Recent theoretical developments on heavy quarkonia in QGP

Miguel Ángel Escobedo

Universitat de Barcelona

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Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

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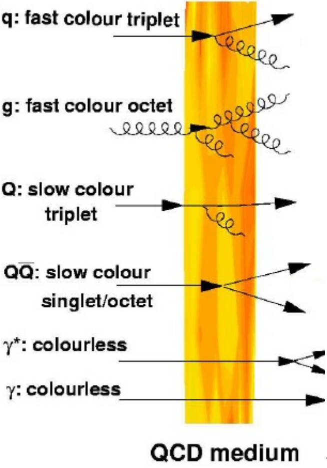
Outline

Hard probes



[1]

Probes that are created at the beginning of the collision (typically because its creation needs a high energy) that get modified in a substantial way and that are relatively easy to detect. In this talk we focus in the ones related with heavy quarkonium

[1]Picture taken from d'Enterria (2007)

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- In the case of quarkonium, other energy scales appear. The inverse of the typical radius $\frac{1}{r}$ and the binding energy E.
- Using heavy quarks we can test the properties of the medium at different energy scales.

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Screening

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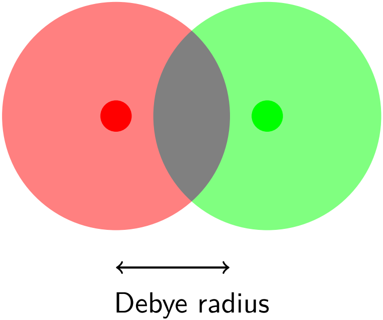
At finite temperature



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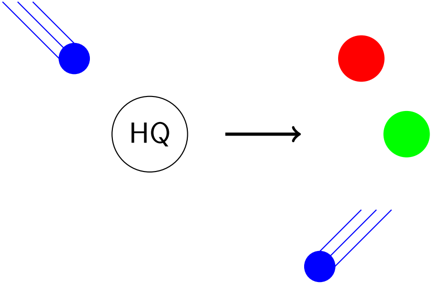
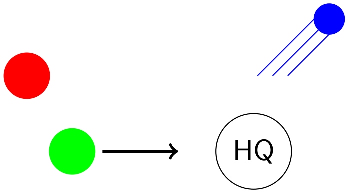


Recombination

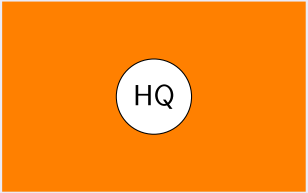


Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

Recombination: Bottomonia vs Charmonia

Bottomonia

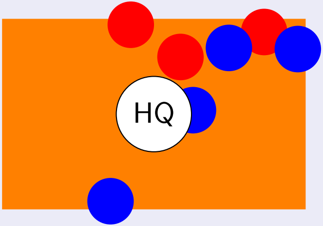
The dilute limit is valid



Recombination from uncorrelated heavy quark is unlikely

Charmonia

The dilute limit is not valid



We need to consider recombination from uncorrelated heavy quarks

Outline

Possible classification of theory approaches

According to how the medium-quarkonium interaction is computed

- Perturbation theory, HTL.
- Input from lattice QCD.
- EFT
- Potential model

According to how the evolution of quarkonium is computed

- Quantum master equation.
- Rate or Boltzmann equation.
- Langevin-like equation.

- Statistical hadronization model.

Decay rate

Boltzmann equation

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{x},\mathbf{p},t) + \mathbf{v}\cdot\nabla_{\mathbf{x}} f_{nl}(\mathbf{x},\mathbf{p},t) = C_{nl}^{(+)}(\mathbf{x},\mathbf{p},t) - C_{nl}^{(+)}(\mathbf{x},\mathbf{p},t)$$

for example in Yao and Mehen (2019). Or rate equation

$$\frac{\partial}{\partial t} p_n = -\Gamma(p_n - p_n^{eq})$$

sometimes assuming that sourcing particles are in equilibrium (Rapp and Zhao (2010), Ferreiro (2014)).
Processes contributing to the decay width are:

- Gluo-dissociation. Dominant for $E \gg m_D$.
- Inelastic parton scattering. Dominant for $E \ll m_D$. Related to the imaginary part of Laine's potential.

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In Boltzmann or rate equation

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- What happens to the wave function when the imaginary part or the decay width is not a perturbation?
- The imaginary part is seen in the time-ordered correlator. What happens with the number of quarkonium states?

