Energy Loss in Small Collision Systems

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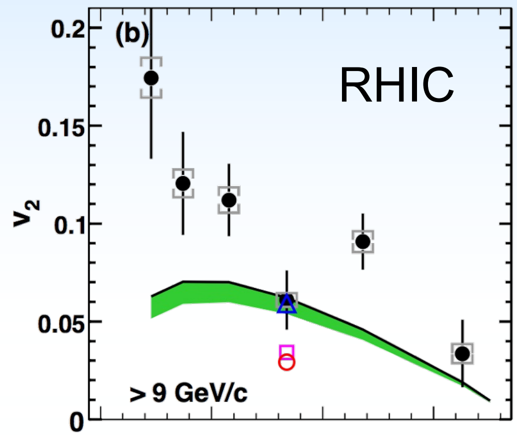
March 29, 2023

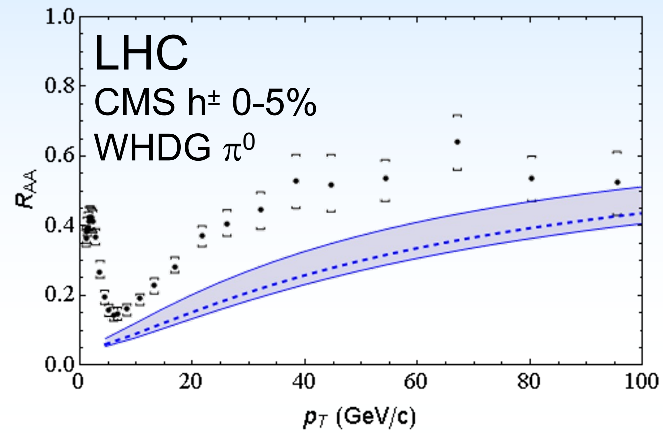
Kolbé and WAH, PRC100 (2019) [1511.09313]
WAH and Du Plessis, PRD105 (2022) [2203.01259]
Faraday and WAH, in prep

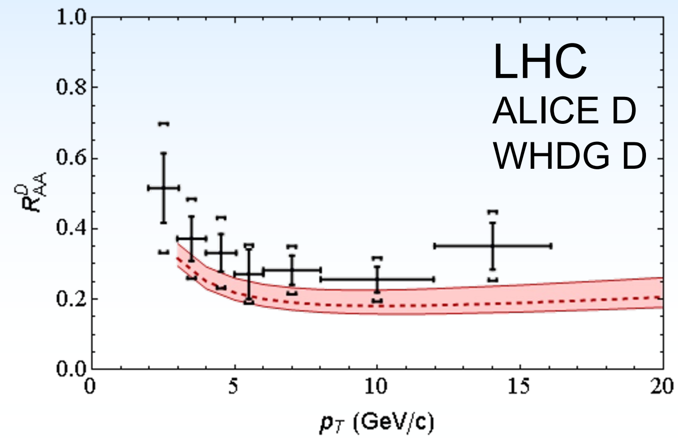
National Research Foundation

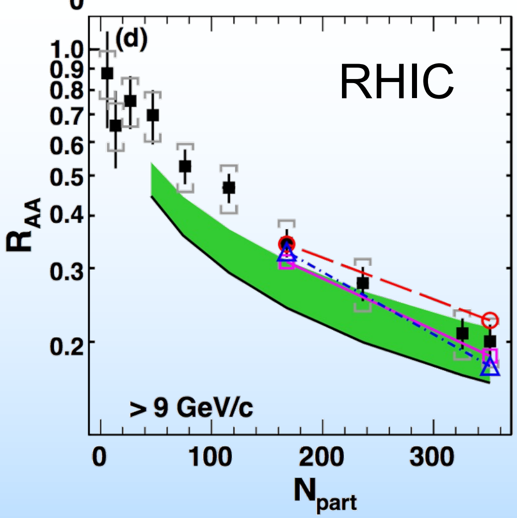
Qual. Success of “LO” Jet Tomo in AA

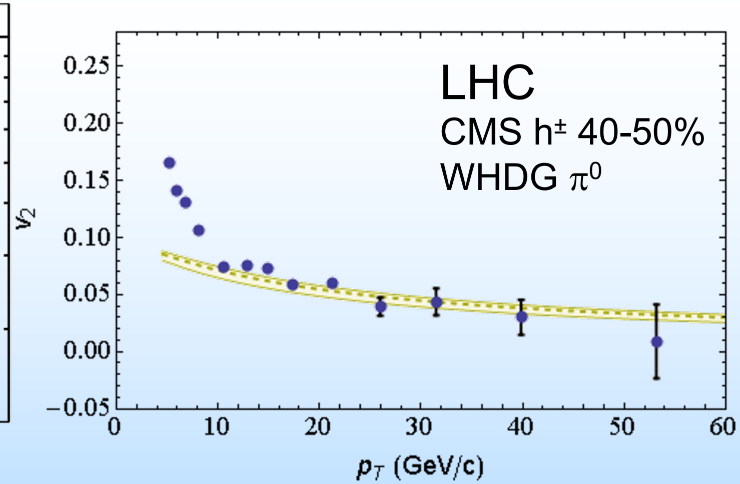
- Rad + El, realistic geom AA correct to factor ~2

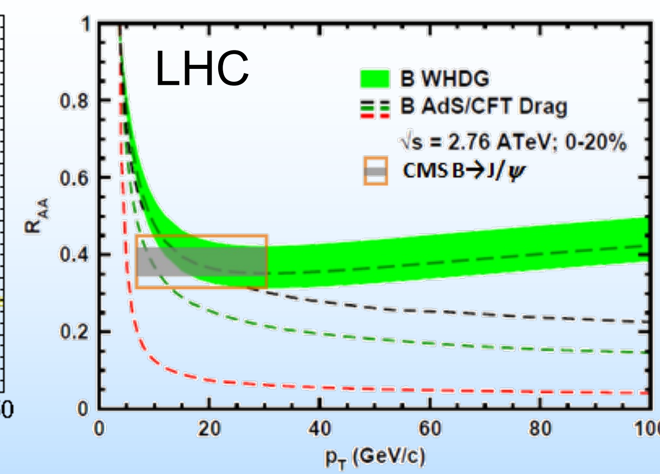












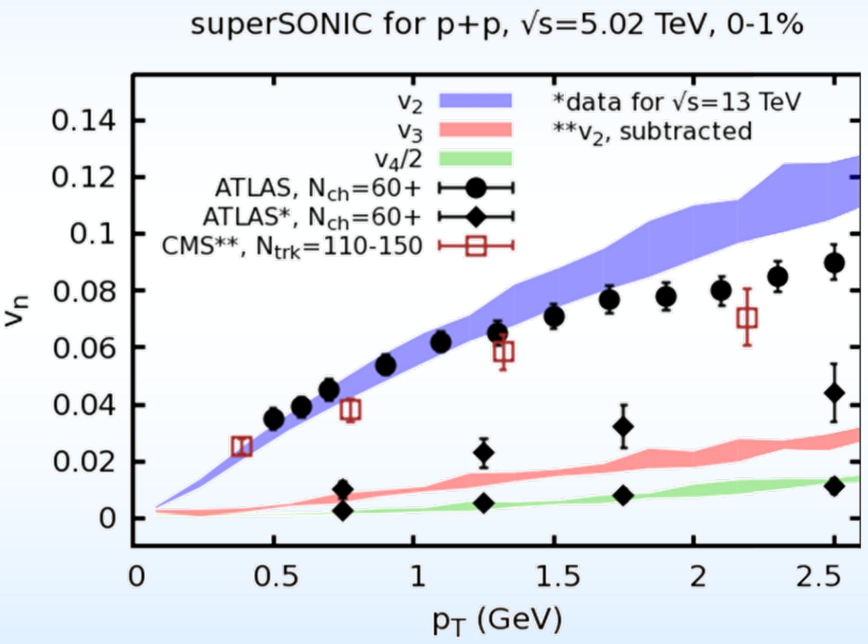
PHENIX PRL105 (2010)

CMS, Eur.Phys.J. C72 (2012)
CMS, PRL109 (2012)

