What is Deep Inelastic Scattering?

Firing small probes like leptons, neutrinos----of high energy and thus small wavelength-----

at big things like protons, neutrons, hadrons, nuclei

To see what is inside them

You could say it started with Rutherford ‘splitting the atom’ with alpha particles

--though that wasn’t very deep—

Deepness is defined by the energy of the probe

As for inelasticity, well an electron at lowish energies could scatter from a proton elastically, ie leaving it intact, but if the electron is high energy it is more likely to break the proton up and that is inelastic

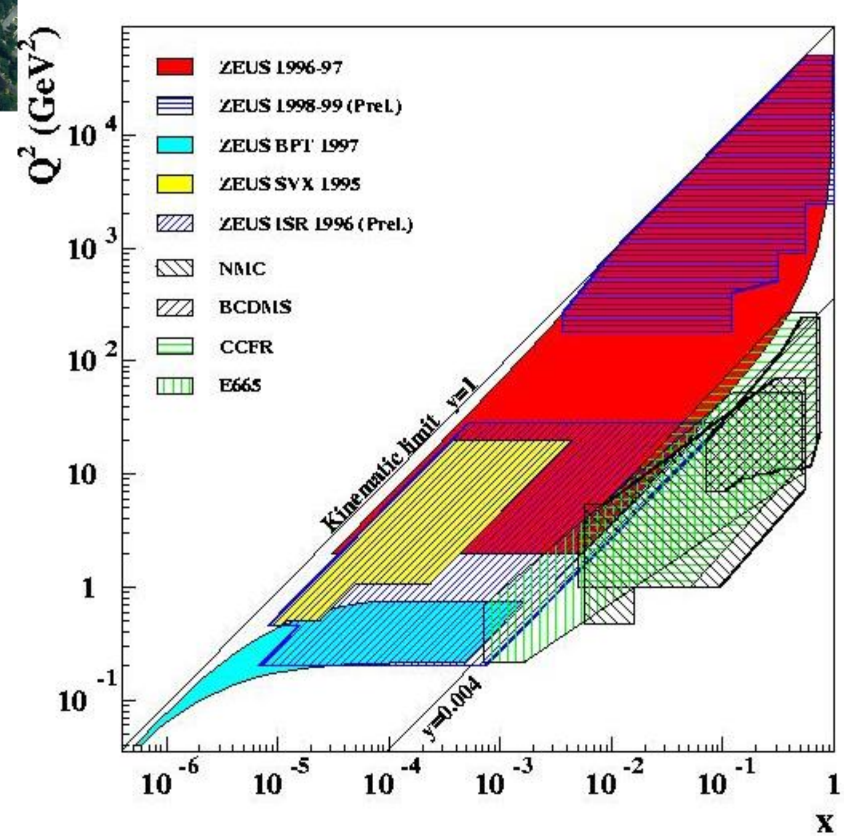
Electron-proton deep-inelastic scattering was first done at SLAC in the late 1960’s

It was also done using muon probes at CERN and at FNAL in the 70’s and 80’s (NMC,EMC)

AND it was done using (anti-)neutrino probes at CERN and Fermilab also in the 70’s and 80’s

WA21,25,47,59,66, CDHS(W), CHARM, CCFR …

BUT these were all ‘fixed target’ experiments



HERA electron-proton collider at DESY Hamburg has provided the most extensive data on Deep Inelastic Scattering, running 1992-2007.

Final inclusive data combination from all HERA-1+11 running

~500pb^{-1} per experiment split ~equally between e$^+$ and e$^-$ beams:arxiv:1506.06042

Running at Ep = 920, 820, 575, 460 GeV
$\sqrt{s}$ = 320, 300, 251, 225 GeV

The lower proton beam energies allow a measurement of F_L and thus give more information on the gluon.

41 input data files to 7 output files with
169 sources of correlated uncertainty

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HERA CC e+p 101 (920)
HERA CC e-p 102 (920)
HERA NC e-p 103 (920)
HERA NC e+p 104 (820)
HERA NC e+p 105 (920)
HERA NC e+p 106 (460)
HERA NC e+p 107 (575)
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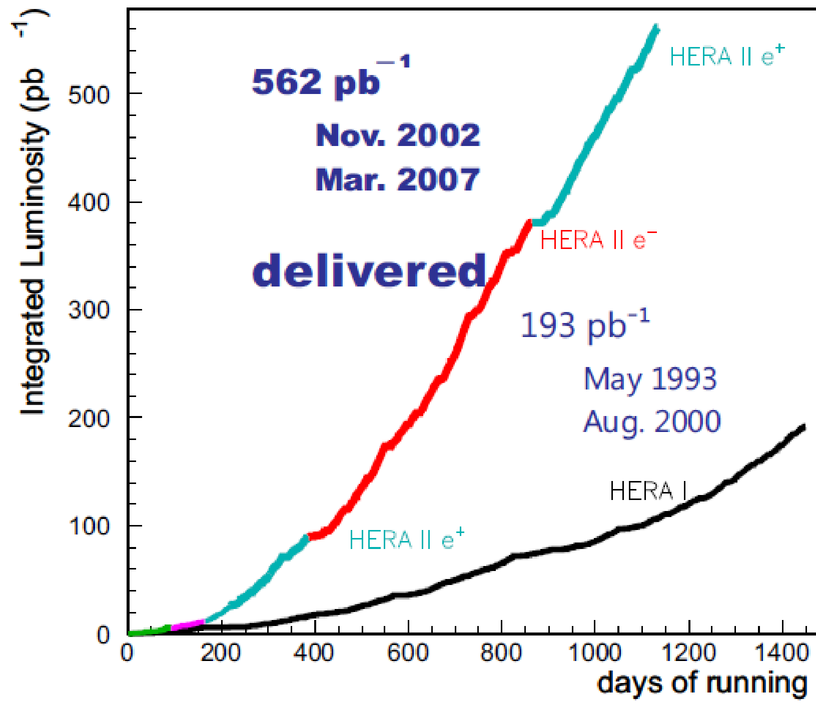
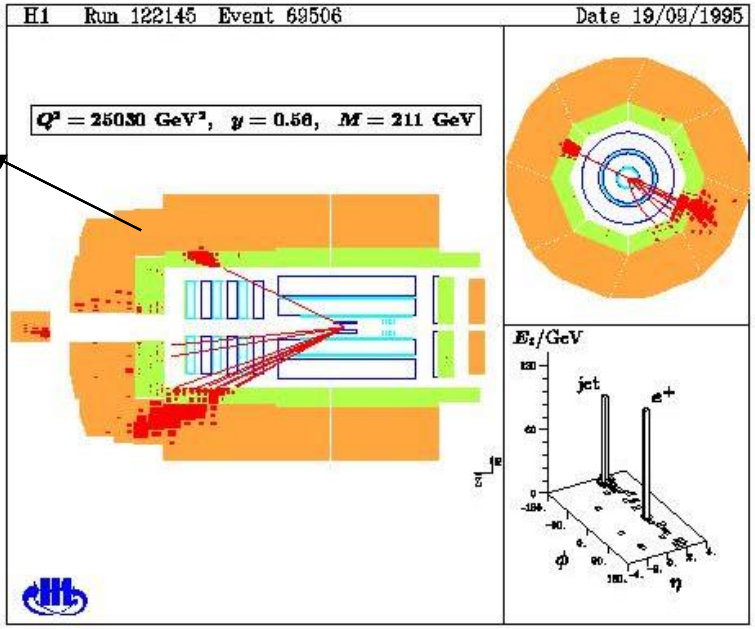


$0.045 < Q^2 < 50000$ GeV^2 $\quad$ $6.\ 10^{-7} < x_{Bj} < 0.65$

Deep inelastic lepton-nucleon scattering -DIS

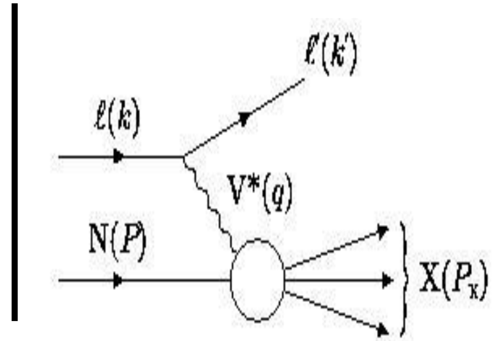


E′

E_e

E_p

dσ ~



2

$L^{\mu\nu}\, W_{\mu\nu}$

Leptonic tensor - calculable

Hadronic tensor- constrained by Lorentz invariance

q = k – k′, Q² = -q²

This is the scale of the vector boson probe

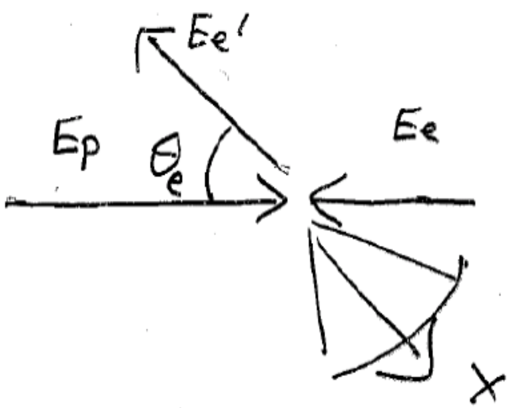
s= (p + k)²

x = Q² / (2p.q)

y = (p.q)/(p.k)

These are 4-vector invariants

Q² = s x y



The kinematic variables are measureable both from electron E', θ_e (NC)
AND from hadron variables (CC)

$$s = 4\,E_e\,E_p$$

$$Q^2 = 4\,E_e\,E'\,\sin^2\frac{\theta_e}{2}$$

$$y = \left(1 - \frac{E'}{E_e}\sin^2\frac{\theta_e}{2}\right)$$

$$x = Q^2 / s\,y$$

The kinematic variables are measurable

Schematically,

$$d\sigma \sim \sum_{x} \left| \right.$$ 

$$\left. \right|^2 \sim L^{\mu\nu} \cdot W_{\mu\nu}$$

Leptonic tensor, calculable ELECTROWEAK

Hadronic tensor, constrained by LORENTZ INVARIANCE

⇒ Charged lepton-neutral current γ, Z

$$\frac{d^2\sigma(\ell^{\pm})}{dx\,dQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\left[y_+ F_2^{NC}(x,Q^2) - y^2 F_L^{NC}(x,Q^2) \mp y_- xF_3^{NC}(x,Q^2)\right]$$