Quarkonium as an Open quantum system

- We consider a *universe* consisting in heavy quarks (system) in a medium of light quarks and gluons (environment). The density matrix $\rho(S, E)$ describes the state of the universe. Its evolution is unitary.

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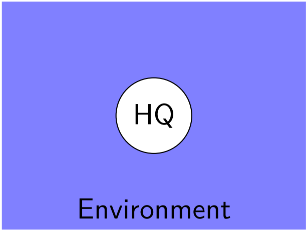

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- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.
- We need to derive the master equation from QCD. This has been done in:
  - Perturbation theory. Akamatsu (2015,2020), Blaizot and Escobedo (2017,2018).
  - Potential non-relativistic QCD (pNRQCD) in the $\frac{1}{r} \gg T$ regime. Brambilla et al. (2016,2017).

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

Can be written as a Lindblad (or GKSL) equation (Lindblad (1976), Gorini, Kossakowski and Sudarshan (1976)).

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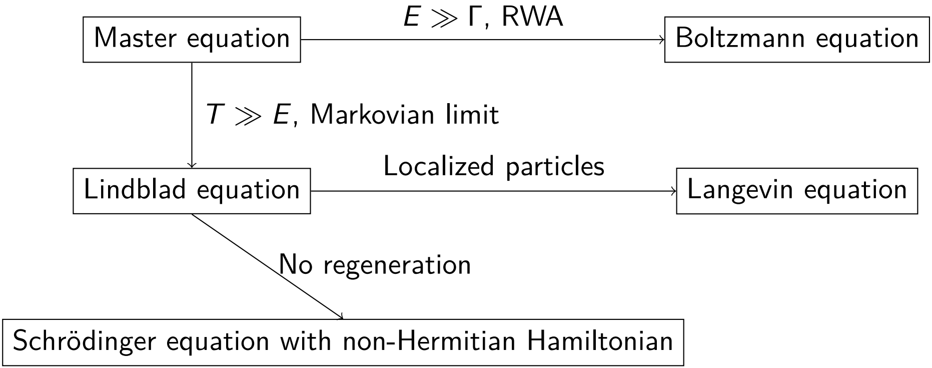
In the case of quarkonium, the Markovian limit corresponds to the case in which the energy of the particles in the environment is larger than the binding energy.

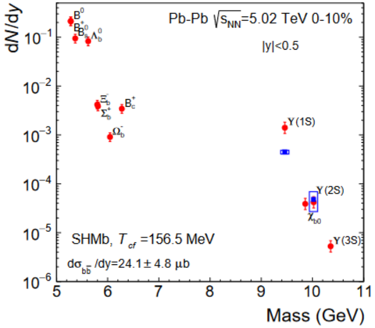
Connections



RWA approximation discussed in Yao and Müller (2019), Blaizot and Escobedo (2018).

Relation to Langevin-like eq. discussed in Blaizot and Escobedo (2018).

Statistical hadronization model



Picture taken from A. Andronic et al. QM 2022 proceedings.

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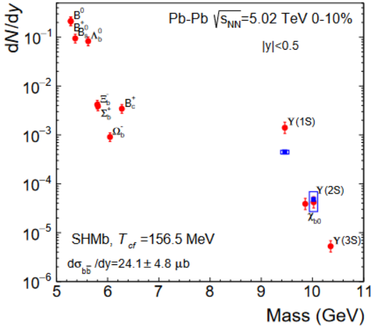

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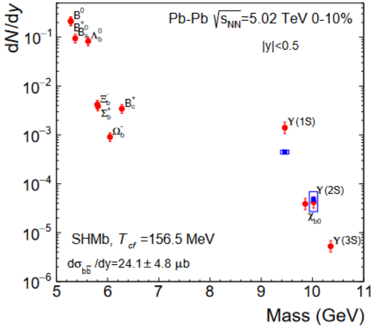

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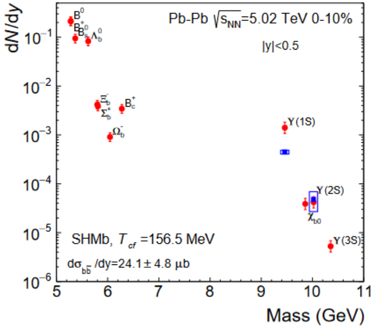

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- In the case of bottomonium, it has to be corrected to include non-thermalized bottom quarks.

