Condensation and early time dynamics in QCD plasmas

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based on work in preparation with J. Berges, K. Boguslavski, T. Butler, J. M. Pawlowski

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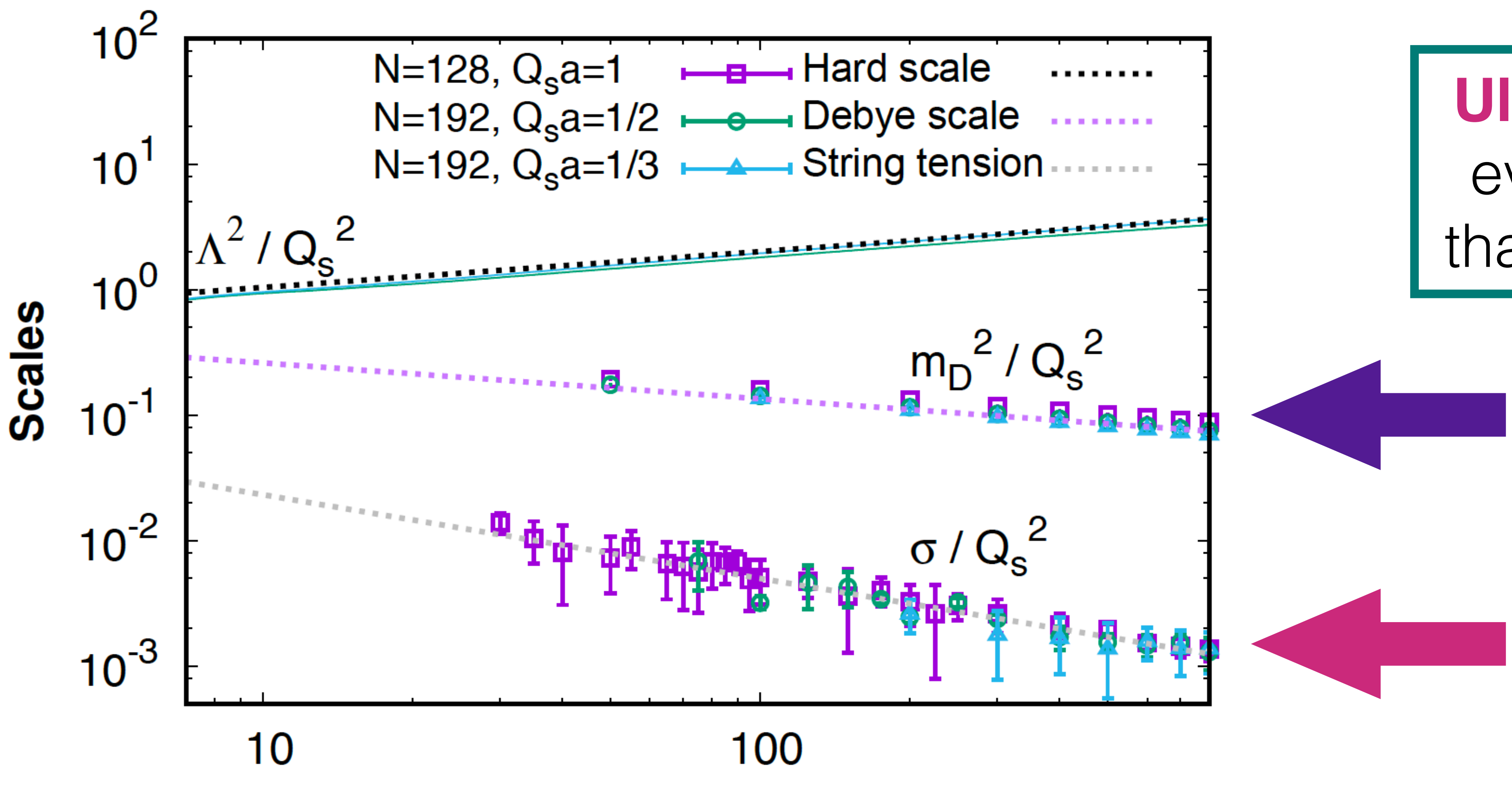
Overview

1. Motivation & Introduction
2. IR order parameters
3. Results: condensation and volume scaling
4. Conclusion and outlook

Thermalization of QGP

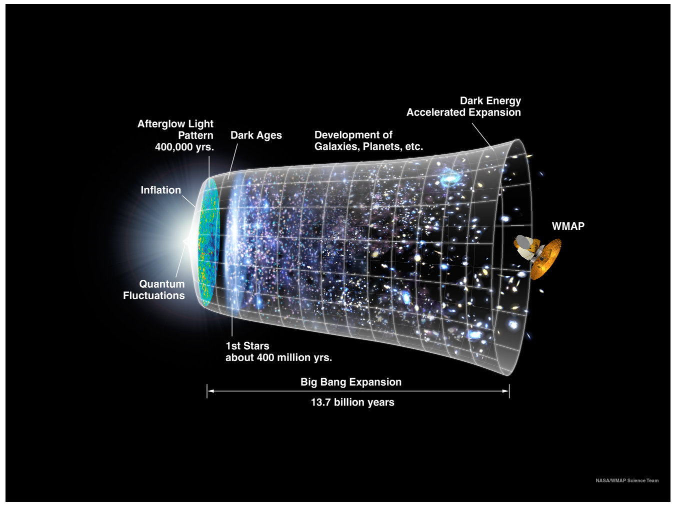
Dynamical separation of scales established for quark gluon plasma far from equilibrium



M. Mace, S. Schlichting, R. Venugopalan, PRD (2016)

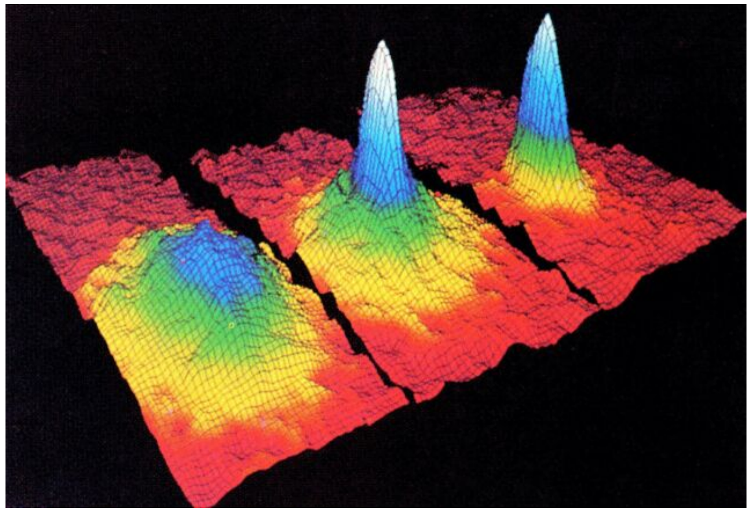
Ultrasoft scale evolves faster than **soft** scale!

