System size dependence of pre-equilibrium and applicability of hydrodynamics in heavy-ion collisions

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based on arXiv:2211.14379 (to appear in PRD), arXiv:2211.14356 (to appear in PRL)

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Standard modelling of heavy ion collisions

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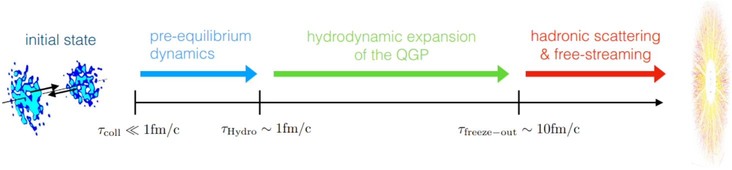
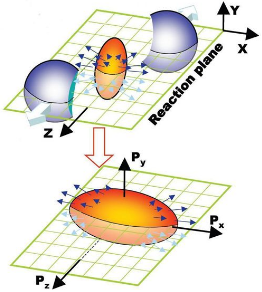
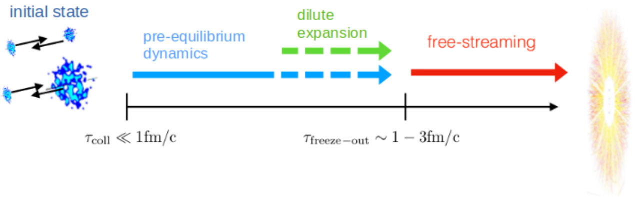


- early stage requires non-equilibrium description, but system quickly equilibrates
- strongly interacting QGP leaves imprints of thermalization and collectivity in final state observables:

  $\mathbf{v_n}$, $\langle p_T \rangle$, particle yields, ...

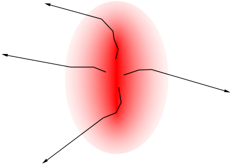


Hiroshi Masui (2008)



Very dilute, hydrodynamics not necessarily applicable

- still collective behaviour is observed!
  Nagle, Zajc Ann.Rev.Nucl.Part. 68 (2018) 211



collectivity can also be explained in kinetic theory, a microscopic description which does not rely on equilibration

- interpolate between free streaming at small opacities and hydrodynamics at large opacities!

Aim

Case study in simplified kinetic theory description on full range from small to large system size with comparison to hydrodynamics for transverse flow observables

Model and Setup

- microscopic description in terms of averaged on-shell phase-space distribution of massless bosons:

$$f(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) = \frac{(2\pi)^3}{\nu_{\mathrm{eff}}} \frac{\mathrm{d}N}{\mathrm{d}^3x\,\mathrm{d}^3p}(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y)$$

  - boost invariance
  - initialized with vanishing longitudinal pressure and no transverse momentum anisotropies

- time evolution: Boltzmann equation in conformal relaxation time approximation

$$p^\mu \partial_\mu f = C_{\mathrm{RTA}}[f] = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\mathrm{eq}}) \ , \quad \tau_R = 5\frac{\eta}{s}T^{-1}$$

results will depend only on initial state and opacity

Parametric dependencies

- dimensionless parameter: opacity $\sim$"total interaction rate"

  Kurkela, Wiedemann, Wu EPJC 79 (2019) 965

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{1}{a\pi}R\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

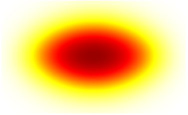
  - encodes dependencies on viscosity, transverse size and energy scale

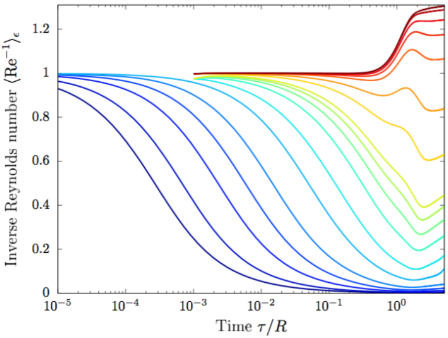
- our initial condition:
  average profiles for centrality classes of Pb+Pb at $5.02\ \mathrm{TeV}$