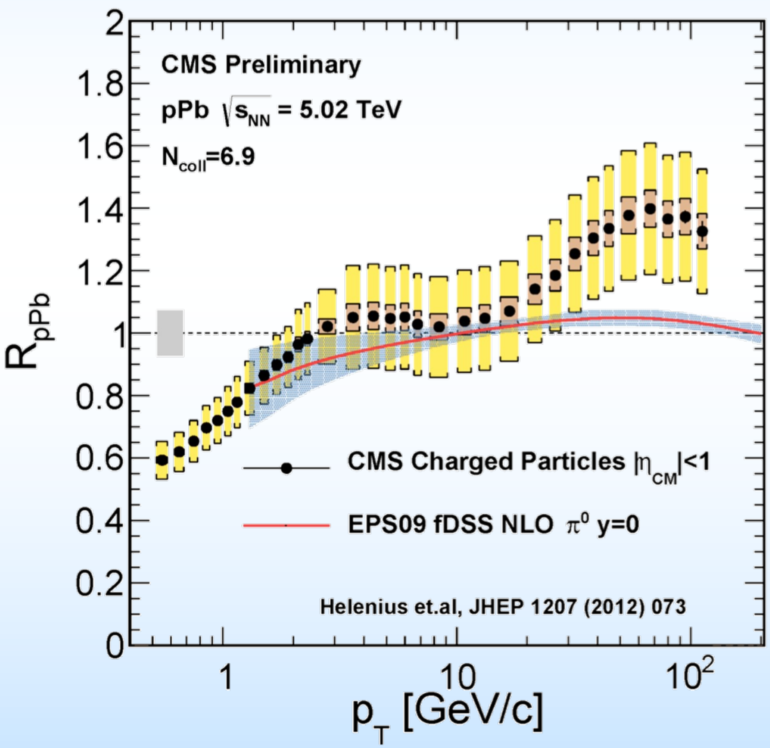
ALICE, JHEP1209 (2012) 112
CMS, JHEP 1205 (2012) 063

Correlations and Suppression in Small Systems

- Flow (?) in p+p
- E-loss (?) in pPb



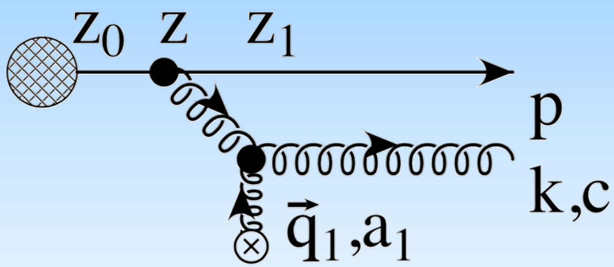
Weller and Romatschke, PLB B 774 (2017)



CMS, HP’13

pQCD E-Loss in pA

- Take seriously possibility of E-loss in small systems
- Wish to apply DGLV to small systems
  - However, DGLV assumes an ordering of scales: $1/\mu_{Debye} << \lambda_{mfp} << \tau_{form} << L_{pathlength}$
- Desire: derive the 1/L corrections to DGLV 1st order in opacity
  - Requires reevaluation of all 11 diagrams, no longer neglecting certain poles originating from the Yukawa scattering potential
  - Calculated by Isobel Kolbé



Neglected Poles

$$M_{1,0,1} \;=\; \int \frac{d^4 q_1}{(2\pi)^4}\, iJ(p+k-q_1)e^{i(p+k-q_1)x_0}\,\Lambda_1(p,k,q_1)V(q_1)e^{iq_1 x_1}\;\times$$
$$\times\; i\Delta_M(p+k-q_1)(-i)\Delta_{m_g}(k-q_1)$$

$$\mathrm{Res}(\bar{q}_1) \approx -v(-\omega_0-\tilde{\omega}_m,\mathbf{q}_1)\frac{e^{i(\omega_0+\tilde{\omega}_m)(z_1-z_0)}}{E^+k^+(\omega_1+\tilde{\omega}_m)}$$

$$\mathrm{Res}(\bar{q}_2) \approx v(\omega_1-\omega_0,\mathbf{q}_1)\frac{e^{i(\omega_0-\omega_1)(z_1-z_0)}}{E^+k^+(\omega_1+\tilde{\omega}_m)}$$

$$\mathrm{Res}(-i\mu_1) \approx \frac{4\pi\alpha_s\, e^{-\mu_1(z_1-z_0)}}{(-2i\mu_1)E^+k^+(-i\mu_1)^2}$$

Summed, Squared Result

$$\Delta E_{ind}^{(1)} = \frac{C_R \alpha_s L E}{\pi \lambda} \int \frac{d^2\mathbf{q}_1}{\pi} \frac{\mu^2}{(\mu^2 + \mathbf{q}_1^2)^2} \frac{d^2\mathbf{k}}{4\pi} \int d\Delta z \bar{\rho}(\Delta z) \times$$

$$\times \left[-2 \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \times \left(1 - \cos\left\{(\omega_1 + \tilde{\omega}_m)\Delta z\right\}\right) \right.$$

$$\times \left(\frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} - \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \right)$$

$$+ \frac{1}{2} e^{-\mu \Delta z)} \left\{ \left(\frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} \right)^2 \times \right.$$

$$\times \left(1 - \frac{2C_R}{C_A}\right) \left(1 - \cos\{(\omega_0 - \tilde{\omega}_m)\Delta z\}\right)$$

$$+ \frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} \cdot \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \times$$

$$\left. \left. \times \left(\cos\{(\omega_0 - \tilde{\omega}_m)\Delta z\} - \cos\{(\omega_0 - \omega_1)\Delta z\}\right) \right\} \right]$$

DGLV

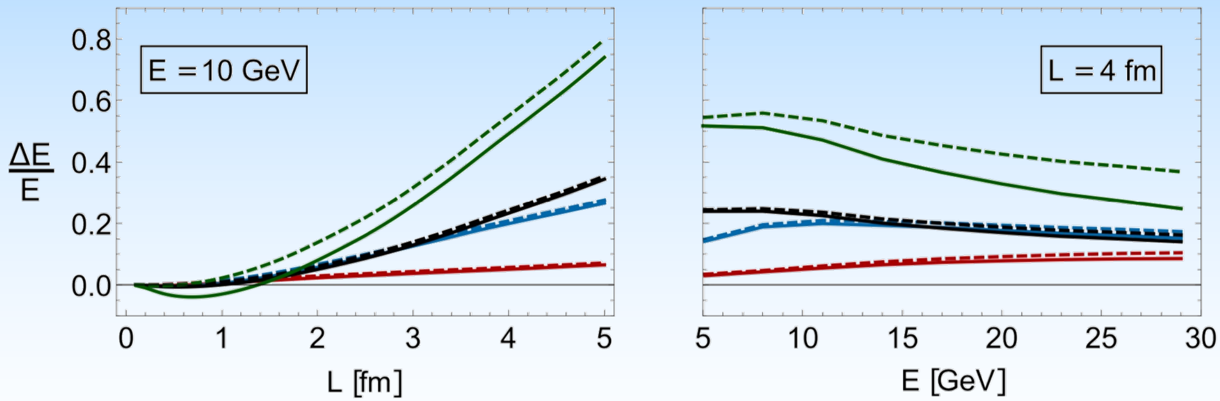
"Small L" Correction

Analyzing the Short Path Correction

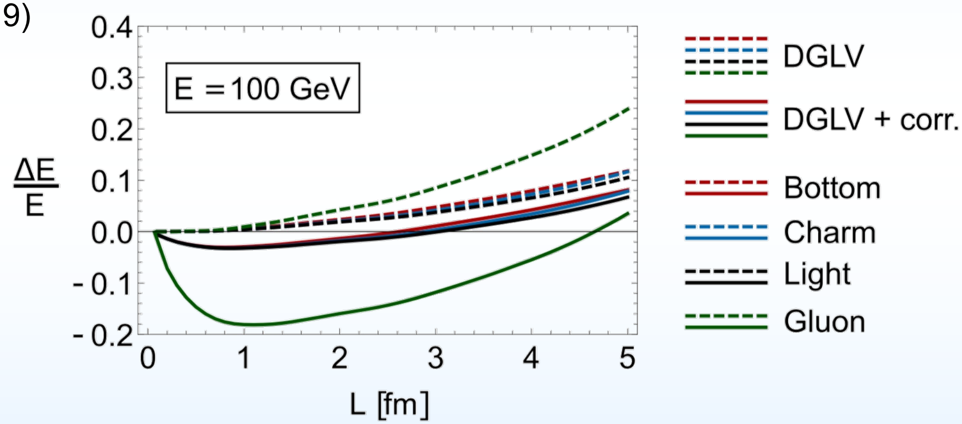
- Correction:
  - => 0 as Δz => 0 (destructive interference)
  - => 0 as μ => ∞ (all paths are long)
  - breaks color triviality

$$\Delta E_{ind}^{(1)} = \frac{C_R \alpha_s L E}{\pi \lambda} \int \frac{d^2\mathbf{q}_1}{\pi} \frac{\mu^2}{(\mu^2 + \mathbf{q}_1^2)^2} \frac{d^2\mathbf{k}}{4\pi} \int d\Delta z \bar{\rho}(\Delta z) \times$$
$$\times \Bigg[-2 \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \times \left(1 - \cos\{(\omega_1 + \tilde{\omega}_m)\Delta z\}\right)$$
$$\times \left(\frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} - \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \right)$$
$$+ \frac{1}{2} e^{-\mu \Delta z)} \Bigg\{ \left(\frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} \right)^2 \times$$
$$\times \left(1 - \frac{2C_R}{C_A}\right) \left(1 - \cos\{(\omega_0 - \tilde{\omega}_m)\Delta z\}\right)$$
$$+ \frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} \cdot \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \times$$
$$\times \left(\cos\{(\omega_0 - \tilde{\omega}_m)\Delta z\} - \cos\{(\omega_0 - \omega_1)\Delta z\} \right) \Bigg\} \Bigg]$$

Num. Investigation of Correction



Kolbé and WAH, PRC100 (2019)



- Surprise 1: Correction leads to *reduction* in E-loss
- Surprise 2: Affects *all* pathlengths L
  - Due to integrating over all distances to scattering Δz in [0,L]
- Surprise 3: Correction *grows* with p_T
- Need to think more carefully re small systems?

