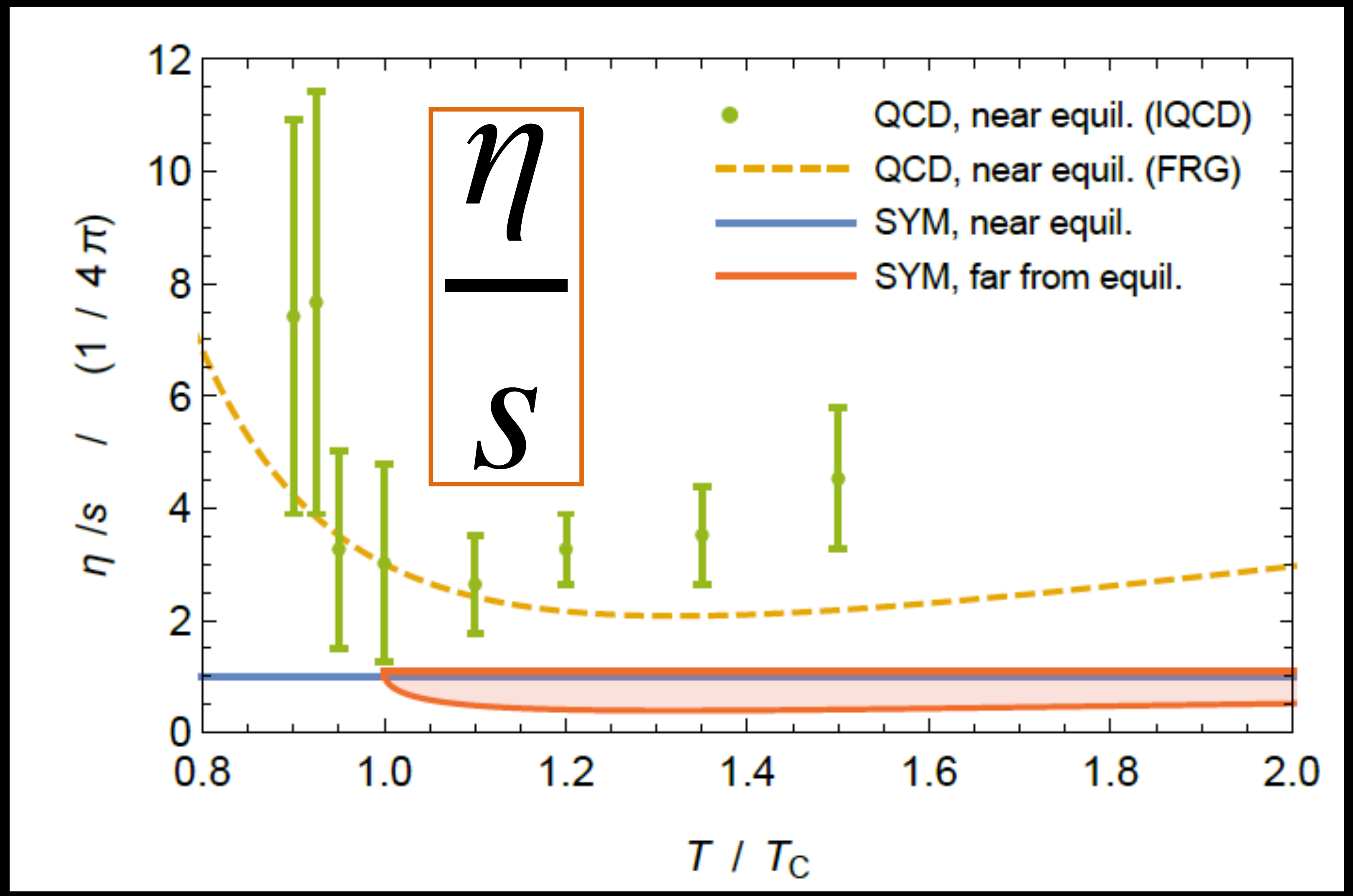
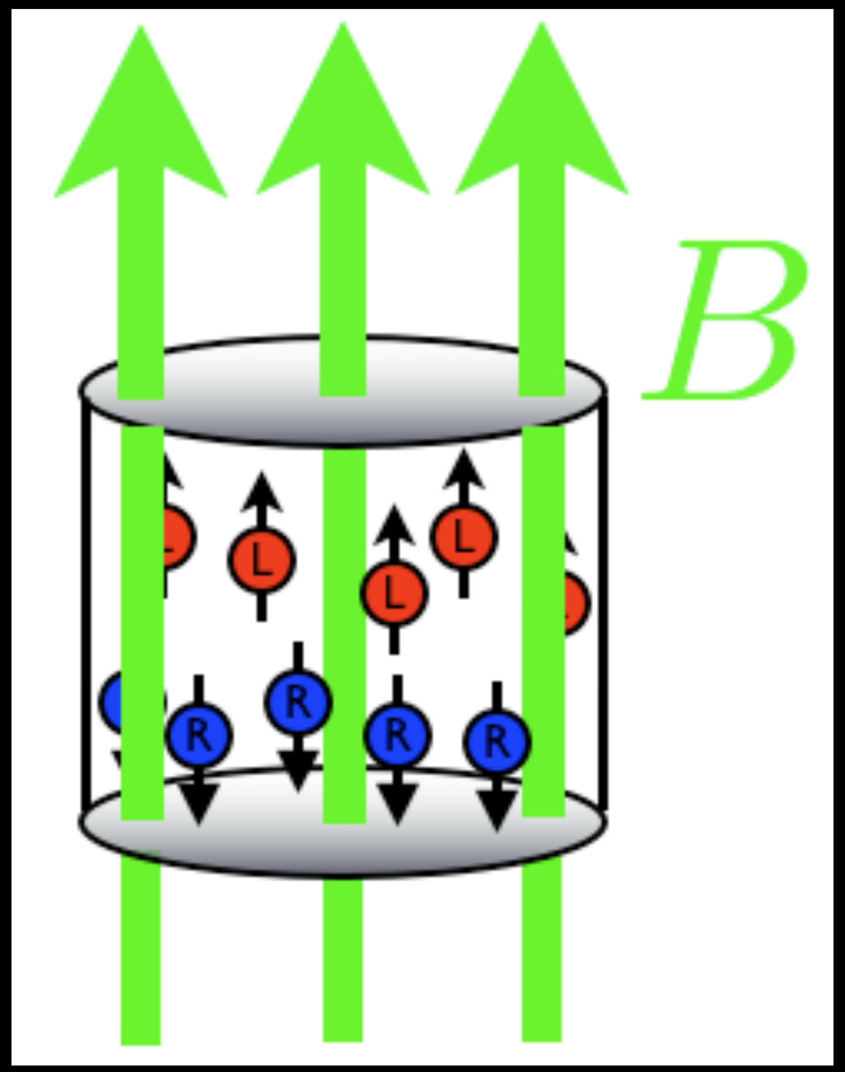
Early time dynamics far from equilibrium via holography

11th International Conference on Hard and Electromagnetic Probes of High-Energy Nuclear Collisions
Aschaffenburg, Germany

March 29th, 2023





[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]
[Bleicher, Kaminski, Wondrak; Quark Matter 2020]
[Cartwright, Kaminski, Schenke; PRC (2022)]
[Cartwright, Kaminski, Knipfer; arXiv:2207.02875]

Matthias Kaminski
University of Alabama

U.S. DEPARTMENT OF ENERGY

Collaborators on these projects

[Cartwright, Kaminski, Knipfer; arXiv:2207.02875]
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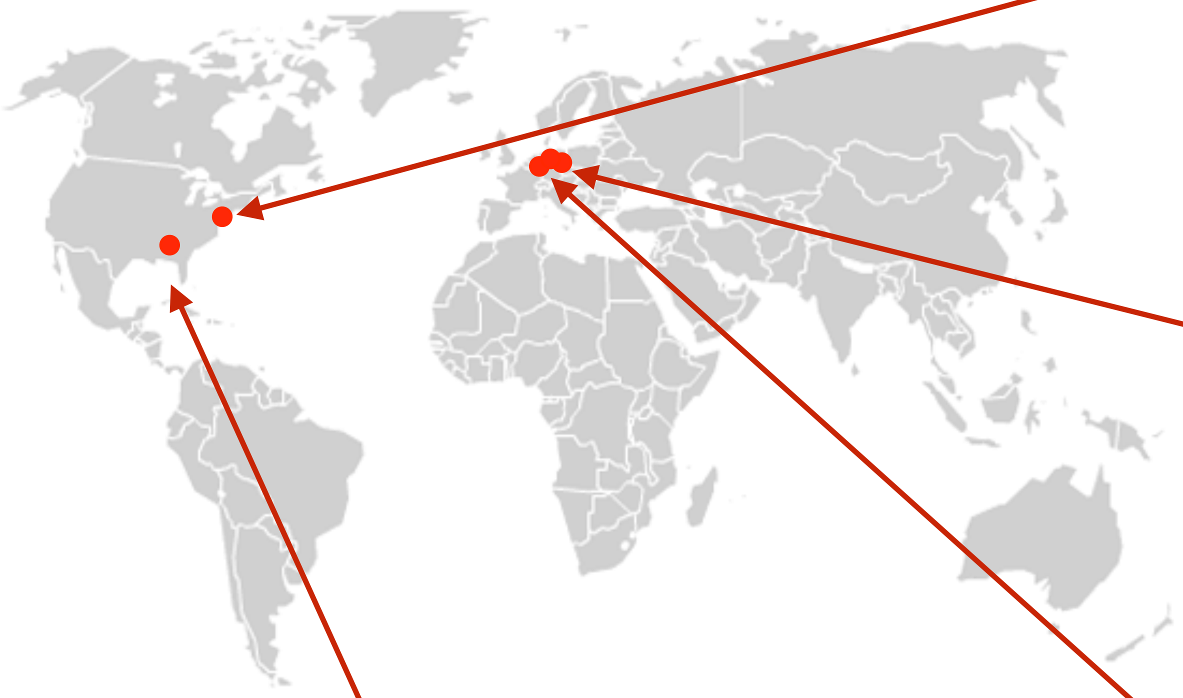
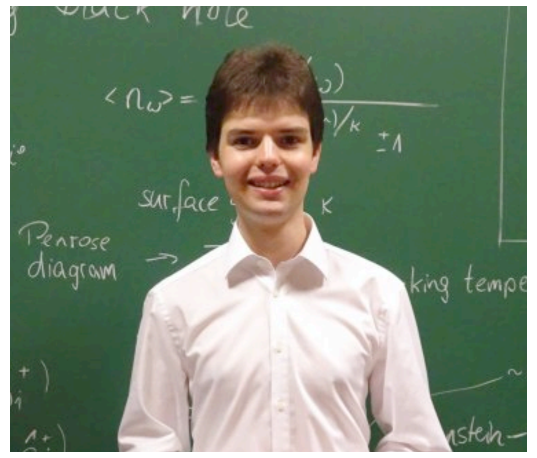