Outline

State-of-the-art, recent works

work	system	quantum	dimensions	dilute limit?	equilibration
Brambilla et al. (2023)	B	✓	3D	Yes	?
Delorme et al. (2023)	B,C	✓	1D	Yes	?
T. Song et al. (2023)	B,C	✓(Remler)	3D	No	✓
Wu et al. (2023)	Bc (B,C,exotics)	X	3D	No	✓
Miura et al. (2022)	B,C	✓	1D	Yes	✓
Yao et al. (2021)	C	X	3D	No	✓

In HF/Quarkonia parallel:

- P. Gossiaux, Tuesday 10:50.
- B. Scheihing, Tuesday 12:10.
- W. Xing, Wednesday 09:20.
- S. Delorme, Wednesday 11:10.
- B. Scheihing, Tuesday 12:10.
- Z. Tang, Tuesday 14:40.

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- Growing interest in studying species other that bottomonium and charmonium. Bc and exotics.

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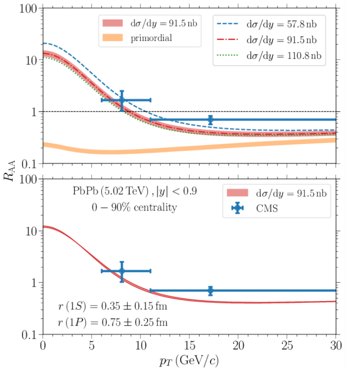

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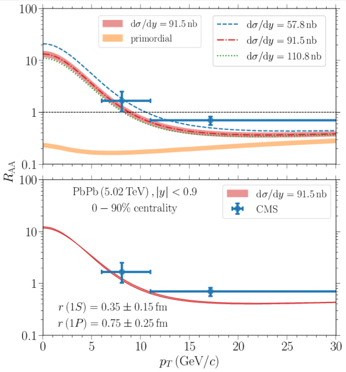

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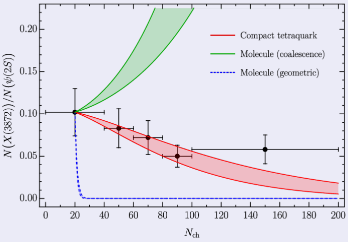
Exotics in high mult. pp



Picture taken from Esposito et al. (2021)

Remmler's approach

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- Wigner distribution is approximated by a classical phase space. It can not be negative.
- Dilute approximation is not needed. uncorrelated recombination can be described.

E/T corrections

- We can efficiently simulate quantum features in the $T \gg E$ limit (Brambilla et al. (2021)).
- In the $T \sim E$ we had a non-Makrovian evolution. Approximations that make it markovian either:
  - Ignore quantum features (Boltzmann-like).
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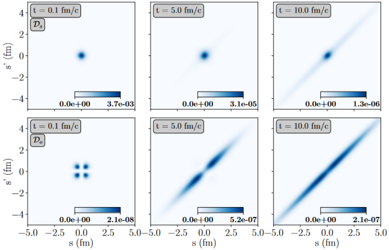
- Computing E/T corrections. An equation that we can solve keeping quantum features and that might get closer to thermalization.
- In QED, the Lindblad equation with E/T is well approximated by a Langevin equation, which leads to the classical equilibrium distribution if it exists.

Nantes approach



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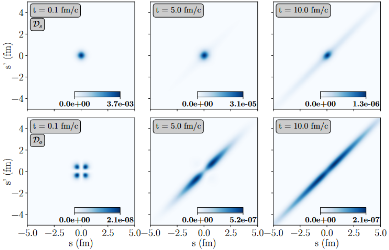

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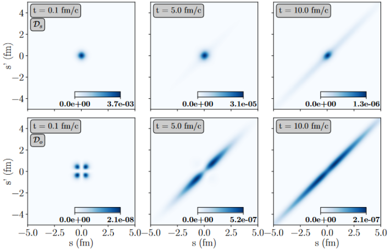

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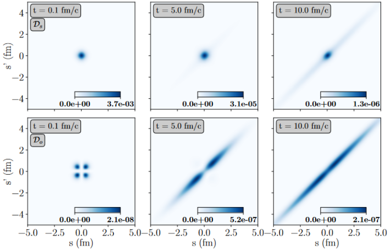