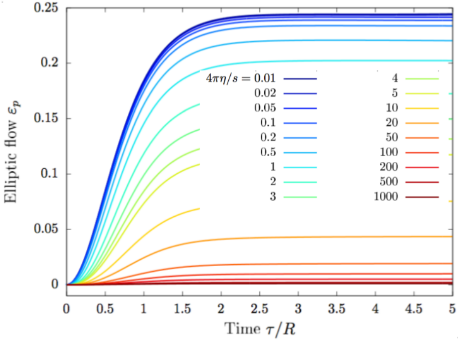
  Borghini, Borrell, Feld, Roch, Schlichting, Werthmann PRC 107 (2023), 034905

  - for fixed profile, vary $\hat{\gamma}$ via η/s: $\hat{\gamma} \approx 11 \cdot (4\pi\eta/s)^{-1}$

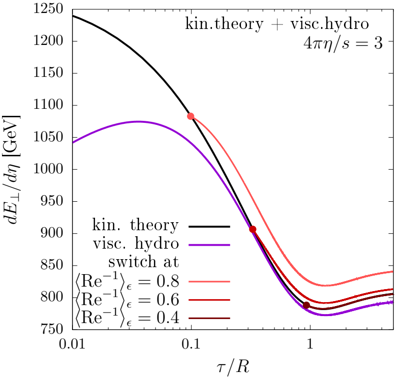
30-40%

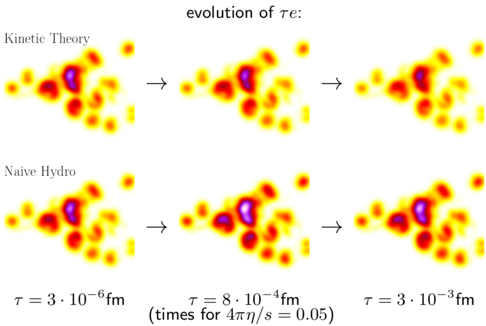






- $\mathrm{Re}^{-1} = \left(\frac{6\pi^{\mu\nu}\pi_{\mu\nu}}{e^2}\right)^{1/2}$ measures relative size of non-equilibrium effects
  - equilibration timescale strongly depends on opacity; smaller systems do not equilibrate before transverse expansion
- elliptic flow on similar timescales; continuously varying strength of response



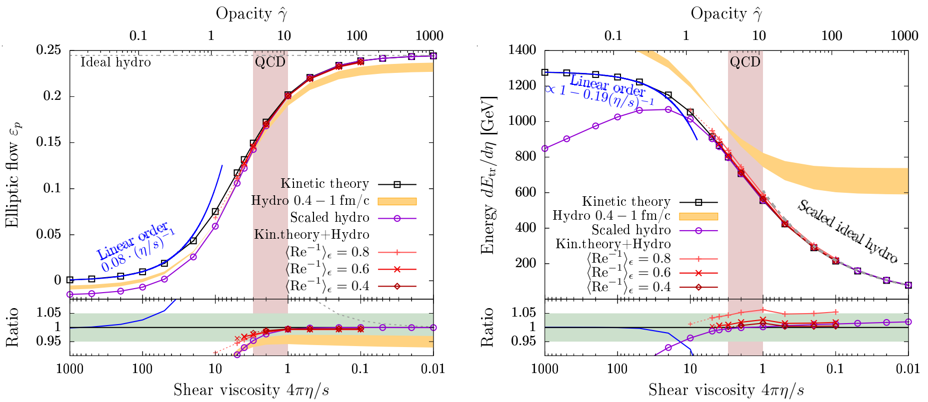


- caveat: even at large opacities, naive hydrodynamics does not accurately describe pre-equilibrium
  - anisotropic flow: inhomogeneous cooling decreases eccentricities

this can be counteracted:

- scaled hydro uses modified initial condition
- hybrid simulations: switching from kinetic theory to hydrodynamics after Re^{-1} has dropped to a specific value

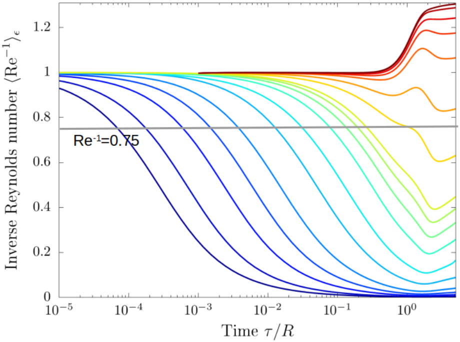
Comparison of improved hydro schemes with kin. theory

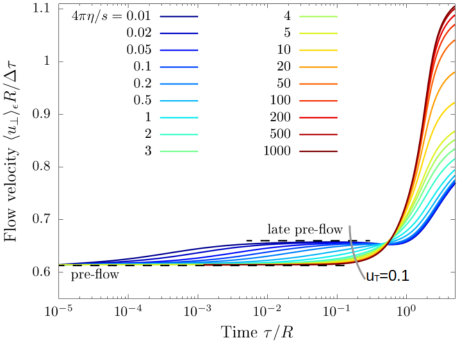


- naive hydro is off; improved schemes in perfect agreement at large $\hat{\gamma}$
- scaled hydro accurate if $\hat{\gamma} \gtrsim 4$
- Hybrid kin. theory scheme can improve on scaled hydro at intermediate opacities

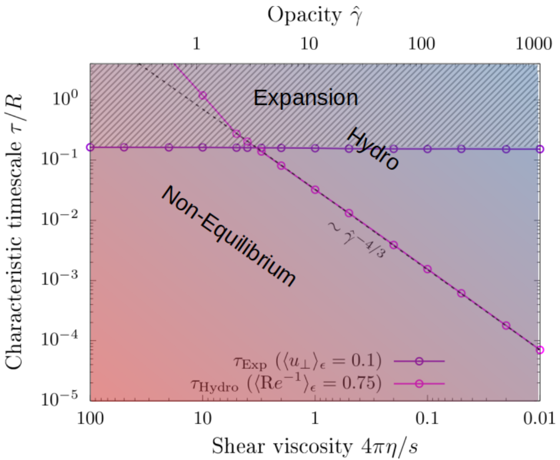
Transition between dynamical regimes

Characterize dynamics by first times certain criteria are met:





- hydro applicable for $\mathrm{Re}^{-1} \lesssim 0.75$
- transverse expansion sets in: $u_\perp \sim 0.1$



- transverse expansion sets in at $\tau_{\text{Exp}} \sim 0.2R$, independent of opacity
- Hydro applicable when $\text{Re}^{-1} < \text{Re}_c^{-1} \sim 0.75$
- $\hat{\gamma} \lesssim 3$ hydrodynamization only after transv. Expansion (if at all)

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Taking the criterion of $\hat{\gamma} \gtrsim 3$ seriously, what does this mean for the applicability of hydrodynamics to "real-life" collisions?

$$\mathrm{p+p}: \quad \overset{\text{min.bias}}{\hat{\gamma}} \sim 0.7 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.12\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{7.1\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{42.25}\right)^{-1/4}$$

far from hydrodynamic behaviour

$$\mathrm{p+Pb}: \quad \overset{\text{min.bias}}{\hat{\gamma}} \sim 1.5 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.81\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{24\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{42.25}\right)^{-1/4} \overset{\text{high mult.}}{\lesssim} 2.7$$

very high multiplicity events approach regime of applicability, but do not reach it