BNL, USA

Prof. Dr. Bjoern Schenke

Frankfurt University



Michael Wondrak (now at Radboud University)

Prof. Dr. Dr. h.c. Marcus Bleicher

University of Alabama, Tuscaloosa, USA

Dr. Marco Knipfer

Dr. Casey Cartwright

(now at Utrecht University, Netherlands)

Motivation: Unreasonable effectiveness of hydrodynamics

Holographic model of heavy ion collision:

[Chesler, Yaffe; PRL (2011)]
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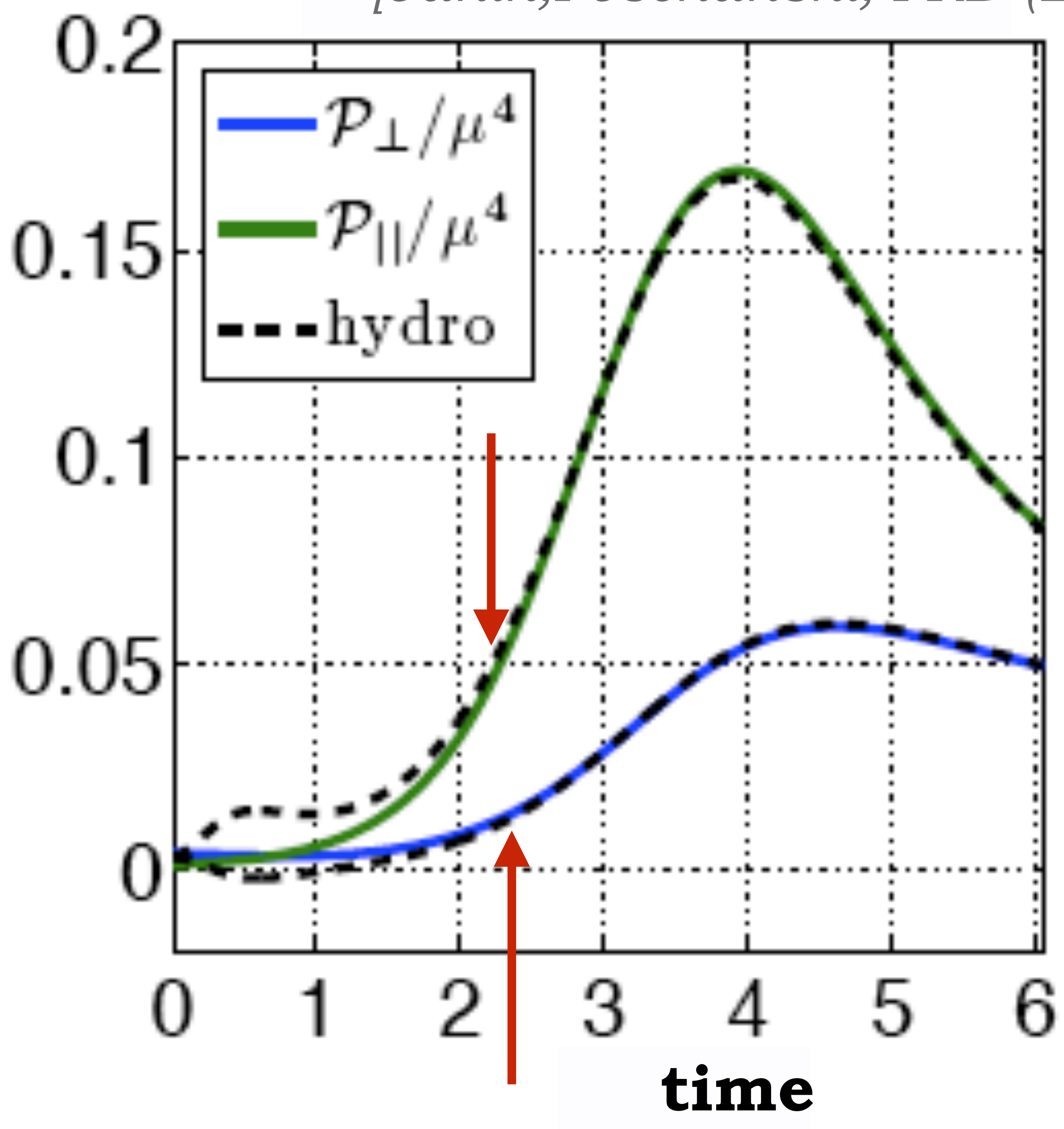
[Janik,Peschanski; PRD (2006)]



Heavy ion collision data:

Experimental data well approximated by assuming nearly perfect fluid dynamics using hydrodynamic equations, for example

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[Romatschke & Romatschke; (2017)]

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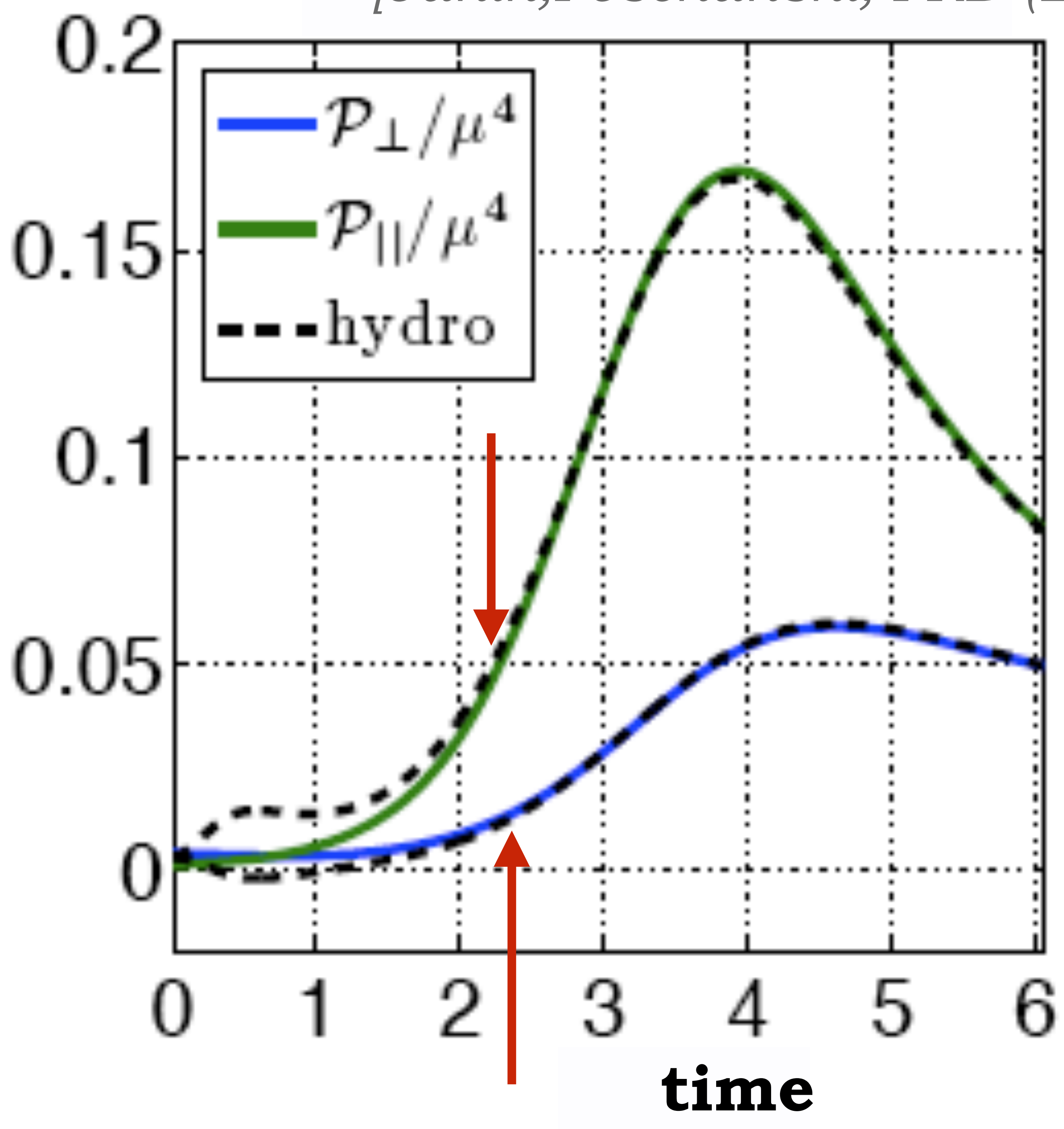

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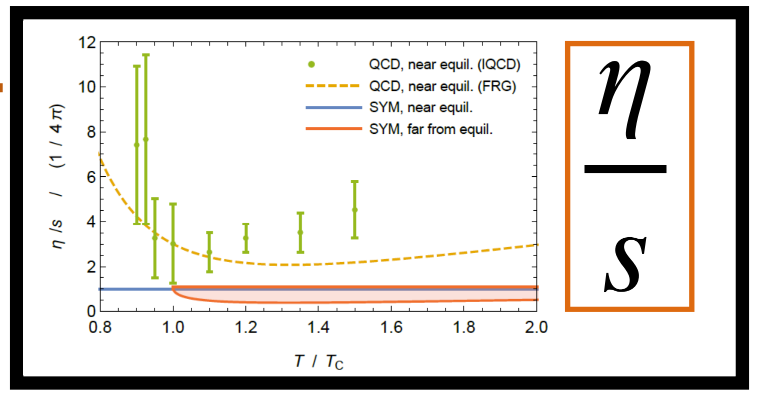
➡**Hydrodynamics valid long before local or global equilibrium and despite anitropies/large gradients**

