Heavy quarks probe the equation of state of QCD matter in heavy-ion collisions

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Outline

• Introduction

• The quasi-particle linear Boltzmann transport (QLBT) model

• Bayesian extraction for QCD EoS and transport coefficients

• Summary and outlook
Hard Probe: Heavy Quark

Ideal clean probe:

\[ m_Q \gg \Lambda_{QCD} \quad m_Q \gg T_{QGP} \]

Jet quenching:

- Interaction with QGP medium
- Gluon radiation

The nuclear modification factor \( R_{AA} \):

\[ R_{AA} = \frac{1}{N_{\text{coll}}} \frac{dN^{AA}/d^2 p_T dy}{dN^{pp}/d^2 p_T dy} \]

Constrain transport properties and EoS of QGP:

- Transverse transport coefficient \( \hat{q} \)
- Spatial diffusion coefficient \( D_s \)
Jet(Bulk) transport coefficients and EoS are constrained separately by jet quenching (bulk observables).

In this work, a direct Bayesian extraction of the QGP EOS using heavy flavor observables based on QLBT model.
The quasi-particle linear Boltzmann transport (QLBT) model


\[ m_q^2(T) = \frac{N_c^2 - 1}{8N_c} g^2(T)T^2 \]

\[ m_g^2(T) = \frac{1}{6} \left( N_c + \frac{1}{2} N_f \right) g^2(T)T^2 \]

The temperature dependent coupling \( g(T) \)

\[ g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left[ \frac{(aT_c + b)^2}{1 + ce^{-d(T/T_c)^2}} \right]} \]

\( T_C \) : the transition temperature between the QGP and the hadronic matter.

Parameter: a, b, c, d
From QPM to EoS

With the temperature-dependent thermal masses of quarks and gluons that are determined by parameters \((a,b,c,d)\) via \(g^2(T)\), one may calculate the pressure of the relativistic gas system as

\[
P(T) = \sum_i \gamma_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_i(p, T)} f_i(p, T) - B(T)
\]

Similarly, the energy density is obtained as

\[
\epsilon(T) = \sum_i \gamma_i \int \frac{d^3p}{(2\pi)^3} E_i(p, T)f_i(p, T) + B(T)
\]

The entropy density

\[
s(T) = \frac{[\epsilon(T) + P(T)]}{T}
\]

Standard process: Lattice QCD EOS \(\rightarrow g^2(T) \rightarrow\) observables

Inverse question: observables \(\rightarrow g^2(T) \rightarrow\) QCD EOS?
Linear Boltzmann transport model

Boltzmann equation: \[ p_1 \cdot \partial f_1 (x_1, p_1) = E_1 \left( C_{el} [f_1] + C_{inel} [f_1] \right) \]

Elastic scattering: \[
\Gamma_{12\rightarrow34} (\vec{p}_1) = \int d^3k w_{12\rightarrow34} (\vec{p}_1, \vec{k}) = \frac{\gamma_2}{2E_1} \int \frac{d^3p_2}{(2\pi)^32E_2} \int \frac{d^3p_3}{(2\pi)^32E_3} \int \frac{d^3p_4}{(2\pi)^32E_4}
\times f_2 (\vec{p}_2) \left[ 1 \pm f_3 (\vec{p}_3) \right] \left[ 1 \pm f_4 (\vec{p}_4) \right] S_2(s, t, u)
\times (2\pi)^4 \delta^{(4)} (p_1 + p_2 - p_3 - p_4) |M_{12\rightarrow34}|^2
\]

\[ P_{el} = 1 - e^{-\Gamma_{el}\Delta t} \]

Inelastic scattering \[ \langle N_g \rangle = \Gamma_g \Delta t = \Delta t \int dxdk_1^2 \frac{dN_g}{dxdk_1^2dt} \]

\[ P_{inel}^a = 1 - e^{-\langle N_g^a \rangle} \]

Elastic+inelastic: \[ P_{el}^a = 1 - e^{-(\Gamma_{el}^a + \Gamma_{inel}^a)\Delta t} \]

QGP background: (3+1)-D CLVisc hydrodynamics model
Linear Boltzmann transport model

Boltzmann equation: \[ p_1 \cdot \partial f_1 (x_1, p_1) = E_1 \left( C_{el} [f_1] + C_{inel} [f_1] \right) \]

Elastic scattering: \[
\Gamma_{12\rightarrow34} (\vec{p}_1) = \int d^3kw_{12\rightarrow34} (\vec{p}_1, \vec{k}) = \frac{r_2}{2E_1} \int \frac{d^3p_2}{(2\pi)^32E_2} \int \frac{d^3p_3}{(2\pi)^32E_3} \int \frac{d^3p_4}{(2\pi)^32E_4} \times f_2 (\vec{p}_2) \left[ 1 \pm f_3 (\vec{p}_3) \right] \left[ 1 \pm f_4 (\vec{p}_4) \right] S_2(s, t, u) \\
\times (2\pi)^4\delta^{(4)} (p_1 + p_2 - p_3 - p_4) \bigg| M_{12\rightarrow34} \bigg|^2 \]

\[ P_{el} = 1 - e^{-\Gamma_{el}\Delta t} \]

