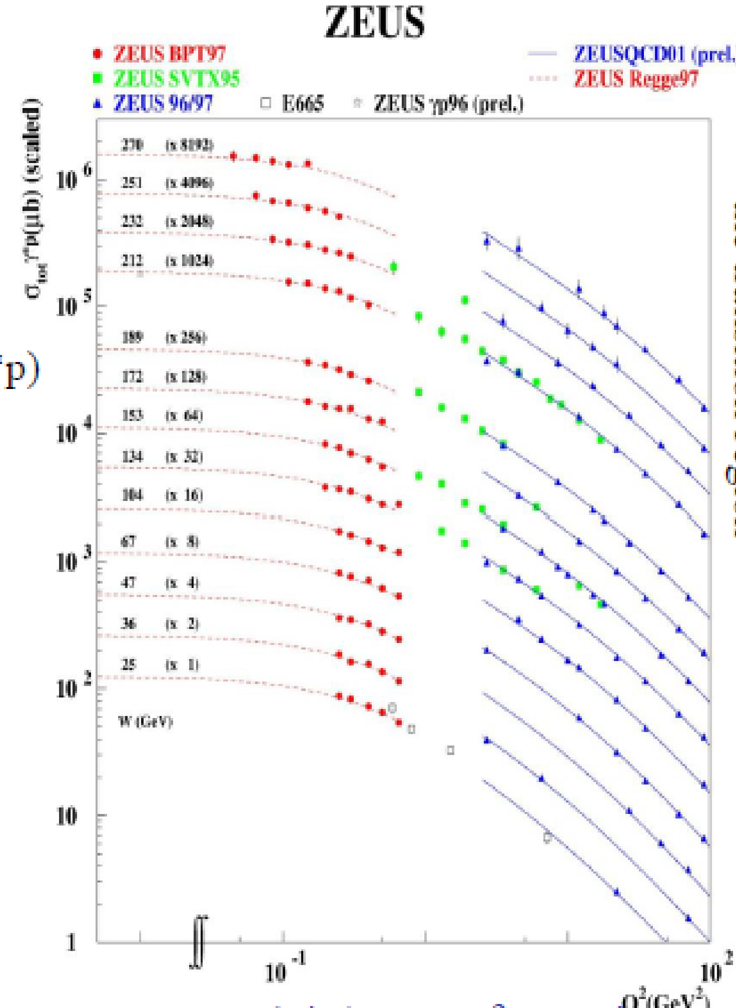
$\sigma = \sigma_0(1 - \exp(-r^2/2R_0(x)^2))$, $R_0(x)^2 \sim (x/x_0)^{\lambda} \sim 1/xg(x)$

r/R_0 small $\rightarrow$ large Q^2, x $\sigma \sim r^2 \sim 1/Q^2$, F2 flat Bjorken scaling

r/R_0 large $\rightarrow$ small Q2, x $\sigma \sim \sigma_0 \rightarrow$ saturation of the dipole cross-section

GBW dipole model

$\sigma(\gamma^* p)$



Now there is HERA data right across the transition region

But $\sigma(\gamma^* p) = \frac{4\pi\alpha^2}{Q^2} F_2$ is general (at small x)

$\sigma(\gamma p)$ is finite for real photons, $Q^2=0$. At high Q^2, $F_2 \sim$flat (weak $\ln Q^2$ breaking) and $\sigma(\gamma^* p) \sim 1/Q^2$