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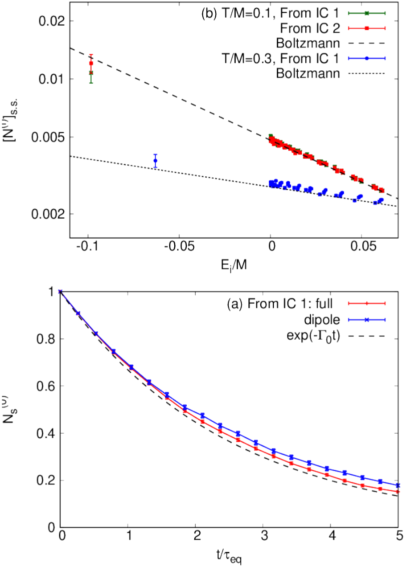
- Confirms that evolution naturally leads to a state in which the density matrix is almost diagonal in coordinate space. The regime in which Langevin-like equations are valid.
- However, we also see that a surviving non-diagonal structure around $\mathbf{r} = \mathbf{0}$.

Osaka approach

Miura, Akamatsu, Asakawa and Kaida, 2022



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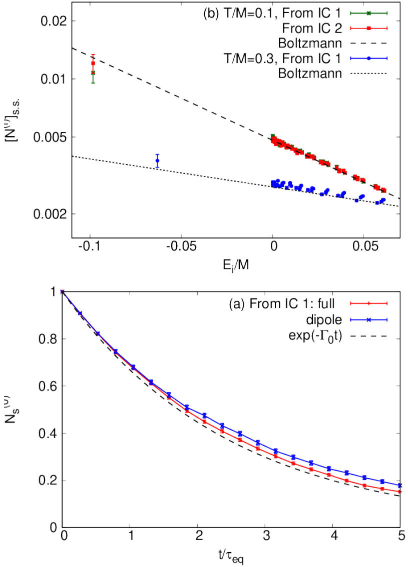

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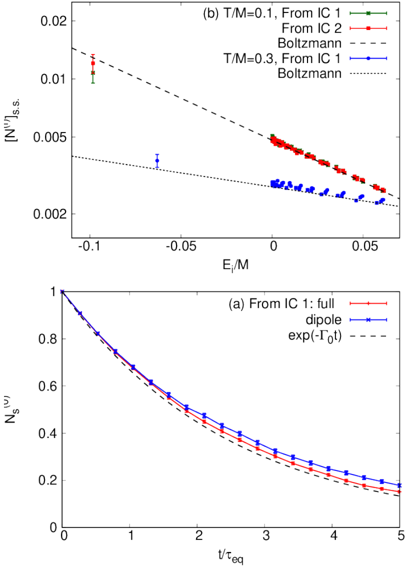

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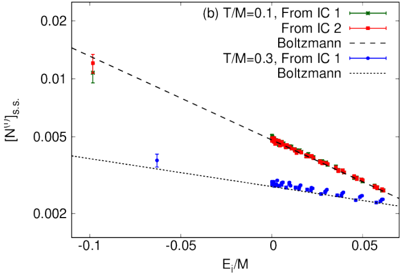

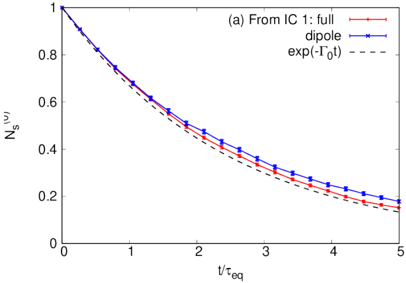

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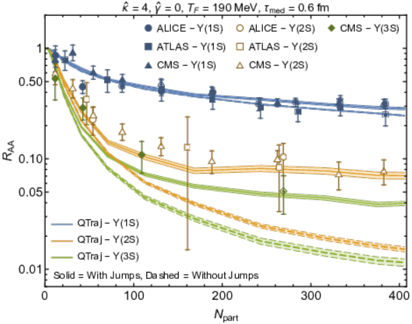
- Their full result and the dipole approximation coincide at early times.