$$\mathrm{O+O}: \quad \overset{30-40\%}{\hat{\gamma}} \sim 2.2 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{1.13\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{55\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{42.25}\right)^{-1/4} \sim \overset{70-80\%}{1.4} - \overset{0-5\%}{3.1}$$

probes transition region to hydrodynamic behaviour

$$\mathrm{Pb+Pb}: \quad \overset{30-40\%}{\hat{\gamma}} \sim 5.7 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{2.78\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{1280\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{42.25}\right)^{-1/4} \sim \overset{70-80\%}{2.7} - \overset{0-5\%}{9.0}$$

hydrodynamic behaviour in all but peripheral collisions

Summary

- kinetic theory description of transverse flow on whole range in system size
- comparison to hydrodynamics: accurate at 5% level if $\mathrm{Re}^{-1} \lesssim 0.75$
- in small systems (p+p, p+Pb) transverse expansion is faster than equilibration; hydro not applicable!
  - O+O covers transition regime to hydro behaviour

Backup

How can hydro still describe small systems?

In theoretical descriptions:

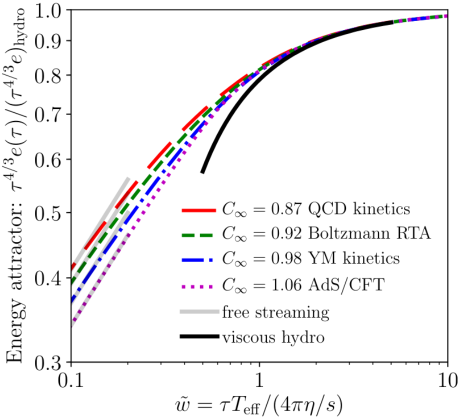
$$v_n = \kappa_{n,n} \cdot \epsilon_n$$

- Flow can be compared to experiment
- Response depends on the dynamical model
- Initial state geometry is poorly constrained in small systems

Varying initial condition in order to fit flow measurements will mask inaccuracies in the description of the dynamical response!

What might happen when going beyond RTA?

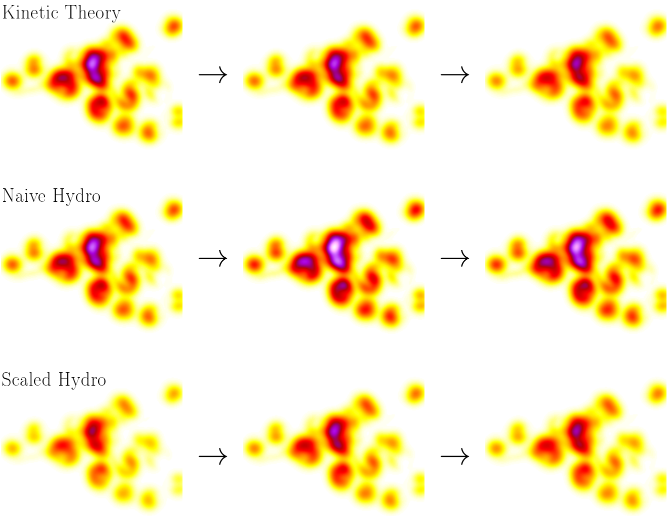
- more complex kernels will introduce further parameter dependence, but opacity dependence might still be "leading order approximation"
- in Bjorken flow, equilibration happens in very similar ways across different model descriptions:



Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301

Early time longitudinal cooling and scaled hydro

evolution of τe:

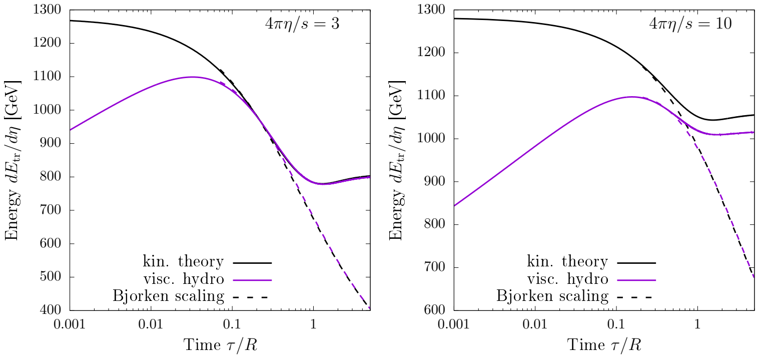


$\tau = 3 \cdot 10^{-6}$fm $\qquad$ $\tau = 8 \cdot 10^{-4}$fm $\qquad$ $\tau = 3 \cdot 10^{-3}$fm

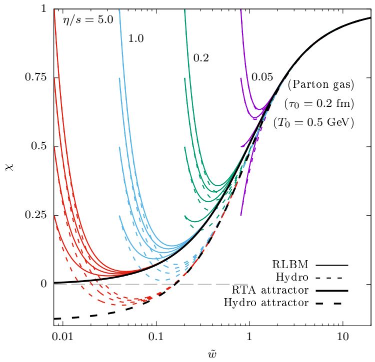
(times for $4\pi\eta/s = 0.05$)

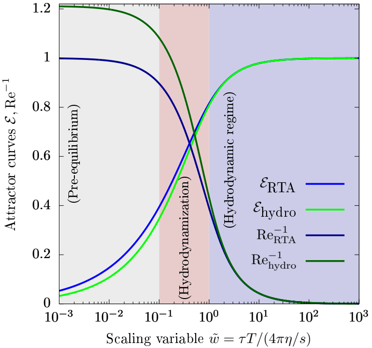
Initializing on the local attractor

- accuracy depends on timescale separation of pre-equilibrium and transv. expansion



- longitudinal boost-invariant Bjorken flow exhibits universal behaviour
- time evolution curves converge to an attractor curve when expressed via the scaling variable $\tilde{w} = \frac{T\tau}{4\pi\eta/s}$
  $\Rightarrow$ expressed via universal scaling functions
  $\chi(\tilde{w}) = p_L/p_T, \quad \mathcal{E}(\tilde{w}) \propto \tau^{4/3} e, \quad f_{E_\perp}(\tilde{w}) \propto \tau^{1/3} \frac{dE_\perp}{dy}, \ldots$
  Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301





Ambruș, Bazzanini, Gabbana, Simeoni, Succi, Tripicione, arXiv:2201.09277

Early time eccentricity decrease

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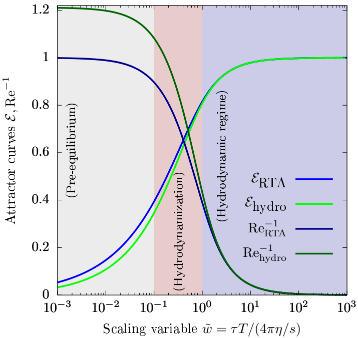
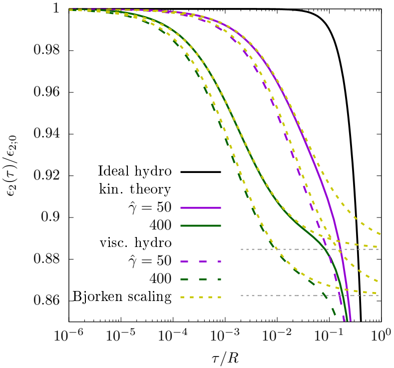
- $\tau \ll R$: no transverse expansion, system locally behaves like 0+1D Bjorken flow
  - universal attractor curve scaling in the variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau,\mathbf{x}_\perp)\tau}{4\pi\eta/s}$
    Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301
  - $\tilde{w} \gg 1$: $\tau^{4/3} e = \text{const.}$, $\tau^{1/3} \frac{dE_\perp}{dy} = \text{const.}$
  - $\tilde{w} \ll 1$: model dependent power law $\tau^{4/3} e \sim \tilde{w}^\gamma$