➡**THIS TALK: three holographic examples far from equilibrium**

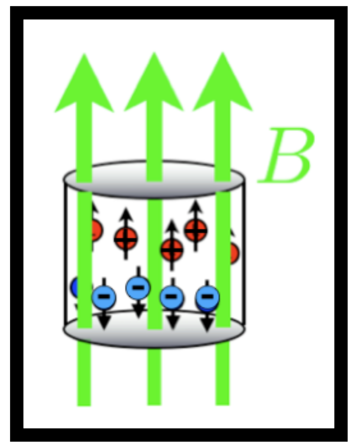
1. **Shear Viscosity**



➡ ***Highlight talk by Kirill Boguslavski***

2. **Speed of Sound**

3. **Chiral Magnetic Effect (CME)**



1. Far from equilibrium shear: Methods

Thermalization in field theory:

[Janik,Peschanski; PRD (2006)]
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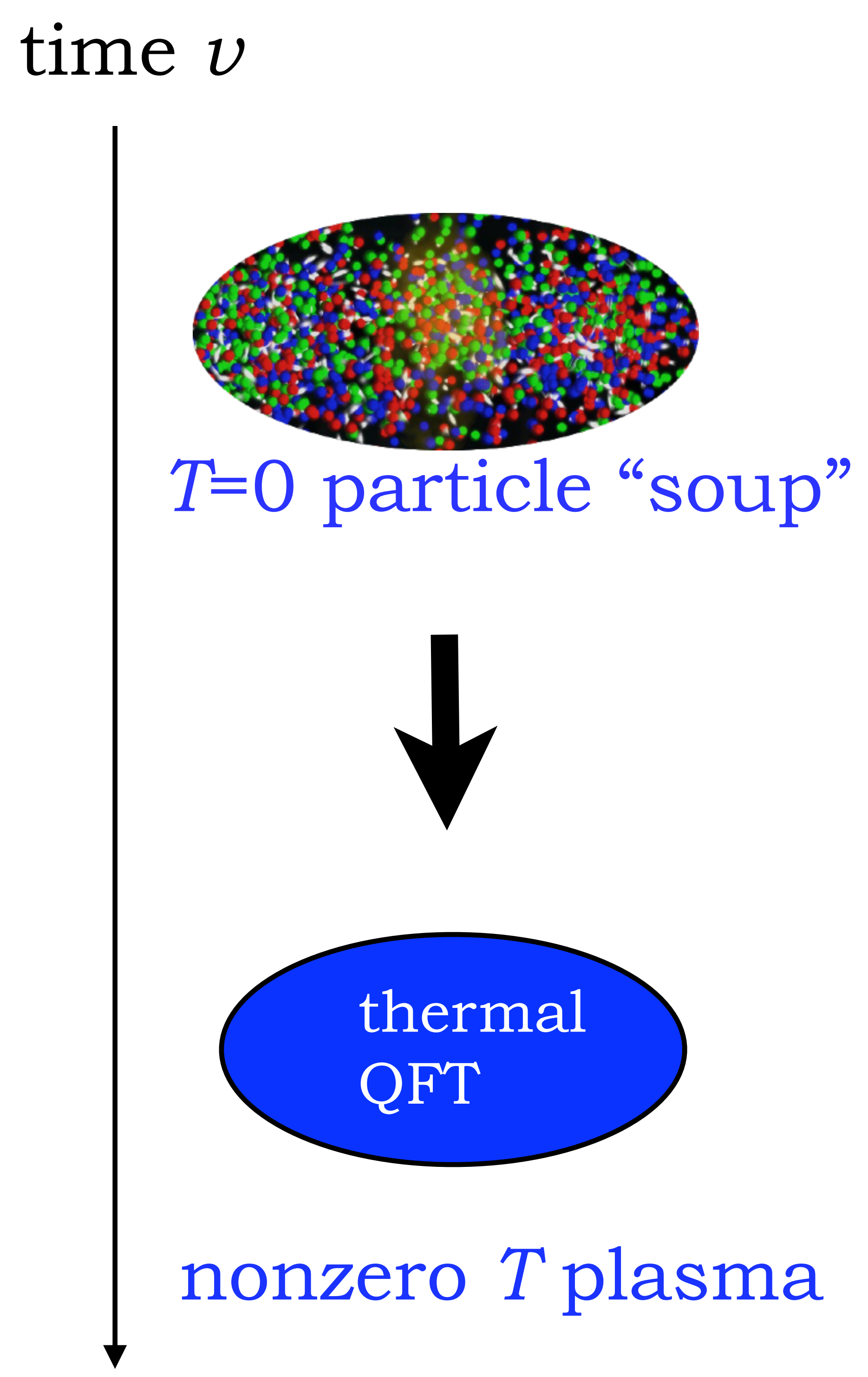

$$\langle T^{\mu\nu} \rangle (v)$$

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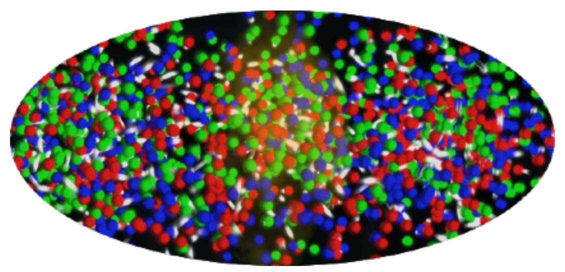
time v



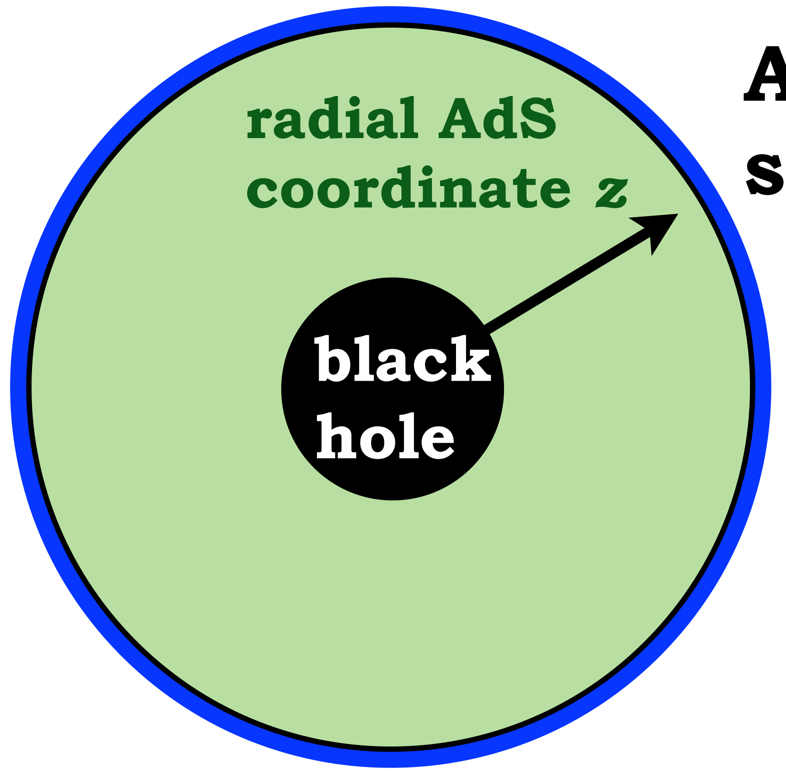
T=0 particle "soup"

thermal QFT

nonzero T plasma

gauge/gravity correspondence

$$\langle T^{\mu\nu}\rangle(v) \longleftrightarrow g_{\mu\nu}(v)$$



Anti-de Sitter space boundary

thermal QFT "lives" here temperature T

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Horizon formation in gravity:

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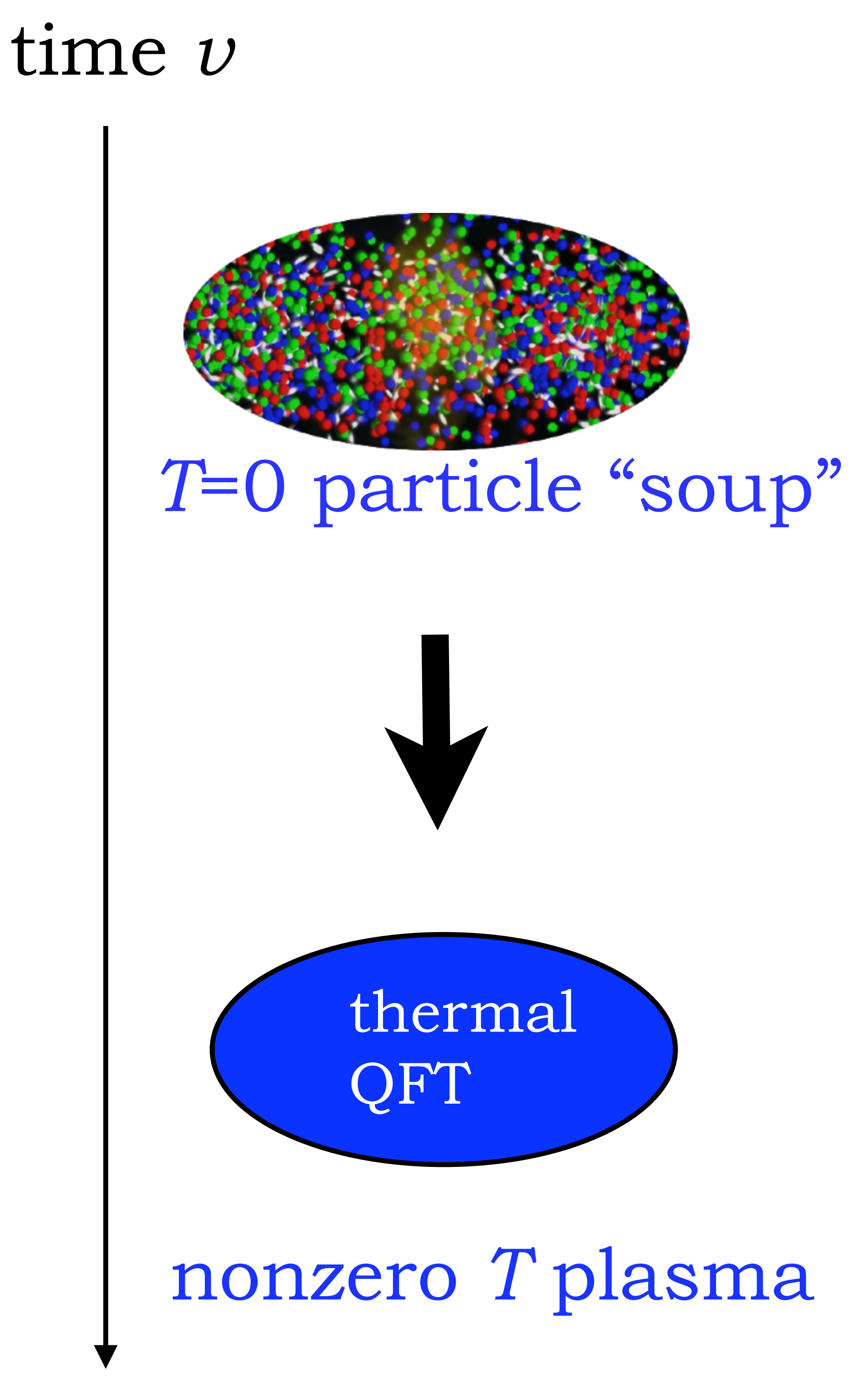