E/T corrections in pNRQCD

Brambilla et al, 2023



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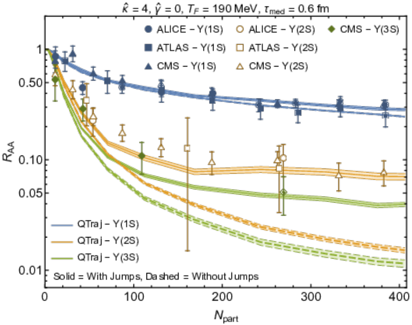

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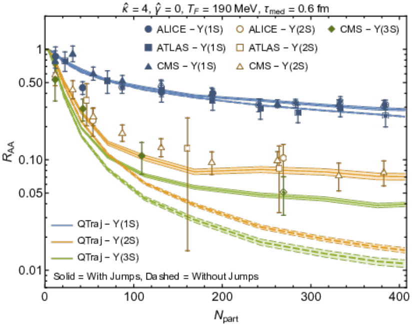

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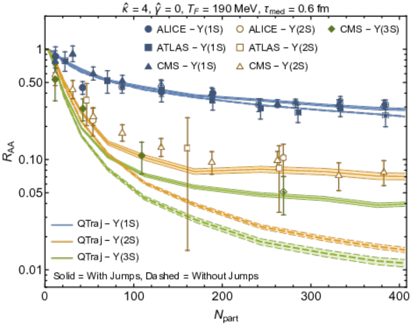

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- Focus on R_{AA}. Thermalization has not been studied yet.

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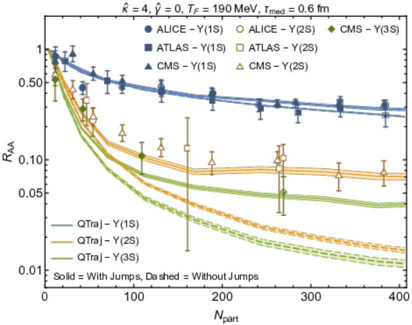

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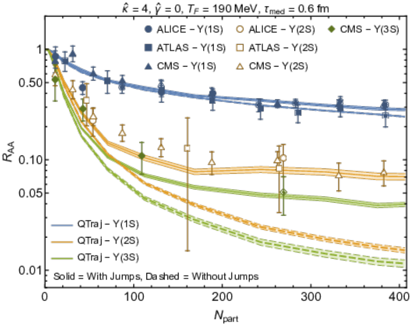

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- Correlated recombination needed to reproduce excited states data.

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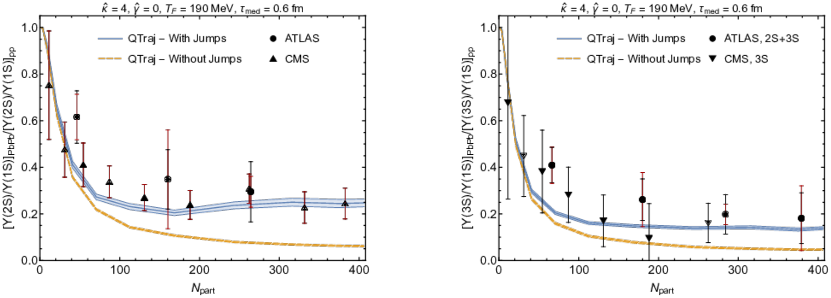

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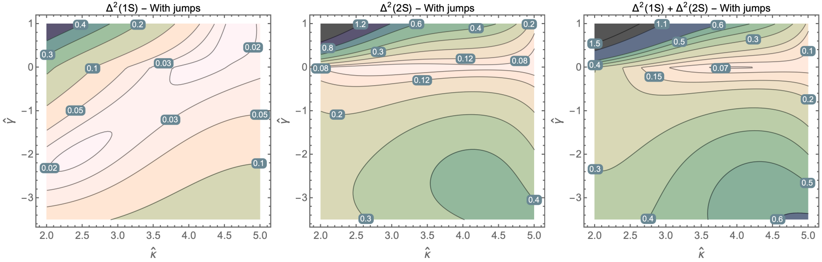
Brambilla et al, 2023

Fit with recombination



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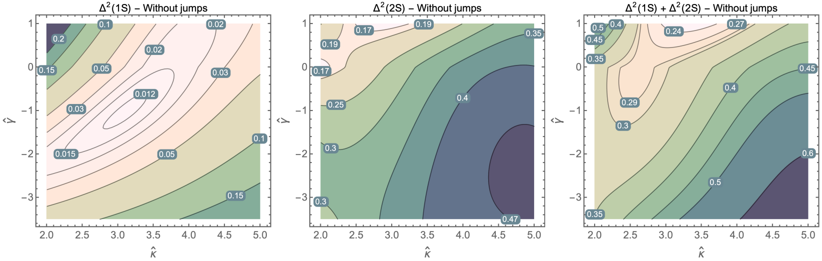

Outline

Conclusions

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- New developments in the open quantum system approach. Improved understanding of the connection with more traditional approaches.
- Recent studies of thermalization and phenomenological consequences when E/T corrections are included.