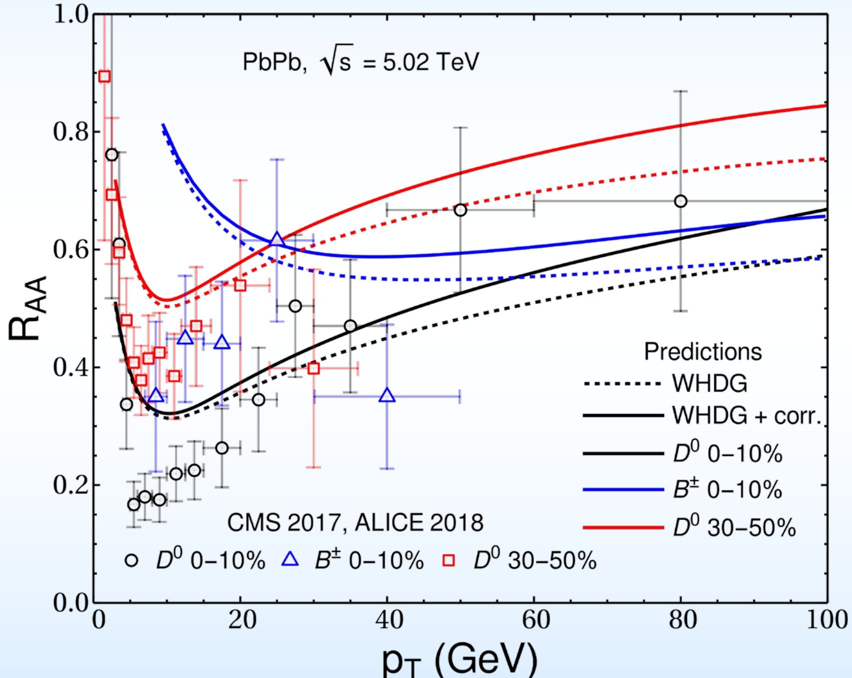
Pheno Investigation

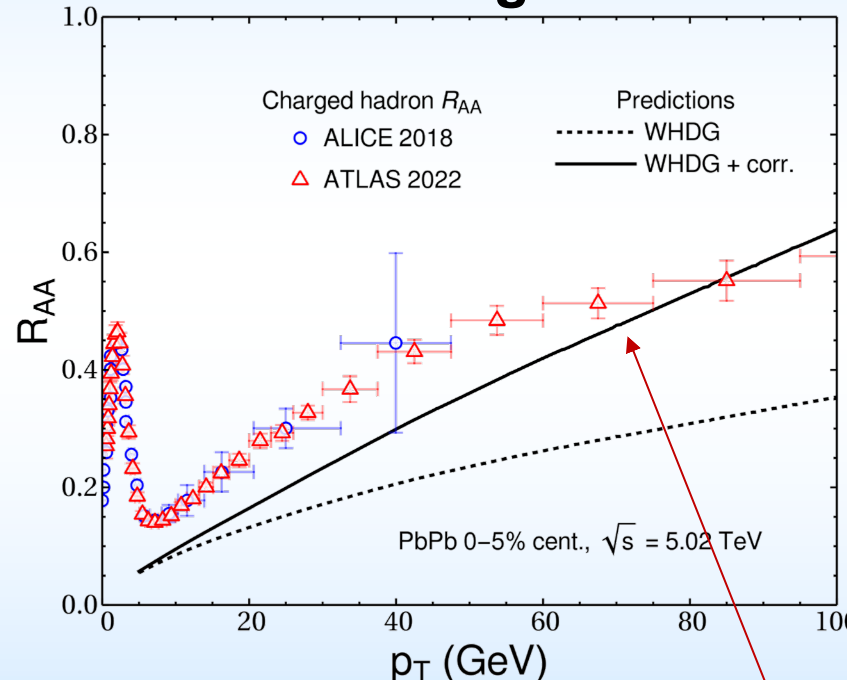
- Reasons to be excited:
  - Destructive interference => $R_{pA} > 1$ similar to data?
  - E (vs ln E) dependence => faster rise in $R_{AA}(p_T)$?
- Modest ambition:
  - How important is short L corr. vs original WHDG in AA? pA? pp?
- Serious comparison to data? Caveats:
  - Literally hot off the presses (plots from >1am *today*)
  - No nPDFs, no NLO, no NLL, no Bayesian, no new parameter extraction, mapping of hydro to bricks, time dependence of expansion, KKP/DSS π FFs not under full control for large p_T, limited by publicly available hydro backgrounds
- Seek ***qualitative*** comparison; is correction important to include? Other important previously neglected physics?

Cole Faraday

Effect on R_{AA}

- *Very preliminary* first results:
  - Full Rad + Coll + Geom (fluc. IP-G into VISHNU)
    - **D and B Mesons**
    - **Pions vs Charged Hadrons**