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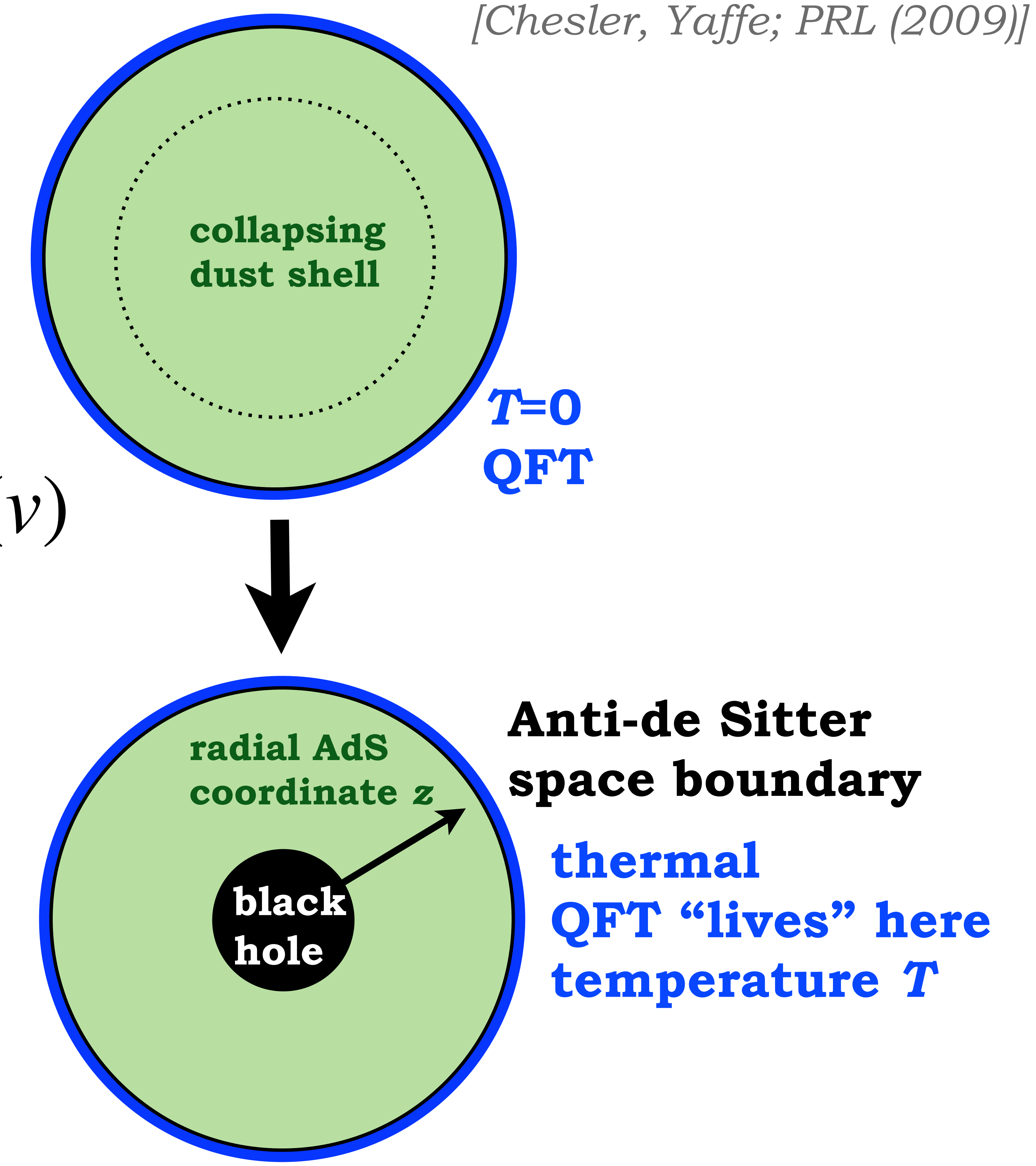

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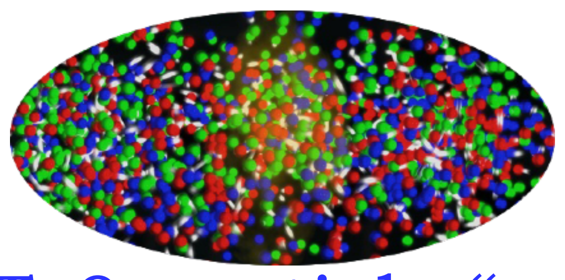

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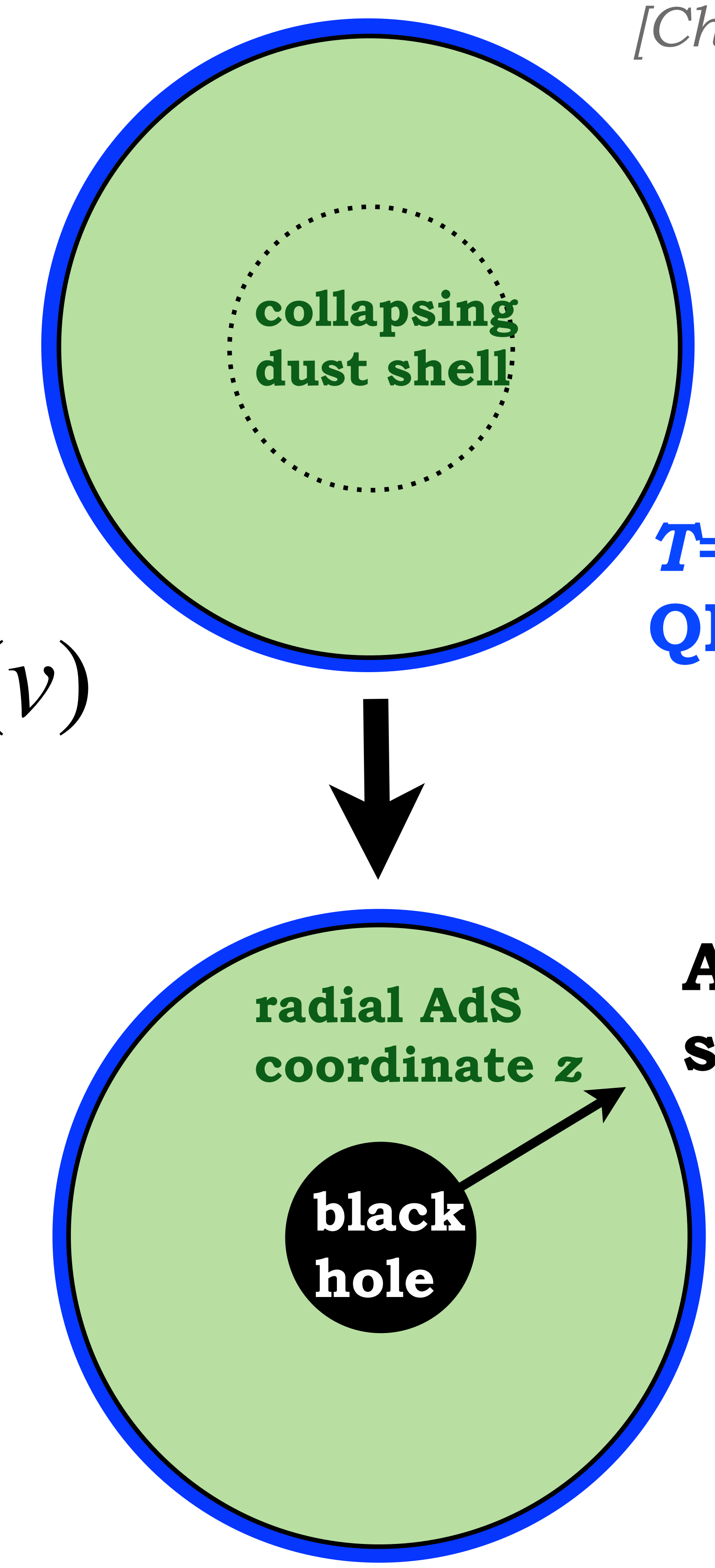
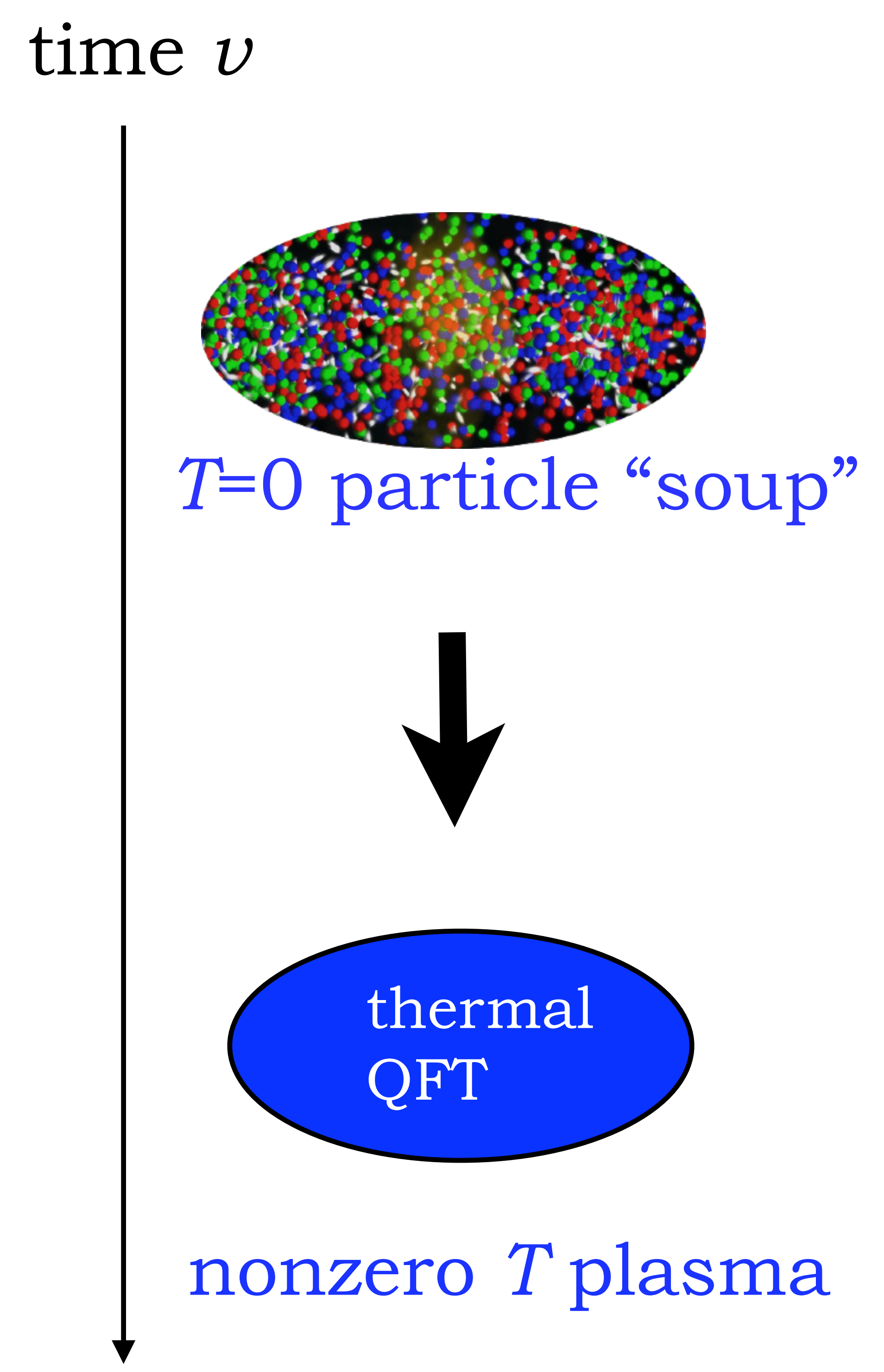


