Heavy quarks probe the equation of state of QCD matter in heavy-ion collisions

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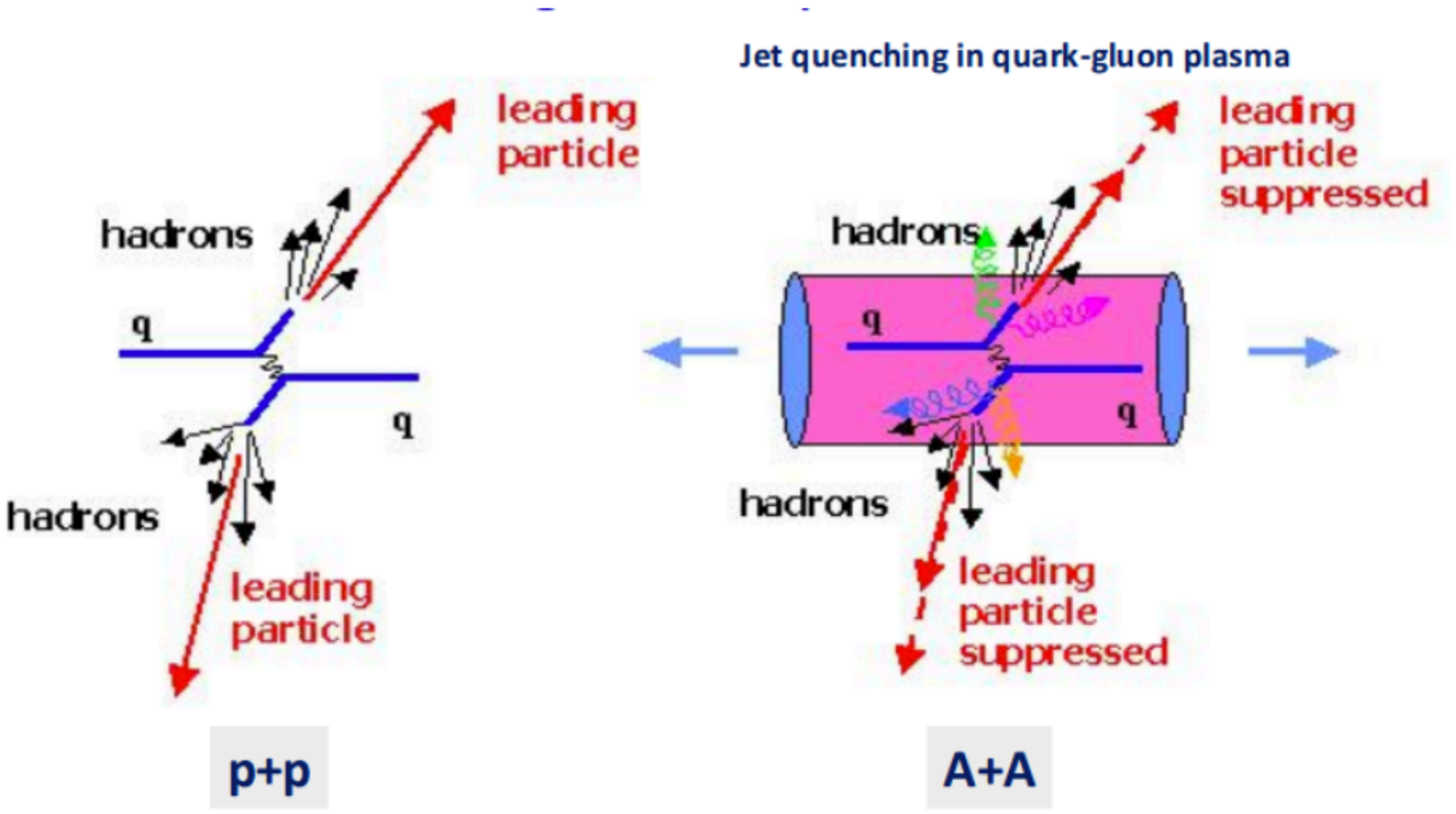
Based on Eur.Phys.J.C 82 (2022) 4, 350 and arXiv: 2304.xxxx

Hard Probe 2023, Aschaffenburg, Germany

Outline

- Introduction
- The quasi-particle linear Boltzmann transport (QLBT) model
- Bayesian extraction for QCD EoS and transport coefficients
- Summary and outlook

Hard Probe: Heavy Quark



Ideal clean probe:

$m_Q \gg \Lambda_{\mathrm{QCD}}$ $m_Q \gg T_{\mathrm{QGP}}$

Jet quenching:

Interaction with QGP medium

Gluon radiation

The nuclear modification factor R_{AA}:

$$R_{AA} = \frac{1}{N_{\mathrm{coll}}} \frac{dN^{AA}/d^2p_T dy}{dN^{pp}/d^2p_T dy}$$

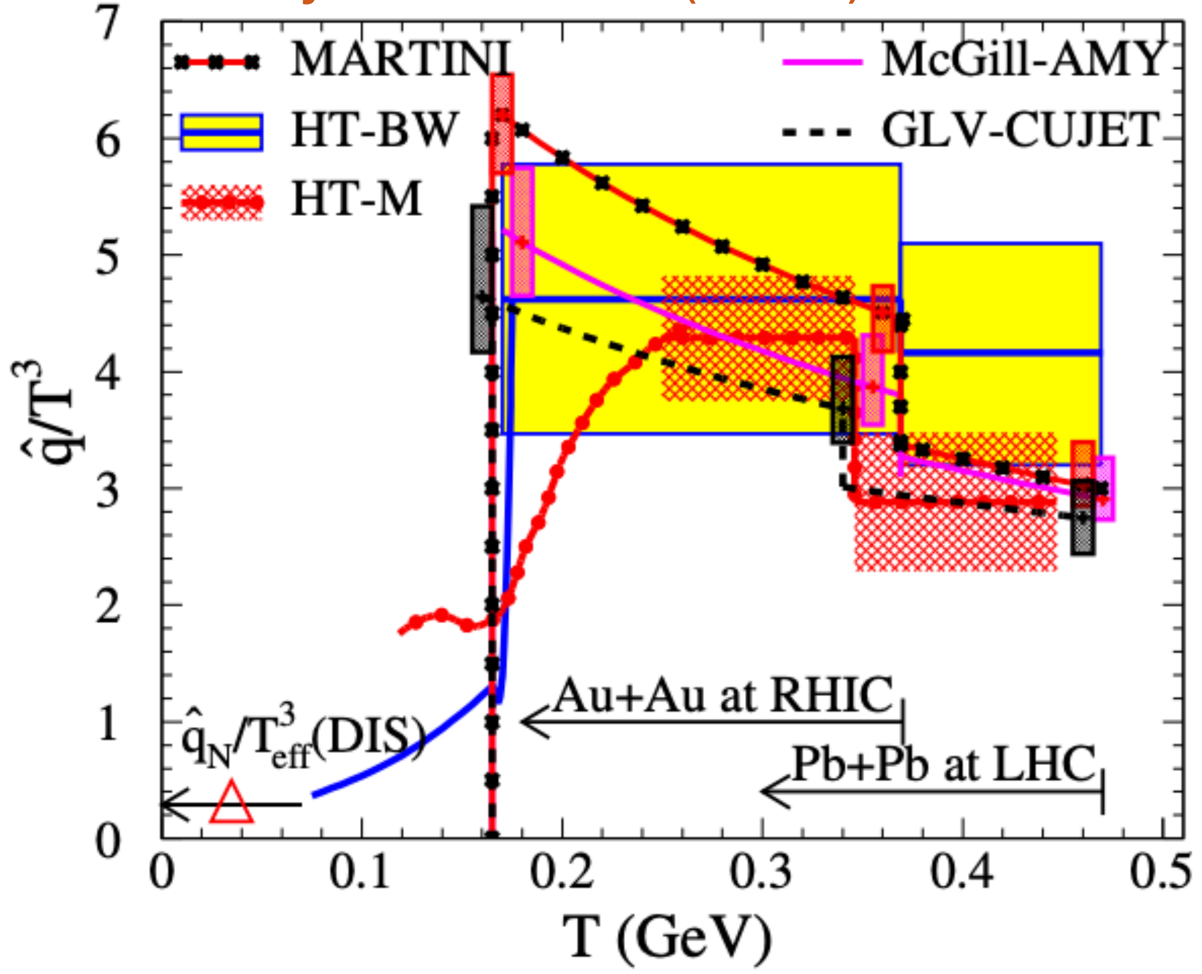
Constrain transport properties and EoS of QGP:

Transverse transport coefficient $\hat{q}$

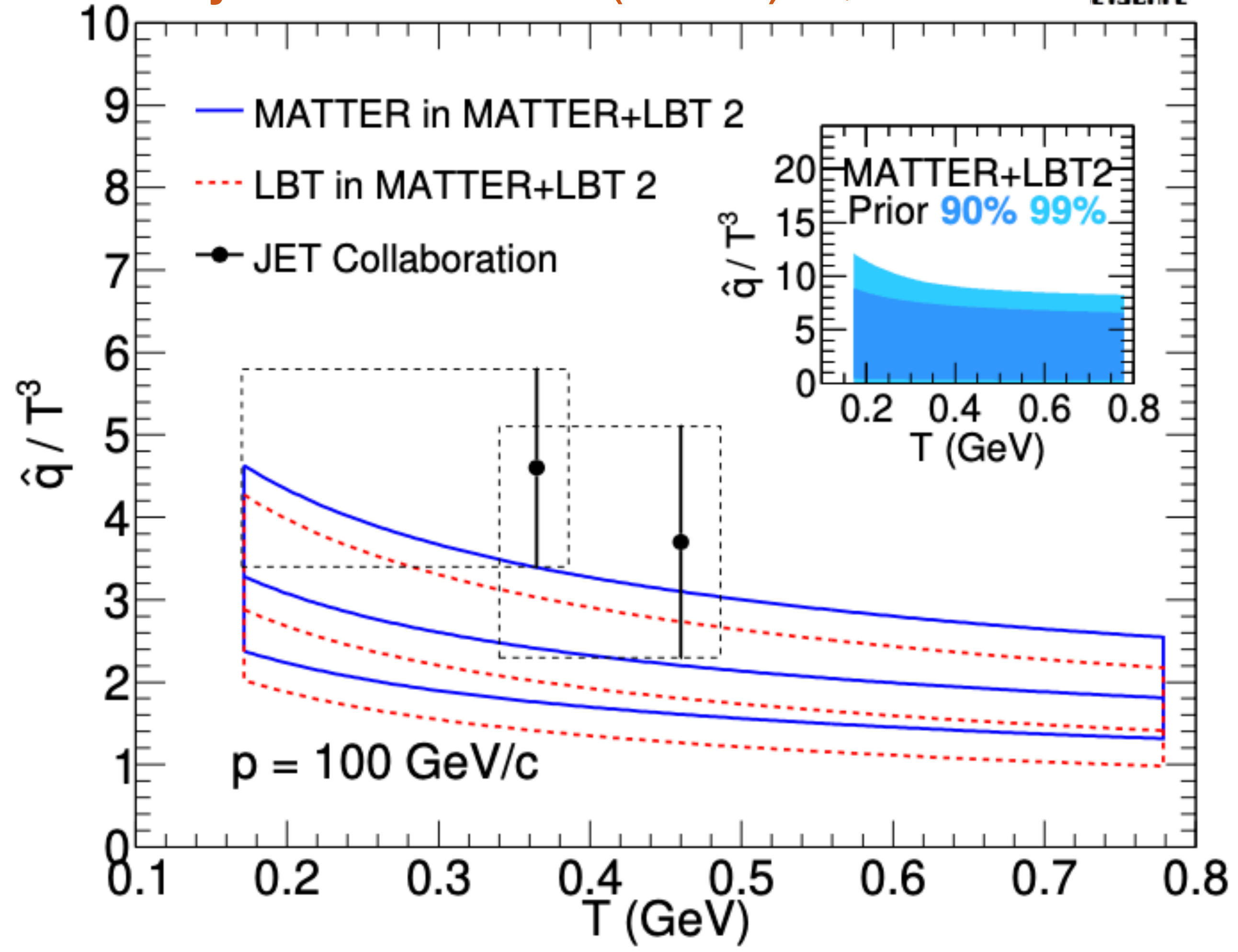
Spatial diffusion coefficient D_S

Extract transport coefficient and EoS

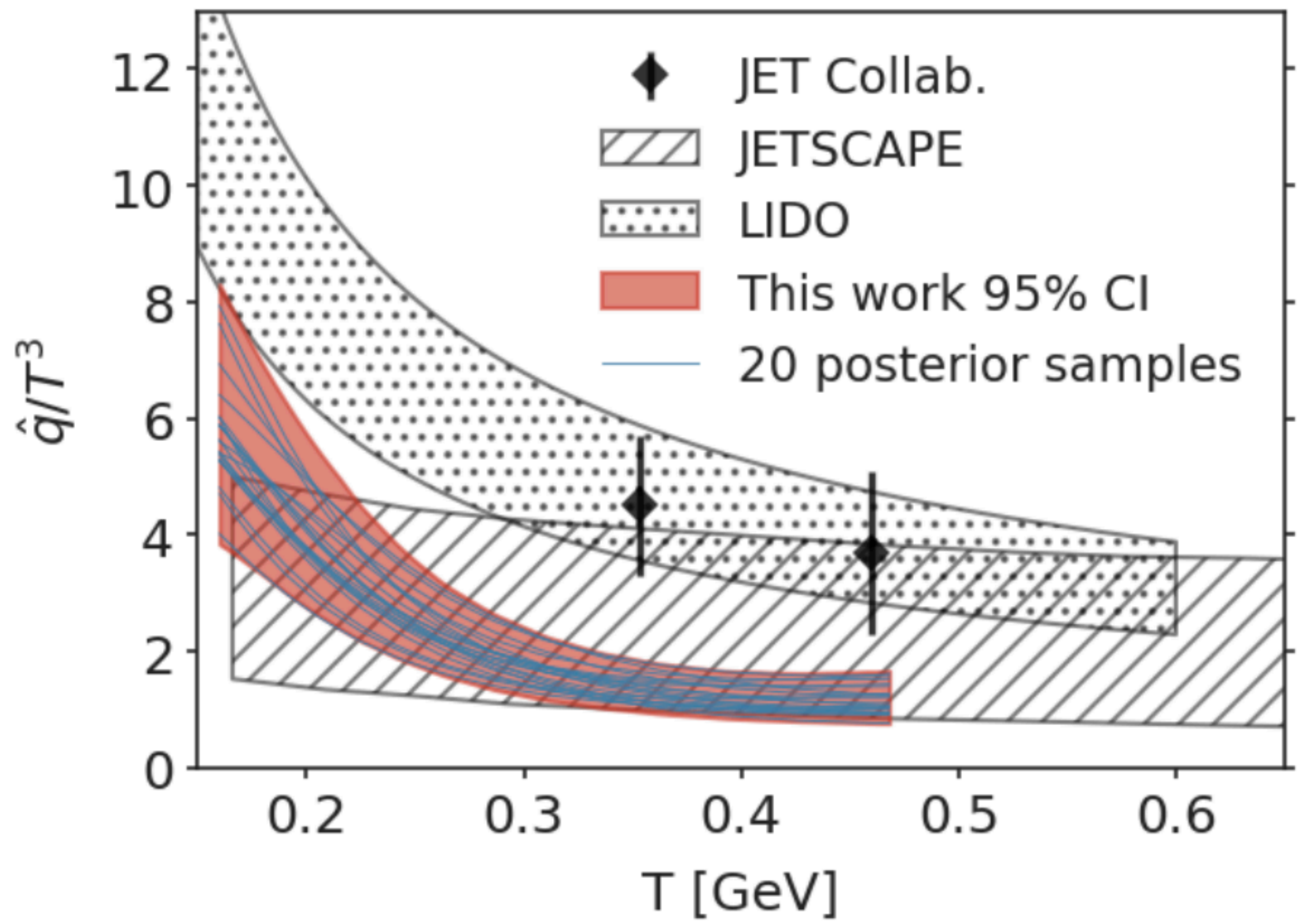
Phys.Rev.C 90 (2014) 1, 014909



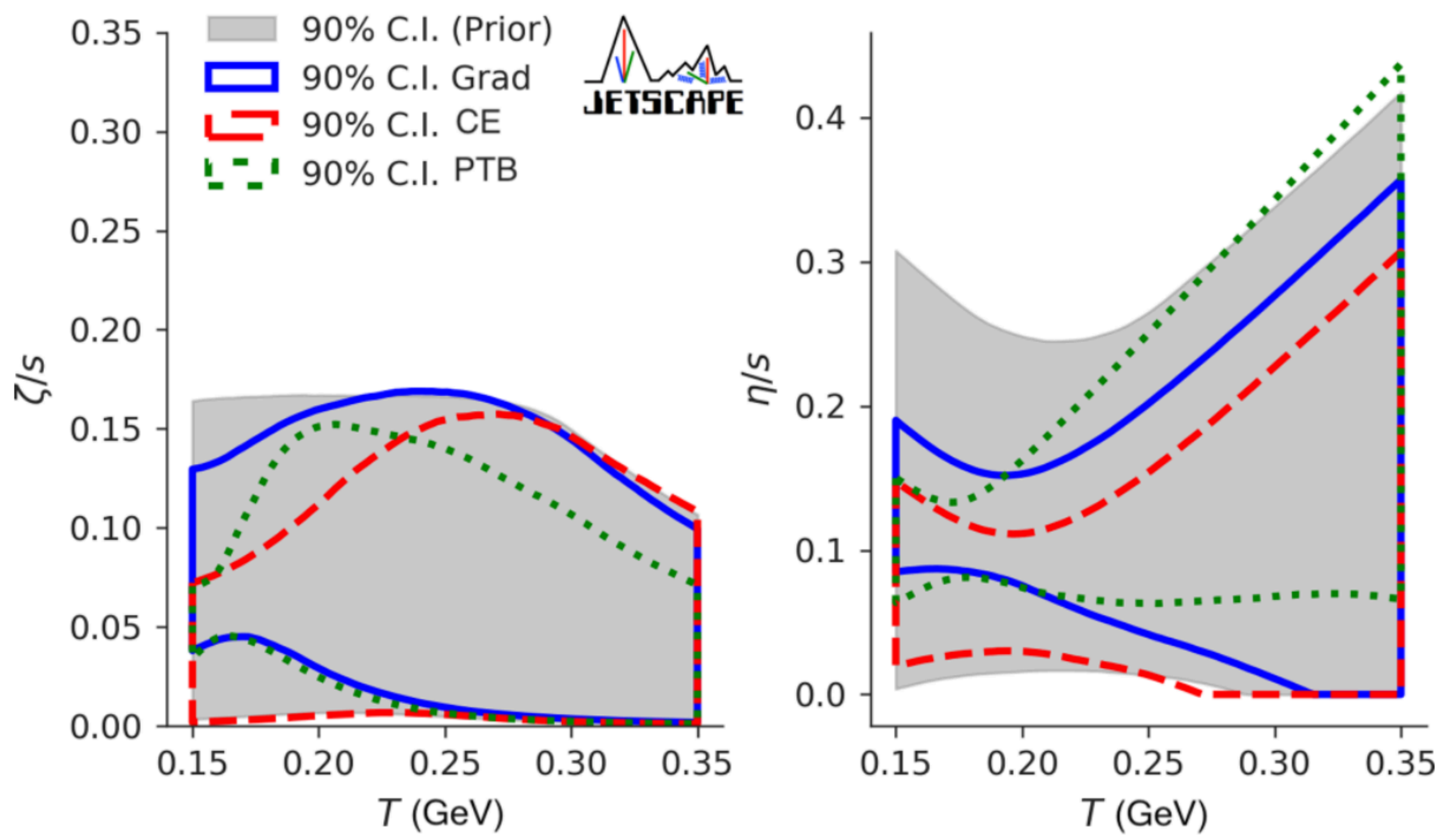
Phys.Rev.C 104 (2021) 2, 024905



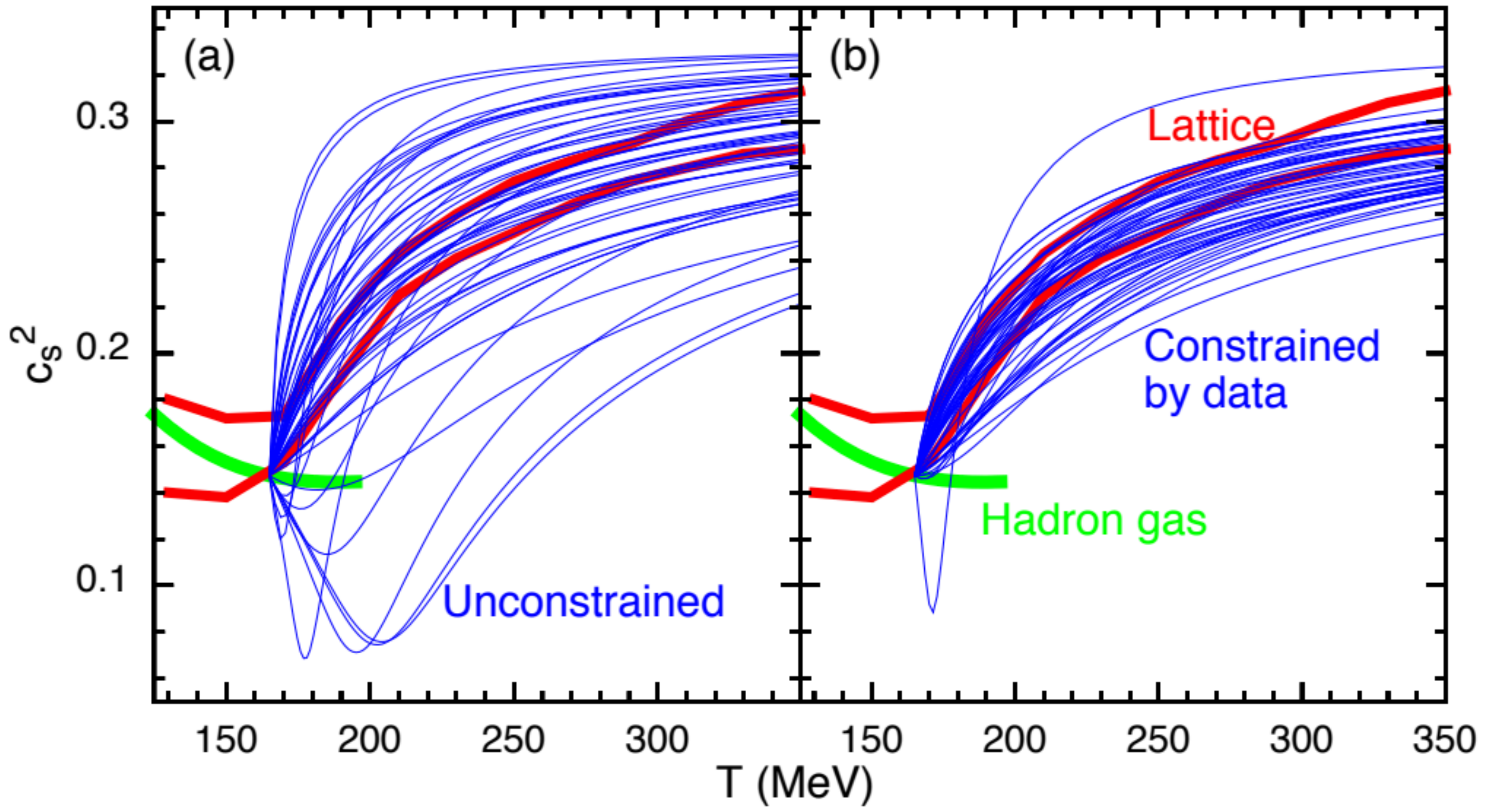
arxiv: 2206.01340



Phys.Rev.C 103 (2021) 5, 054904



Phys.Rev.Lett. 114 (2015) 202301



Jet(Bulk) transport coefficients and EoS are constrained separately by jet quenching (bulk observables).

In this work, a direct Bayesian extraction of the QGP EOS using heavy flavor observables based on QLBT model.

The quasi-particle linear Boltzmann transport (QLBT) model

QLBT model improve the linear Boltzmann transport (LBT) model for heavy quark evolution in the QGP by modeling the QGP as a collection of thermalized quasi-particles (quasi particle model, QPM). The temperature dependent effective masses of quarks and gluons among medium. Phys.Rev.D 84 (2011) 094004

$$m_q^2(T) = \frac{N_c^2 - 1}{8N_c} g^2(T) T^2$$

$$m_g^2(T) = \frac{1}{6}\left(N_c + \frac{1}{2}N_f\right) g^2(T) T^2$$