Leptonic part hadronic part

$$y_{\pm} = 1 \pm (1-y)^2$$

Charged lepton- charged current $W^{\pm}$

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{G_F^2}{4\pi x}\frac{M_W^4}{(Q^2+M_W^2)^2}\left[y_+ F_2^{CC}(x,Q^2) - y^2 F_L^{CC}(x,Q^2) \mp y_- xF_3^{CC}(x,Q^2)\right]$$

Leptonic part hadronic part

F_2 , F_L , xF_3 are STRUCTURE FUNCTIONS

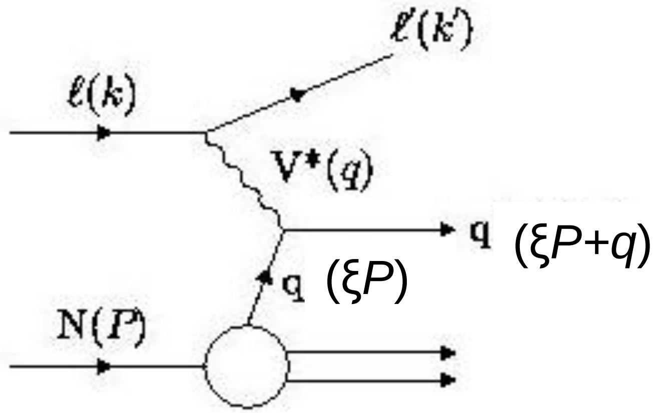
which parameterise our ignorance of the hadronic sector

⇒ MEASUREABLE as functions of x, Q^2

What do the structure functions mean?

Naïve approach – Quark Parton Model

What if, when the lepton strikes the proton, it is actually not the whole proton that is hit but one of its constituents that Feynman called a parton, which may be identified with constituents that were being called quarks (although there were supposed to be only 3 of those) AND the electron scatters off these point-like constituents elastically



Elastically precisely because they ARE point-lilke, they have no structure to make it inelastic
Or rather, that is our hypothesis…

ξ - Fraction of momentum of incoming nucleon taken by the struck quark

At large Q^2 ignore terms of order M^2 and put quarks on mass-shell

$$(\xi p + q)^2 = \xi^2 p^2 + q^2 + 2\xi\, p\cdot q = 0$$

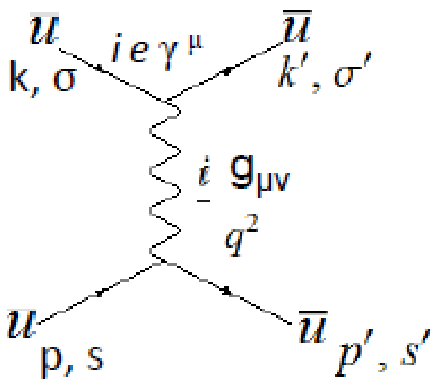
$$\rightarrow \xi = \frac{Q^2}{2\, p\cdot q} = x$$

In other words the fractional momentum of a struck quark is a measurable kinematic variable x

See lepton-hadron scattering as a sum over lepton-quark scatters.
Structure functions are then sums over quark momentum distributions within the hadron.

Now maybe we can calculate electron quark scattering.. It must be like elctron-muon

Consider elastic electron-muon scattering in γ exchange



$$j_e^{\nu} = i\, e\, \overline{u}\,(k', \sigma')\, \gamma^{\nu}\, u(k, \sigma)$$

$$j_{\text{muon}}^{\mu} = i\, e' \overline{u}\,(p', s')\, \gamma^{\mu}\, u(p, s)$$

Matrix element is
Current . propagator. current

$$\mathrm{M} = -\frac{e\, e'}{q^2}[\overline{u}(k', \sigma')\, \gamma_{\mu}\, u(k, \sigma)][\overline{u}(p', s')\, \gamma_{\mu}\, u(p, s)]$$

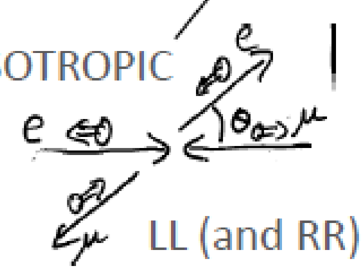
Note I am calling the muon charge *e'* not *e* deliberately

sum + average over spins $\sigma', \sigma\ \&\ s, s'$

$$\frac{1}{4}\sum_{\text{spins}} |\mathrm{M}|^2 = \frac{e^2\, e'^2}{q^4} L_e^{\mu\nu}\, L_{\mu\nu}^{\text{muon}} = \frac{e^2\, e'^2}{q^4}\, 2\, s^2[1 + (1-y)^2] \qquad Q^2 = s\, y$$

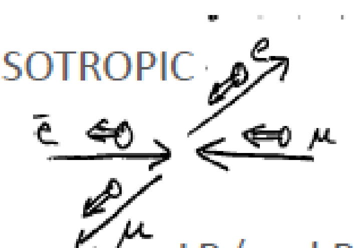
$$\Rightarrow \frac{d\sigma}{d\,y} = \frac{2\,\pi\alpha^2}{Q^4}\left(\frac{e'}{e}\right)^2[1 + (1-y)^2]\, s$$

ISOTROPIC



LL (and RR)

NON-ISOTROPIC



LR (and RL)

$$y = \frac{(1-\cos\theta)}{2}$$

As
$y \to 1,\ \theta \to \pi$

Scattering from ANY fermion (e.g. quarks) similar → depends on fermion charge e'

Consider a collection of quarks and antiquarks (in a hadron)
→ any one can be struck
→ let this one have *x* of the protons momentum, so *s* → *xs*

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4}(e^i)^2\left[1+(1-y)^2\right]x\,s$$

for a quark of charge (e^ie)

So e^i = 2/3 or -1/3

so for the HADRON

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4}\left[1+(1-y)^2\right]s\sum_i (e_i)^2[x\,q_i(x) + x\,\overline{q}_i(x)]$$

where q(x) is the PROBABILITY of the quark having the momentum fraction *x* the momentum distribution and *xq(x)* is called a PARTON DISTRIBUTION FUNCTION (PDF)

Now compare the above quark parton model $\frac{d^2\sigma}{dx\,dy}$ to the general predictions

QPM
$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\left[1+(1-y)^2\right]\sum_i (e_i)^2\left[x\,q_i(x) + x\,\overline{q}_i(x)\right]$$

General
$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\left[Y_+ F_2(x,Q^2) - y^2 F_L(x,Q^2) \mp Y_- x F_3(x,Q^2)\right]$$

remember $Y_+ = \left[1+(1-y)^2\right]$

⇒ QPM predictions

$$F_2(x,Q^2) = \sum_i e_i^2\, x\left[q_i(x) + \overline{q}_i(x)\right]$$

i.e. F_2 is only a function of Q^2 ⟶ BJORKEN SCALING

contrast elastic scattering $F(q^2) \sim \dfrac{1}{(1+q^2/m^2)^2}$

$F_L(x,Q^2) = 0$ because quarks are spin ½ fermions

$xF_3(x,Q^2) = 0$ because only γ exchange considered

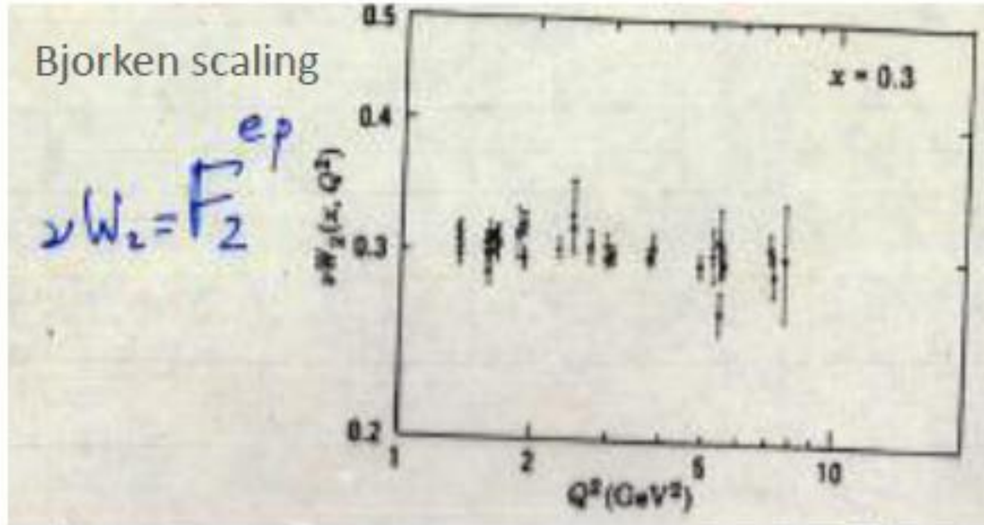
The results are for charged-lepton/hadron scattering via γ exchange and are independent of lepton charge

⇒ BUT there's more!

(what made me include antiquarks in this sum?)