“Condensation” and initial over-occupation



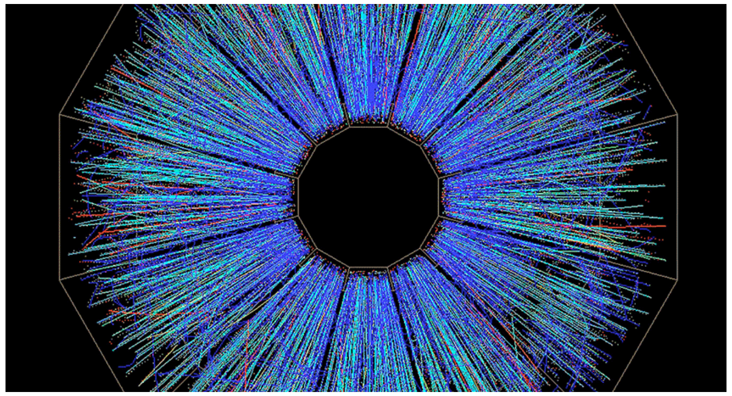
NASA/WMAP

inflation dynamics



JILA/NIST

ultracold Bose gas

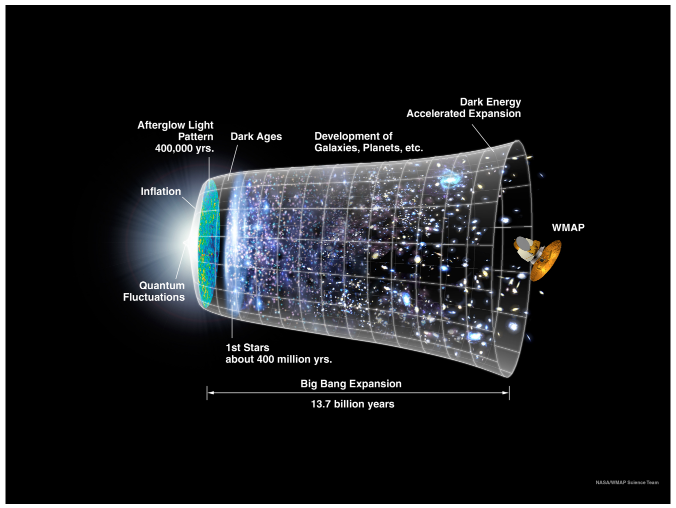


STAR

heavy ion collisions

- far from equilibrium
- instabilities
- “overpopulation”

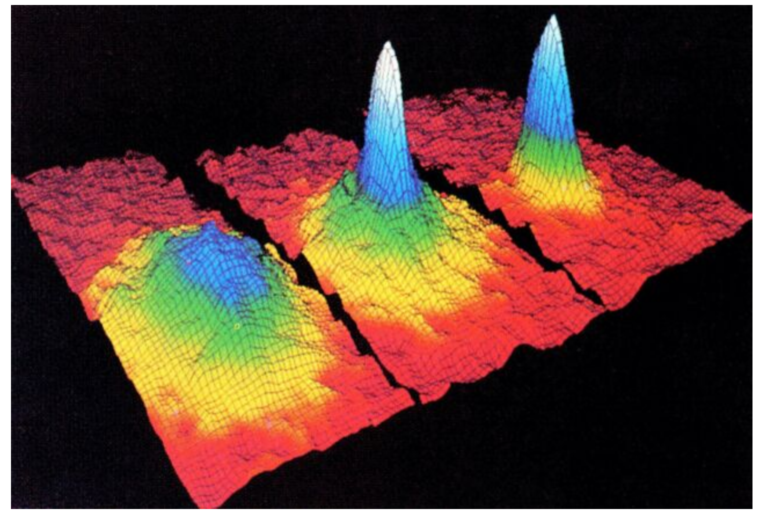
“Condensation” and initial over-occupation



NASA/WMAP

inflation dynamics

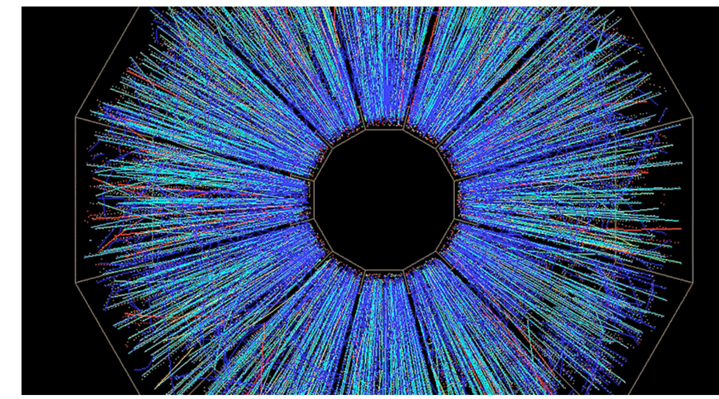
relativistic scalar inflaton



JILA/NIST

ultracold Bose gas

non-relativistic Bose field



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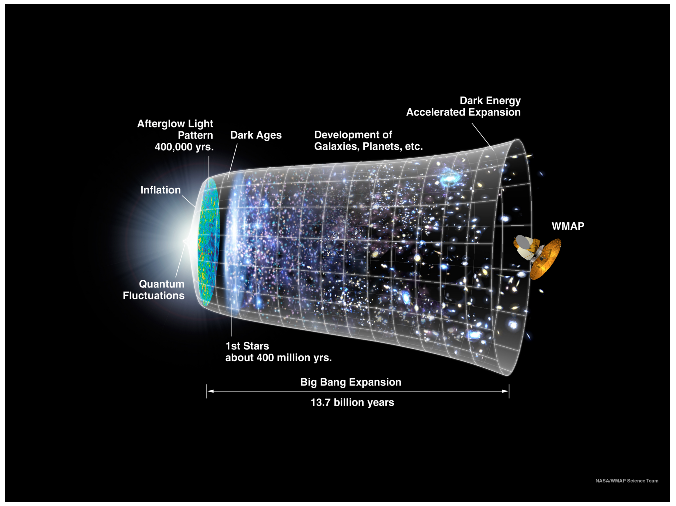
heavy ion collisions

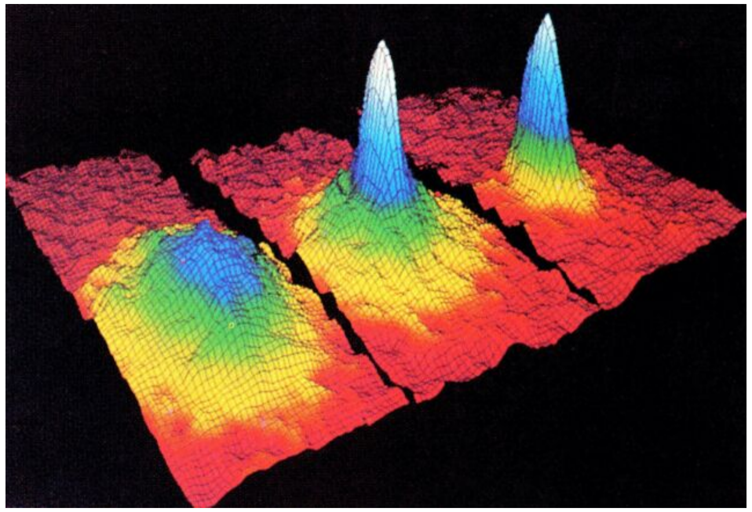
Scalar fields — Genuine BEC:
showed with volume scaling

“Condensation” and initial over-occupation



NASA/WMAP

inflation dynamics



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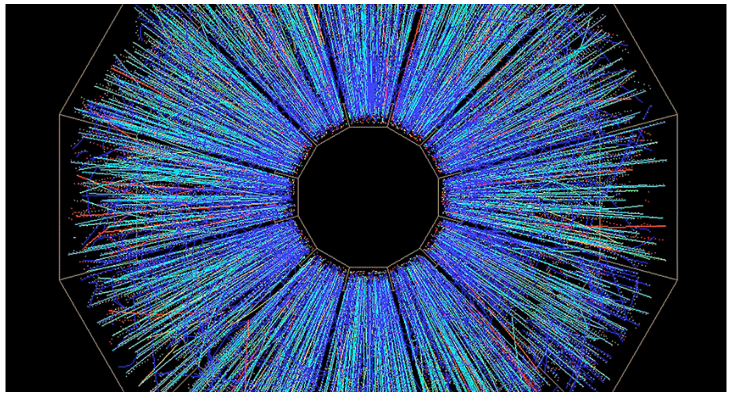

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heavy ion collisions

Condensation in a non-Abelian gauge theory?