Back up slides

Quarkonium observables

Nuclear modification factor

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All of this might depend on centrality, transverse momentum and rapidity

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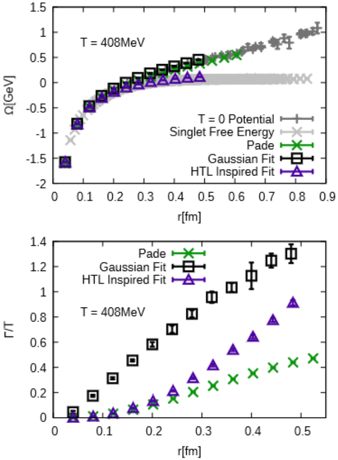
Lattice, static limit



Picture taken from Parkar et al. (2022).

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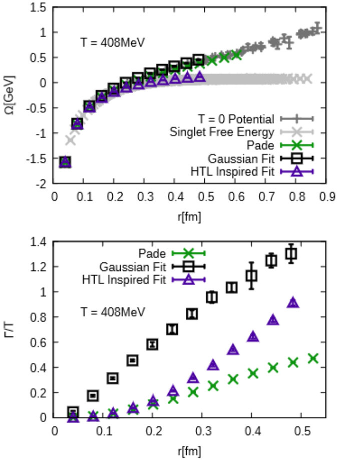

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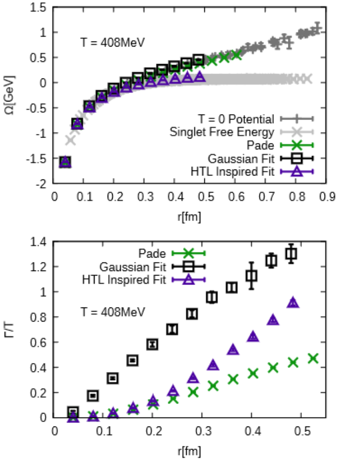

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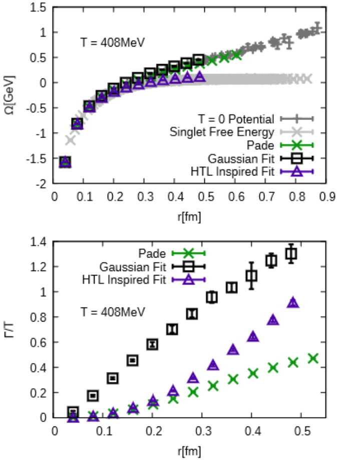

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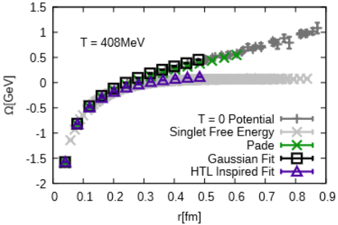

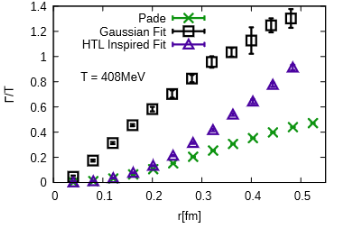

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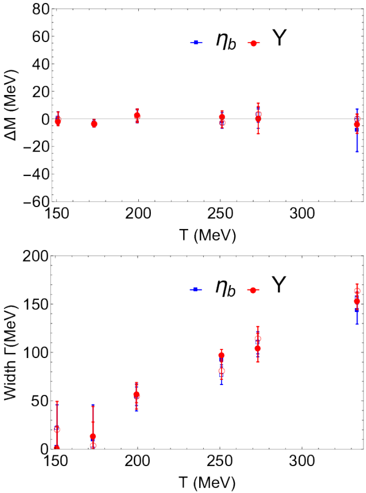
- For example, one can relate it with Wilson coefficients of non-relativistic EFTs.