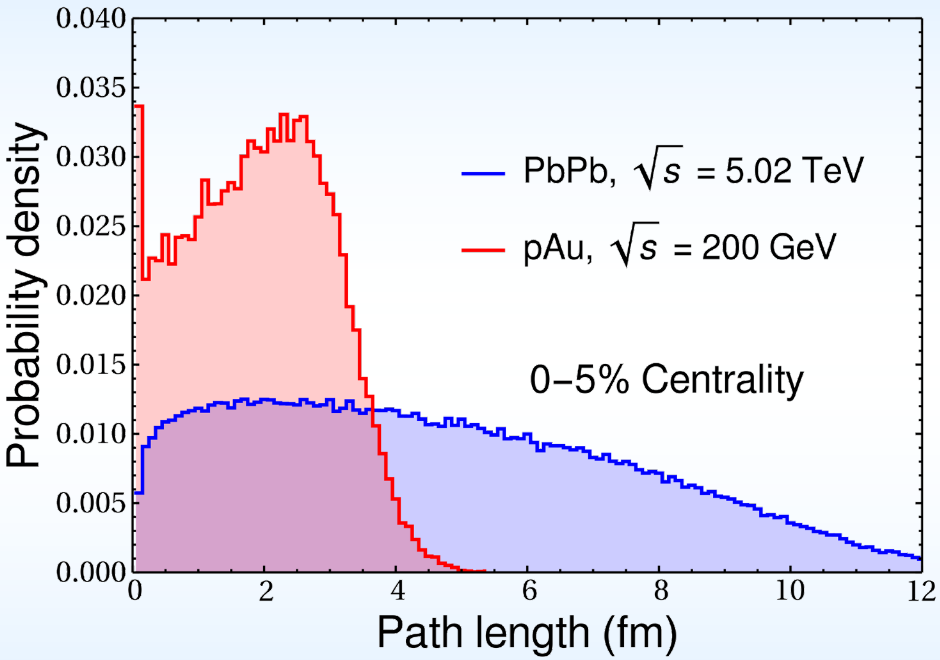
Faraday and WAH, *in prep*

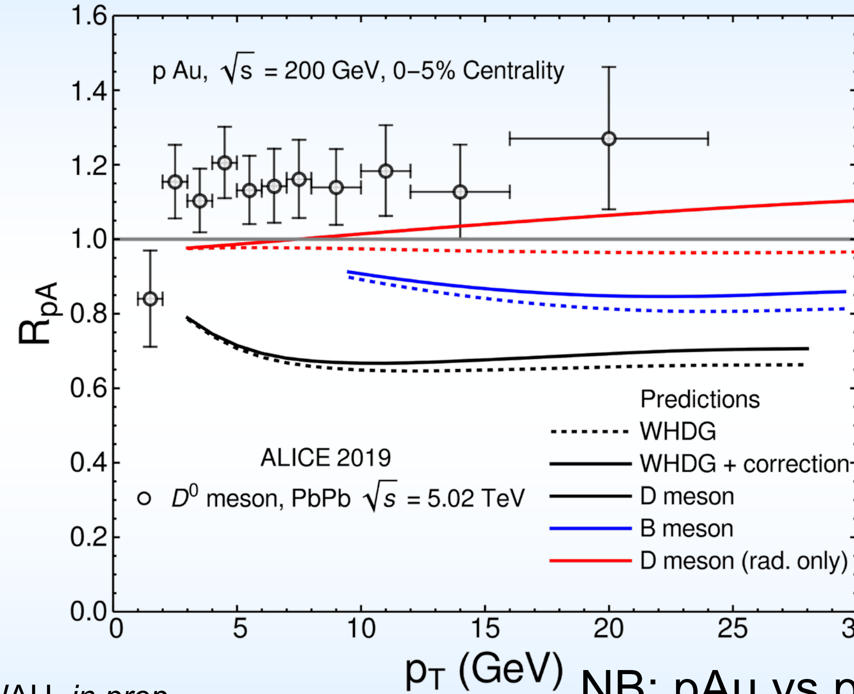
FFs becoming questionable

  - Short path length correction to ΔE_{rad}:
    - reduces suppression
    - increases energy dependence

Effect on R_{pA}

- Non-trivial pathlengths in pA collisions





Faraday and WAH, *in prep*

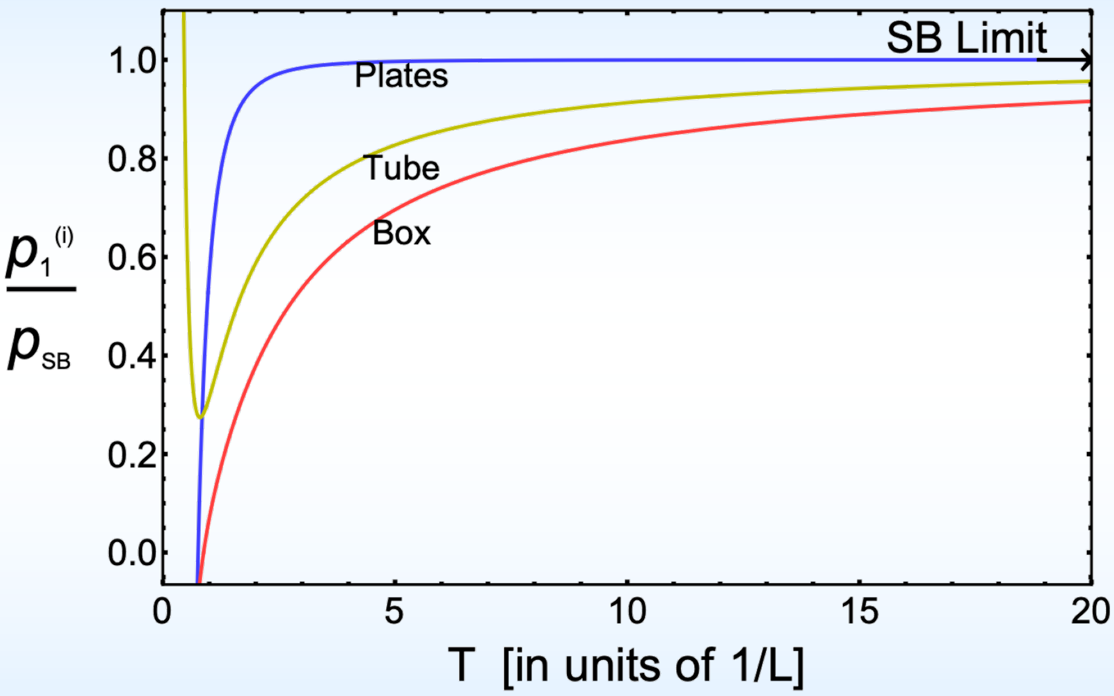
NB: pAu vs pPb

  - Average elastic energy loss inappropriate for small systems
  - Tantalizing hints of $R_{pA} > 1$ and rising

Thermodynamics of Small Systems

Does Finite Size Affect Thermodyn.?

- Test using free scalar field theory



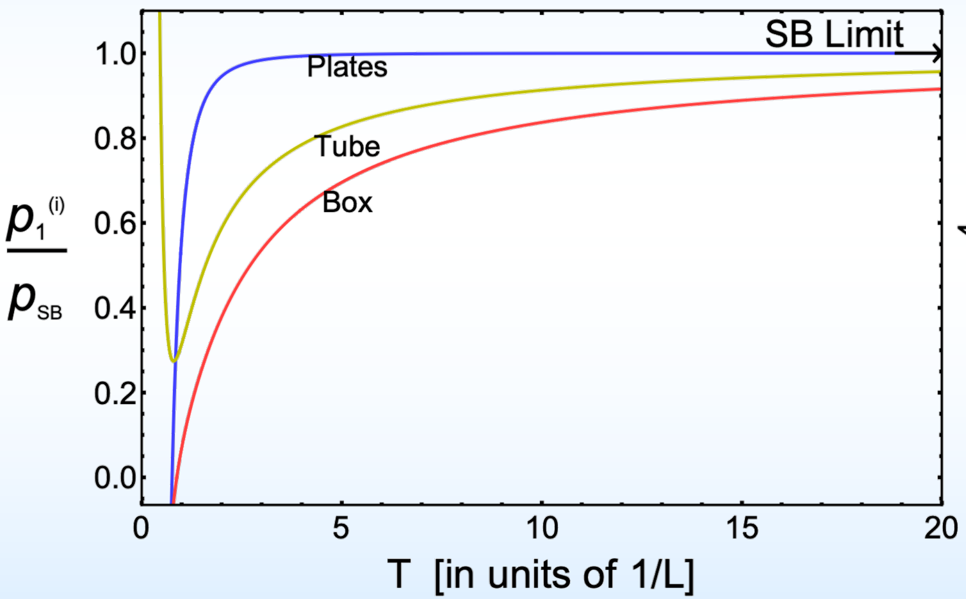
~10% effect at 400 MeV for ~1 fm system

Mogliacci, Kolbé, and WAH, PRD102 (2020)

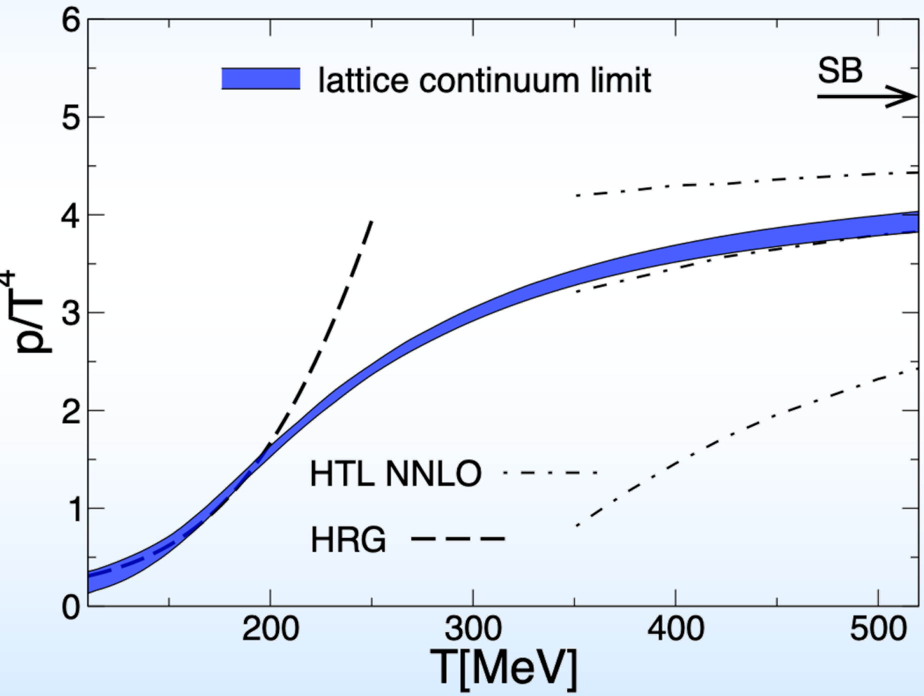
– p decreases significantly as T decreases for fixed L, converging to T = 0 Casimir effect

Does Finite Size Affect Thermodyn.?