The temperature dependent coupling $g(T)$

$$g^2(T) = \frac{48\pi^2}{\left(11N_c - 2N_f\right) \ln\left[\frac{\left(aT/T_c + b\right)^2}{1 + ce^{-d\left(T/T_c\right)^2}}\right]}$$

T_C : the transition temperature between the QGP and the hadronic matter.

Paramater: a, b, c, d

From QPM to EoS

With the temperature-dependent thermal masses of quarks and gluons that are determined by parameters (a,b,c,d) via $g^2(T)$, one may calculate the pressure of the relativistic gas system as

$$P(T) = \sum_i \gamma_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_i(p,T)} f_i(p,T) - B(T)$$

$$E_i(p,T) = \sqrt{p^2 + m_i^2(T)}$$

$$g^2(T) = \frac{48\pi^2}{\left(11N_c - 2N_f\right) \ln\left[\frac{(aT/T_c + b)^2}{1 + ce^{-d(T/T_c)^2}}\right]}$$

Similarly, the energy density is obtained as

$$\epsilon(T) = \sum_i \gamma_i \int \frac{d^3p}{(2\pi)^3} E_i(p,T) f_i(p,T) + B(T)$$

The entropy density

$$s(T) = [\varepsilon(T) + P(T)]/T$$

Standard process: Lattice QCD EOS $\rightarrow g^2(T) \rightarrow$ observables

Inverse question: observables $\rightarrow g^2(T) \rightarrow$ QCD EOS ?