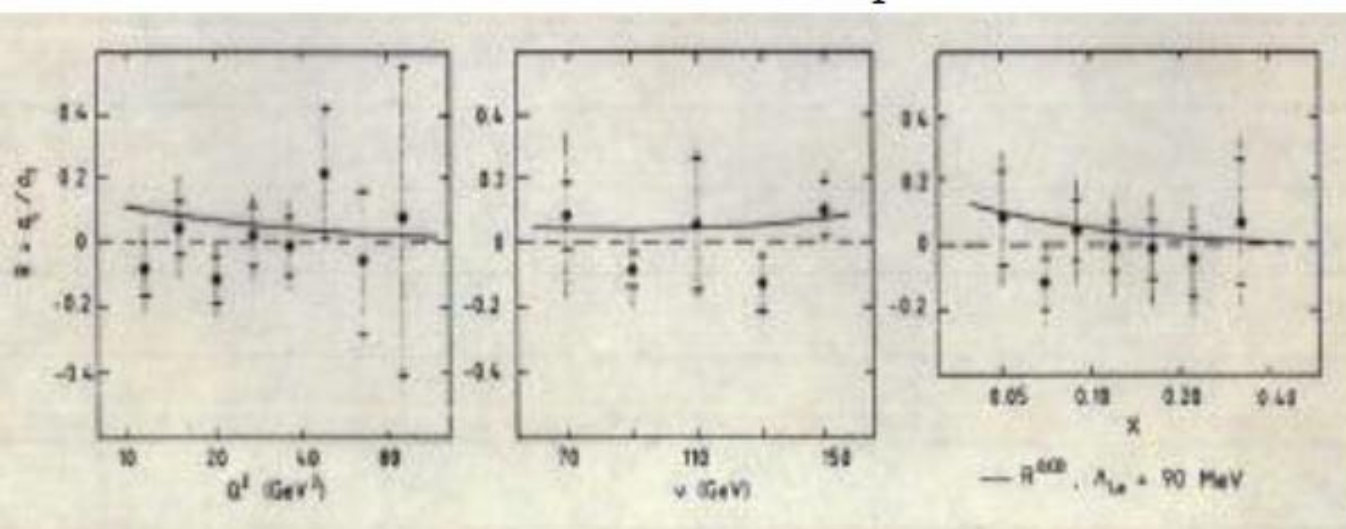
BUT FIRST, the early SLAC results

Early observations that F_2 is independent of $Q^2 \to$ point-like scattering centres "partons" in the nucleon (otherwise $F_2 \sim 1/Q^2$)



Let's see the early data on Bjorken scaling, and on $F_L = 0$

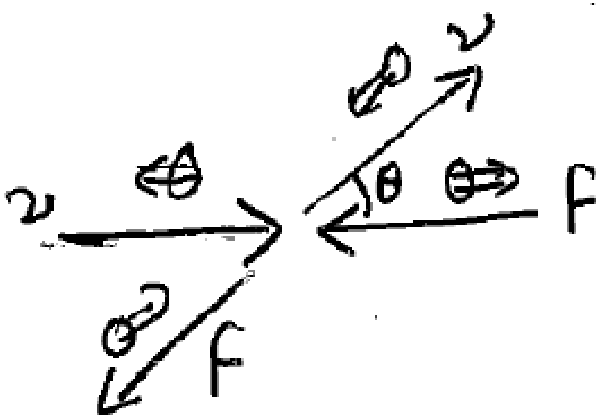
Early observations that $R = \dfrac{\sigma_L}{\sigma_T} \approx 0 \;\; \to R = \dfrac{F_L}{F_2 - F_L} = \dfrac{F_L}{2\,x\,F_1} \approx 0$



Establish spin $^1/_2$ nature of partons

Why did I include antiquarks?
Neutrinos can also be used as the Deep Inelastic probe AND….

Consider $\nu, \overline{\nu}$ which are left/right handed at low energy $M_W >> Q$



νf scattering - both left handed, NO net spin along beam direction → ISOTROPIC

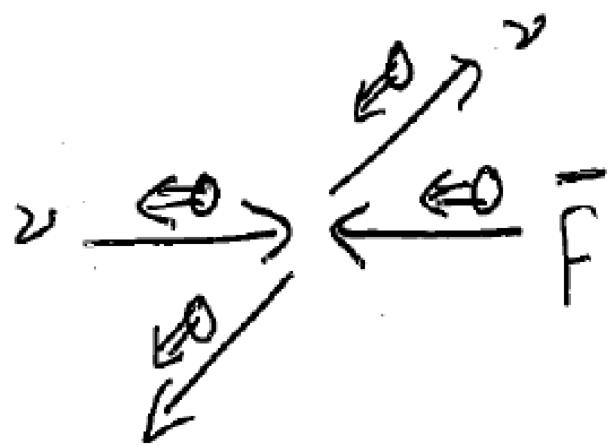
$$\frac{d\sigma}{dy} = \frac{G_F^2 s}{\pi} \sim \text{ independent of } y = \left(\frac{1-\cos\theta}{2}\right)$$

Similarly, $\overline{\nu}\,\overline{f}$ both right handed

$\nu\,\overline{f}$ left-right → net spin along the beam direction
→ NOT ISOTROPIC

$$\frac{d\sigma}{dy} = \frac{G_F^2 s}{\pi}(1-y)^2$$

Similarly, $\overline{\nu}\, f$ (right-left)



Consider $\nu, \bar{\nu}$ scattering via the $W^{\pm}(\mathrm{cc}^{\nu})$

AND at low enough energy (Q^2) that $\dfrac{M_W^4}{\left(Q^2 + M_W^2\right)^2} = 1$

The early neutrino scattering experiments were not at very high energy

The equivalent result for νq scattering is $\dfrac{d\sigma}{dy} = \dfrac{G_F^2}{\pi} x s$ ISOTROPIC

and for $\bar{\nu} q$ scattering $\dfrac{d\sigma}{dy} = \dfrac{G_F^2}{\pi} x s(1-y)^2$ NON-ISOTROPIC

because $\nu, \bar{\nu}$ are LEFT, RIGHT handed.

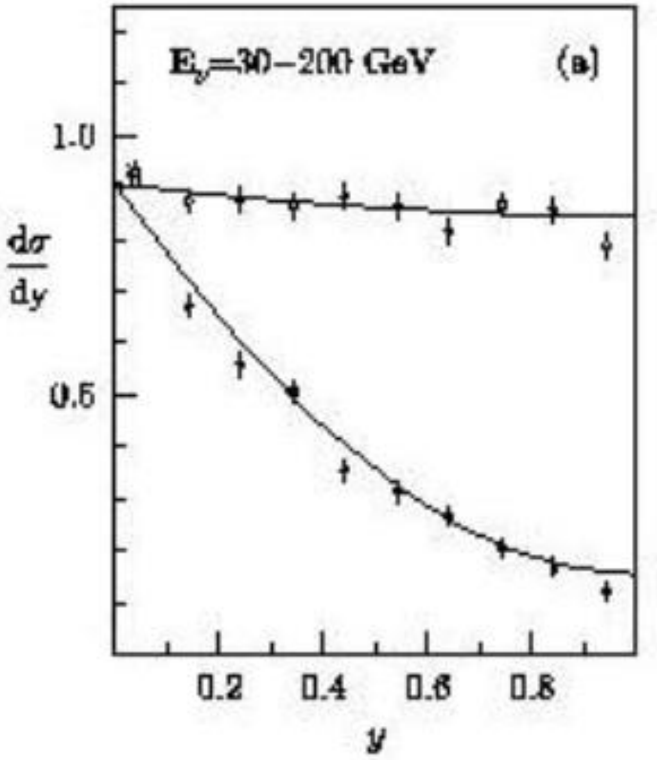
For $\nu \bar{q}$, $\bar{\nu} \bar{q}$ these results are oppositely handed.

So for a HADRON,

$$\frac{d^2\sigma^{(\nu)}}{dx\,dQ^2} = \frac{G_F^2}{\pi} x s\left[\sum_i q_i(x) + (1-y)^2 \sum_i \bar{q}_i(x)\right]$$

$$\frac{d^2\sigma^{(\bar{\nu})}}{dx\,dQ^2} = \frac{G_F^2}{\pi} x s\left[\sum_i \bar{q}_i(x) + (1-y)^2 \sum_i q_i(x)\right]$$

See next page



So for neutrino proton scattering with no antiquarks the cross section vs y should be flat--- and it isn't QUITE

And for antineutrino-proton scattering with no antiquarks the cross section vs y should be pure $(1-y)^2$– and there is definitely a flat offset

There is clearly a need for the qbar term

This leads to the idea of 3-valence quarks PLUS a $q\,\overline{q}$ Sea

$$q = q_{\text{valence}} + q_{\text{sea}}$$
$$\overline{q} = \overline{q}_{\text{sea}}$$

Where does the Sea come from?

Anticipating what is to come..

$$q \rightarrow q\,g \qquad\qquad g \rightarrow q\,\overline{q}$$

For $\nu \bar{q}$, $\bar{\nu} q$ these results are oppositely handed.

So for a HADRON,

$$\frac{d^2\sigma^{(\nu)}}{dx\,dQ^2} = \frac{G_F^2}{\pi}\, x\, s\left[\sum_i q_i(x) + (1-y)^2 \sum_i \bar{q}_i(x)\right]$$

$$\frac{d^2\sigma^{(\bar{\nu})}}{dx\,dQ^2} = \frac{G_F^2}{\pi}\, x\, s\left[\sum_i \bar{q}_i(x) + (1-y)^2 \sum_i q_i(x)\right]$$

See previous page

Now compare these to the general formula

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{G_F^2\, s}{4\pi}\left[Y_+ F_2\left(x, Q^2\right) - y^2 F_L\left(x, Q^2\right) \pm Y_-\, x F_3\left(x, Q^2\right)\right]$$

$$Y_+ = \left[1 - (1-y)^2\right] \qquad Y_- = \left[1 + (1-y)^2\right]$$

$$F_L\left(x, Q^2\right) = 0$$

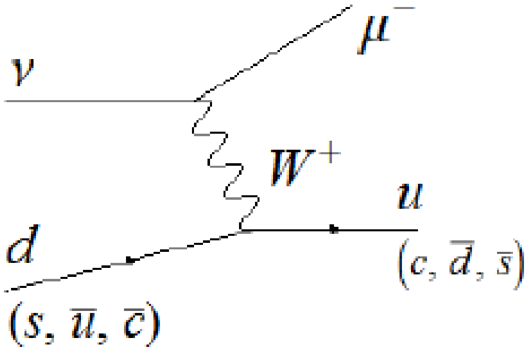
Bjorken scaling as before

$$F_2\left(x, Q^2\right) = 2\sum_i x\left[q_i(x) + \bar{q}_i(x)\right] \qquad x F_3\left(x, Q^2\right) = 2\sum_i x\left[q_i(x) - \bar{q}_i(x)\right]$$

- Clearly a relationship between F_2's for ν, $\bar{\nu}$ and charged lepton scattering
- More information from xF_3

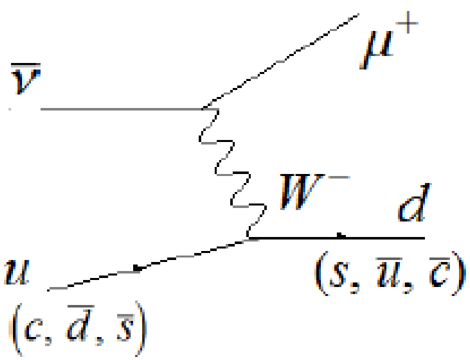
FURTHERMORE….