***T*=0**
QFT

Anti-de Sitter space boundary

thermal QFT “lives” here temperature *T*

Thermalization in field theory:



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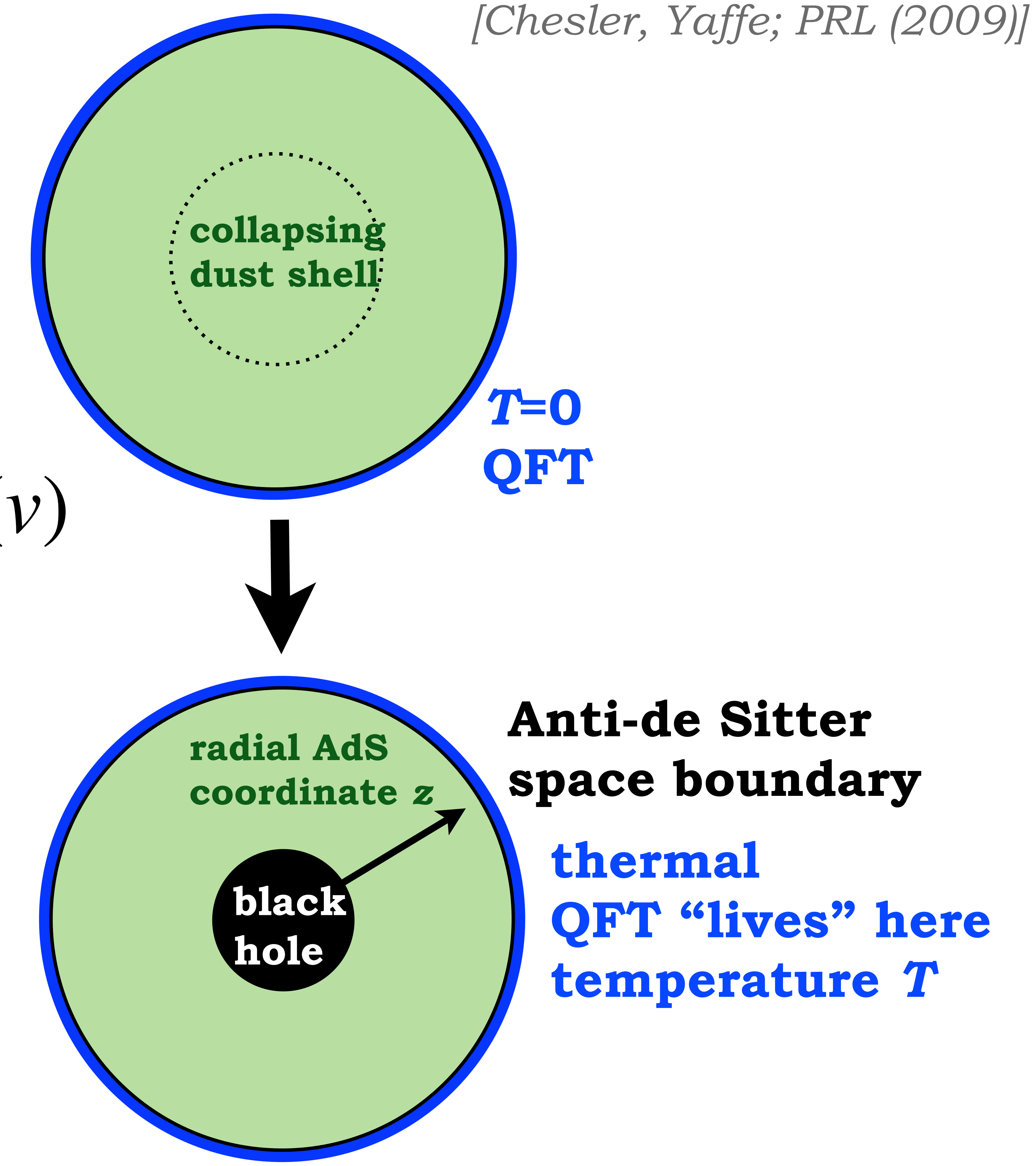

This work: Einstein gravity, time-dependent Vaidya metric
[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]

$$ds^2 = \frac{1}{z^2}\left(-f(v,z)\,\mathrm{d}v^2 - 2\,\mathrm{d}v\,\mathrm{d}z + \mathrm{d}x^2 + \mathrm{d}y^2\right) \qquad f(v,z) = 1 - 2G_{\mathrm{N}}M(v)\,z^3 + G_{\mathrm{N}}Q(v)^2\,z^4$$

With time-dependent black hole mass $M(v) = m + m_{\mathrm{s}}\ \left(1 + \tanh\left(v/\Delta t\right)\right)/2$

1. Far from equilibrium shear: Perturbations

$\frac{\eta}{s}$

Perturb the background metric $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$

→ ***Talk by Travis Dore***
→ ***Talk by Xiaojian Du***

[Son, Starinets; JHEP (2002)]
[Iqbal, Liu; Fortschr.Phys. (2008)]
[van Rees, Skenderis; PRL (2008)]

$$ds^2 = \frac{1}{z^2}\left(-f(v,z)\,dv^2 - 2\,dv\,dz + dx^2 + dy^2\right) = g_{\mu\nu}\,dx^\mu dx^\nu \Rightarrow$$ **linearized Einstein equations for** $h_{\mu\nu}$

Near-boundary expansion

$$h_{\mu\nu} \sim h^{(0)}_{\mu\nu} + \langle T_{\mu\nu}\rangle\, z^4 + \dots$$

metric perturbation ($h_{\mu\nu}$), ***source*** ($h^{(0)}_{\mu\nu}$), ***one-point function*** ($\langle T_{\mu\nu}\rangle$)

Equilibrium result

[Kubo formula]

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\Im\,\tilde{G}_R^{xy,xy}(\omega)$$

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Linear response: retarded correlator from metric fluctuation (only shear perturbation h_{xy}**)**

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]
[Ishii; arXiv:1605.08387]

$$\langle T^{xy}(t_2)\rangle_h = \int \mathrm{d}\tau\, G_{\mathrm{R}}^{xy,xy}(\tau,\, t_2)\, \underbrace{h^{(0)}_{xy}(\tau)}_{\propto\,\delta(\tau - t_{\mathrm{P}})} \propto G_{\mathrm{R}}^{xy,xy}(t_{\mathrm{P}},\, t_2)$$

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Wigner transform $G_R^{xy,xy}(t_p, t_2) \to G_R^{xy,xy}(t_{avg}, t_{rel}) \sim \tilde{G}_R^{xy,xy}(t_{avg}, \omega)\, e^{-i\omega t_{rel}}$

$$t_{avg} = (t_p + t_2)/2$$
$$t_{rel} = t_p - t_2$$

Equilibrium result

[Kubo formula]

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Equilibrium result
[Kubo formula]

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Generalized Kubo formula for "shear viscosity" far from equilibrium
[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]

$$\eta\,(t_{\mathrm{avg}}) = -\lim_{\omega\to 0} \frac{1}{\omega}\,\Im\,\tilde{G}_{\mathrm{R}}^{xy,xy}(t_{\mathrm{avg}}, \omega)$$

$\frac{\eta}{s}$

1. Far from equilibrium shear: Results

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]