Inelastic scattering \[ \langle N_g \rangle = \Gamma_g \Delta t = \Delta t \int dx dk_1^2 \frac{dN_g}{dx dk_1^2 dt} \]

\[ P_{a\text{inel}} = 1 - e^{-\langle N_g \rangle} \]

Elastic+inelastic: \[ P_{a\text{el}} = 1 - e^{-(\Gamma_{a\text{el}} + \Gamma_{a\text{inel}})\Delta t} \]

QGP background: (3+1)-D CLVisc hydrodynamics model

Kinematic cut:
\[ S_2(s, t, u) = \theta (s \geq 2\mu_D^2) \theta (t \leq -\mu_D^2) \theta (u \leq -\mu_D^2) \]

Debye mass \[ \mu_D^2(T) = 2m_g^2(T) \]

Thermal distribution: \[ f_2 (\vec{p}_2), f_4 (\vec{p}_4) \]

\[ E_i(p, T) = \sqrt{p^2 + m_i^2(T)} \]
Two types of coupling vertices

cg → cg

\[ \alpha_s(T) = g^2(T)/(4\pi) \]

The vertices that connect to the thermal patrons:

\[ g^2(T) = \frac{48\pi^2}{\left(11N_c - 2N_f\right) \ln \left[ \frac{(aT/T_c + b)^2}{1 + ce^{-d(T/T_c)^2}} \right]} \]

The vertices that connect to the heavy quarks:

\[ g^2(E) = \frac{48\pi^2}{\left(11N_c - 2N_f\right) \ln \left[ (AE/T_c + B)^2 \right]} \]

Paramater: a, b, c, d, A, B
Bayesian analysis

Parameters \((\theta = a, b, c, d, A, B)\) → Latin-Hypercube

QLBT

Gaussian process emulator

LHC & RHIC data \(R_{AA}, v_2\) and \(T_c\)

\[ P(\text{data} | \theta) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(y_i - y_{\text{exp}})^2}{2\sigma_i^2}} \]

\(\sigma_i\): Experimental error and the interpolation error from GP

MCMC

\[ P(\theta | \text{data} ) \propto P(\text{data} | \theta)P(\theta) \]

Equilibrium

Posterior \(P(\theta | \text{data} )\)
Evaluate the EoS and transport coefficient
Calibration of the QLBT calculation (WB $T_c = 150$ MeV)

One observes that after the Bayesian calibration, our QLBT model is able to provide a reasonable description of the D meson observables in heavy-ion collisions.
Calibration of the QLBT calculation (HotQCD $T_c = 154$ MeV)

One observes that after the Bayesian calibration, our QLBT model is able to provide a reasonable description of the D meson observables in heavy-ion collisions.

Similar results can be obtained for $T_c = 154$ MeV compared to $T_c = 150$ MeV.
Posterior distributions of the model parameters

HotQCD $T_c = 154$ MeV

WB $T_c = 150$ MeV

Reasonable constraints on these model parameters have been obtained for same $T_c$. 
Posterior distributions of the model parameters

Reasonable constraints on these model parameters have been obtained for same $T_c$.

The extracted parameters is sensitive to $T_c$.

HotQCD $T_c = 154$ MeV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WB</th>
<th>HotQCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.1060</td>
<td>1.0936</td>
</tr>
<tr>
<td>B</td>
<td>0.0867</td>
<td>0.1131</td>
</tr>
<tr>
<td>a</td>
<td>2.0634</td>
<td>3.0117</td>
</tr>
<tr>
<td>b</td>
<td>0.7148</td>
<td>0.3362</td>
</tr>
<tr>
<td>c</td>
<td>4.7047</td>
<td>8.2149</td>
</tr>
<tr>
<td>d</td>
<td>0.5105</td>
<td>0.4524</td>
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</table>
The extract EoS ($T^3$-rescaled entropy density)

One observes that the heavy flavor observables do provide constraints on the QGP EoS.

The extracted EoS with $T_c = 150$ MeV agrees well with the WB lattice data that shares the same $T_c$.

Some deviation can be observed from HQ data: a larger $T_c$, terminates early, higher entropy density.
The transport coefficient
Summary

We have carried out a Bayesian analysis of the experimental data on D meson spectra and anisotropy $v_2$ at both RHIC and LHC based on QLBT.