—> Look for the build up of a macroscopic zero mode that scales with volume

Objectives

Basic questions:

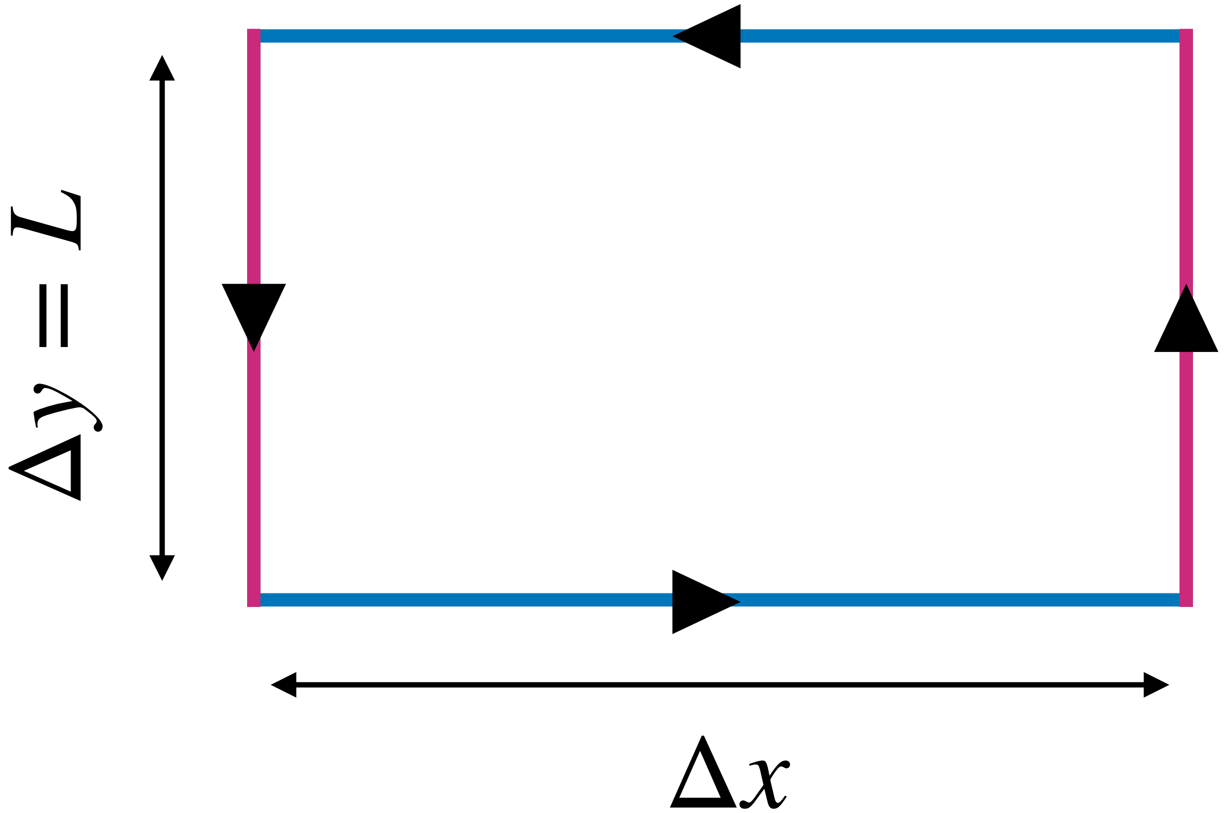
- What's driving dynamics in the deep IR?
- What's the best order parameter for condensation in non-Abelian gauge theories?
- What are the consequences of possible condensation in heavy ion collisions?

Infrared order parameters

Spatial Wilson loop

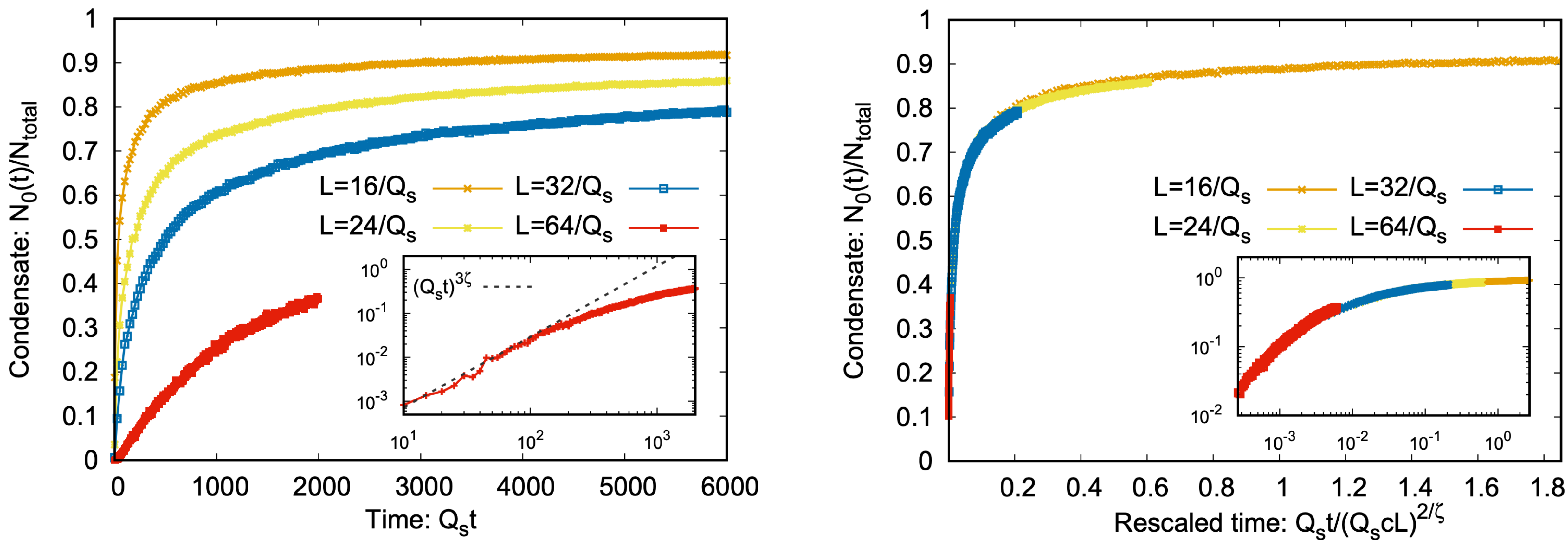
Infrared excitations of non-Abelian gauge theories are extended objects, which can be computed from Wilson loops

Wilson loops are a gauge invariant quantity that captures long distance behavior of gauge fields



$$W[t, \Delta x, \Delta y] = \frac{1}{N_c} \mathrm{tr} \mathscr{P} e^{-ig \int_{\mathscr{C}} A_i(t, \mathrm{x}) dx_i}$$

Spatial Wilson loop and condensation



J. Berges, K. Boguslavski, M. Mace, J. M. Pawlowski, PRD (2020)

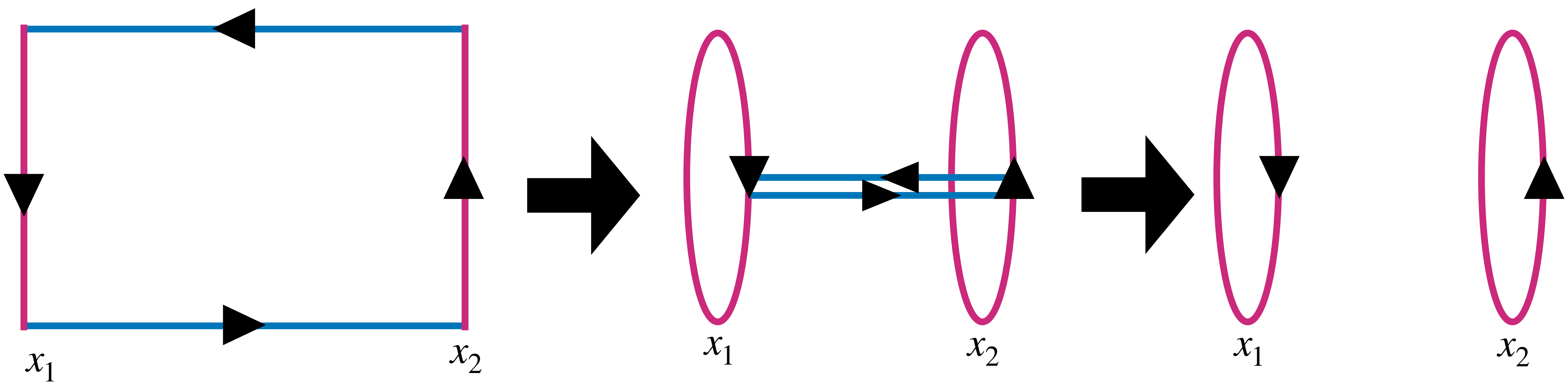
$$\frac{N_0}{N_{\text{total}}} = \frac{1}{V}\int_0^L d^d x \, \langle W(\Delta x, L, t) \rangle$$

$$t_{\text{cond}} \sim L^{2/\zeta}$$

$$\zeta = 0.54 \pm 0.04$$

Spatial "Polyakov loop" correlator

$$\langle W(\Delta x, L, t)\rangle \approx \langle P(x_1, L, t)P^\dagger(x_2, L, t)\rangle$$