Linear Boltzmann transport model

Boltzmann equation: $p_1 \cdot \partial f_1 (x_1, p_1) = E_1 \left(C_{\mathrm{el}} [f_1] + C_{\mathrm{inel}} [f_1] \right)$

Elastic scattering:

$$\Gamma_{12\to34}(\vec{p}_1) = \int d^3k w_{12\to34}(\vec{p}_1, \vec{k}) = \frac{\gamma_2}{2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4}$$
$$\times f_2(\vec{p}_2) \left[1 \pm f_3(\vec{p}_3)\right] \left[1 \pm f_4(\vec{p}_4)\right] S_2(s,t,u)$$
$$\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \left| M_{12\to34} \right|^2$$

$$P_{\mathrm{el}} = 1 - e^{-\Gamma_{\mathrm{el}} \Delta t}$$

LO pQCD

Inelastic scattering

$$\langle N_g \rangle = \Gamma_g \Delta t = \Delta t \int dx dk_\perp^2 \frac{dN_g}{dx dk_\perp^2 dt}$$

$$P^a_{\mathrm{inel}} = 1 - e^{-\langle N^a_g \rangle}$$

Higher-twist formalism

Elastic+inelastic: $P^a_{\mathrm{el}} = 1 - e^{-(\Gamma^a_{\mathrm{el}} + \Gamma^a_{\mathrm{inel}})\Delta t}$

QGP background: (3+1)-D CLVisc hydrodynamics model

Linear Boltzmann transport model

Boltzmann equation: $p_1 \cdot \partial f_1 \left(x_1, p_1\right) = E_1 \left(C_{el} \left[f_1\right] + C_{\text{inel}} \left[f_1\right]\right)$

Elastic scattering:

$$\Gamma_{12\to34}\left(\vec{p}_1\right) = \int d^3k w_{12\to34}\left(\vec{p}_1, \vec{k}\right) = \frac{\gamma_2}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\int \frac{d^3p_3}{(2\pi)^3 2E_3}\int \frac{d^3p_4}{(2\pi)^3 2E_4}$$
$$\times f_2\left(\vec{p}_2\right)\left[1 \pm f_3\left(\vec{p}_3\right)\right]\left[1 \pm f_4\left(\vec{p}_4\right)\right] S_2(s,t,u)$$
$$\times (2\pi)^4 \delta^{(4)}\left(p_1 + p_2 - p_3 - p_4\right)\left|M_{12\to34}\right|^2,$$

$$P_{\text{el}} = 1 - e^{-\Gamma_{\text{el}}\Delta t}$$

Inelastic scattering

$$\langle N_g \rangle = \Gamma_g \Delta t = \Delta t \int dx dk_\perp^2 \frac{dN_g}{dx dk_\perp^2 dt}$$

$$P^a_{\text{inel}} = 1 - e^{-\langle N^a_g \rangle}$$

Elastic+inelastic: $P^a_{\text{el}} = 1 - e^{-\left(\Gamma^a_{\text{el}} + \Gamma^a_{\text{inel}}\right)\Delta t}$

QGP background: (3+1)-D CLVisc hydrodynamics model

Kinematic cut:

$S_2(s,t,u) = \theta\left(s \geq 2\mu_D^2\right)\theta\left(t \leq -\mu_D^2\right)\theta\left(u \leq -\mu_D^2\right)$

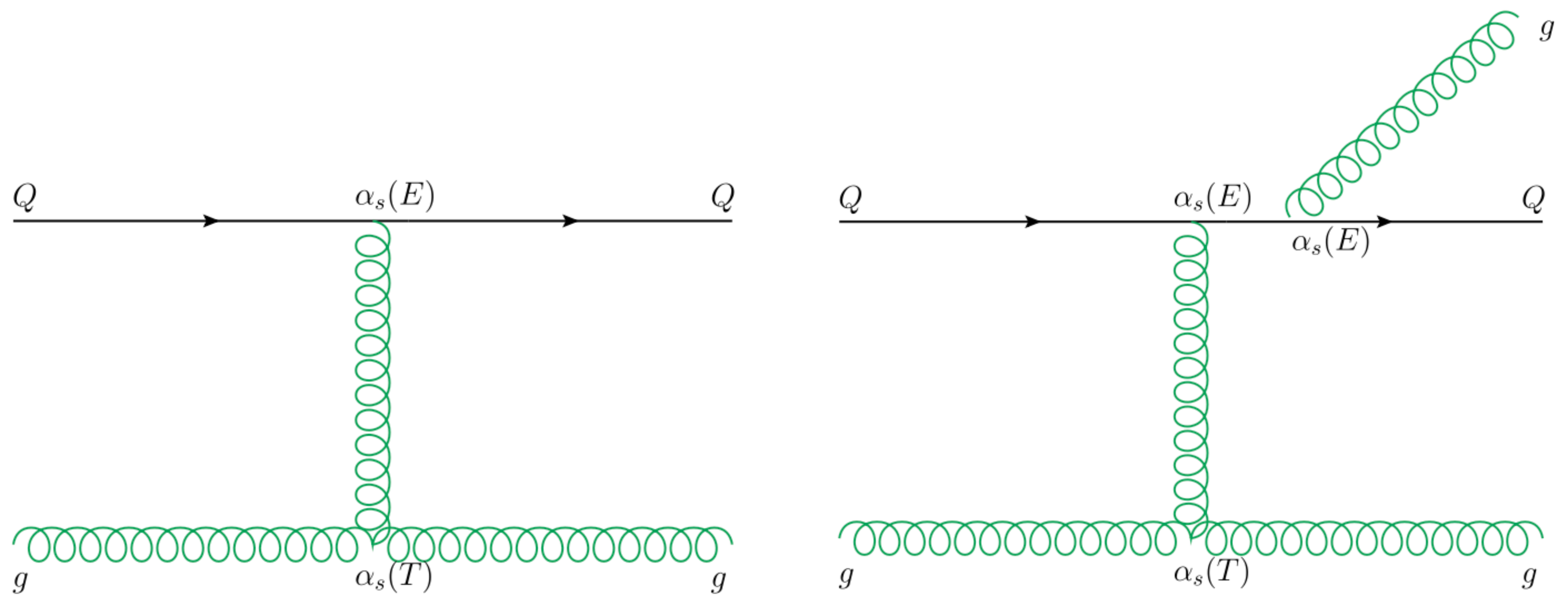
Debye mass $\mu_{\text{D}}^2(T) = 2m_g^2(T)$

Thermal distribution: $f_2\left(\vec{p}_2\right),\ f_4\left(\vec{p}_4\right)$

$E_i(p,T) = \sqrt{p^2 + m_i^2(T)}$

Two types of coupling vertices

$cg \to cg$



$$\alpha_s(T) = g^2(T)/(4\pi)$$

The vertices that connect to the thermal patrons:

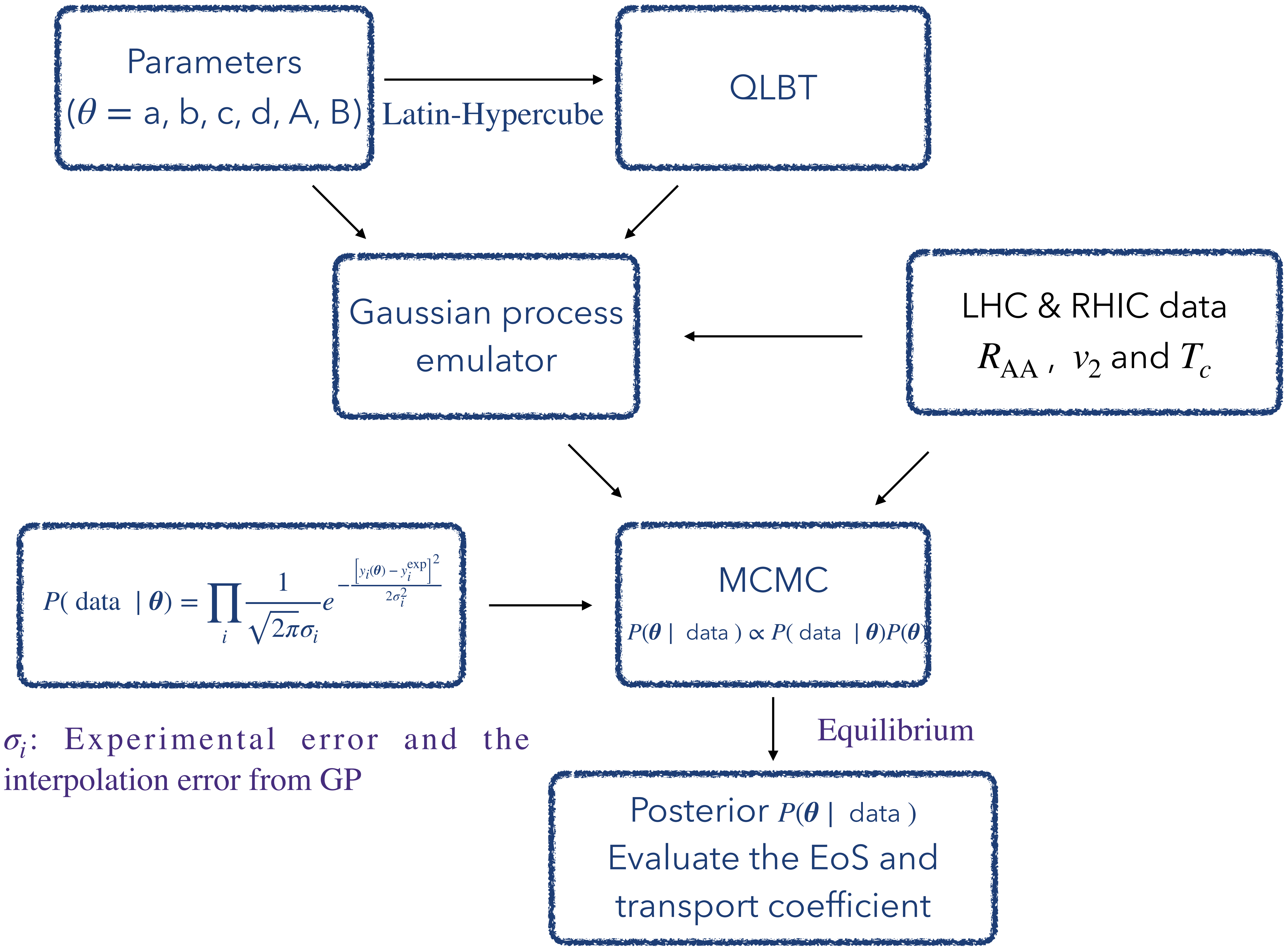
$$g^2(T) = \frac{48\pi^2}{\left(11N_c - 2N_f\right) \ln \left[\frac{\left(aT/T_c + b\right)^2}{1 + ce^{-d(T/T_c)^2}}\right]}$$

Paramater: a, b, c, d, A, B

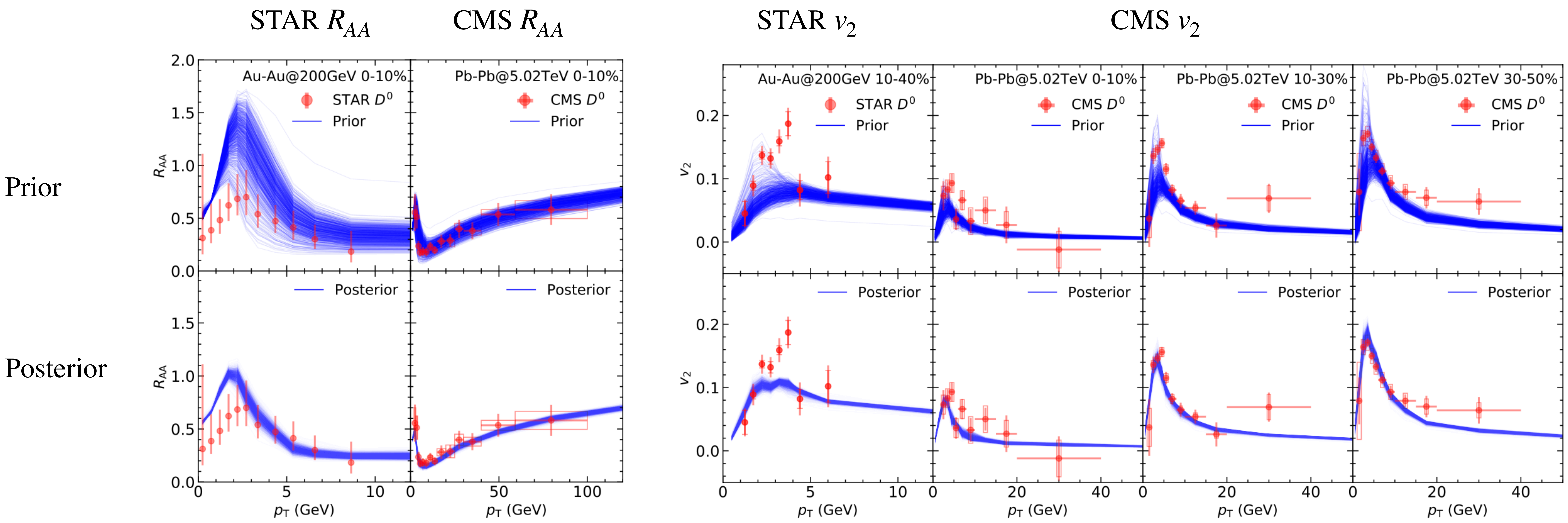
The vertices that connect to the heavy quarks:

$$g^2(E) = \frac{48\pi^2}{\left(11N_c - 2N_f\right) \ln \left[\left(AE/T_c + B\right)^2\right]}$$

Bayesian analysis

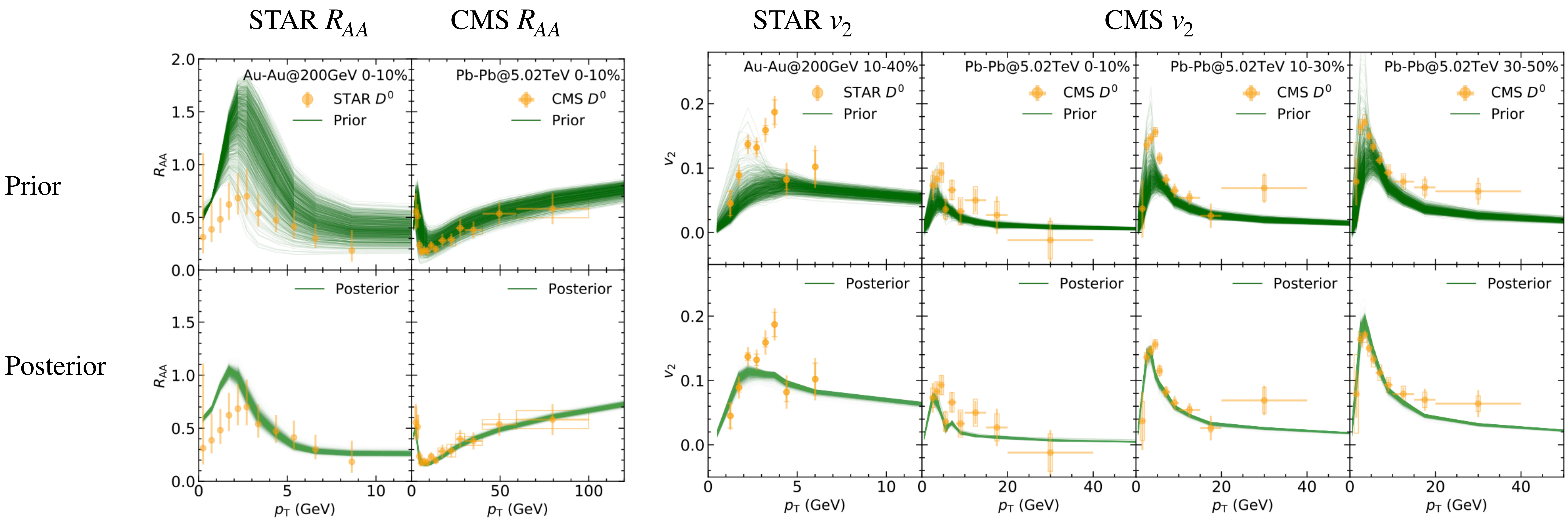


Calibration of the QLBT calculation (WB $T_c = 150$ MeV)



One observes that after the Bayesian calibration, our QLBT model is able to provide a reasonable description of the D meson observables in heavy-ion collisions.