- $\nu, \overline{\nu}$ scattering is FLAVOUR SENSITIVE



W^+ MUST hit:
q charge $-{}^1/_3$e or
$\overline{q}$ charge $-{}^2/_3$e

$$\frac{d^2\sigma^{(\nu)}}{dx\,dy} = \frac{G_F^2\, s}{\pi}\left[\,(d(x)+s(x)) + (1-y)^2\,(\overline{u}(x)+\overline{c}(x))\right]$$



W^- MUST hit:
q charge ${}^2/_3$e or
$\overline{q}$ charge ${}^1/_3$e

$$\frac{d^2\sigma^{(\overline{\nu})}}{dx\,dy} = \frac{G_F^2\, s}{\pi}\left[\,(u(x)+c(x))\,(1-y)^2 + \left(\overline{d}(x)+\overline{s}(x)\right)\right]$$

The flavours were written down assuming a proton target,

$$F_2^{(\nu p)} = 2\,x(d + s + \overline{u} + \overline{c})$$

$$x\,F_3^{(\nu p)} = 2\,x(d + s - \overline{u} - \overline{c})$$

So write down F2 and xF3 for neutrino scattering on a proton target

$$F_2^{(\nu p)} = 2x(d + s + \bar{u} + \bar{c})$$

$$xF_3^{(\nu p)} = 2x(d + s - \bar{u} - \bar{c})$$

For a neutron target, SWAP d $\rightarrow$ u and $\bar{u} \rightarrow \bar{d}$ STRONG ISOSPIN

$$F_2^{\nu n} = 2x\left(u + s + \bar{d} + \bar{c}\right)$$

$$xF_3^{\nu n} = 2x\left(u + s - \bar{d} - \bar{c}\right)$$

Finally MOST $\nu, \bar{\nu}$ data are taken on ISOSCALAR targets $\frac{n+p}{2}$

$$F_2^{(\nu N)} = x\left(u + d + \bar{u} + \bar{d} + 2s + 2\bar{c}\right)$$

$$xF_3^{(\nu N)} = x\left(u + d - \bar{u} - \bar{d} + 2s - 2\bar{c}\right)$$

Similarly for $\bar{\nu}$

$$F_2^{(\bar{\nu} N)} = x\left(u + d + \bar{u} + \bar{d} + 2s + 2\bar{c}\right)$$

$$xF_3^{(\bar{\nu} N)} = x\left(u + d - \bar{u} - \bar{d} - 2\bar{s} + 2\bar{c}\right)$$

Since the contribution of s, c is small and $s = \bar{s}$, $c = \bar{c}$

Well almost!!

$$F_2^{(\nu N)} = F_2^{(\bar{\nu} N)} \qquad xF_3^{(\nu N)} \approx xF_3^{(\bar{\nu} N)}$$

Now go further,

$$u = u_{\text{valence}} + u_{\text{sea}} = u_v + u_{\text{sea}}$$

$$\bar{u} = \bar{u}_{\text{sea}} \quad \text{and} \quad \bar{q}_{\text{sea}} = q_{\text{sea}}$$

Similarly $d = d_v + d_{\text{sea}}$

So,

$$x F_3^{(\nu,\bar{\nu}\, N)} = x\left(u - \bar{u} + d - \bar{d}\right) = x(u_v + d_v) = x(\text{valence})$$

$$F_2^{(\nu,\bar{\nu}\, N)} = x\left(u_v + d_v + u_{\text{sea}} + d_{\text{sea}} + \bar{u} + \bar{d} + s + \bar{s} + c + \bar{c}\right)$$

$$= x(\text{valence} + \text{sea})$$

Measuring F_2 and xF_3 in ν, $\bar{\nu}$ scattering separates valence and sea. AND

Compare $F_2^{(\nu,\bar{\nu})}$ to $F_2^{(e^{\pm})}$

$$F_2^{(\text{ep})} = x\left[\frac{4}{9}(u + \bar{u}) + \frac{1}{9}\left(d + \bar{d}\right) + \frac{1}{9}(s + \bar{s}) + \frac{4}{9}(c + \bar{c})\right.$$

for isoscalar targets,

$$F_2^{(\text{eN})} = \frac{5}{18}\, x\left[(u + \bar{u}) + \left(d + \bar{d}\right) + \frac{2}{5}(s + \bar{s}) + \frac{8}{5}(c + \bar{c})\right.$$

$$F_2^{(\text{eN})} = \frac{5}{18} F_2^{(\nu,\bar{\nu}\, N)}$$ again assuming *s*, *c* small $\longrightarrow$ OBSERVATIONS

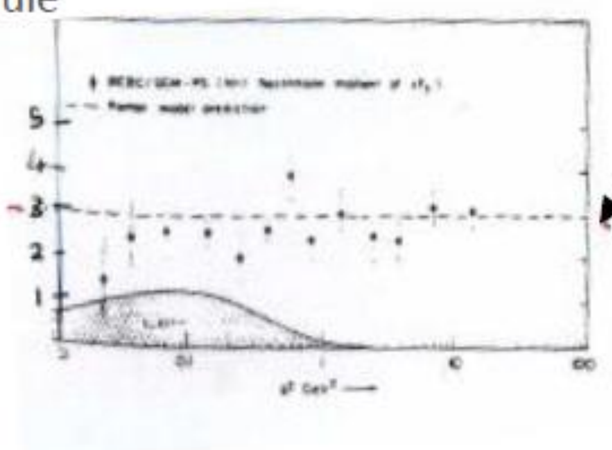
$$x F_3^{(\bar{\nu} N)} = x V(x) \rightarrow$$ momentum distribution of valence quarks

$$= F_3^{(\bar{\nu} N)} = V(x) \rightarrow$$ number distribution of valence quarks

$$\Rightarrow \int \frac{x F_3^{(\bar{\nu} N)}}{x}\, dx = \int_0^1 V(x)\, dx$$ Total number of valence quarks in nucleon

GLS sum-rule

$$\int F_3 \, dx$$

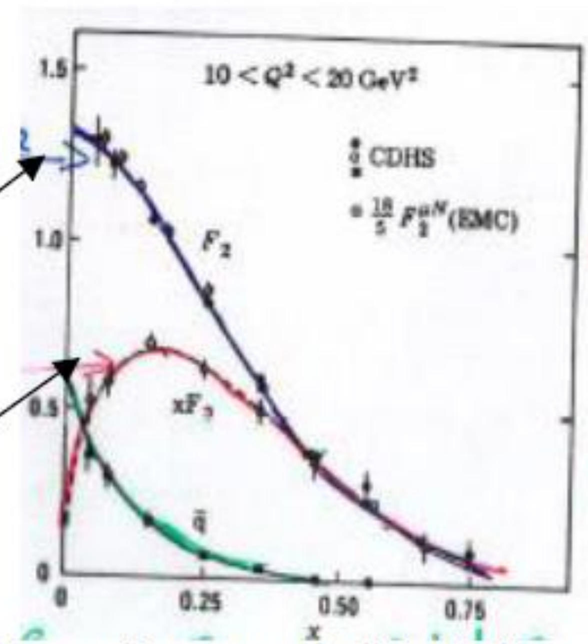


=3 ! Parton model

$\rightarrow$ Quark-parton model

shape of valence
+ sea quark distributions

shape of valence quark
momentum distribution



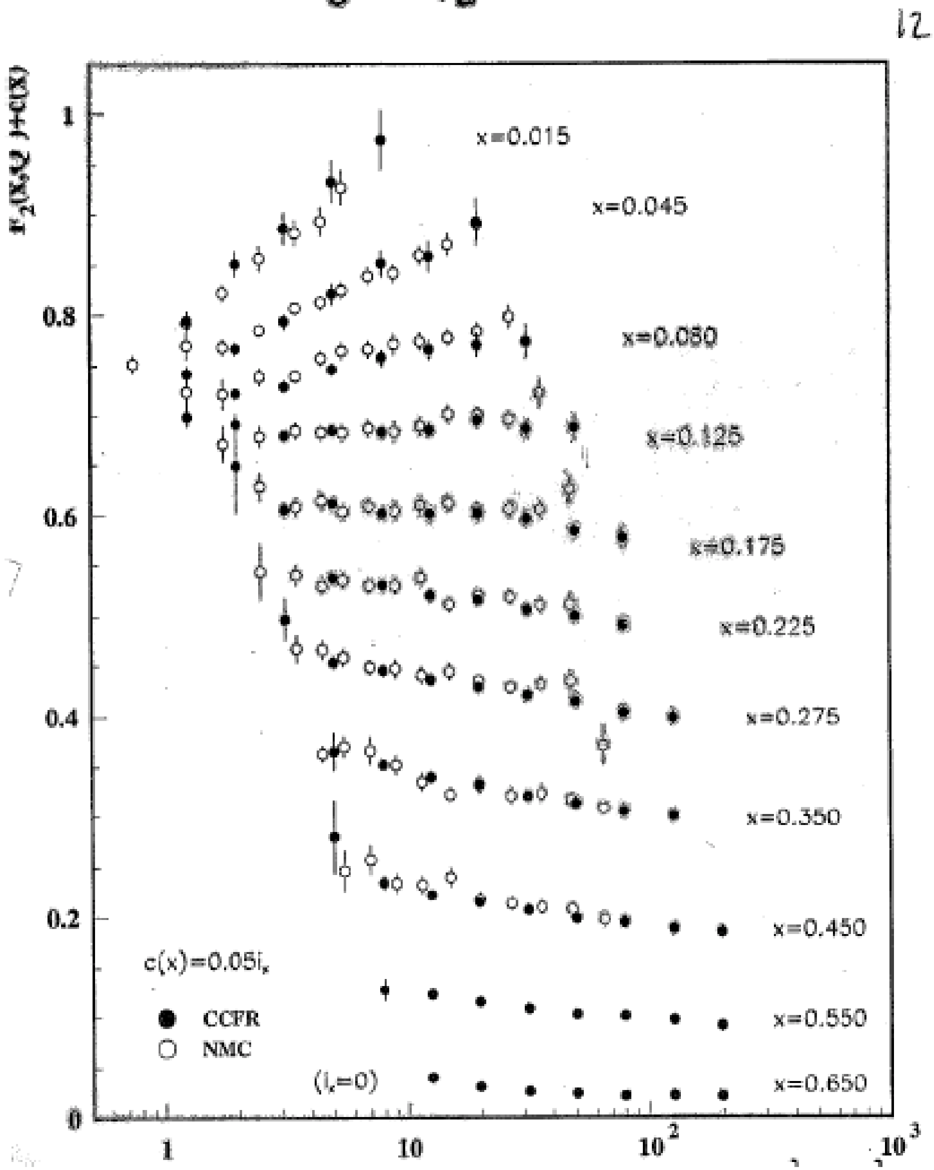
$x^a(1-x)^b$

shape of sea quark distribution $\sim (1-x)^b$

at high x $\rightarrow$ all valence

• is $F_2^{\nu N} \times \frac{5}{18}$

o is F_2^{eD}



So hopefully you now believe that the Quark Parton Model has some basis in fact

BUT but you might notice that The F2 structure functions being shown here are not flat with Q2..ie Bjorken scaling does not hold