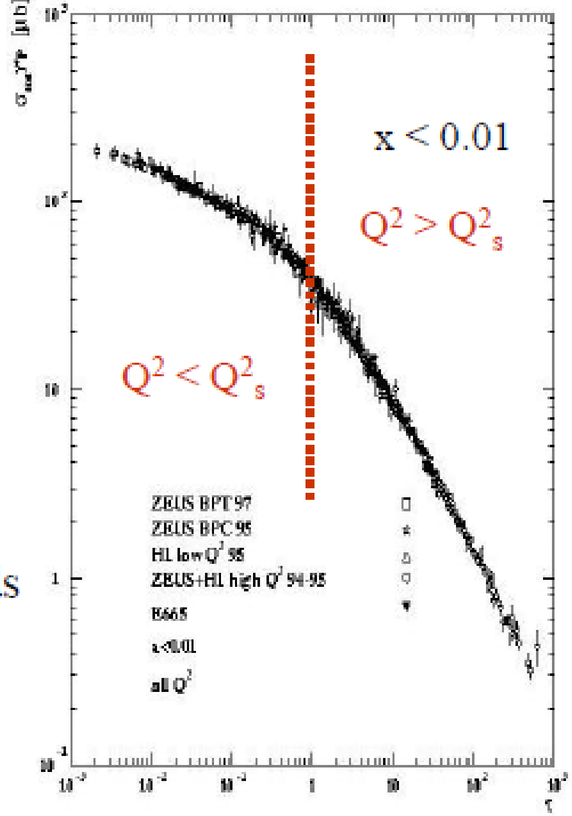
$\sigma = \sigma_0\,(1 - \exp(-1/\tau))$

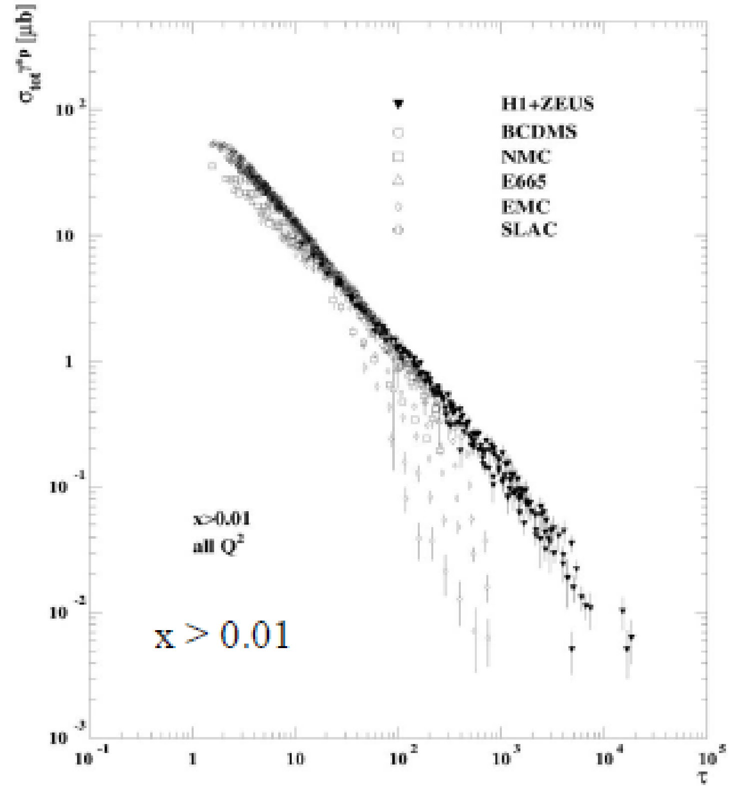
Involves only

$\tau \;\; = Q^2 R_0^{\,2}(x)$

$\tau = Q^2/Q_0^{\,2}\,(x/x_0)^{\lambda}$

And INDEED, for $x<0.01$, $\sigma(\gamma^* p)$ depends only on τ, not on x, Q^2 separately





τ is a new scaling variable, applicable at small x

It can be used to define a `saturation scale' , $Q^2_s = 1/R_0^{\,2}(x) \sim x^{-\lambda} \sim x\,g(x)$, gluon density