- Provocative qualitative T dependence:



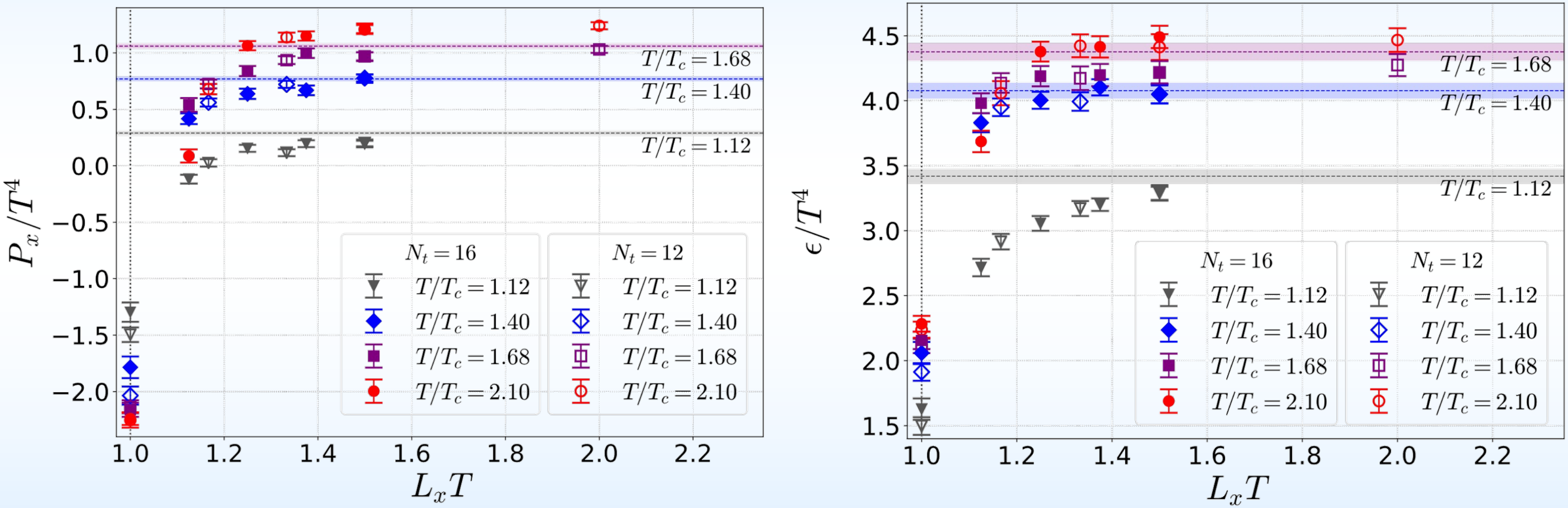
Mogliacci, Kolbé, and WAH, PRD102 (2020)



Borsanyi et al., PLB730 (2014)

FS Effects in Quenched QCD

- Thermodynamic quantities on anisotropic lattice with periodic boundary conditions

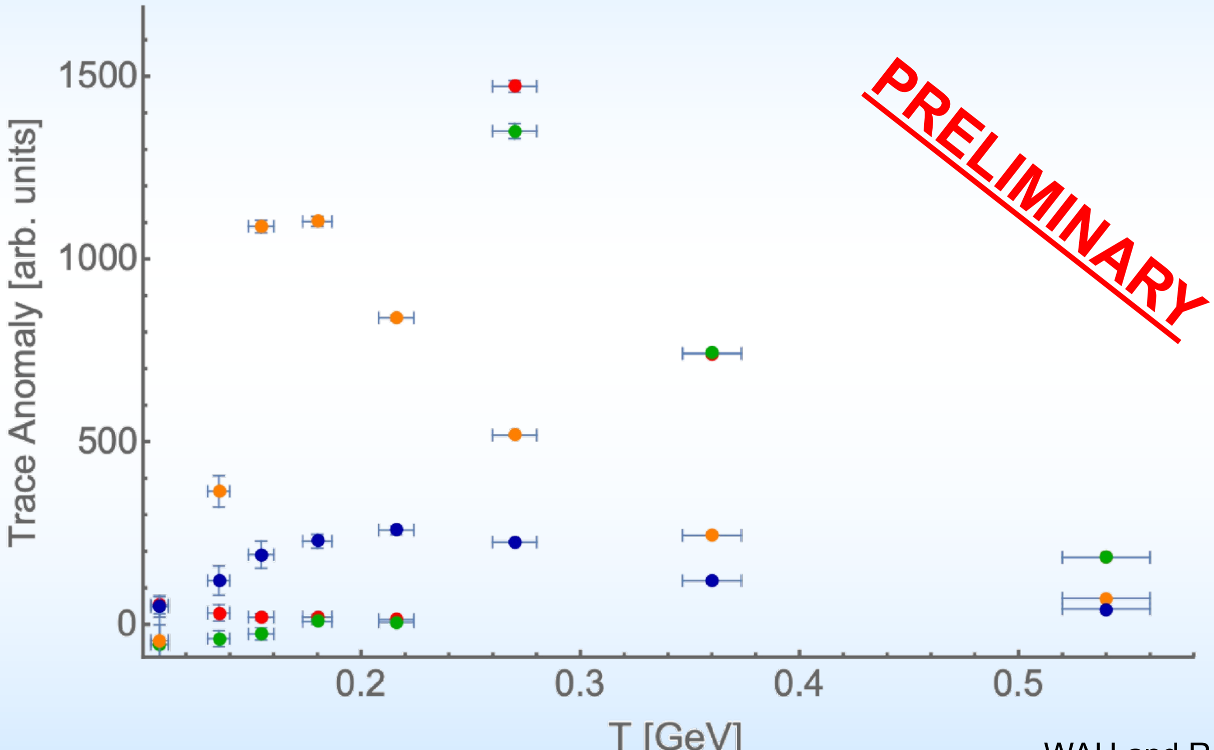


Kitazawa, Mogliacci, Kolbé, and WAH, PRD99 (2019)

  – Reduction of p, e with T*L qualitatively similar to free, massless scalar theory

What about Trace Anomaly Δ?

- $\Delta \Rightarrow c_s \Rightarrow \eta/s$
- From lattice QCD:



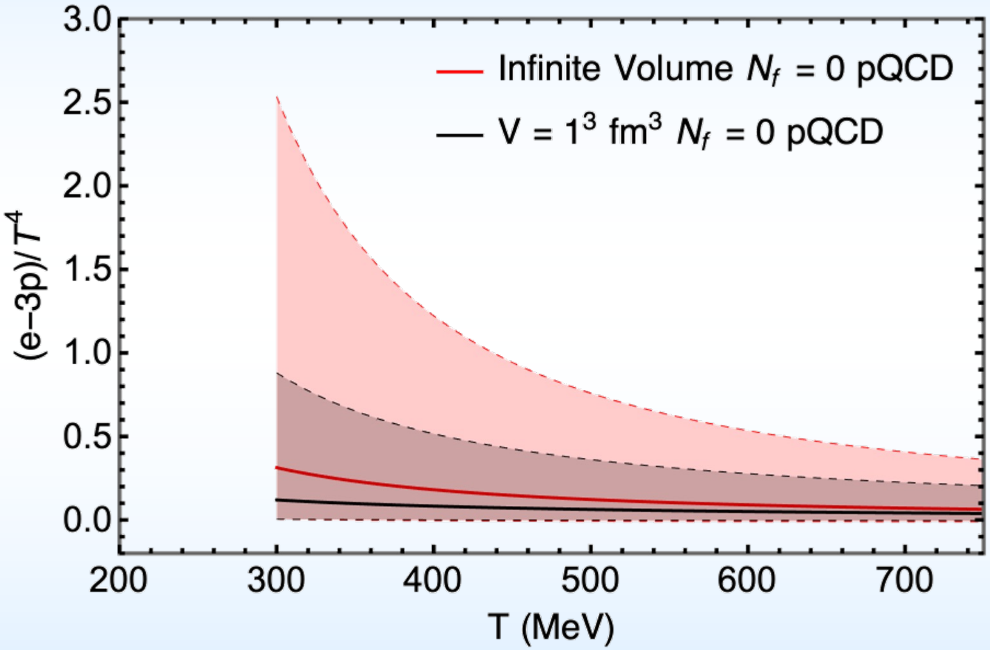
Quenched QCD
Periodic BCs

WAH and Rothkopf, *in progress*

  – Decreasing system size:
    - decreases Δ
    - washes out phase transition

Analytic Results for Δ

- Δ = 0 for massless, free scalar theory
  - Even when in a box!
- Δ = 0 for HTL QCD
  - Even to g^5!
- Δ != 0 *only* when coupling runs
- Estimate: use HTL QCD with lattice scalar running coupling
  - λ decreases and Δ *significantly* decreases from F.S.