And that's because the world does not adhere to the simple quark-parton model completely –there are slow logarithmic scaling violations because of Quantum Chromodynamics…..

BUT…

The observation that $\int_0^1 dx\, F_3^{\nu N} = \int_0^1 dx\,(u_v + d_v) = 3$

in early neutrino data was crucial for the parton model.

But there was another more worrying sum rule,

$$\int_0^1 dx\, F_2^{\nu N} = \int_0^1 dx \cdot x\left[u + \bar{u} + d + \bar{d} + s + \bar{s} + c + \bar{c}\right]$$

This is the total momentum in the proton SO

so QPM predicts,

$$\int_0^1 dx\, F_2^{\nu N} = 1 \quad \text{but} \quad \int_0^1 dx\, F_2^{\nu N} \sim 0.5 \quad \text{was observed.}$$

- Where has the momentum gone? GLUONS
- QPM treats partons as non-interacting.
- Cannot be true, they are bound in hadrons.
- QCD says that quarks interact with gluons

with interaction strength ~ α_S

- $\alpha_S \downarrow$ as $Q^2 \uparrow$ "Asymptotically free"

  $\Rightarrow$ Modify the QPM

So what are gluons?

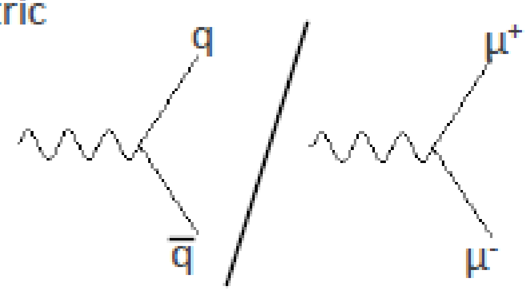
The force carrier of QCD

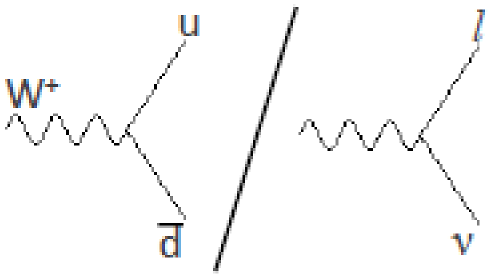
They couple to colour charge rather than electric charge
They are also coloured themselves so they are self-interacting

EVIDENCE?

1. Pauli statistics $\Omega^-(s^\uparrow s^\uparrow s^\uparrow)$ Symmetric we can get around this if colour wave-fn is anti symmetric

2. $$\frac{\sigma(e^+ e^- \to \text{hadrons})}{\sigma(e^+ e^- \to \mu^+ \mu^-)} = 3 \sum e_q^2$$



3. 

In W decay quark channel branching fractions are a factor of 3 times larger than lepton channels

W data:

$$\left.\begin{array}{l} e\nu = \mu\nu = \tau\nu \simeq 11\,\%\ (10.8 \pm 0.1\,\%) \\ c\bar{s} \simeq u\bar{d} = 32.8 \pm 0.3\,\% \end{array}\right\} \approx 3$$

Similarly for the Z decays: ee,μμ,ττ +ν's =6 lepton channels, (uu,dd,ss,cc,bb)*3 colours=15 hadron channels, so ~5% each for leptons, ~15%each for quarks: but Z couplings differ between up-type and down-type flavours, down couple more strongly

$$e^+ e^- = \mu^+ \mu^- = \tau^+ \tau^- = 3.3\,\%$$

$$\frac{u\bar{u} + c\bar{c}}{2} = 11.6 \pm 0.6\,\%$$

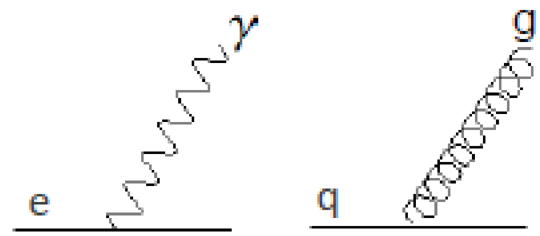
$$\frac{d\bar{d} + s\bar{s} + b\bar{b}}{3} = 15.6 \pm 0.4\,\%$$

4. Profound (avoid field theory anomalies)

$$\sum Q_{\text{lepton}} + Q_{\text{quark}} = 0$$

-1 $\quad$ $(^1/_3) \times 3$ $\longleftrightarrow$ q and l unification

5. Gluons exist



spacetime	γ_μ	γ_μ
strength	$\sqrt{\alpha}$	$\sqrt{\alpha_s}$
number	1	8

The same $d\sigma / \mathrm{dq}$

e.g. $e^+ e^- \rightarrow q\, q / \mu\, \mu$

$1 + \cos^2\theta$

$$\frac{e\, q \rightarrow e\, q}{e\, \mu \rightarrow e\, \mu}$$ Flat

$$\frac{e^+ e^- \rightarrow q\, \bar{q}\, g}{e^+ e^- \rightarrow \mu^+ \mu^- \gamma}$$ Similar

$\alpha_s \simeq \dfrac{1}{10}$ at LEP-LHC

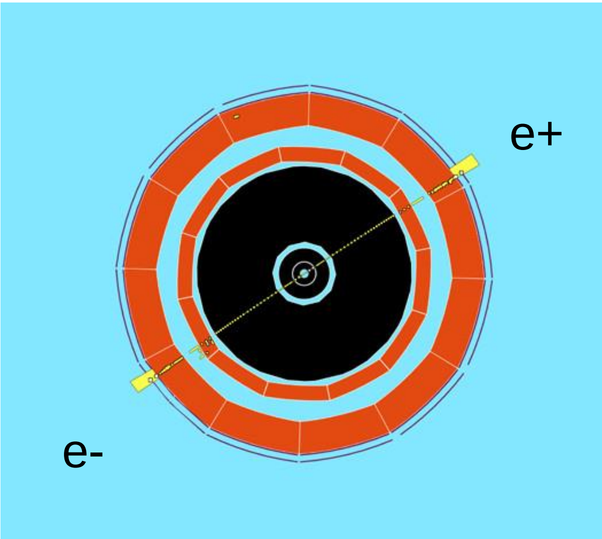
$\alpha_s \simeq 1$ at LowEnergy

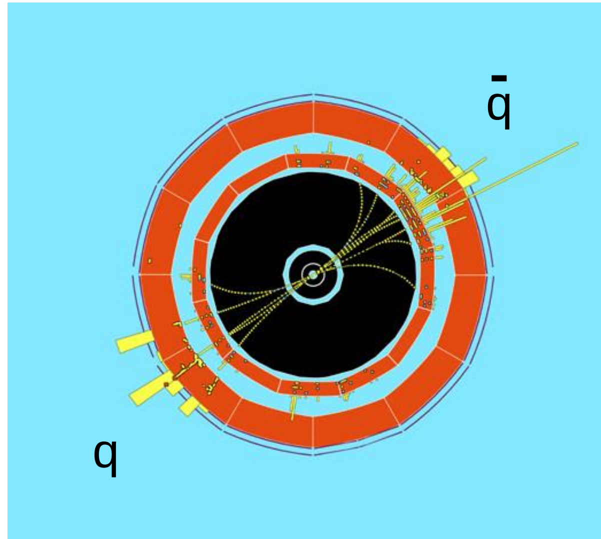
$\alpha \simeq \dfrac{1}{137}$ (but not always!! – see later..)