Rectangular Wilson loop:
Extended object/non-local

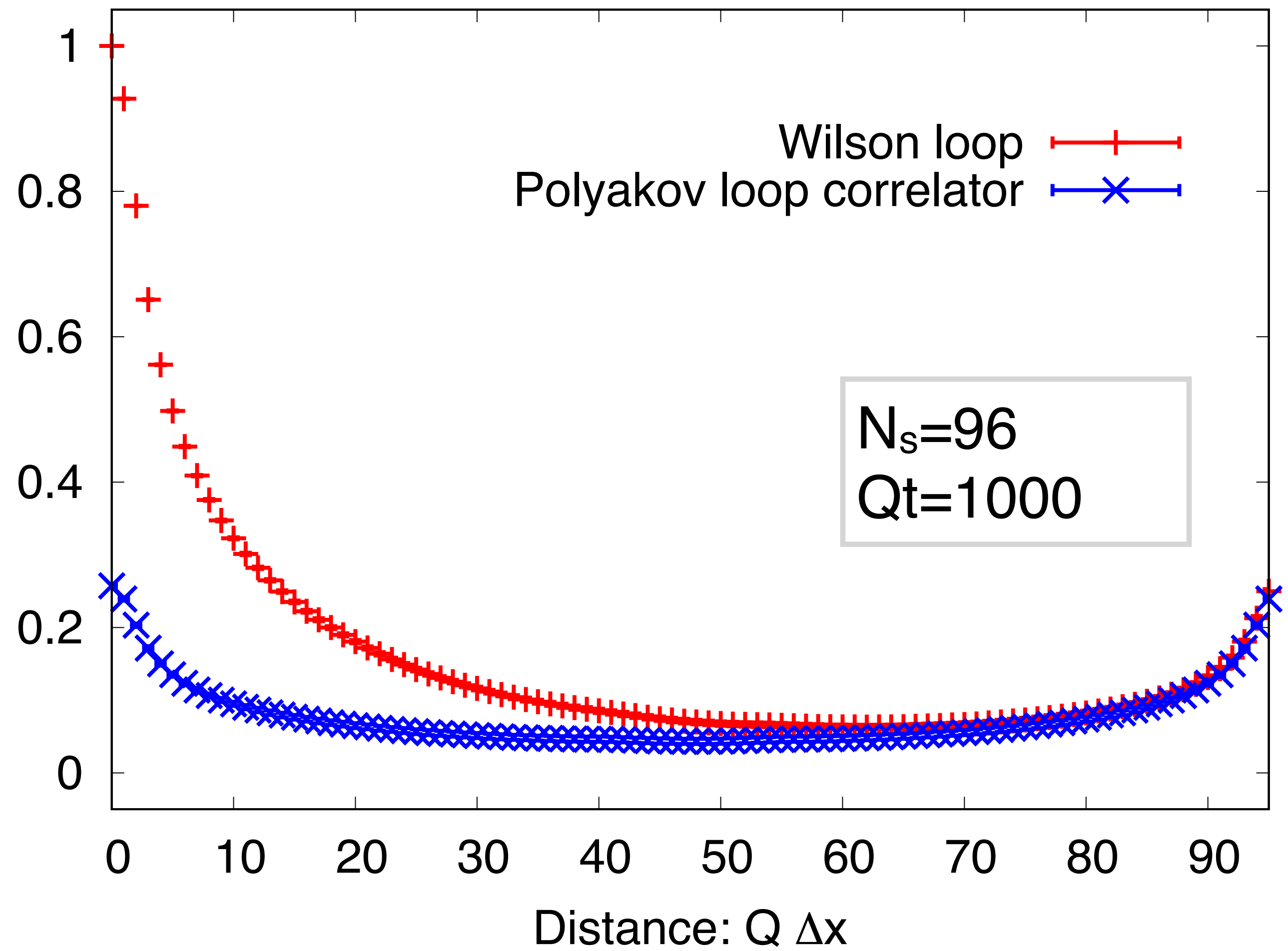
"Polyakov loop" correlator: Local correlation function of non-local loops

$$P_i(t, \mathrm{x}) = \frac{1}{N_c}\mathrm{tr}\mathscr{P}e^{-ig\int_0^L A_i(t,\mathrm{x})dx_i}$$

Spatial Polyakov loop

Wilson loop and spatial Polyakov loop correlator show same dynamics at long distances

Spatial Polyakov loop correlator is *symmetric, local, gauge invariant*



Criteria met, but can we do better?

Holonomous eigenvalue field

The untraced spatial Polyakov loop can be rewritten:

$$\tilde{P}_i(x) \equiv \mathscr{P} e^{-ig\int_0^L A_i(t,x)dx_i} = e^{i\phi_i(x)}$$

such that $\phi_i(x) = \phi_i^a(x) t^a, \qquad t^a = \sigma^a/2, \qquad \tilde{P}_i \in SU(N)$

We can then define a gauge invariant scalar field φ

via the relation:

$$\frac{1}{N_c}\mathrm{tr}\tilde{P}_i \equiv \cos\varphi_i$$

this is the holonomous eigenvalue field

Diagonalization of *P(x)*

The untraced Polyakov loop transforms covariantly under the gauge transformation:

$$\tilde{P}_i(x) \rightarrow U(x)\tilde{P}_i(x)U^\dagger(x), \qquad U(x) \in SU(N)$$

It follows for the algebra field:

$$\phi_i(x) \rightarrow U(x)\phi_i(x)U^\dagger(x), \qquad U(x) \in SU(N)$$

This *SU(N)* rotation diagonalizes the quantity. Hence, this fixes the gauge freedom to a diagonalization gauge such that

$$\phi_i(x) = \varphi_i(x)t^i$$

Condensation and volume scaling

Characteristics of over-occupied QGP

Gluons produced in heavy ion collisions are expected to have typical momenta on order of saturation scale $Q_s \sim 1/\alpha_s$

—> Over-occupation of gluons at time $t \sim 1/Q_s$

Running gauge coupling is small: $\alpha_s(Q_s) \ll 1$

System is considered strongly correlated due to high gluon occupancy

Non-perturbative quantum problem can be mapped to classical-statistical lattice gauge theory

Numerical implementation

Real-time classical-statistical lattice simulations for $SU(N=2)$ gauge theory discretized on 3-dimensional periodic spatial lattice of length L and spacing a_s:

- Fields are initialized as a superposition of transversely polarized gluon fields
- Characteristic initial over-occupation is translated into energy density and fluctuations to initialize LGT evolution
- For real-time evolution, classical Heisenberg EOMs are solved in the temporal axial gauge: $A_0 = 0$

Comparison of order parameters

We compare the fraction of the volume that is correlated for both the spatial Polyakov loop and the scalar holonomous eigenvalue field

The observable for the Polyakov loop is the connected correlation function:

$$\left\langle PP^{\dagger}\right\rangle_{c}(t, L, \Delta x) = \left\langle PP^{\dagger}\right\rangle(\Delta x) - \left\langle P\right\rangle\left\langle P^{\dagger}\right\rangle(\Delta x)$$