- inhomogeneous cooling changes energy density profile

Bjorken flow universal attractor curve in scaling variable $\tilde{w}(\tau,\mathbf{x}_\perp) = \frac{T(\tau,\mathbf{x}_\perp)\tau}{4\pi\eta/s}$:

$$\epsilon(\tau)\tau^{4/3} = (4\pi\eta/s)^{4/9} a^{1/9} (\epsilon\tau)_0^{8/9}\ C_\infty\ \mathcal{E}(\tilde{w})\ ,$$

$$\tau^{1/3} \frac{\mathrm{d}E_\perp}{d^2\mathbf{x}_\perp \mathrm{d}\eta} = (4\pi\eta/s)^{4/9} a^{1/9} (\epsilon\tau)_0^{8/9}\ C_\infty\ f_{E_\perp}(\tilde{w})$$

- using $\epsilon = aT^4$, recast first eq. into self consistency eq. for $\tilde{w}$
- use this togehter with initial cond. for $\epsilon\tau$ to relate differentials of $\mathrm{d}\tilde{w}$ and $\mathrm{d}x_\perp$
- integrate second equation to find scaling of $\mathrm{d}E_\perp/\mathrm{d}\eta$
- use $\frac{(4\pi\eta/s)^4 a}{dE_\perp^0/d\eta\ \ R} = \frac{1}{\pi}\left(\frac{4\pi}{5\hat{\gamma}}\right)^4$ to identify $\hat{\gamma}$

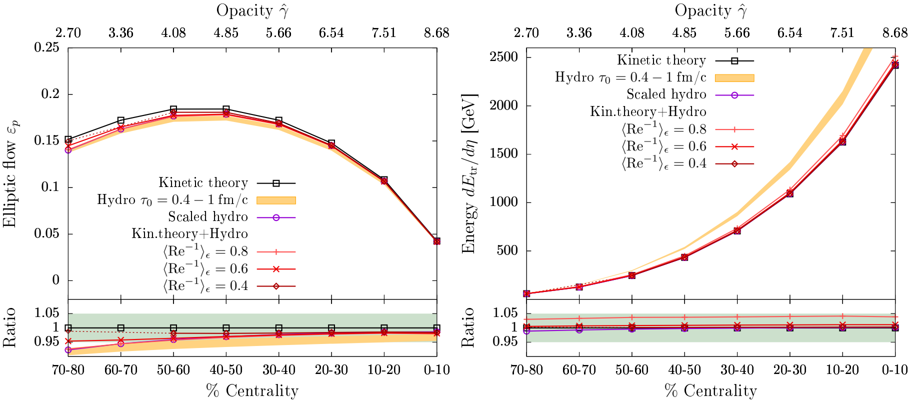
$$\frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = \frac{9}{2}\ \left(\frac{4\pi}{5\hat{\gamma}}\right)^4\ \left(\frac{R}{\tau}\right)^3 \int_0^{\tilde{w}(\tau,\mathbf{x}_\perp=0)} \frac{\tilde{w}^3 d\tilde{w}}{\mathcal{E}(\tilde{w})} \left[1 - \frac{\tilde{w}}{4}\frac{\mathcal{E}'(\tilde{w})}{\mathcal{E}(\tilde{w})}\right]\ f_{E_\perp}(\tilde{w})\ ,$$

$$\tilde{w}(\tau,\mathbf{x}_\perp = 0) = \left(\frac{5\hat{\gamma}}{4\pi}\right)^{8/9} \left(\frac{\tau}{R}\right)^{2/3} [C_\infty \mathcal{E}(\tilde{w})]^{1/4}$$