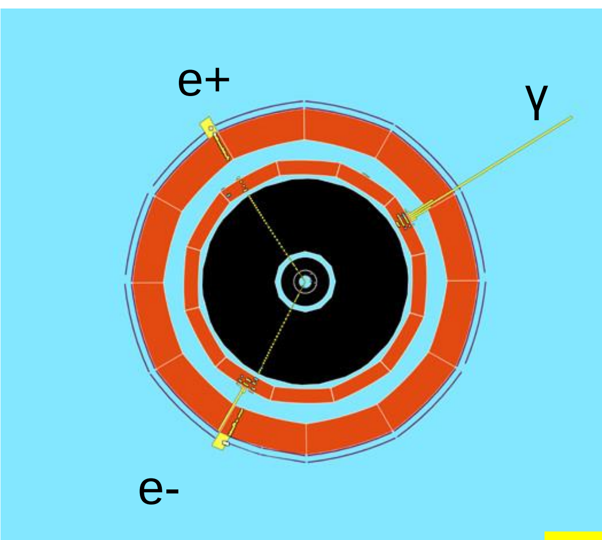
8 coloured gluons from SU(3) combinations of red,green,blue (quark colours) with anti-red, anti-green and anti-blue (antiquark colours)
3 * 3 = 8 +1

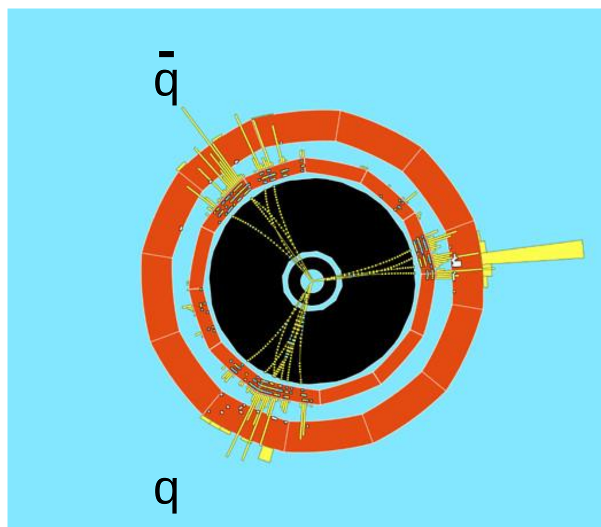
QED (electrons and photons)

QCD (quarks and gluons)



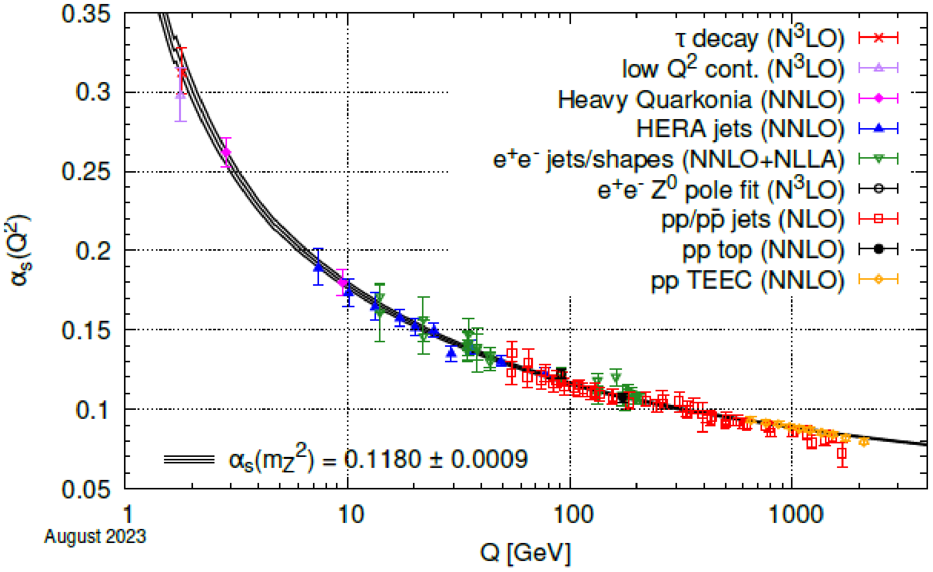






And here are some pictures of quarks, gluons, electrons and photons from LEP @ CERN 1989-2000 e+ e- annihilation. Illustrating that partons behave similarly to the point-like electron and photon

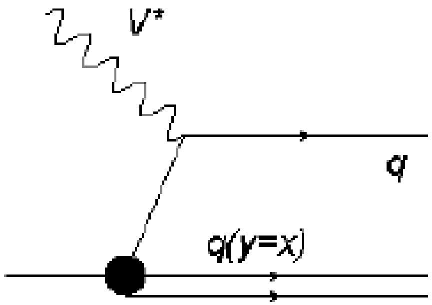
You don’t see the quark or gluon emerge instead you see a **jet**: a “blast” of particles, all going in roughly the same direction.



The running of the strong coupling with scale

Let us apply some of this to the parton model

Formally the QPM calculate the cross section in terms of a convolution of the point-like V*q scattering and the parton distribution function.

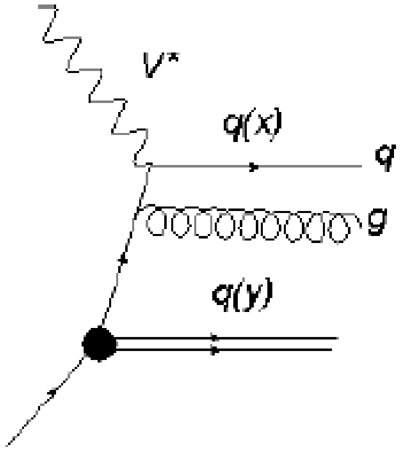


$$\frac{F_2(x)}{x} = \int dy\, dz\, \delta(x - zy)\, \sigma^{\text{point}}(z)\, q(y)$$

$$\sigma^{\text{point}} = q_i^2\, \delta(1-z) \qquad z = x/y$$

$$\Rightarrow \frac{F_2(x)}{x} = q_i^2\, q(x) \Rightarrow y = x$$

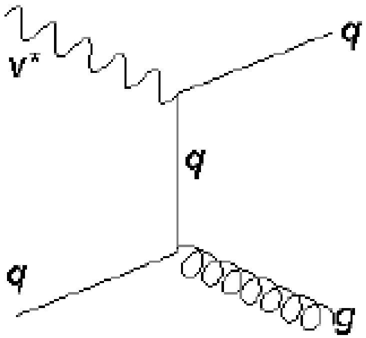
QCD adds to this extra diagrams. E.g. a quark of momentum fraction y emits a gluon to become a quark of momentum fraction x, (y > x) before it interacts with V*.



$$\frac{F_2(x)}{x} = \int dy\, dz\, \delta(x - zy)\, q(y)\left[\sigma^{\text{point}}(z) + \sigma^{V*q\to qg}\right]$$

A new term: $\sigma^{V*q\to qg}$
Is added to the point-like cross section.

$\sigma(V^* q - q g)$ is calculable from QCD Feynman rules.



$$V^* q q \text{ vertex} \Rightarrow e_i^2$$

$$g q q \text{ vertex} \Rightarrow \alpha_s$$

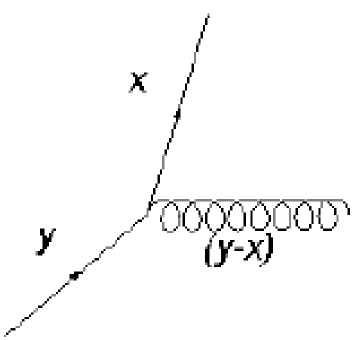
Quark "exchanged" propagator $\sim \dfrac{1}{p_t^{\,2}}$

Quark acquires $\boldsymbol{p_t}$ w.r.t proton because of the gluon emission

Until now we had been considering it collinear

$$\int_{\min}^{\max} \frac{d\, p_t^{\,2}}{p_t^{\,2}} \Rightarrow \ln \frac{Q^2}{Q_0^2}$$

$$\sigma(V^* q \to q g) = e_i^2 \frac{\alpha_s}{2\pi} P_{qq}(z) \ln \frac{Q^2}{Q_0^2}$$



$$P_{qq}(z) = C_F \frac{1 + z^2}{1 - z}$$

Is the SPLITTING FUNCTION

Probability that quark of momentum ***yP*** splits to quark momentum ***xP*** and gluon of momentum ***(y-x)P***

z = x/y

$$\frac{F_2(x)}{x} = \int_x^1 \frac{dy}{y} q(y)\left[e_i^2 \delta\left(1 - \frac{x}{y}\right) + \sigma(V^* q - q g)\left(\frac{x}{y}, Q^2\right)\right]$$

$$= e_i^2 \int_x^1 \frac{dy}{y}\left[q(y)\,\delta\left(1 - \frac{x}{y}\right)\right] + e_i^2\,\Delta\mathrm{q}\left(x, Q^2\right)$$

$$= e_i^2\left[q(x) + \Delta\mathrm{q}\left(x, Q^2\right)\right] = e_i^2\, q\left(x, Q^2\right)$$

where $\Delta\mathrm{q}\left(x, Q^2\right) = \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{Q_0^2} \int_x^1 \frac{dy}{y} q(y)\, P_{\mathrm{qq}}\left(\frac{x}{y}\right)$

The Q^2 dependence of $\sigma(V^* q - q g)$

has been transferred into the parton distribution function $q(x) \to q\left(x, Q^2\right)$

$$F_2(x) = \sum_i e_i^2\, x\left[q\left(x, Q^2\right) + \overline{q}\left(x, Q^2\right)\right]$$

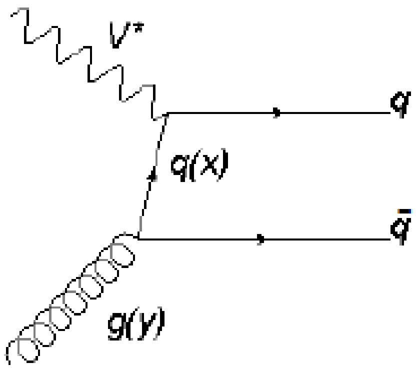
Bjorken scaling violation is predicted by QCD, but only ~lnQ^2

$$\frac{d\,q\left(x,\,Q^2\right)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 q\left(y,\,Q^2\right) P_{qq}\left(\frac{x}{y}\right)\frac{d\,y}{y}$$

Quark distribution evolve with Q^2

Shape of $q\left(x,\,Q^2\right)$ is **NOT** predicted, but its evolution **IS** (and is measurable).