WAH and Rothkopf, *in progress*

Analytic FS Δ in QCD

- What happens in QCD?
  - Does α_s grow or shrink as L => 0?
- Non-trivial conceptual issues:
  - How to regularize and renormalize?
    - Dim reg difficult to generalize in F.S. setting
  - Torons
- First examine F.S. effects for running coupling in 2 => 2 scattering in ϕ^4 theory

Define New Regularization Scheme

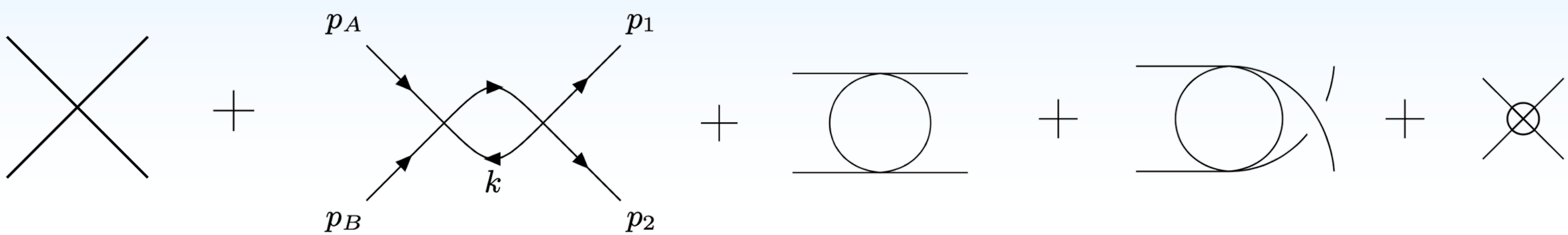
- Denominator regularization (den reg) instead of dimensional regularization (dim reg)
  - ***Number of dimensions fixed***
  - Feynman *x* combine propagators
  - Analytically continue the power of the single denominator
  - Introduce fictitious scale μ to maintain dimensions

$$V(p^2;\mu,\epsilon) = -\frac{1}{2}\int_0^1 dx \int \frac{d^{4-\epsilon}k}{(2\pi)^{4-\epsilon}} \frac{i\mu^\epsilon}{(k^2-\Delta^2)^2}$$

Use instead: $\Rightarrow -\frac{1}{2}\int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{i(-\mu^2)^{2\epsilon}}{(k^2-\Delta^2)^{2+\epsilon}}$

2 => 2 at NLO in ϕ^4 in Finite System

- Feynman Diagrams:



- Define $(-i\lambda)^2 iV(p^2) \equiv$ 

- Impose Periodic B.C.'s

$$V(p^2,\{L_i\};\mu,\epsilon) = -\frac{1}{2}\int_0^1 dx \int \frac{dk^0}{2\pi} \sum_{\vec{k}\in\mathbb{Z}^3} \frac{1}{(2\pi)^3 L_1 L_2 L_3} \frac{\mu^{2\epsilon}}{[k^2-\Delta^2]^{2+\epsilon}}$$

Capture the Divergence

- Result is a generalized Epstein Zeta fcn

$$V(p^2,\{L_i\};\mu,\epsilon) = -\frac{1}{2}\frac{1}{2\pi}\frac{1}{(2\pi)^3 L_1 L_2 L_3}\frac{\sqrt{\pi}\Gamma\left(\frac{3}{2}+\epsilon\right)}{\Gamma(2+\epsilon)}\int_0^1 dx \sum_{\vec{k}\in\mathbb{Z}^3}\frac{\mu^{2\epsilon}}{\left(\sum_{i=1}^3\left(\frac{k^i}{L_i}+x p^i\right)^2+\Delta^2\right)^{\frac{3}{2}+\epsilon}}$$

- Poisson Summation Formula: $\sum_{\vec{n}\in\mathbb{Z}^p} f(\vec{n}) = \sum_{\vec{m}\in\mathbb{Z}^p}\tilde{F}(\vec{m})$

- Yields new analytic continuation for g.E.Z:

$$\sum_{\vec{n}\in\mathbb{Z}^p}(a_i^2 n_i^2 + b_i n_i + c - i\varepsilon)^{-s} = \frac{1}{a_1\cdots a_p}\frac{1}{\Gamma(s)}\Bigg[\pi^{p/2}\Gamma\left(s-\frac{p}{2}\right)\left(c-\sum\frac{b_i^2}{4a_i^2}-i\varepsilon\right)^{\frac{p}{2}-s}$$

$$+2\pi^s \sum_{\vec{m}\in\mathbb{Z}^p}{}' e^{-2\pi i\sum\frac{m_i b_i}{2a_i^2}}\left(\frac{c-\sum\frac{b_i^2}{4a_i^2}-i\varepsilon}{\sum\frac{m_i^2}{a_i^2}}\right)^{\frac{p}{4}-\frac{s}{2}} K_{s-\frac{p}{2}}\left(2\pi\sqrt{\left(c-\sum\frac{b_i^2}{4a_i^2}-i\varepsilon\right)\left(\sum\frac{m_i^2}{a_i^2}\right)}\right)\Bigg]$$

Finite Size Result at NLO

- Found the pole! And the F.S. correction!

$$V(p^2,\{L_i\};\mu,\epsilon) = -\frac{1}{2}\frac{1}{(4\pi)^2}\int_0^1 dx\left\{\frac{1}{\epsilon} - 1 + \ln\frac{\mu^2}{\Delta^2} + 2\sum_{\vec{m}\in\mathbb{Z}^3}{}' e^{-2\pi i x \sum m_i p^i L_i} K_0\left(2\pi|\Delta|\sqrt{\sum m_i^2 L_i^2}\right)\right\}$$

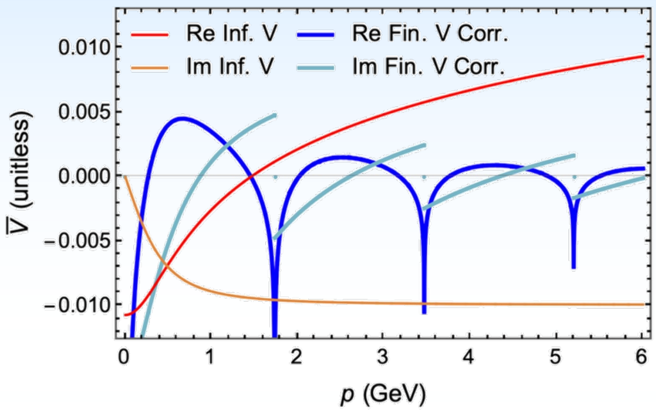
  - Correction
    - goes to 0 as L_i, p => infinity
    - satisfies unitarity/optical theorem

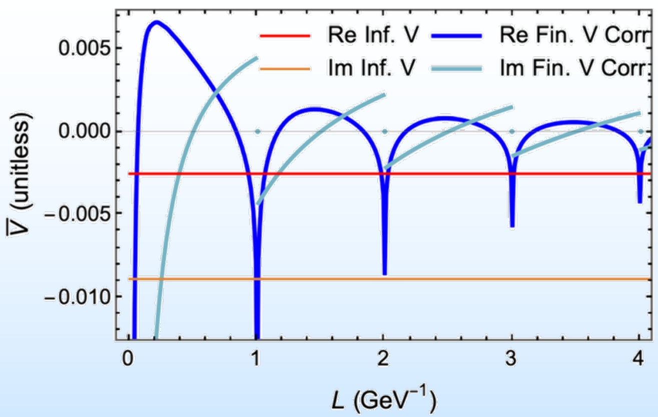
Jean du Plessis

- Optical thm check highly non-trivial
  - Requires generalization of a number theory result from Hardy/Ramanujan