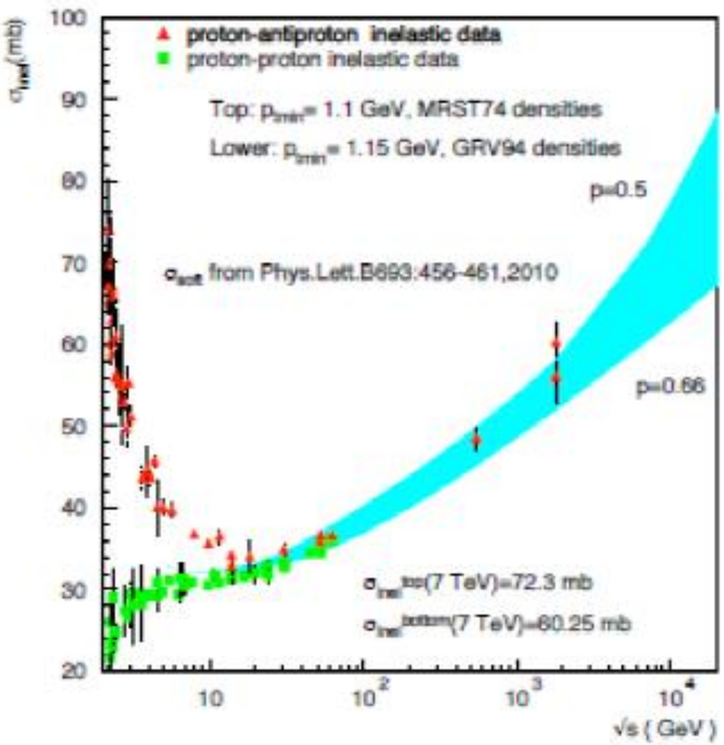
- such that saturation extends to higher Q^2 as x decreases

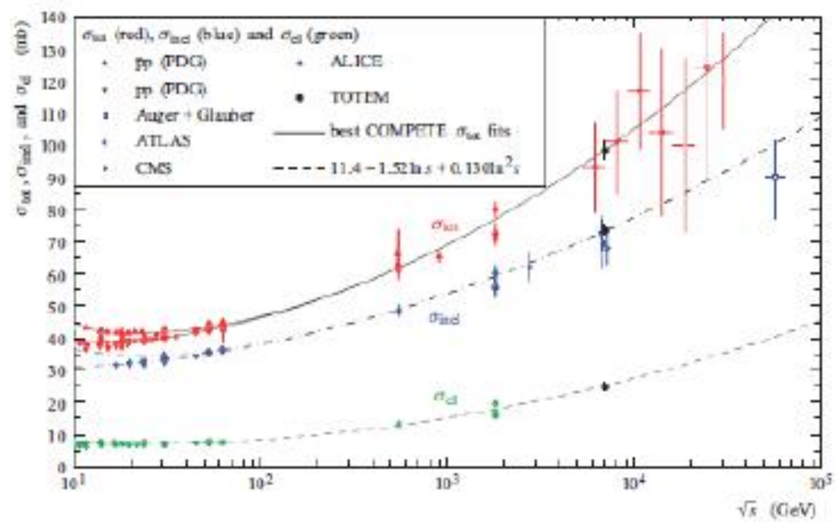
Some understanding of this scaling, of saturation and of dipole models is coming from work on non-linear evolution equations applicable at high density– Colour Glass Condensate, JIMWLK, Balitsky-Kovchegov. There can be very significant consequences for high energy cross-sections e.g. neutrino cross-sections – also predictions for heavy ions- RHIC, diffractive interactions – Tevatron, HERA and the LHC- even some understanding of soft hadronic physics

Do we understand the rise of hadron-hadron cross-sections at all?

Could there always have been a hard Pomeron- is this why the effective Pomeron intercept is 1.08 rather than 1.00?

Does the hard Pomeron mix in more strongly at higher energies? What about the at the LHC?





Pre ATLAS prediction with uncertainty from assumptions on mixing in of hard Pomeron

If anything TOTEM result looks even steeper

What about the Froissart bound?