Extend the formalism,



Maybe a gluon of momentum ***yP*** splits into a quark of momentum ***xP*** and an antiquark of momentum ***(y-x)P***

Splitting function $P_{qg}(z)$

$$\frac{d\,q\left(x,\,Q^2\right)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\,y}{y}\left[P_{qq}\left(\frac{x}{y}\right) q\left(y,\,Q^2\right) + P_{qg}\left(\frac{x}{y}\right) g\left(y,\,Q^2\right)\right)$$

Quark evolution equation DGLAP

$$\frac{dg\left(x,\,Q^2\right)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\,y}{y}\left[P_{gq}\left(\frac{x}{y}\right) q\left(y,\,Q^2\right) + P_{gg}\left(\frac{x}{y}\right) g\left(y,\,Q^2\right)\right)$$

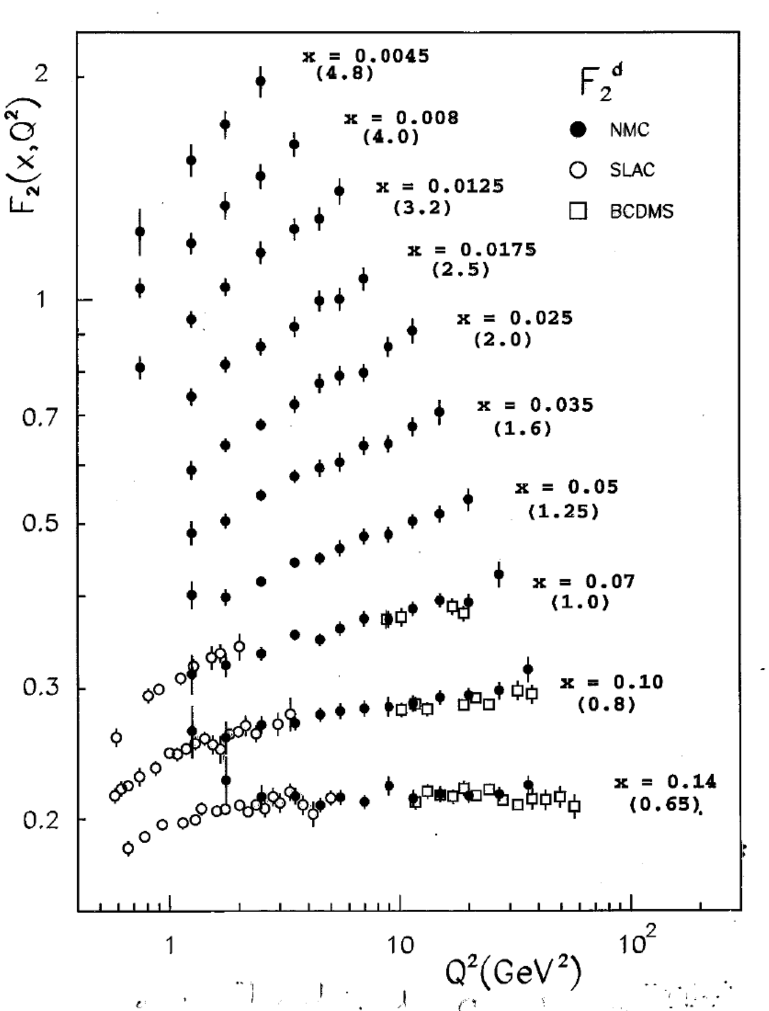
gluon evolution equation

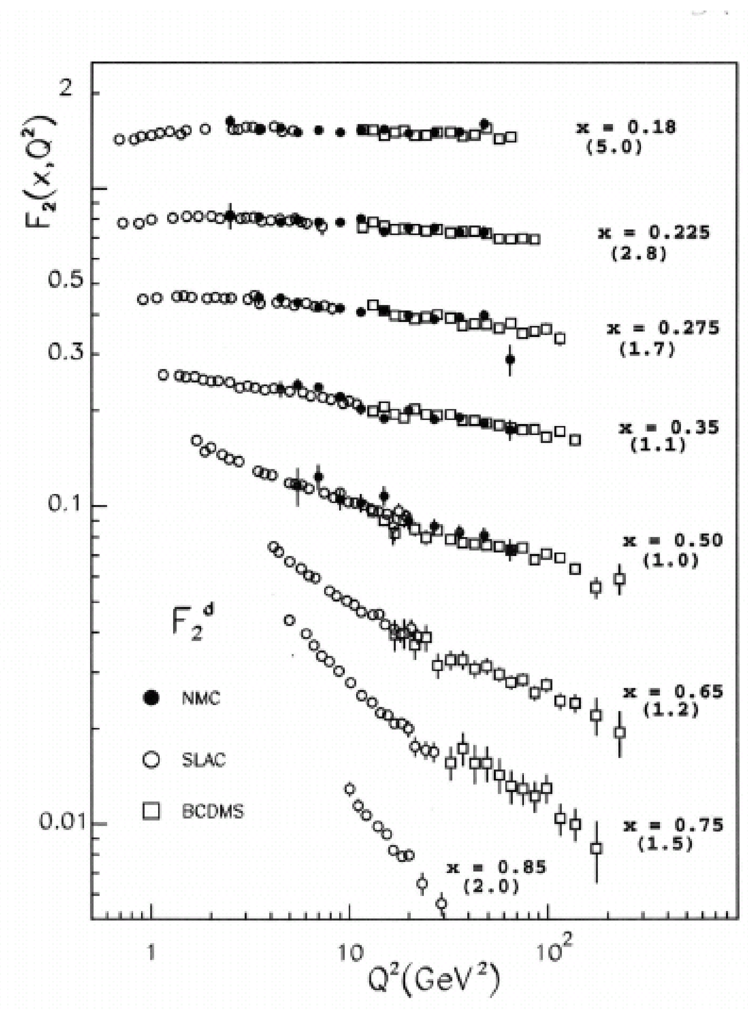
COUPLED differential equations

So $\mathbf{F_2}(x,Q^2) = \Sigma_i\, e_i^2(xq(x,Q^2) + x\bar{q}(x,Q^2))$

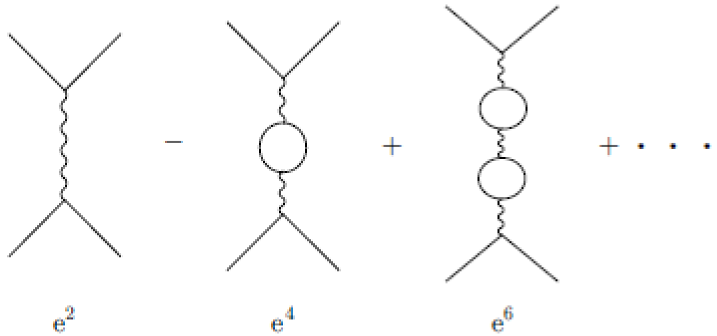
in LO QCD

And the theory predicts the rate at which the parton distributions (both quarks and gluons) evolve with Q^2- (the energy scale of the probe) -BUT it does not predict their starting shape





Contributions to the perturbative expansion of scattering amplitudes beyond the leading order are often divergent, e.g for QED



The loops are divergent beacuse of unrestricted integration over momentum in these loops
We have to renormalize the theory. This is done by making constants of the theory like the coupling α become dependent on the scale of the process. It is sucessful if this takes care of ALL the infinities to all orders.

For one loop the fermion propagator $\frac{-ig_{\mu\nu}}{q^2}$ becomes

$$\frac{-ig_{\mu\nu}}{q^2}\left[1-\Pi(q^2)\right].$$ where $$\Pi(Q^2) \approx \frac{\alpha_0}{3\pi}\ln\left(\frac{\Lambda^2}{Q^2}\right),$$

Λ is a high momentum cut-off and α_0 is the bare e.m. charge

For many loops the effect of summing the 'leading logs' can be accounted for by redefining the coupling

$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha_0} + \frac{1}{3\pi}\ln\left(\frac{\Lambda^2}{Q^2}\right)$$

We can remove the dependence on Λ and α_0 by defining the coupling at some scale μ^2 and writing its value at all other scales Q^2 in terms of this   ('renomalisation')

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1-\frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}.$$

Thus for $Q^2>\mu^2$ the coupling increases
And indeed the 1/137 you are used to at low energies becomes 1/125 at the scale of M_Z^2

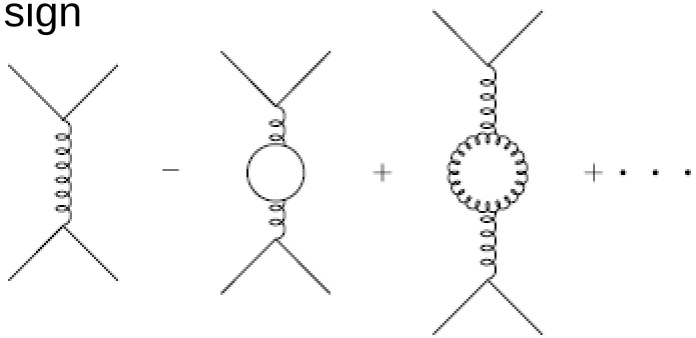
This can be understood qualitatively in terms of charge screening

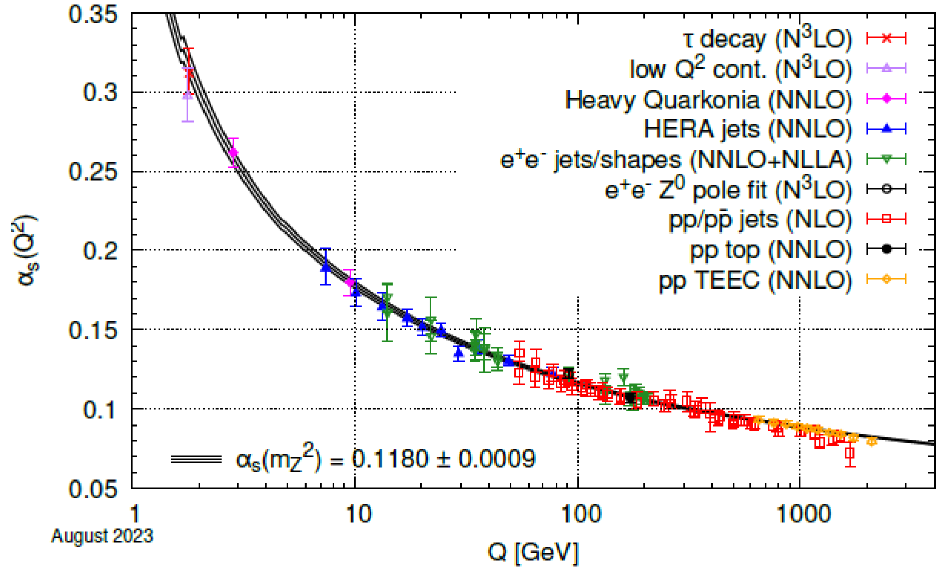
What of QCD?

What of QCD?

There is another type of loop diagram.

Both contributions diverge logarithmically but with opposite sign





The QCD coupling is renormalised as

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + \alpha(\mu^2) b_0 \ln\left(\frac{Q^2}{\mu^2}\right)}.$$

Where $b_0 = (33 - 2n_f)/12\pi.$ at leading order.