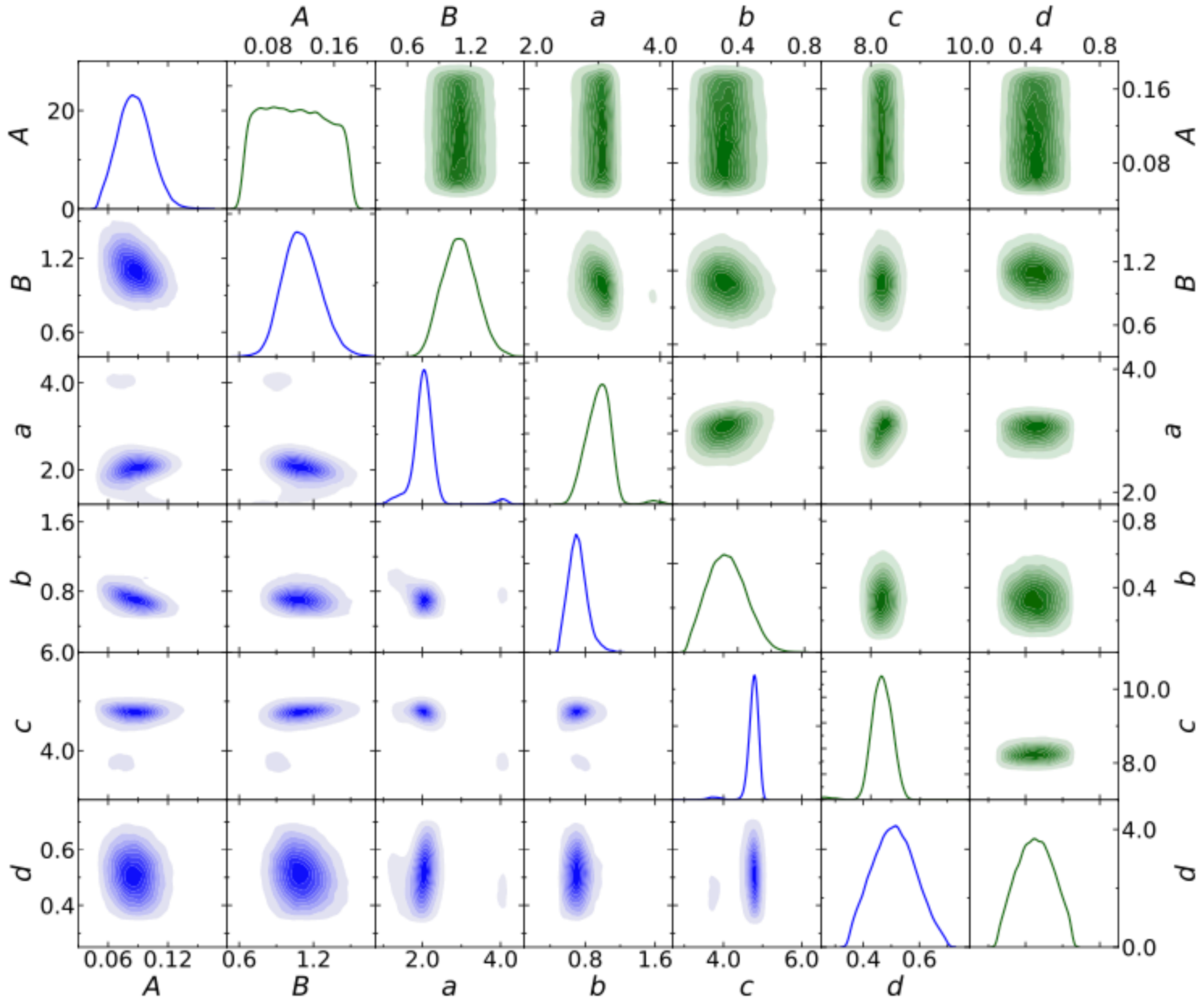
Calibration of the QLBT calculation (HotQCD $T_c = 154$ MeV)



One observes that after the Bayesian calibration, our QLBT model is able to provide a reasonable description of the D meson observables in heavy-ion collisions.

Similar results can be obtained for T_c = 154 MeV compared to T_c = 150 MeV.

Posterior distributions of the model parameters

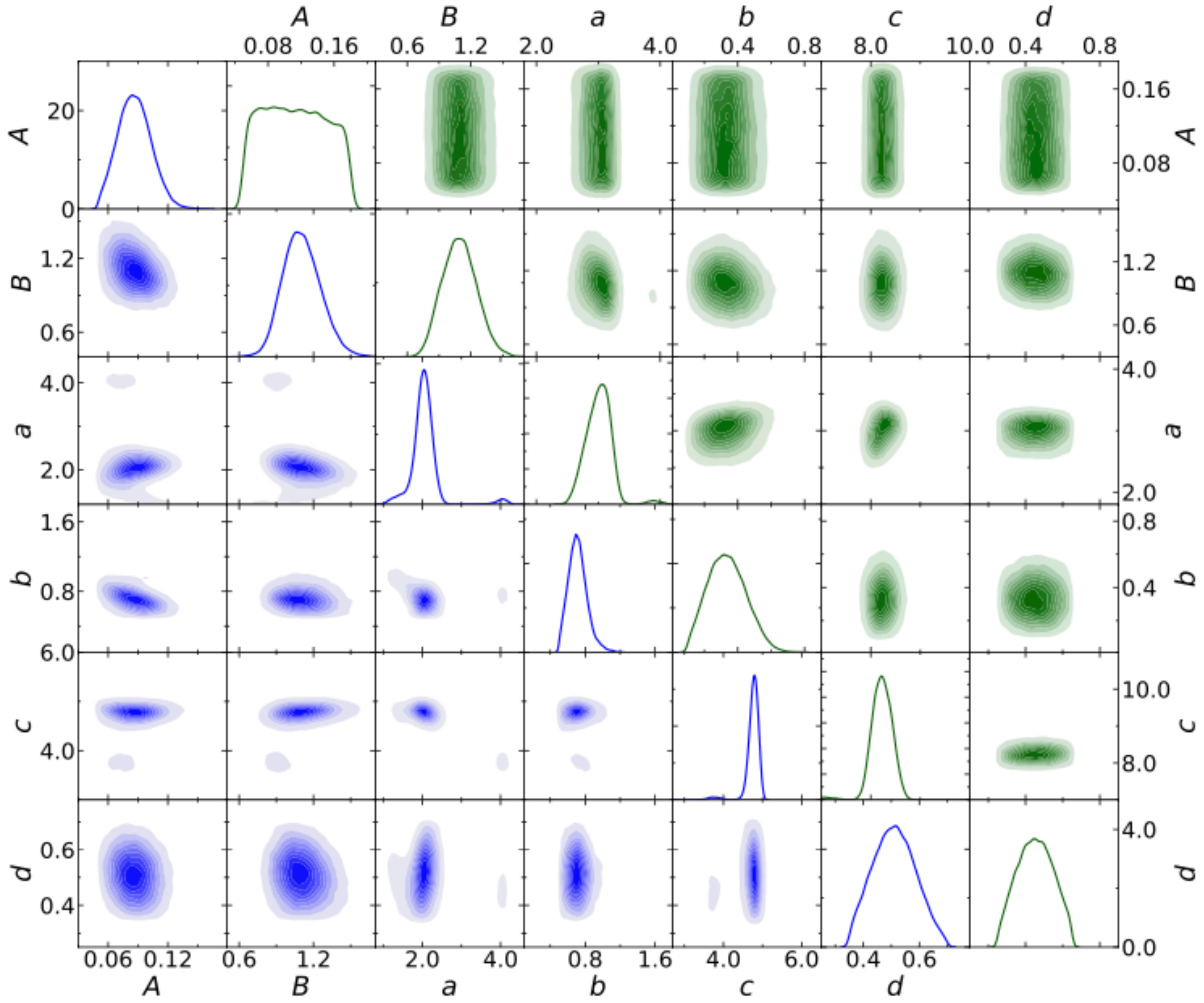


HotQCD $T_c = 154$ MeV

WB $T_c = 150$ MeV

Reasonable constraints on these model parameters have been obtained for same T_c .