Lattice, quarkonium state



Plots from Larsen et al. (2019)

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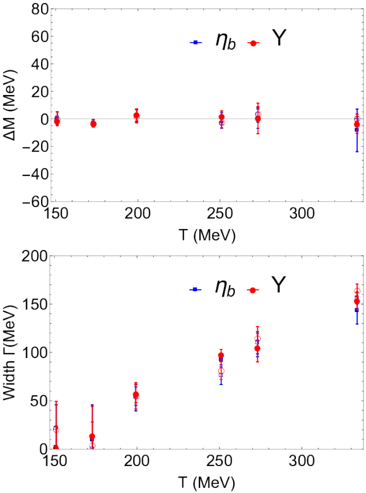

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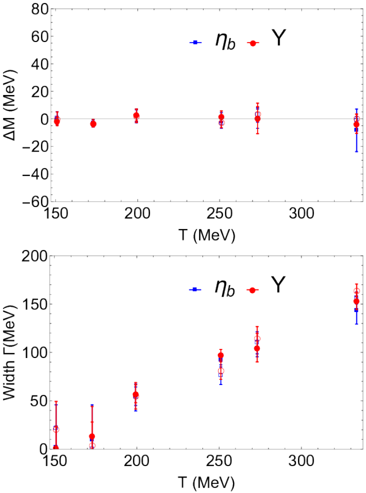

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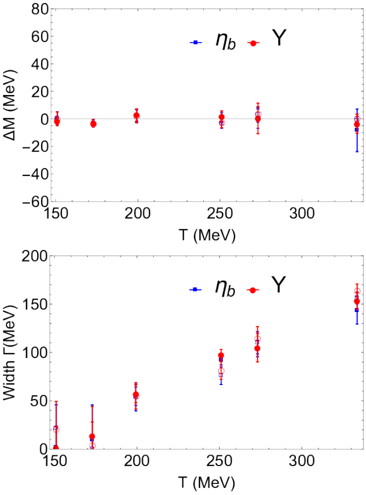

Plots from Larsen et al. (2019)

- Extract thermal mass shift and decay width from lattice QCD.
- Recent unquenched computations suggest a very small mass shift. Indication of no screening?
- Lattice data can be used on the EFT framework to constraint needed transport coefficients.

Problems of the Lindblad equation

- *Computational cost*. Since the state of the system is represented by a density matrix, if we use a lattice with the double of points, the cost is multiplied by four.

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  - Quantum state diffusion. Discussed for HQ in Akamatsu and Rothkopf (2012).
  - Quantum trajectories method or Monte Carlo wave function method. Discussed for HQ in Brambilla et al. (2021).

The Monte-Carlo Wave Function method

Take the Lindblad equation

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa) C_k(\kappa), \rho\})$$

Let us define

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

and

$$H_{eff} = H - i\frac{\Gamma}{2}$$

$\rho(t) = \sum_n p_n |\Psi_n(t)\rangle\langle\Psi_n(t)|$. If we know how to evolve the case $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, it is straightforward to generalize.

The Monte-Carlo Wave Function method

The algorithm to evolve from t to $t + dt$

- With probability $1 - \langle\Psi(t)|\Gamma|\Psi(t)\rangle dt$.
  - Evolve the wave-function with $(1 - iH_{eff}\,dt)|\Psi(t)\rangle$. In our case, this implies solving a 1D Schrödinger equation because H_{eff} does not mix states with different color or angular momentum.
- With probability $\langle\Psi(t)|\Gamma_n|\Psi(t)\rangle dt$.
  - Take a quantum jump, $|\Psi(t)\rangle \to C_n|\Psi(t)\rangle$.
  - Only here transitions between different color and angular momentum are allowed.
- Normalize the resulting wave-function.

The average of this stochastic evolution of the wave-function is equivalent to the Lindblad equation for the density matrix.

How does the quantum trajectory method encode each effect?

- Screening. Through the Hermitian part of the Hamiltonian. If there are no bound states the heavy quark and antiquark will separate.
- Decay width. Through the Non-hermitian part of the Hamiltonian. Possibility to take a quantum jump to an unbound state.
- Recombination. Through jump operator. Finite probability to jump back to a bound state.

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- In a more general master equation (non-Markovian) the decay width would depend on the whole history of the wave-function.