Note the quark loop gives 1/6π for each flavour of fermion (n_f) – this is like the 1/3π of QED bar a conventional factor of 2. The new feacture is the 33/12π of the gluon loop which swaps the sign

Gives us anti-screening

Or ASMYPTOTIC FREEDOM

The coupling decreases as the scale goes up

At high energies we may make perturbative caluclations

At low- energies we can't and we have CONFINEMENT

QCD is a locally gauge invariant field theory like QED

For QED this means $\psi(x) \to \psi'(x) = e^{iq\theta(x)}\psi(x)$

Where q is the charge and θ is a space-time dependent phase. The QED Lagrangian is

$$L_{QED} = \sum_f \bar{\psi}_f(i\gamma_\mu D^\mu - m_f)\psi_f - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

U(1)

with $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and $D^\mu = \partial^\mu + iqA^\mu$

The interaction between the fermions and the field is in the covariant derviative

For QCD $\psi(x) \to \psi'(x) = e^{igt\cdot\theta(x)}\psi(x)$

Where g is the strong charge and t.θ is the product of the colour group generators with a vector of space-time phase functions in colour space.

SU(3)

The group generators t satisfy $[t^a, t^b] = if^{abc}t^c$

Where f^{abc} are SU(3) structure constants.

$f^{123}=1$, $f^{147}=f^{246}=f^{257}=f^{345}=1/2$, $f^{156}=f^{367}=1/2$, $f^{458}=f^{678}=\sqrt{3}/2$

and t^a are hermitian matrices = λ^a /2 where λ are the Gell-Mann matrices given by

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

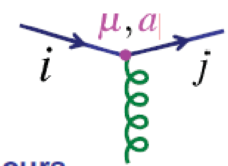
The quark part of theQCD Lagrangian is

$$L_{QCD} = \sum \bar{\psi}^i_f(i\gamma_\mu D^\mu - m_f)_{ij}\psi^j_f$$

With $D^\mu_{ij} = \delta_{ij}\partial^\mu + ig(t^a)_{ij}A^\mu_a$ where t^a_{ij} are hermitian matrices = λ^a_{ij} /2.

This describes the qqg interaction

$\boxed{-ig_s\frac{1}{2}\lambda^a_{ji}\gamma^\mu}$



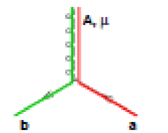
i, j = 1,2,3 are quark colours,

λ^a a = 1,2,..8 are the Gell-Mann SU(3) matrices

Where quarks now have colour indices i=1,2,3 (R,G,B) as well as flavour indices f.

The gluon fields A have a=1-8 flavour indices......

The colour exchange in q q g
diagram can be thought of like this



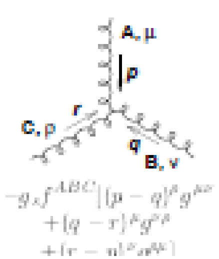
SO the gluon has colour red- antigreen: r-gbar
Obviously r-bbar, g-rbar, g-bbar, b-gbar, b-rbar are also possible
and the combinations
(r-rbar - g-gbar)/√2, (r-rbar + g-gbar – 2b-bar)/√6 and (r-rbar + b-bar + g-gbar)/√3

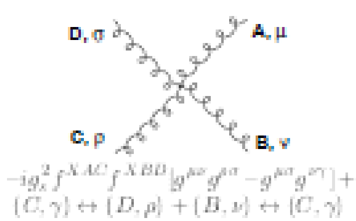
In the mathematics of SU(3) this is 3 * 3 = 8 + 1
And the last combination is the singlet– which is not coloured at all,
hence eight coloured gluons
(Think of SU(2) 2*2=3+1 -triplets and singlets- in atomic physics if this puzzles you)

Gluons: $r\bar{g}, g\bar{r}$	$r\bar{b}, b\bar{r}$	$g\bar{b}, b\bar{g}$	$\frac{1}{\sqrt{2}}(r\bar{r}-g\bar{g})$ $\frac{1}{\sqrt{6}}(r\bar{r}+g\bar{g}-2b\bar{b})$

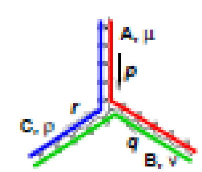
The second part of the QCD Lagrangian is purely gluonic $-\frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}$

Where $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f^{abc} A_b^\mu A_c^\nu,$
The difference from QED is the A A term which is what makes gluons interact with gluons
(NON-Abelian) with both a g-g-g vertex and a g-g-g-g vertex





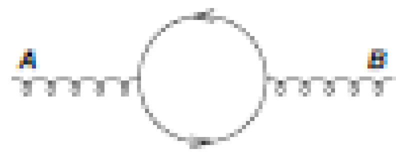
The colour flow is more
complex for these vertices ~
twice as strong as for q-q-g



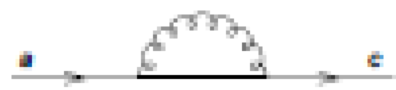
This extra term is also what makes QCD gauge invariant under local SU(3) transformation

Perturbative QCD involves an order by order expansion in a small coupling $\alpha_S = g^2/4\pi << 1$ and calculations are made using Feynman diagrams. The rules for the vertices have already been shown.
The main complication in comparison to QED is the need for colour factors.
After squaring an amplitude and summing over colours of incoming and outgoing particles the colour factors often appear in one or other of the following combinations:

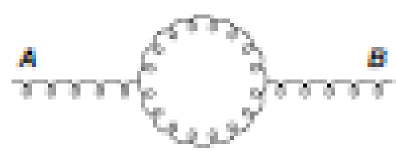
$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}, \quad T_R = \frac{1}{2} \qquad (8a)$$



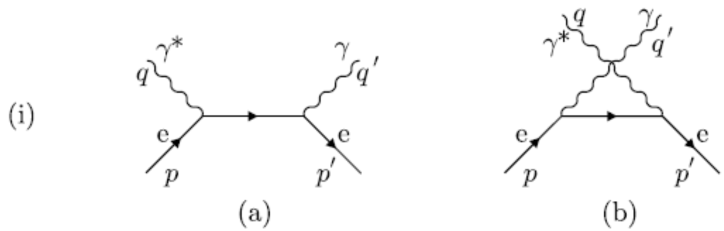
$$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}, \quad C_F = \frac{N_C^2 - 1}{2N_C} = \frac{4}{3} \qquad (8b)$$



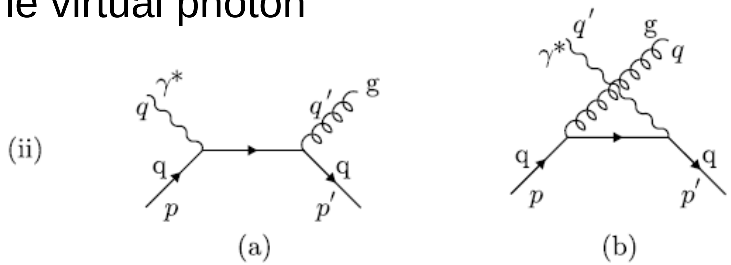
$$\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}, \quad C_A = N_C = 3 \qquad (8c)$$



See page 38/39 Devenish and Cooper-Sarkar for a simple colour factor calculation



Consider QED Compton scattering with one virtual photon



The QCD analogue is QCDC and the kinematic invariants are

$$
\begin{aligned}
s &= (q+p)^2 = (q'+p')^2 = 2q.p - Q^2 \\
t &= (q-q')^2 = (p'-p)^2 = -2p.p' \\
u &= (q-p')^2 = (q'-p)^2 = -2q'.p
\end{aligned}
$$

The amplitudes for the two QED diagrams are

$$
\begin{aligned}
M_a &= e^2 \varepsilon'^*_\nu \varepsilon_\mu \bar{u}(p')\gamma^\nu(\not{q}+\not{p})\gamma^\mu u(p)/s \\
M_b &= e^2 \varepsilon'^*_\nu \varepsilon_\mu \bar{u}(p')\gamma^\mu(\not{p}-\not{q}')\gamma^\nu u(p)/u.
\end{aligned}
$$

Adding squaring and taking care of spins

$$\frac{1}{4}\sum_{spins}|M_a+M_b|^2 = 2e^4\left[-\frac{u}{s}-\frac{s}{u}+\frac{2tQ^2}{su}\right].$$

Devenish and Cooper-Sarkar p40-43

To go to QCD e^4 must be replaced

$$e^4 \to e^2 e_i^2 g^2 \to (4\pi)^2 \alpha\alpha_s e_i^2.$$

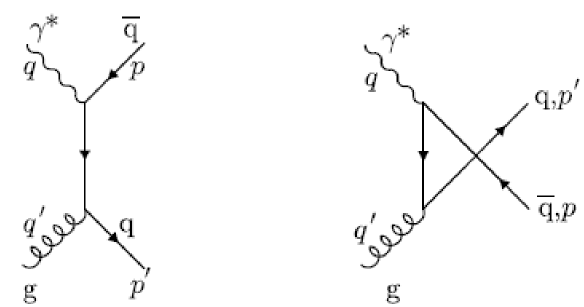
And the colour factor from the loop insertion C_F=4/3

$$\overline{|M_{QCDC}|^2} = \frac{1}{4}\sum_{spins}|M_a+M_b|^2 = \frac{8}{3}(4\pi)^2 e_i^2\alpha\alpha_s\left[-\frac{u}{s}-\frac{s}{u}+\frac{2tQ^2}{su}\right]$$

And using the Golden rule to go to the cross-section via the phase space factors

$$\left.\frac{d\sigma}{d\Omega}\right|_{QCDC} = \frac{2}{3}\frac{e_i^2\alpha\alpha_s}{s}\left[-\frac{u}{s}-\frac{s}{u}+\frac{2tQ^2}{su}\right].$$

A further important process is Boson-Gluon Fusion BGF



which, similarly, has the cross-section

$$\left.\frac{d\sigma}{d\Omega}\right|_{BGF} = \frac{1}{4}\frac{e_i^2\alpha\alpha_s}{s}\left[\frac{u}{t}+\frac{t}{u}-\frac{2sQ^2}{tu}\right].$$