Posterior distributions of the model parameters



HotQCD $T_c = 154$ MeV

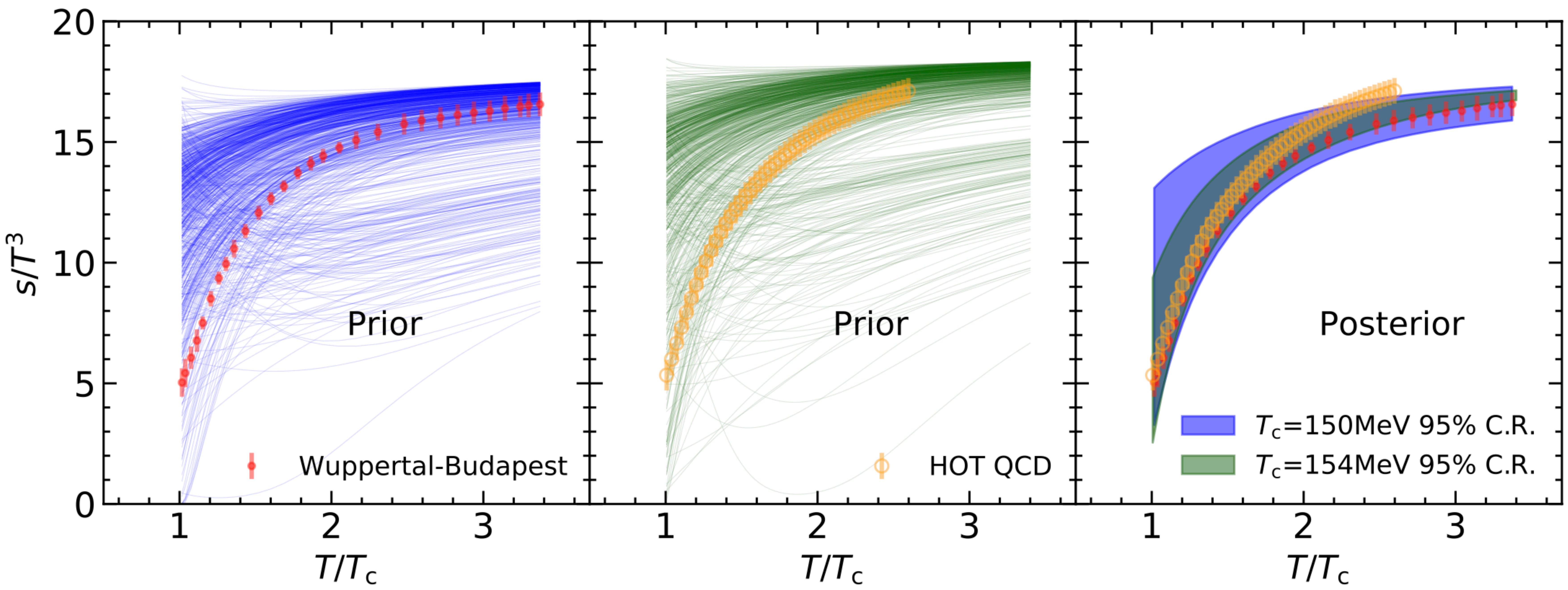
WB $T_c = 150$ MeV

mean	WB	HotQCD
A	1.1060	1.0936
B	0.0867	0.1131
a	2.0634	3.0117
b	0.7148	0.3362
c	4.7047	8.2149
d	0.5105	0.4524

Reasonable constraints on these model parameters have been obtained for same T_c .

The extracted parameters is sensitive to T_c.

The extract EoS (T^3- rescaled entropy density)

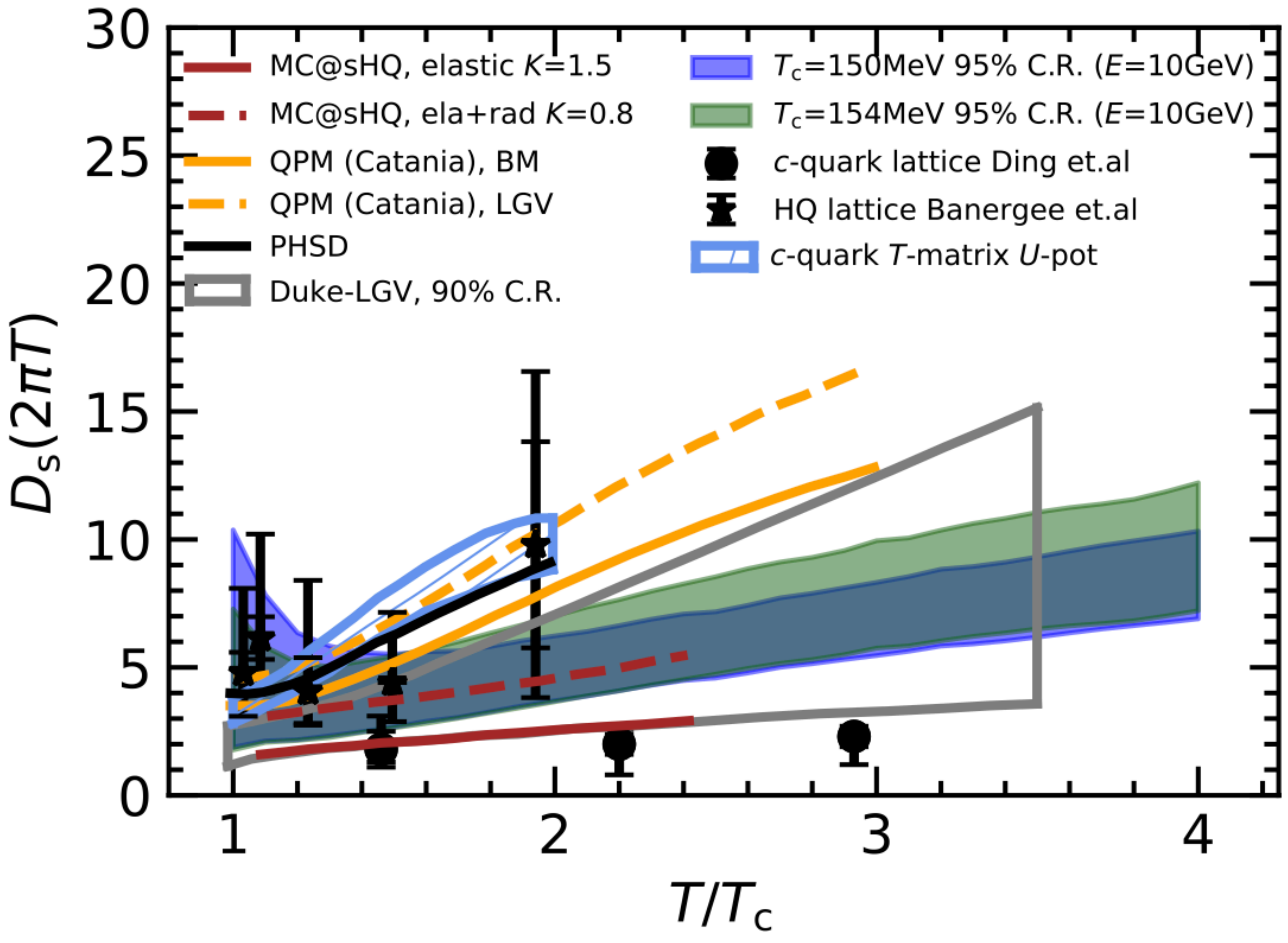


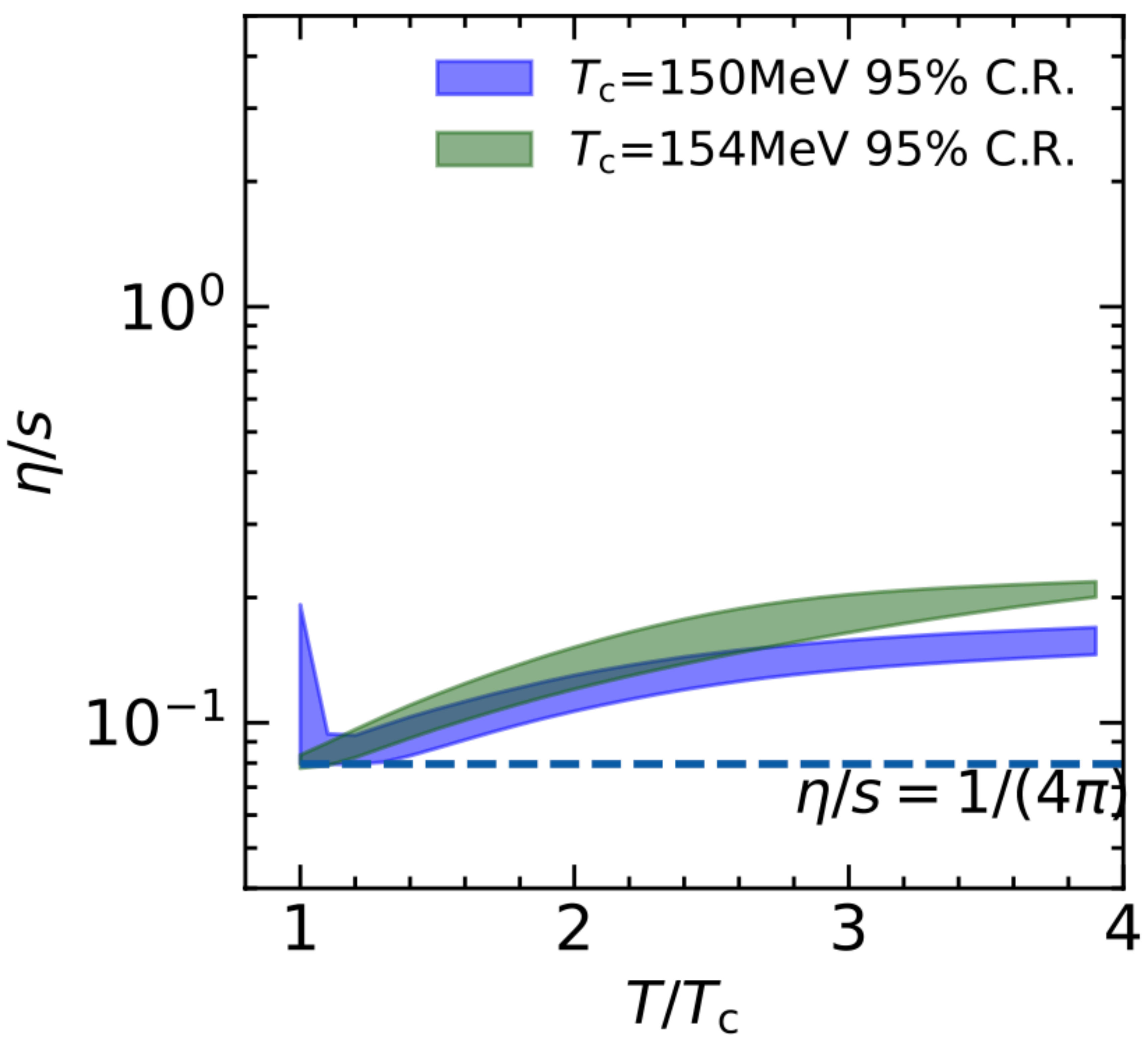
One observes that the heavy flavor observables do provide constraints on the QGP EoS.