We realized a simultaneous constraint on the properties of the QGP and heavy quark probes:
- The QGP EOS we extract is consistent with the lattice QCD results.
- The heavy quark diffusion coefficient we obtain agrees with results from other model and lattice calculations.

Outlook

Incorporate a more extensive set of parameters, e.g. phase transition temperature $T_c$.

Involve a broader range of jet observables and soft hadron spectra in order to accomplish the goal of constraining the properties of nuclear matter using hard probes.

Thank you for your attention!
Back up
Backup: Shear viscosity

The formulas for the viscosities $\eta$ are derived for a quasi-particle description with bosonic and fermionic constituents S. Plumari, W. M. Alberico, V. Greco, C. Ratti, Phys. Rev. D84 (2011)

$$\eta = \frac{1}{15 T} \sum_i d_i \int \frac{d^3 p}{(2\pi)^3} \frac{\vec{p}^4}{E_i^2} f_i (1 \mp f_i)$$

$d^3 p = p^2 dp \sin \theta d\theta d\psi$.

In relaxtime approximation, shear viscosity depends on collision relax time $\tau_i$ given by (HTL):

$$\tau_q^{-1} = 2 \frac{N_C^2 - 1}{2N_C} \frac{g^2 T}{8\pi} \ln \frac{2k}{g^2}, \quad \tau_g^{-1} = 2N_C \frac{g^2 T}{8\pi} \ln \frac{2k}{g^2},$$

where $g$ is the coupling obtained and $k$ is a parameter which is fixed by requiring that $\tau_i$ yields a minimum of one for the quantity $4\pi \eta/s$.

- $k$ if fixed to have a minimum $\eta/s = 1/4\pi$. For WB: $k=23.3$ and For HQ: $k=22.7$
Spatial diffusion coefficient

\[ \hat{q} = \sum_{bcd} \frac{\gamma_b}{2E_a} \left[ \prod_{i=b,c,d} \frac{d^3 p_i}{2E_i(2\pi)^3} f_b \right] \left| M_{ab\rightarrow cd} \right|^2 S_2(\hat{s}, \hat{t}, \hat{u}) \]

\[ \times (2\pi)^4 \delta^4 (p_a + p_b - p_c - p_d) \left[ \vec{p}_c - (\vec{p}_c \cdot \hat{p}_a) \hat{p}_a \right]^2 \]

\[ D_s(2\pi T) = 8\pi T^3 / \hat{q} \]
Bayesian analysis

Parameters
$(\theta = a, b, c, d, A, B)$

QLBT

Gaussian process emulator

LHC & RHIC data $R_{AA}, v_2$ and $T_c$

MCMC

$P(\theta | \text{data}) \propto P(\text{data} | \theta)P(\theta)$

Posterior $P(\theta | \text{data})$

Evaluate the EoS and transport coefficient

$P(\text{data} | \theta) = \prod_i \frac{1}{\sqrt{2\pi\sigma_i}} e^{-\frac{[y_i(\theta) - y_{\text{exp}}i]^2}{2\sigma_i^2}}$

$\sigma_i$: Experimental error and the interpolation error from GP

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau_f (T_c = 150 \text{ MeV})$</th>
<th>$\tau_f (T_c = 154 \text{ MeV})$</th>
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<tbody>
<tr>
<td>$a$</td>
<td>178.242</td>
<td>33.871</td>
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<td>$b$</td>
<td>16.175</td>
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<td>$c$</td>
<td>310.174</td>
<td>40.206</td>
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<td>$d$</td>
<td>1.329</td>
<td>0.967</td>
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<td>$A$</td>
<td>3.764</td>
<td>1.069</td>
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<tr>
<td>$B$</td>
<td>14.365</td>
<td>1.119</td>
</tr>
<tr>
<td>Parameters</td>
<td>Prior Range ((T_c = 150 \text{ MeV}))</td>
<td>Prior Range ((T_c = 154 \text{ MeV}))</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>(a)</td>
<td>([0.18, 4.5])</td>
<td>([0.26, 15.0])</td>
</tr>
<tr>
<td>(b)</td>
<td>([0.5, 1.2])</td>
<td>([0.1, 0.8])</td>
</tr>
<tr>
<td>(c)</td>
<td>([-2.0, 5.0])</td>
<td>([-4.0, 20.0])</td>
</tr>
<tr>
<td>(d)</td>
<td>([0.35, 0.7])</td>
<td>([0.25, 0.65])</td>
</tr>
<tr>
<td>(A)</td>
<td>([0.05, 0.16])</td>
<td>([0.05, 0.18])</td>
</tr>
<tr>
<td>(B)</td>
<td>([0.6, 3.0])</td>
<td>([0.6, 3.2])</td>
</tr>
</tbody>
</table>

TABLE I: The ranges of model parameters used in the prior distributions, for two different values of \(T_c\).