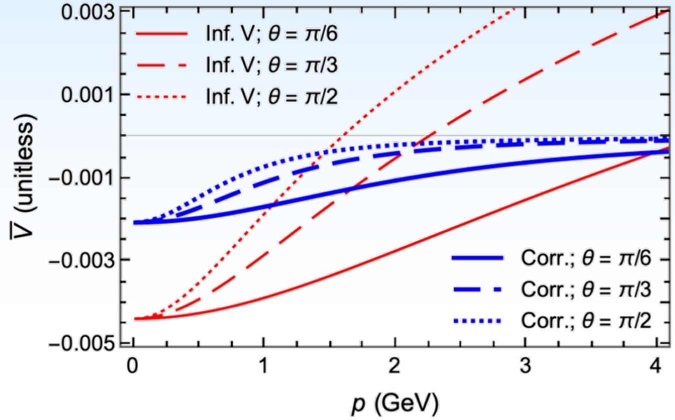
Numerical Results with 1 Compact D

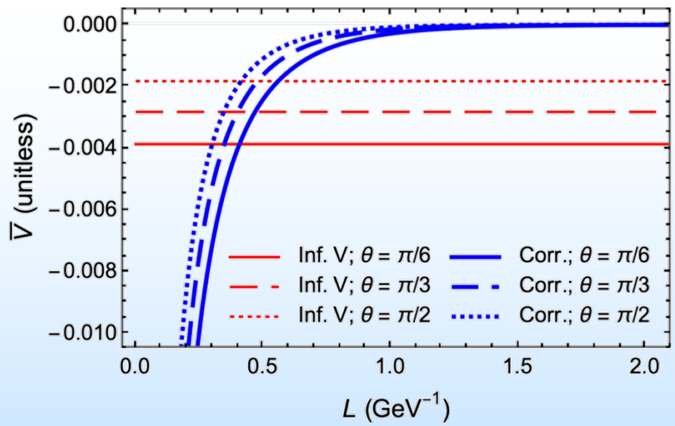
- s channel





- t channel

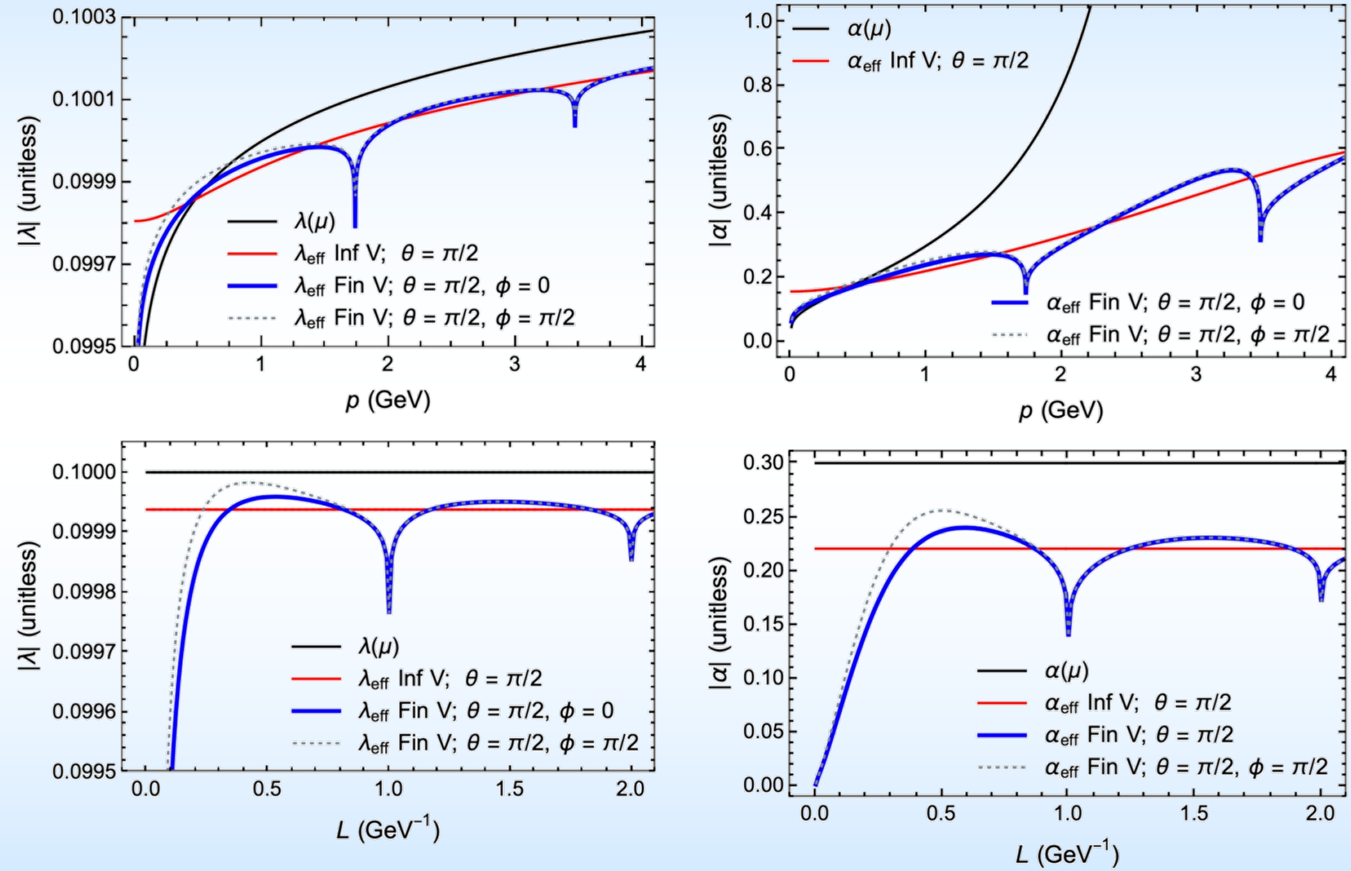




  - Correction => 0 in L and p
  - Non-trivial behavior from x int over BesselK

Running Coupling in 1 Compact D

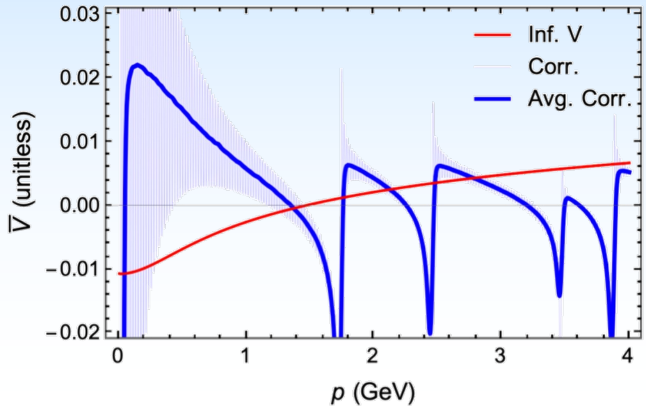
– Sum the geometric series of bubbles

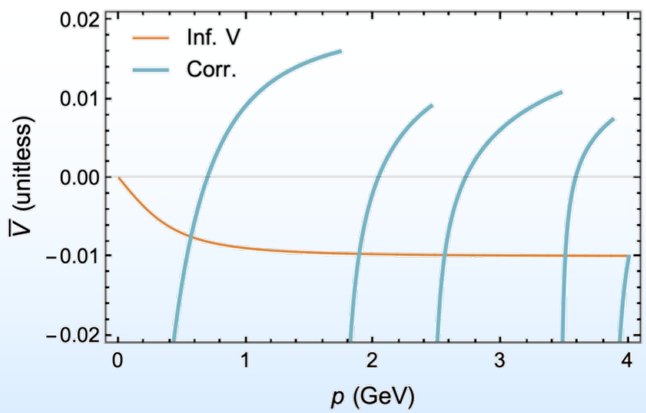


- Result depends on coupling at initial scale
- No pole in full series; cf LL from beta fcn