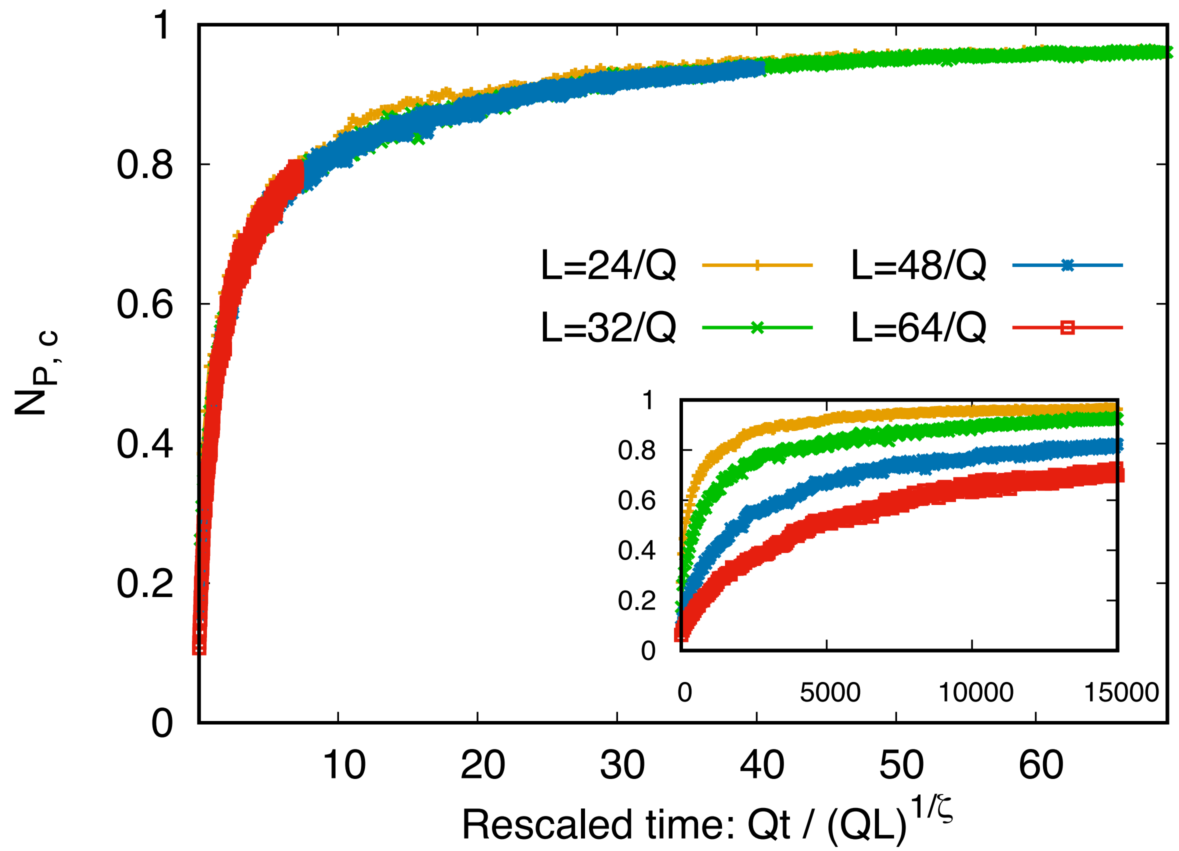
And for the holonomous eigenvalue field:

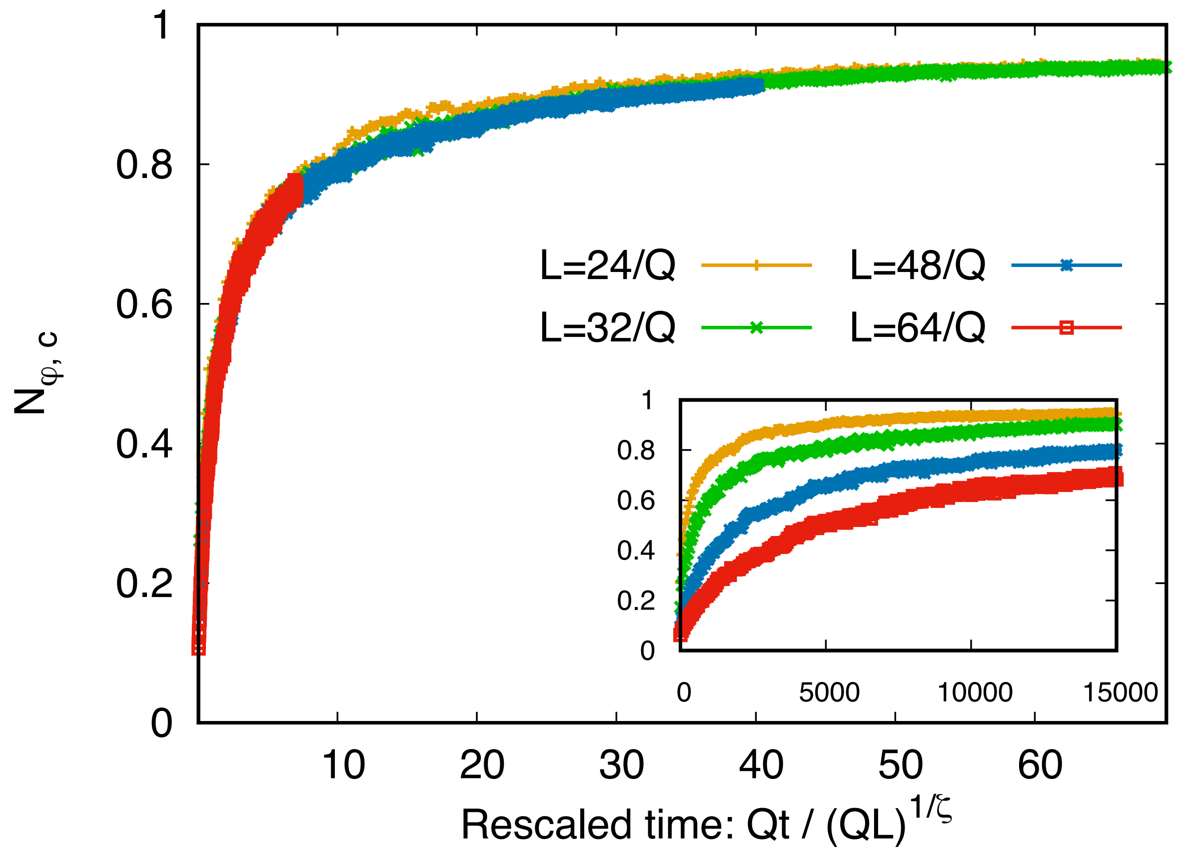
$$\langle \varphi\varphi^{*}\rangle_{c}(t, L, \Delta x) = \langle \varphi\varphi^{*}\rangle(\Delta x) - \langle \varphi\rangle\,\langle \varphi^{*}\rangle(\Delta x)$$

Condensate fractions

$$N_{\mathcal{O},c} \equiv \frac{1}{L}\int_0^L d\Delta x \frac{< \mathcal{O}\mathcal{O}^\dagger >_c (t, \Delta x, L)}{< \mathcal{O}\mathcal{O}^\dagger >_c (t, \Delta x = 0, L)}$$