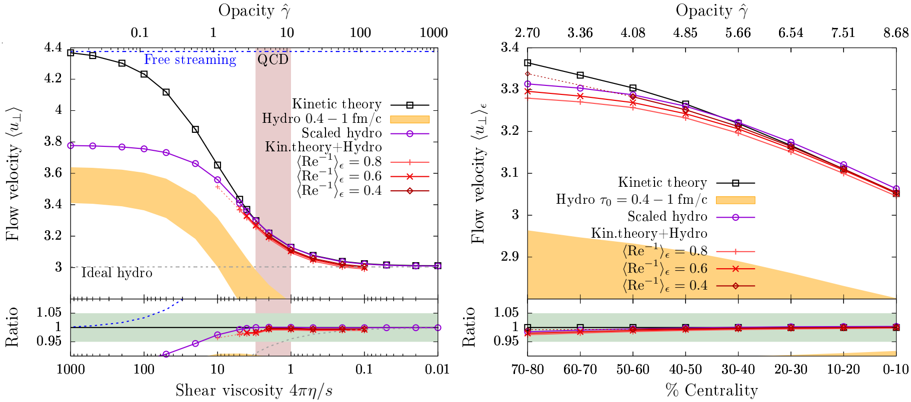
Limits of this scaling law:

- $\hat{\gamma}\left(\frac{\tau}{R}\right)^{3/4} \ll 1 \Rightarrow \tilde{w} \ll 1\ \Rightarrow\ \mathcal{E}(\tilde{w}) \approx f_{E_\perp}(\tilde{w}) \approx C_\infty^{-1}\tilde{w}^{4/9}\ \Rightarrow\ \frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = 1$
- $\hat{\gamma}^{3/4}\left(\frac{\tau}{R}\right) \gg 1\ \Rightarrow\ \tilde{w} \gg 1\ \Rightarrow\ \mathcal{E}(\tilde{w}) \approx 1, f_{E_\perp} \approx \frac{\pi}{4}$
  $\Rightarrow\ \frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = \frac{9\pi}{32}\left(\frac{4\pi}{5\hat{\gamma}}\right)^{4/9}\left(\frac{R}{\tau}\right)^{1/3} C_\infty$

Centrality dependence



Transverse flow velocity



Hydrodynamization in viscosity and centrality dependence

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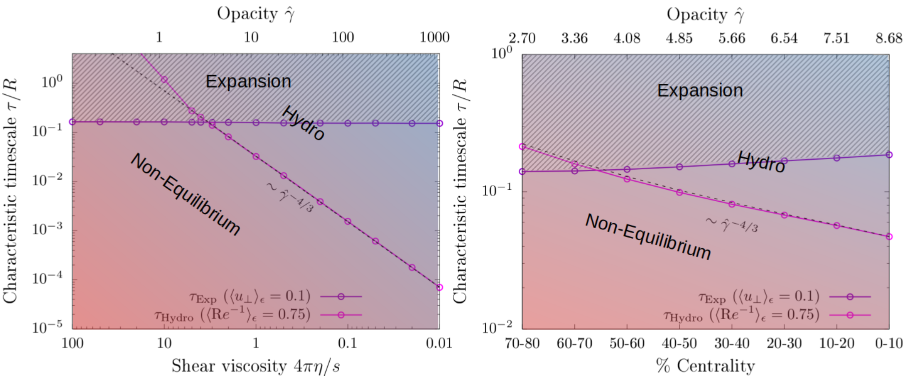

- transverse expansion sets in at $\tau_\perp \sim 0.2R$, independent of opacity
- Hydro appicable when $\mathrm{Re}^{-1} < \mathrm{Re}_c^{-1} \sim 0.75$ after timescale

$$\tau_{\mathrm{Hydro}}/R \approx 1.53\ \hat{\gamma}^{-4/3} \left[(\mathrm{Re}_c^{-1})^{-3/2} - 1.21(\mathrm{Re}_c^{-1})^{0.7}\right]$$

- hydrodynamization before transv. Expansion for $\hat{\gamma} \gtrsim 3$