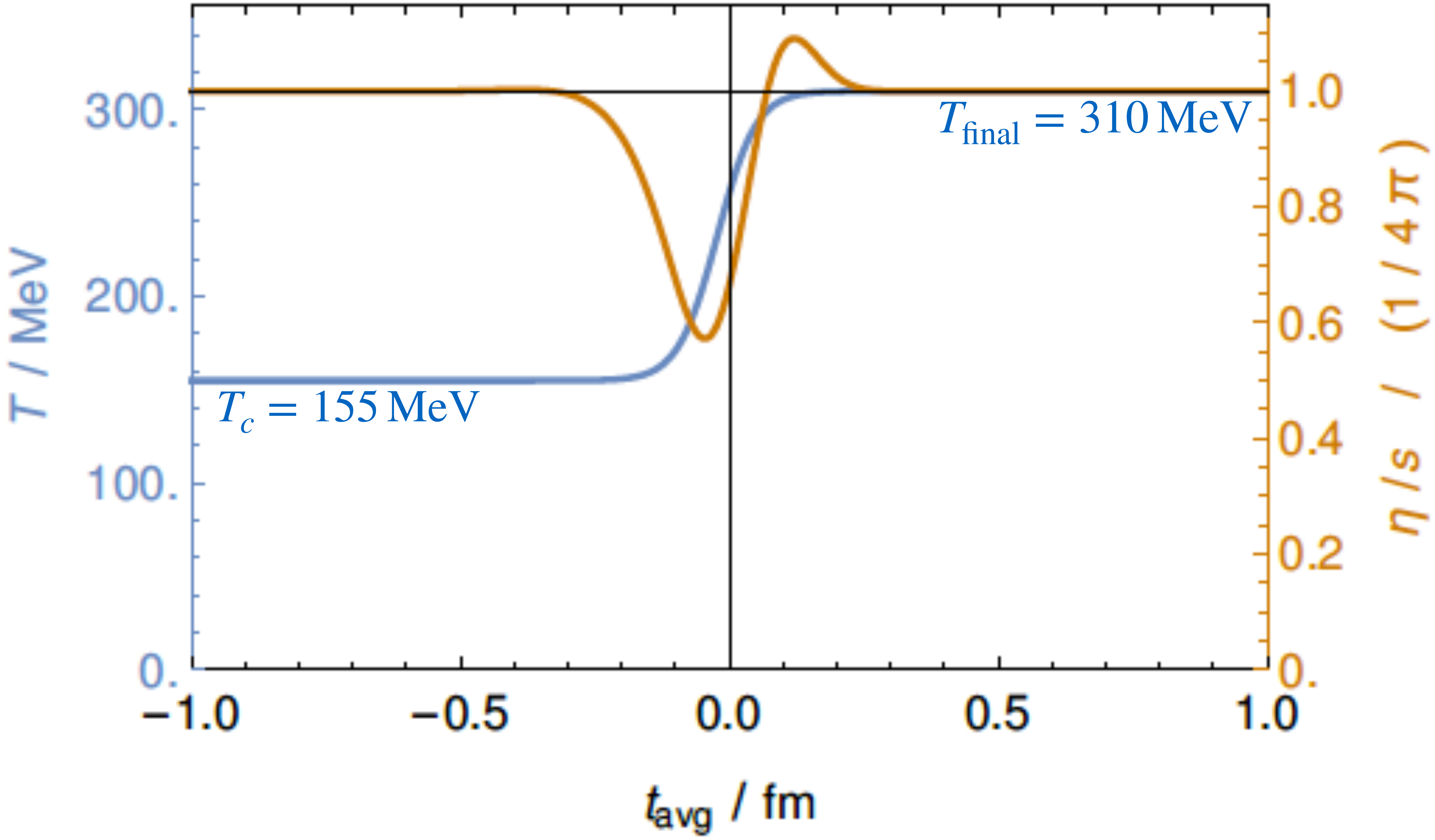
RHIC parameters: $\sqrt{s_{NN}} = 200\,\text{GeV}$ $\quad \Delta t = 0.3\,\text{fm}$

Temperature

$$T = T_{\text{Hawking}}$$

Entropy density from generating functional

$$s \sim \frac{\partial S^{\text{on-shell}}}{\partial T}$$



KSS equilibrium result

[Kovtun,Son, Starinets; PRL (2005)]

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

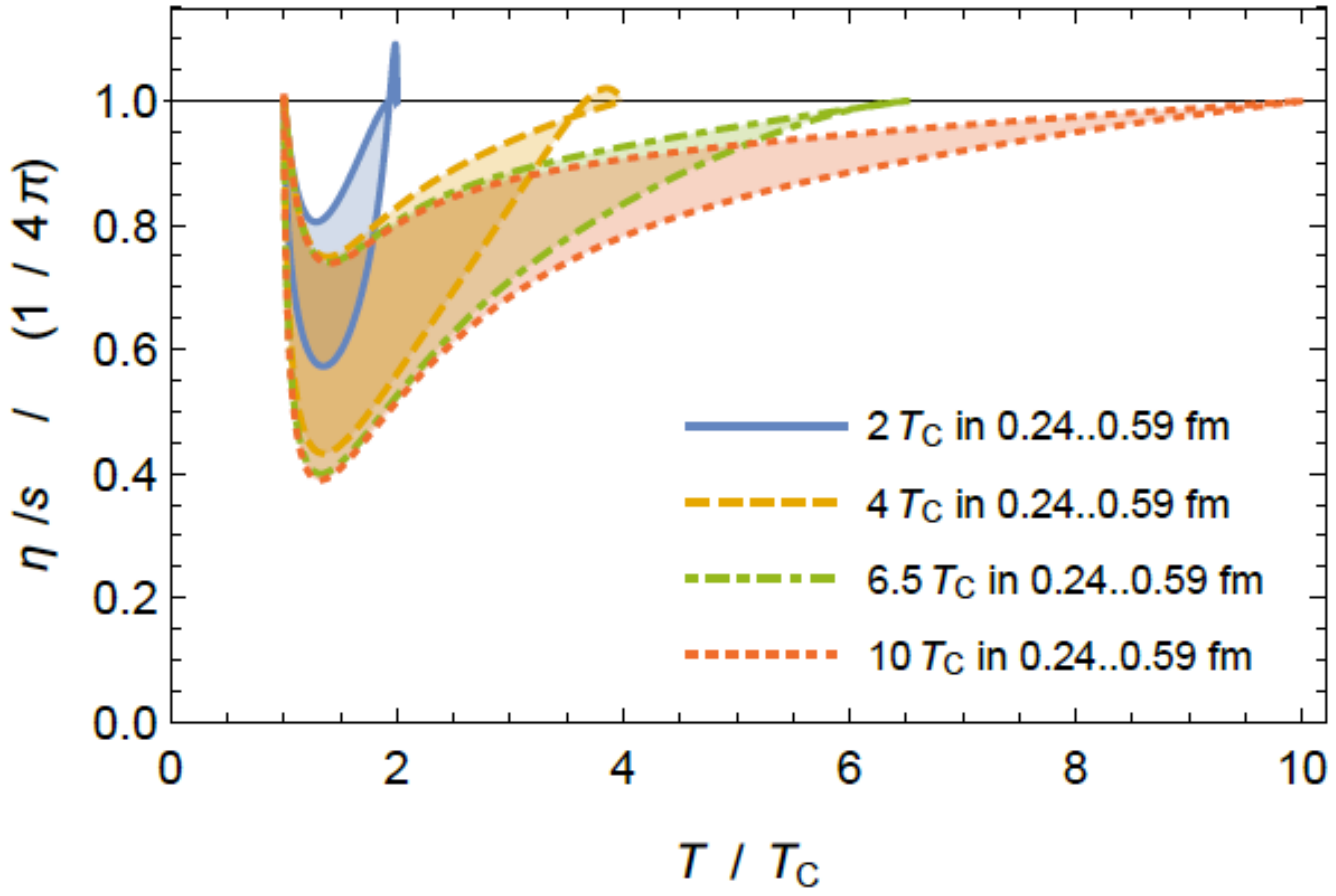
No universal bound

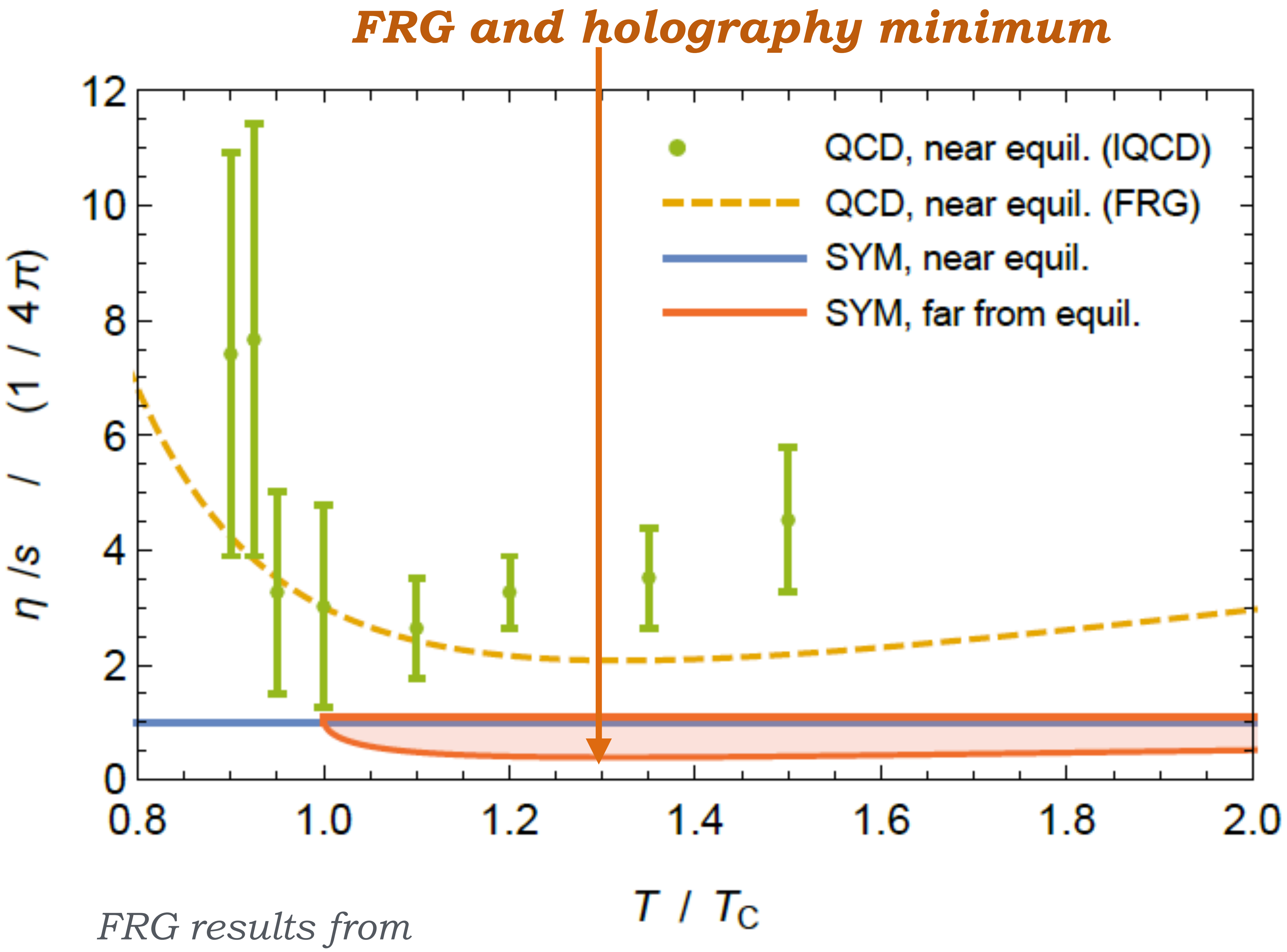
[Buchel, Myers, Sindha; JHEP (2008)]