$$t_{\text{cond}} \sim L^{1/\zeta}$$

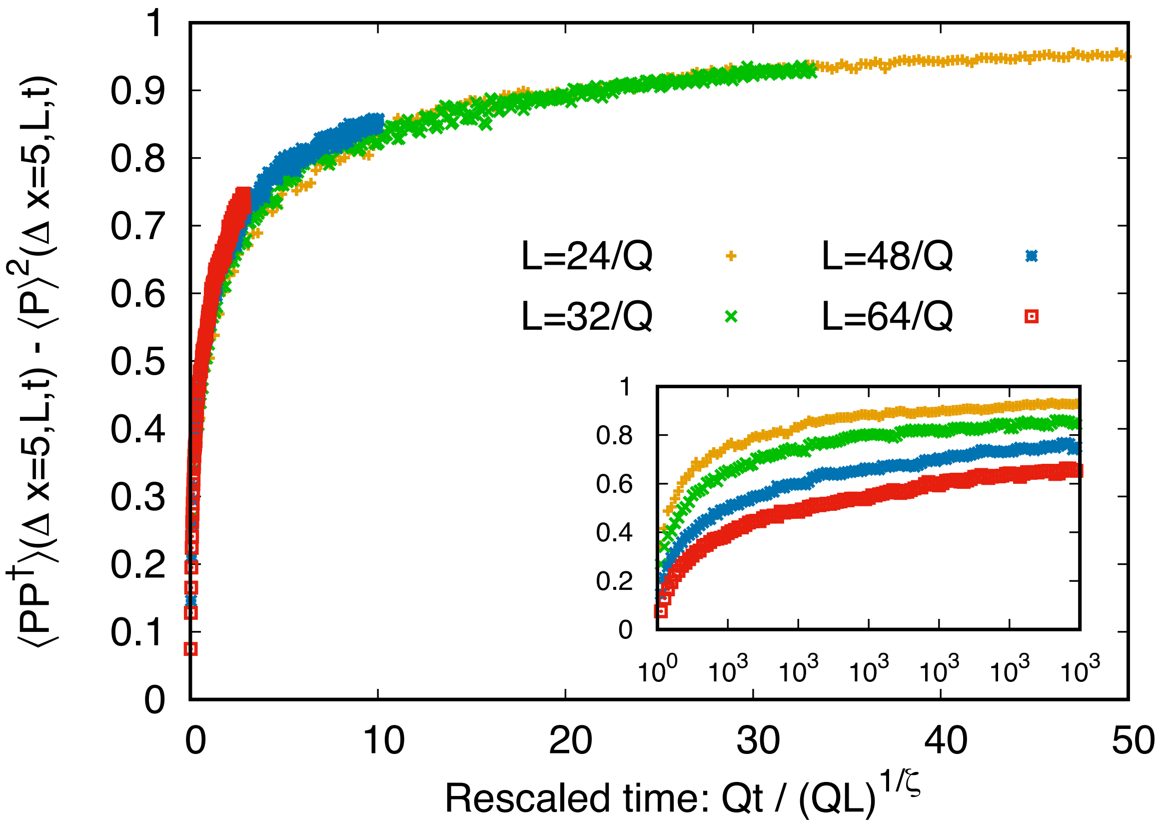


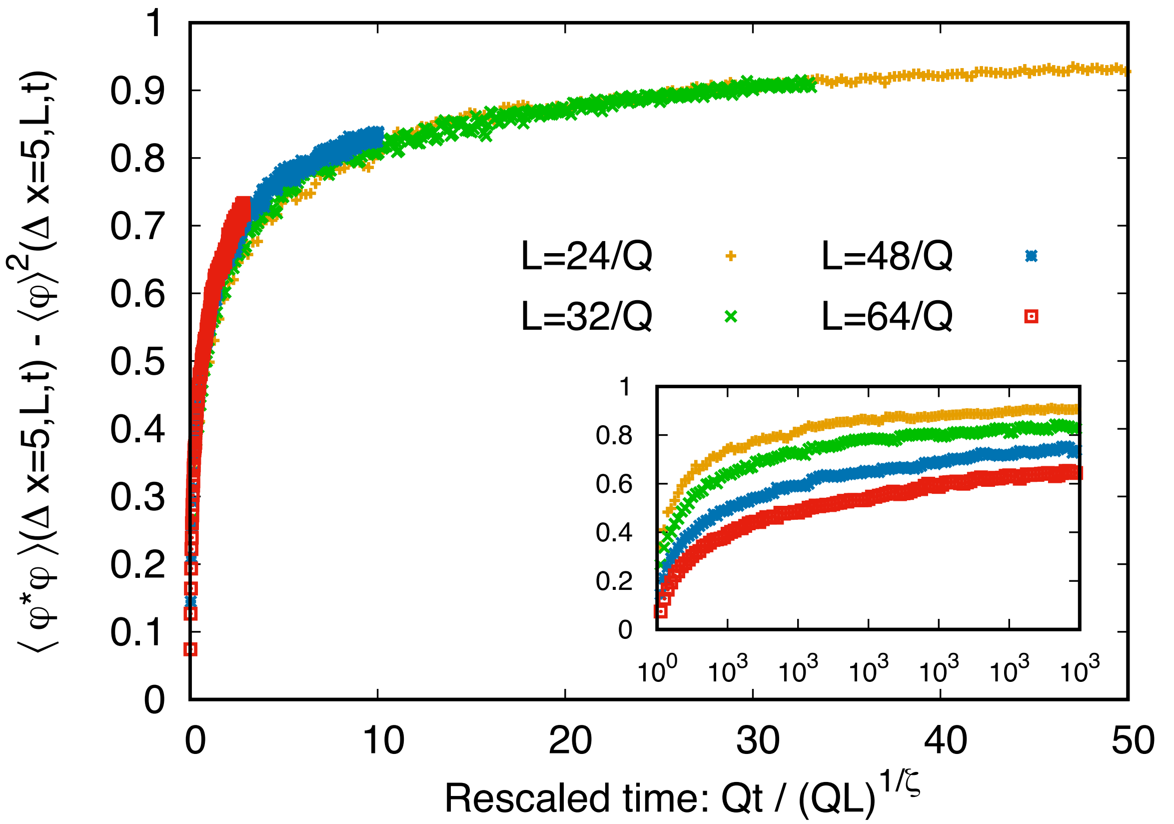


$\zeta = 0.31 \pm 0.09$

$\zeta = 0.34 \pm 0.03$

Scaling at fixed x-coordinate



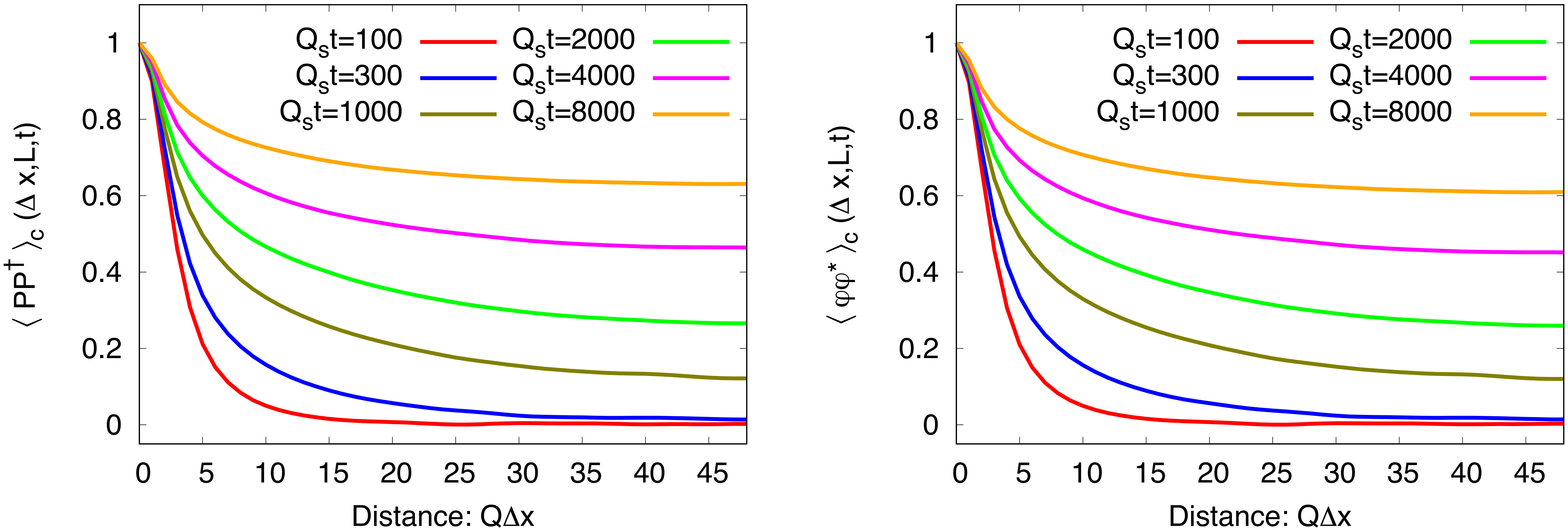


$\zeta = 0.31 \pm 0.09$

$\zeta = 0.34 \pm 0.03$

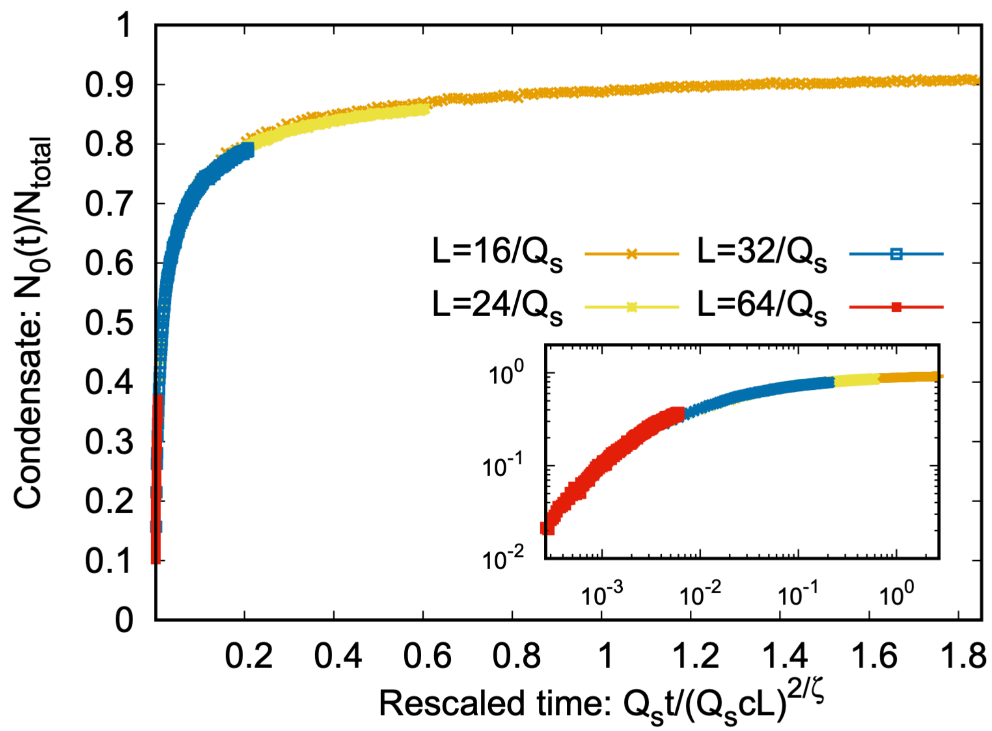
Rescales with the same scaling exponent as the integrated CF!

Time evolution of OPs

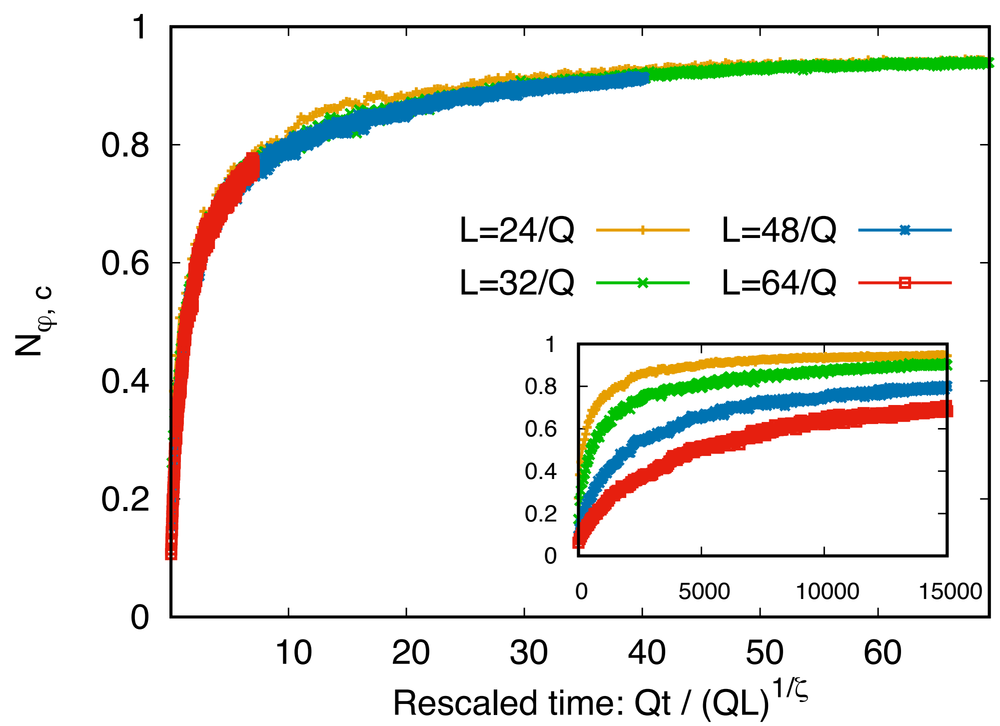


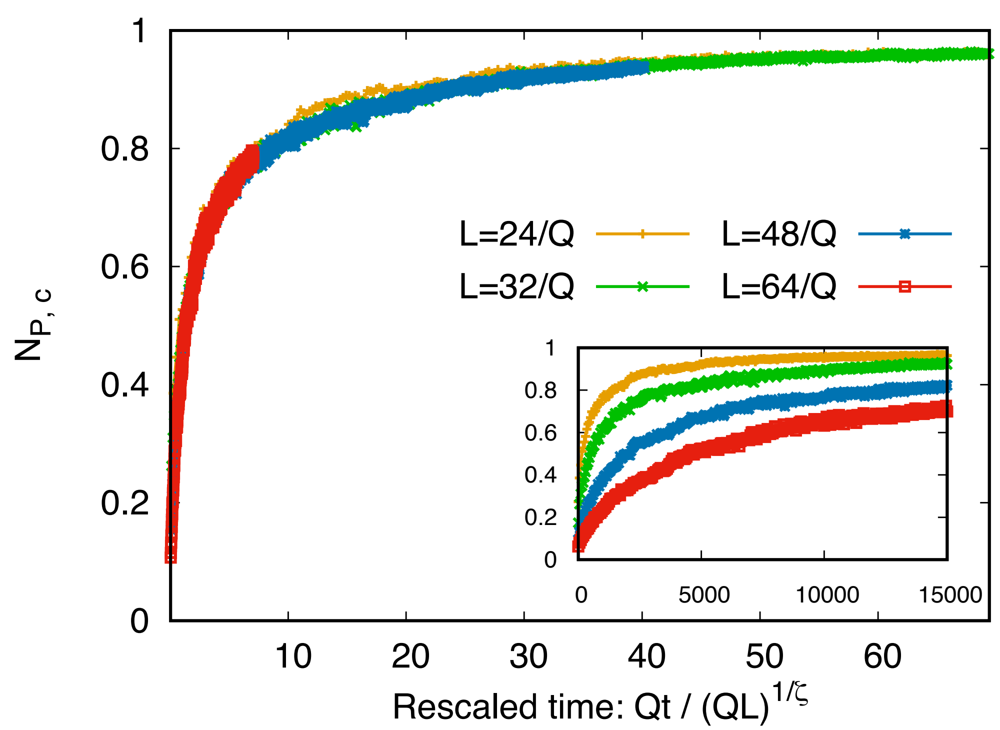
Time evolution of both correlators demonstrates condensate build up over time

Comparison with Wilson loop results



J. Berges, K. Boguslavski, M. Mace, J. M. Pawlowski, PRD (2020)





Observable	ζ
$\langle W \rangle$	0.27 ± 0.06
$\langle PP^\dagger \rangle_c$	0.31 ± 0.09
$\langle \varphi\varphi^* \rangle_c$	0.34 ± 0.03

Conclusions

We have identified two local order parameters for gauge-invariant condensation—agree with previous Wilson loop studies within error

One of these is a local scalar field that can be related to scalar Bose condensation