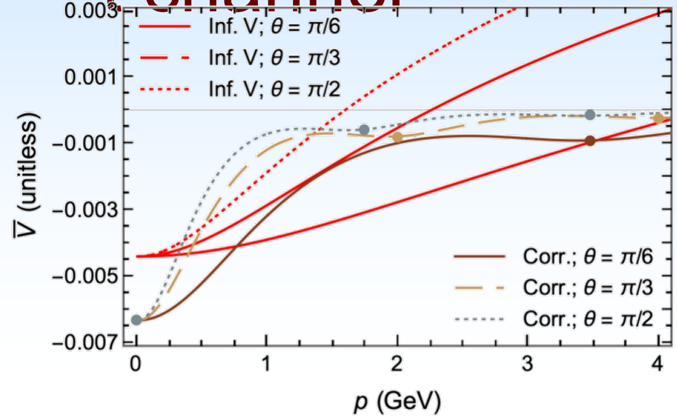
Numerical Results with 2 Compact D’s

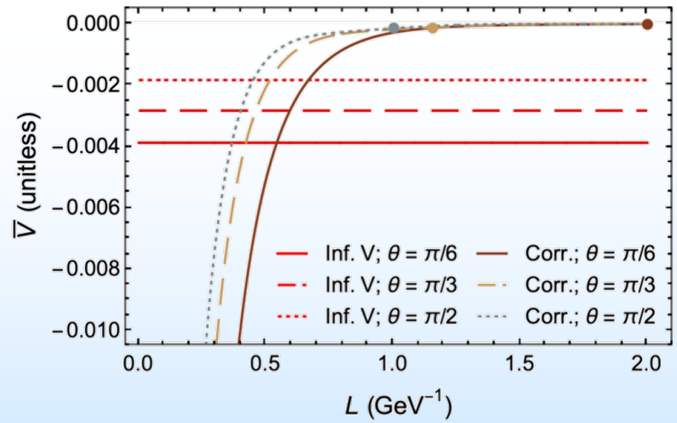
- s channel





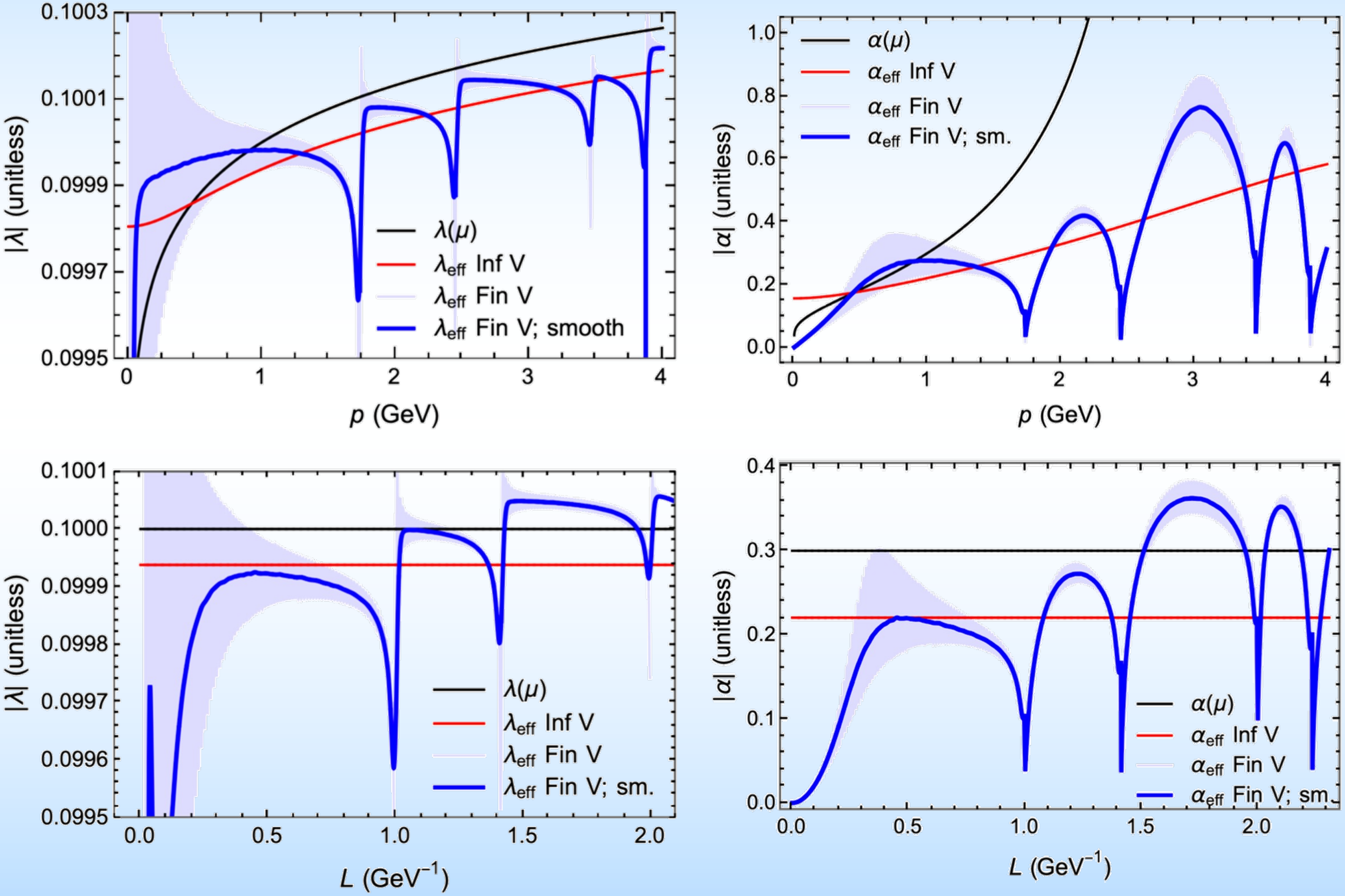
- t channel





  - Extremely difficult numerical problem for Re of s channel
  - 3D compact is trivial

Running Coupling in 2 Compact D's



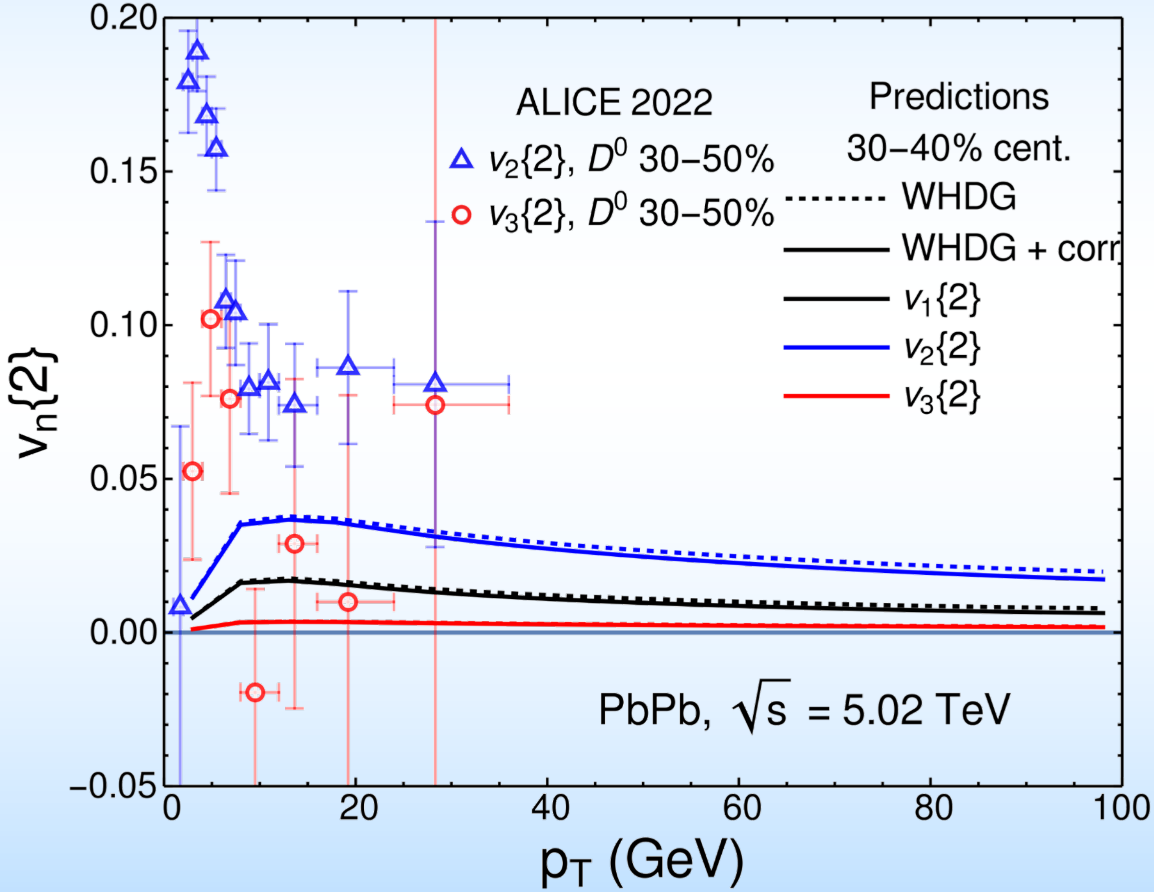
Conclusions

- Presented first (*and only*) analytic small L correction to E-loss derivation
  - Correction grows w 1/L, E; breaks color triviality
- First pheno results. Small L correction:
  - Reduces suppression
  - Increases rise with p_T
  - ~10% effect for heavy flavor R_{AA}
  - Broken color triviality => large effect for light hadrons
  - Significant pathlenths in pA; hints of R_{pA} > 1; ***must*** compute small pathlength corrections for elastic energy loss
- To do: see caveats; then v_n's for AA and pA, high multiplicity pp, …

Bonus Conclusions

- Massive, free scalar field in a box
  - Thermodynamics significantly altered; mimics QCD
  - Quenched lattice confirms qualitative physics
- NLO 2 => 2 scattering in ϕ^4 in 1, 2, and 3 compact dim's
  - Analytic continuation of the generalized Epstein zeta function
  - Captured finite size corrections
  - Checked unitarity
  - Showed first results for running coupling in 1, 2, and 3 compact dim's
- A lot of interesting work to do!!

Very Preliminary v_n's



Faraday and WAH, *in prep*