➡ Shear transport ratio first drops below 60%, then rises above 110% of KSS value $1/(4\pi)$

1. Far from equilibrium shear: Results - continued

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]





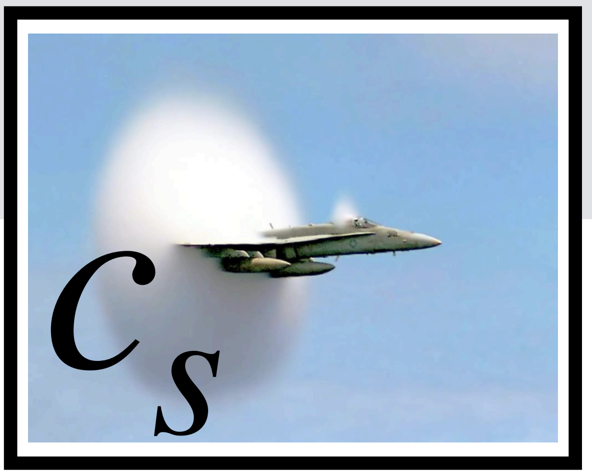
FRG results from
[Christiansen, Haas, Pawlowski, Strodthoff; PRL (2015)]
Lattice QCD data from
[Astrakhantsev, Braguta, Kotov; JHEP (2017)]

➡ stark contrast: near equilibrium lattice QCD / FRG suggest $\eta/s > 1/(4\pi)$

whereas far from equilibrium Super-Yang-Mills (SYM) plasma suggests $\eta/s < 1/(4\pi)$

➡ currently underestimating flow generated at early times *[Bernhard, Moreland, Bass, Nature (2019)]*

➡ Talk by Travis Dore



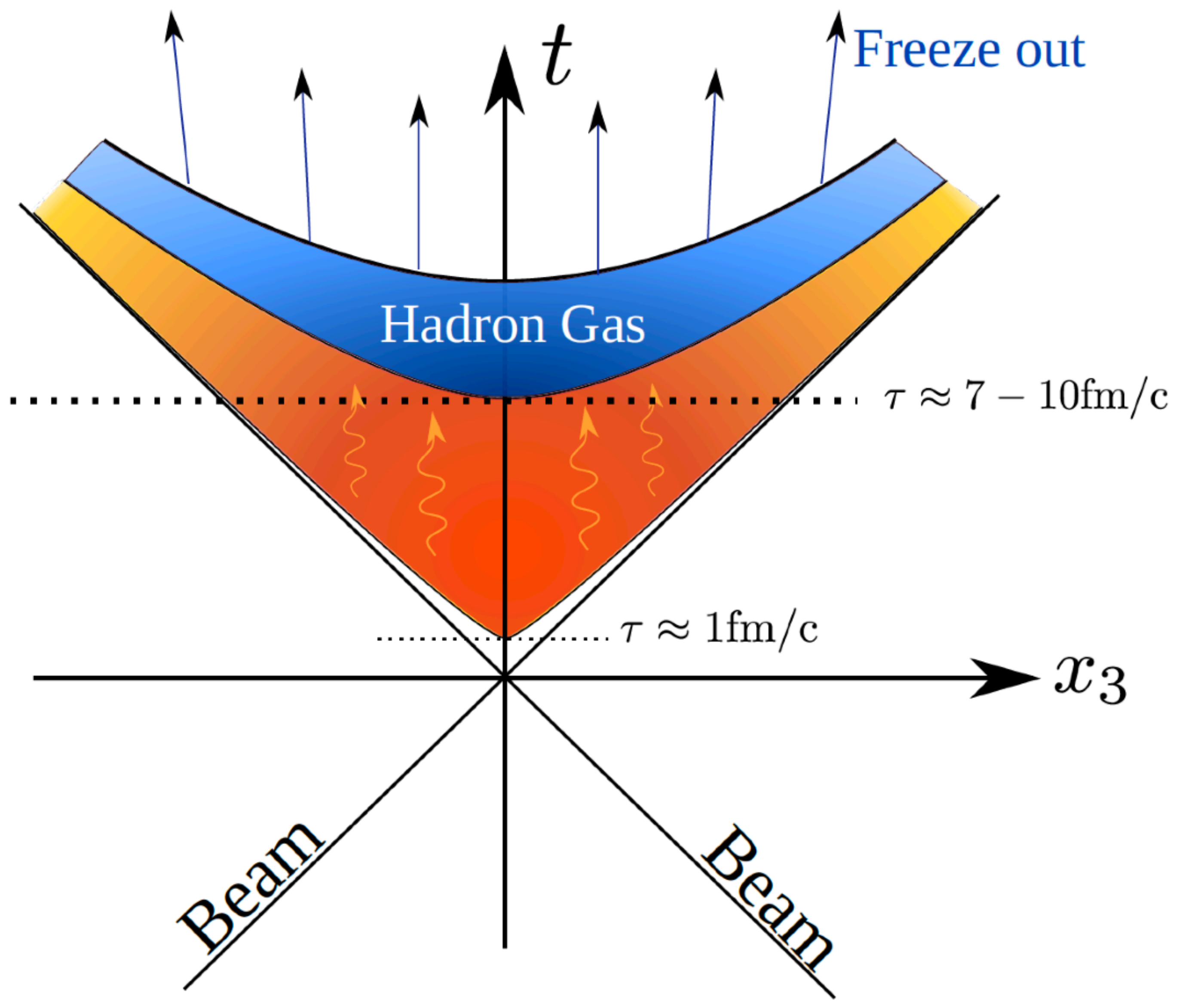
2. Bjorken-expanding plasma

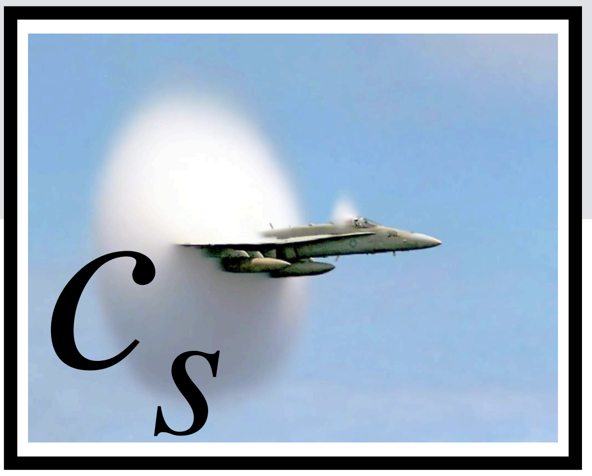
[Cartwright,Kaminski,Knipfer; (2022)]

- **far away from equilibrium thermodynamic quantities are not well-defined**
- **plasma is approximately boost invariant along the beam-line**
- **initially large anisotropy between that direction and the transverse plane**

proper time $\tau = \sqrt{t^2 - x_3^2}$

rapidity $\xi = \frac{1}{2}\ln[(t+x_3)/(t-x_3)]$





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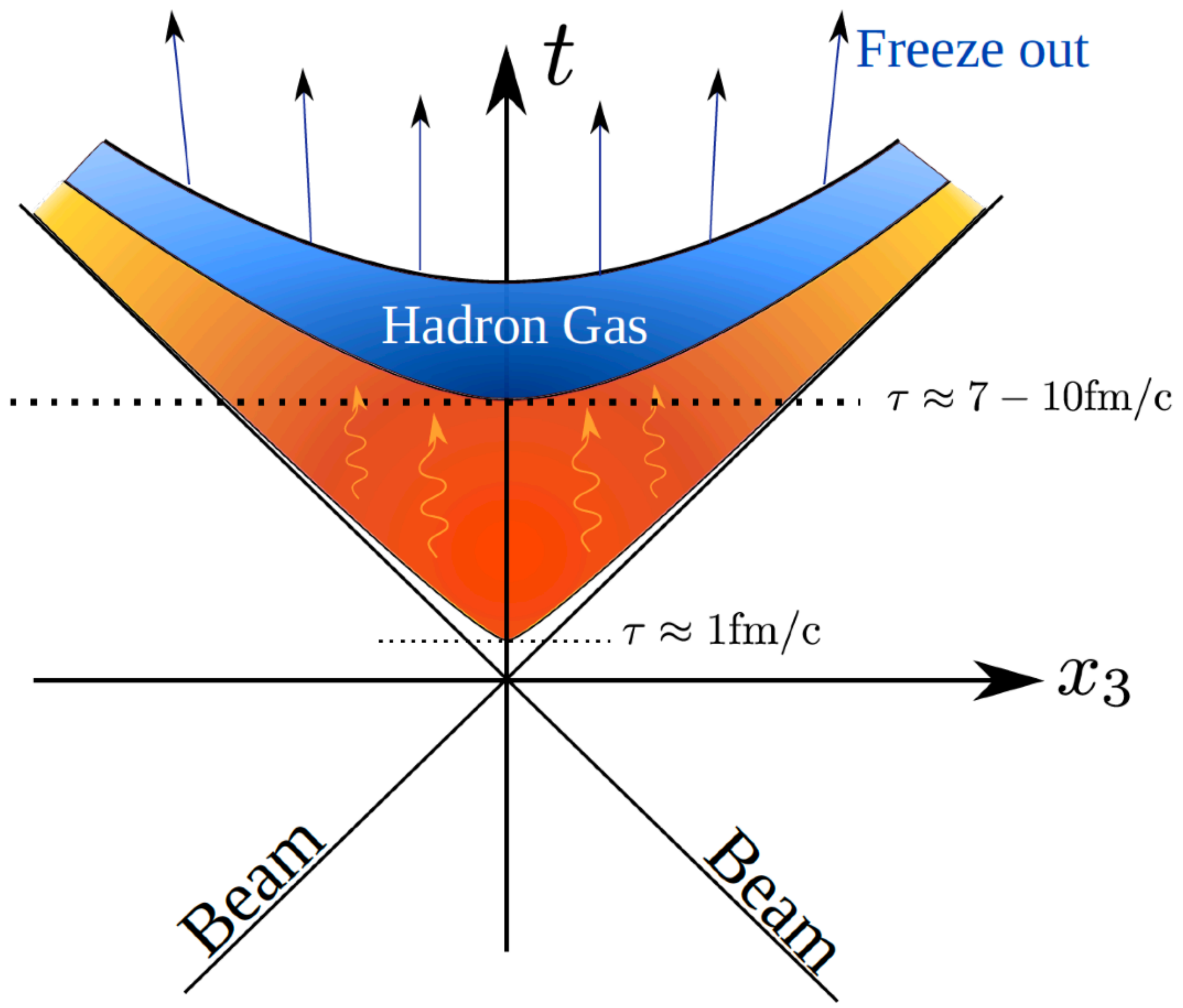