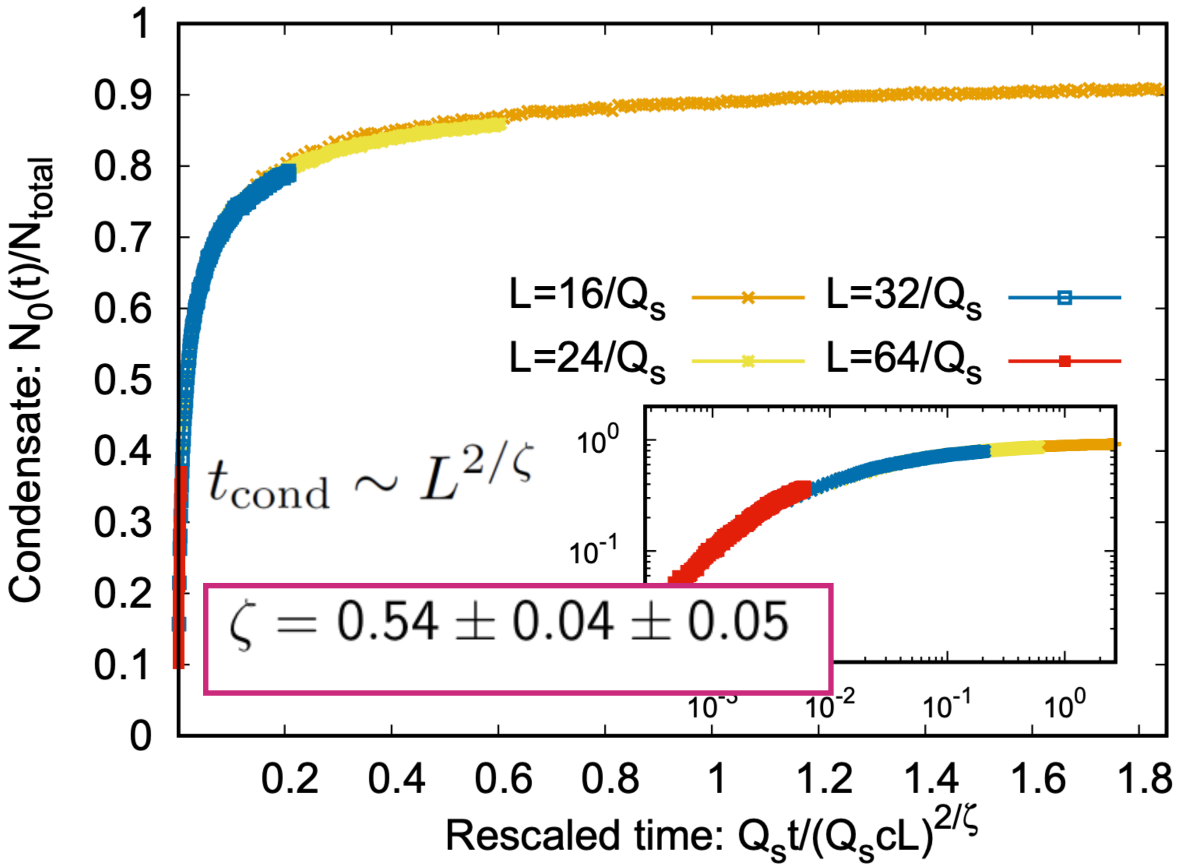
Allows for the construction of effective actions in terms of the holonomous eigenvalue field, naturally suited to describe IR dynamics far-from-equilibrium

Important step for investigation of and understanding universal features out of equilibrium

Back-up slides

vs. previous results

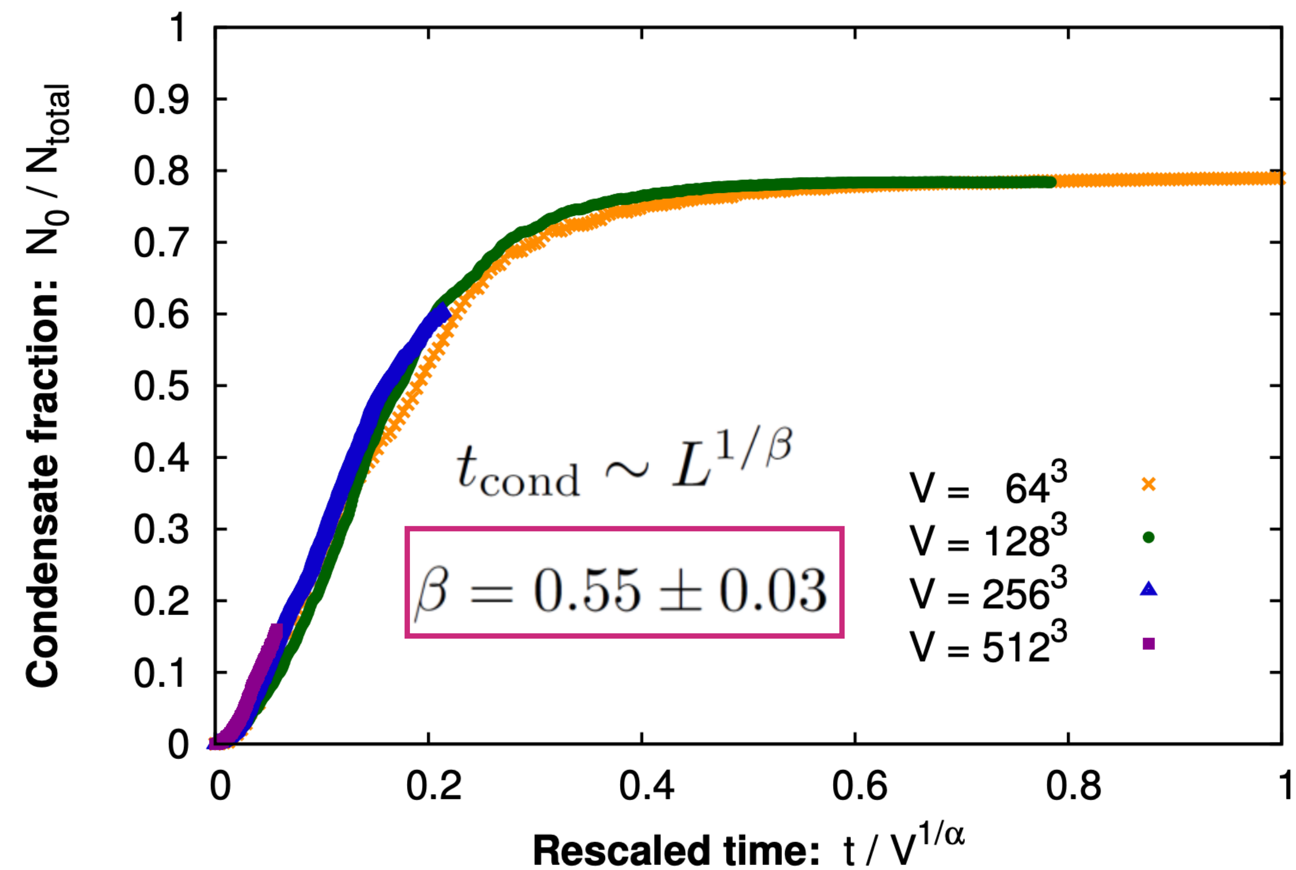
Gauge fields



$$\frac{N_0(t,L)}{N_{\rm total}} = \frac{1}{V}\int_0^L d^d x \, \langle W(\Delta x, L, t)\rangle = \frac{1}{V}\int_0^L d^d \Delta x \, \omega_S(\Delta x \, L/t^{\zeta})$$

J. Berges, K. Boguslavski, M. Mace, J. M. Pawlowski, PRD (2020)

Scalar fields



A. Piñeiro Orioli, J. Berges, K. Boguslavski PRD (2015)

$$N^{\phi}_{\rm total} = \langle \phi(\mathbf{x},t)\,\phi^\dagger(\mathbf{x},t)\rangle$$

$$\frac{N^{\phi}_0(t)}{N^{\phi}_{\rm total}} = \frac{1}{N^{\phi}_{\rm total} V}\int_0^L d^d x \, \langle \phi(x)\phi^\dagger(0)\rangle = \frac{1}{N^{\phi}_{\rm total} V}\int_0^L d^d \Delta x \, f_S(\Delta x/t^{\beta})$$