The extracted EoS with T_c = 150 MeV agrees well with the WB lattice data that shares the same T_c.

Some deviation can be observed from HQ data: a larger T_c, terminates early, higher entropy density.

The transport coefficient





Summary

We have carried out a Bayesian analysis of the experimental data on D meson spectra and anisotropy v_2 at both RHIC and LHC based on QLBT.

We realized a simultaneous constraint on the properties of the QGP and heavy quark probes:
- The QGP EOS we extract is consistent with the lattice QCD results.
- The heavy quark diffusion coefficient we obtain agrees with results from other model and lattice calculations.

Outlook

Incorporate a more extensive set of parameters, e.g. phase transition temperature T_c.

Involve a broader range of jet observables and soft hadron spectra in order to accomplish the goal of constraining the properties of nuclear matter using hard probes.

Thank you for you attention!

Back up

Backup: Shear viscosity

The formulas for the viscosities η are derived for a quasi-particle description with bosonic and fermionic constituents S. Plumari, W. M. Alberico, V. Greco, C. Ratti, Phys. Rev. D84 (2011)

$$\eta = \frac{1}{15T}\sum_i d_i \int \frac{d^3p}{(2\pi)^3}\tau_i \frac{\vec{p}^4}{E_i^2} f_i(1 \mp f_i)$$

$d^3p = p^2 dp \sin\theta d\theta d\psi$.
In relaxtime approximation, shear viscosity depends on collision relax time τ_i given by (HTL):

$$\tau_q^{-1} = 2\frac{N_C^2-1}{2N_C}\frac{g^2T}{8\pi}\ln\frac{2k}{g^2}, \quad \tau_g^{-1} = 2N_C\frac{g^2T}{8\pi}\ln\frac{2k}{g^2},$$

where g is the coupling obtained and k is a parameter which is fixed by requiring that τ_i yields a minimum of one for the quantity $4\pi\eta/s$.

- k if fixed to have a minimum $\eta/s = 1/4\pi$. For WB: k=23.3 and For HQ: k=22.7

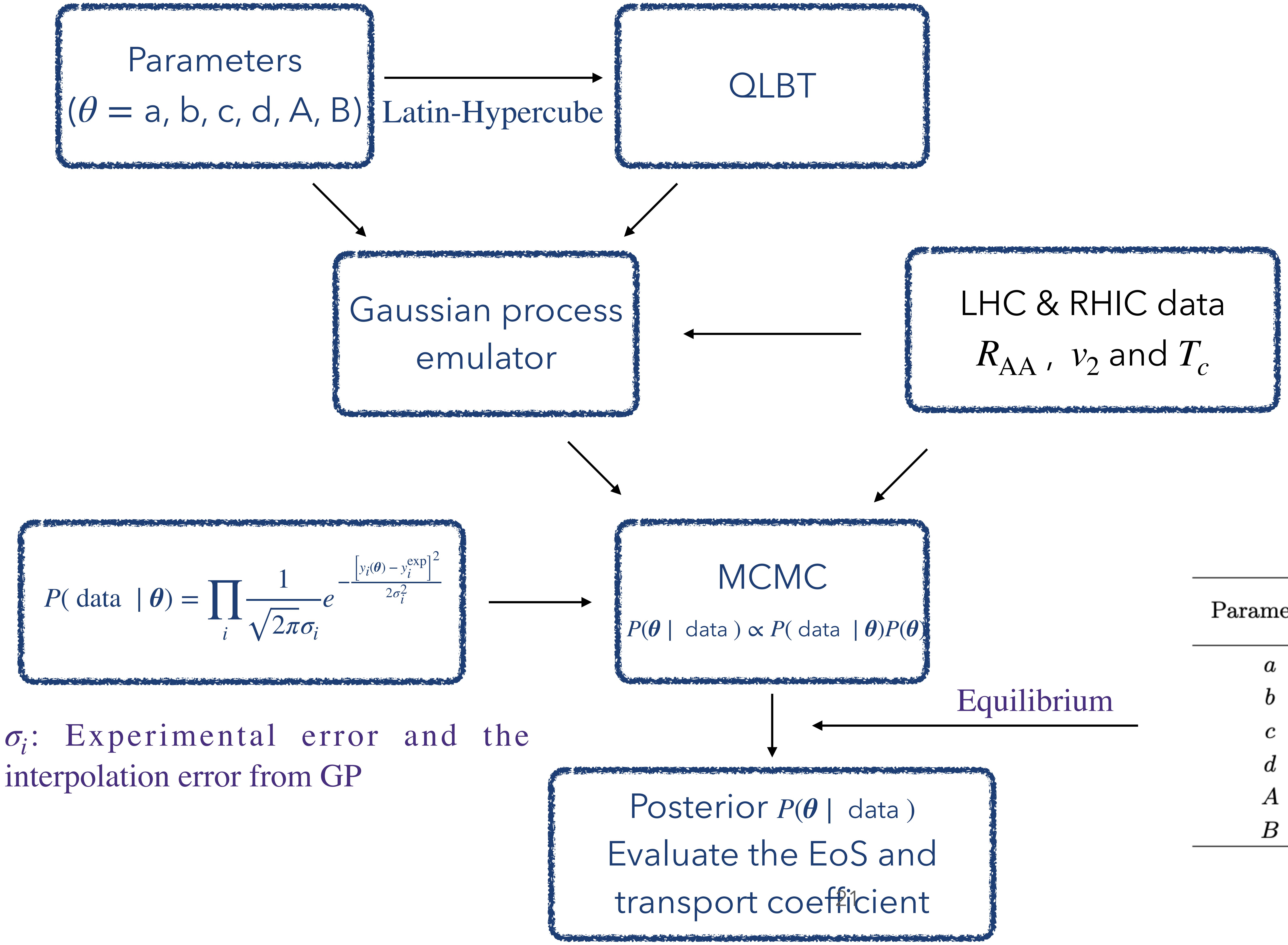
Spatial diffusion coefficient

$$\hat{q} = \sum_{bcd} \frac{\gamma_b}{2E_a} \int \prod_{i=b,c,d} \frac{d^3 p_i}{2E_i (2\pi)^3} f_b \left| \mathscr{M}_{ab\to cd} \right|^2 S_2(\hat{s}, \hat{t}, \hat{u})$$

$$\times (2\pi)^4 \delta^4 \left(p_a + p_b - p_c - p_d\right) \left[\vec{p}_c - \left(\vec{p}_c \cdot \hat{p}_a\right) \hat{p}_a\right]^2$$

$$D_s(2\pi T) = 8\pi T^3 / \hat{q}$$

Bayesian analysis



Parameters	τ_f ($T_c = 150$ MeV)	τ_f ($T_c = 154$ MeV)
a	178.242	33.871
b	16.175	2.187
c	310.174	40.206
d	1.329	0.967
A	3.764	1.069
B	14.365	1.119

Parameters	Prior Range ($T_c = 150$ MeV)	Prior Range ($T_c = 154$ MeV)
a	[0.18, 4.5]	[0.26, 15.0]
b	[0.5, 1.2]	[0.1, 0.8]
c	[-2.0, 5.0]	[-4.0, 20.0]
d	[0.35, 0.7]	[0.25, 0.65]
A	[0.05, 0.16]	[0.05, 0.18]
B	[0.6, 3.0]	[0.6, 3.2]

TABLE I: The ranges of model parameters used in the prior distributions, for two different values of T_c.

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