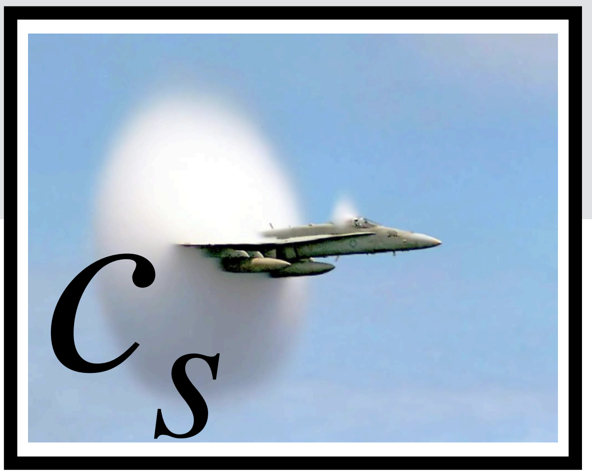
Gravity dual: Einstein Gravity, anisotropic metric ***AdS radial coordinate*** $r = 1/z$

$$\mathrm{d}s^2 = 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + e^{B(v,r)}S(v,r)^2(\mathrm{d}x_1^2 + \mathrm{d}x_2^2) + S(v,r)^2 e^{-2B(v,r)}\mathrm{d}\xi^2$$

boundary at $r = \infty$ ***has Milne metric:*** $\lim_{r\to\infty}\frac{1}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$

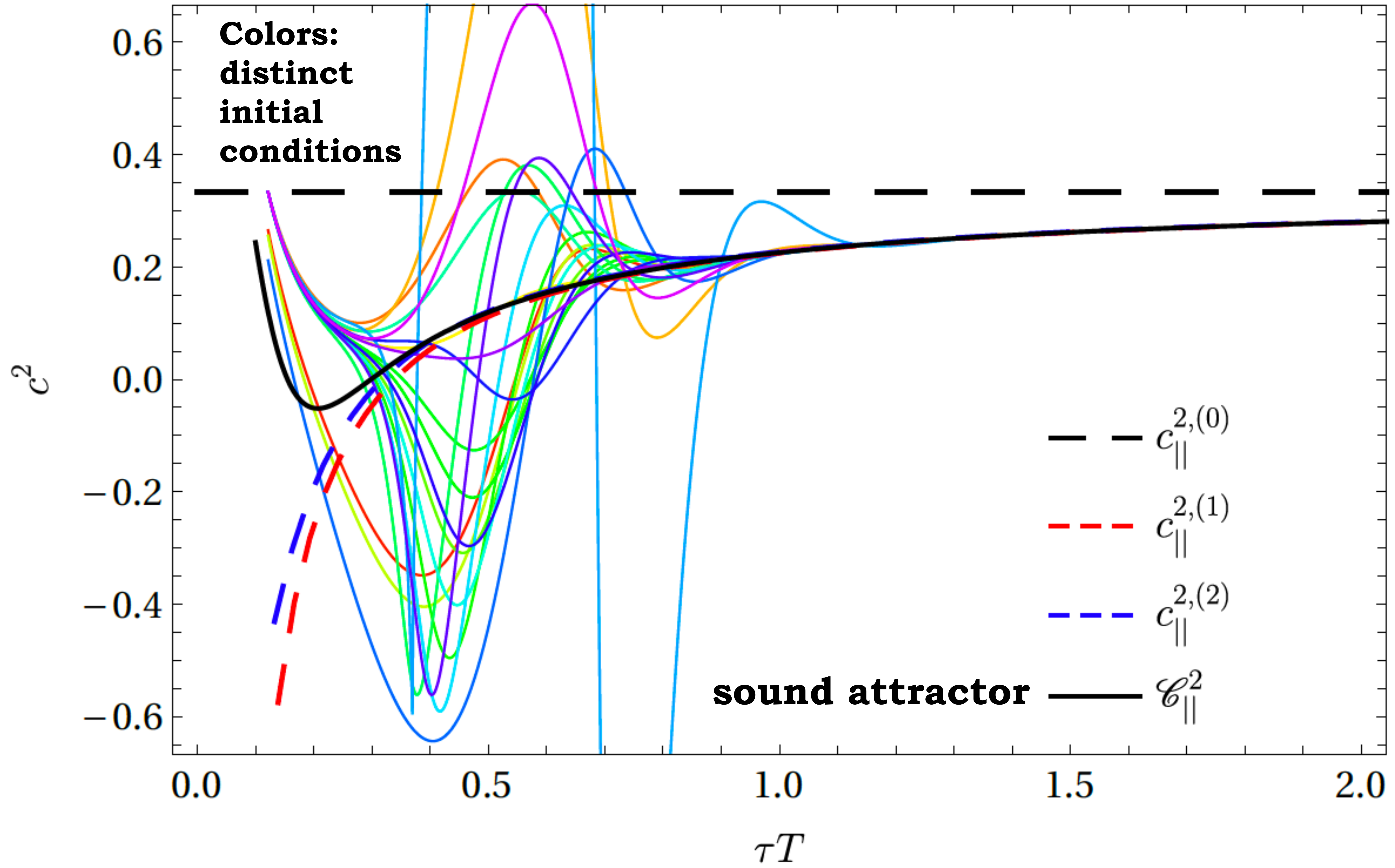
➡**Late times: system still expanding but approximately isotropic.**

➡**Far from equilibrium at early times: define a speed of sound via holography.**



2. "Speed of sound" in Bjorken-expanding QGP

[Cartwright,Kaminski,Knipfer; (2022)]

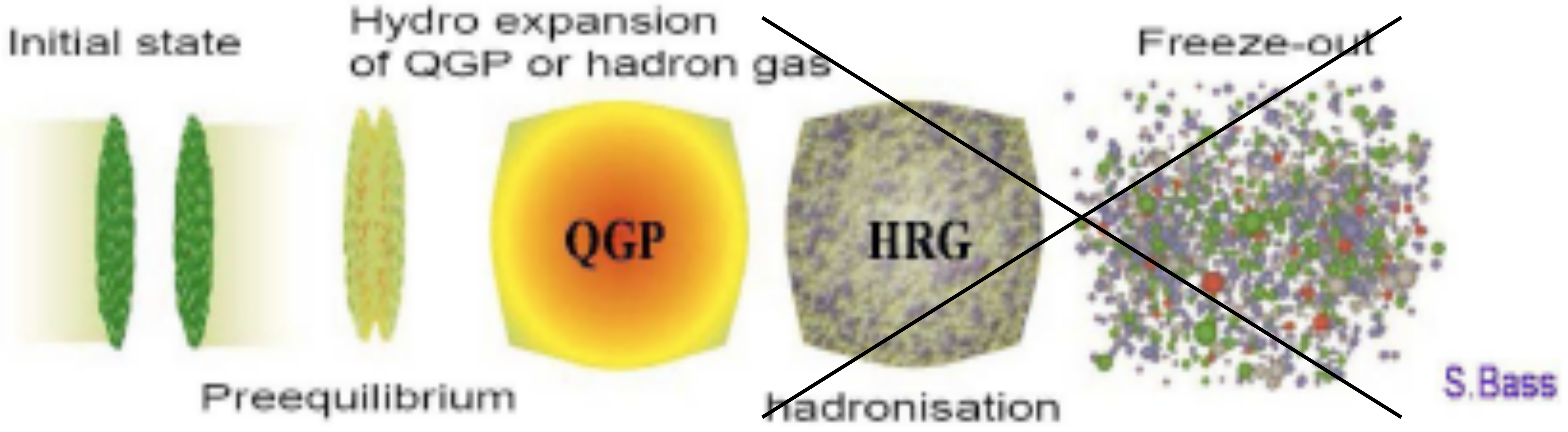


Temperature from energy:

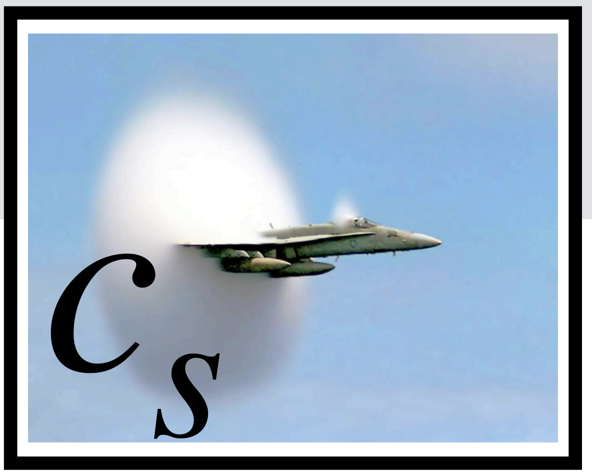
$$T = (\epsilon/\sigma_{SB})^{1/4}$$

Equilibrium speed of sound

$$c_s^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_s$$



[Spalinski; PLB (2018)]



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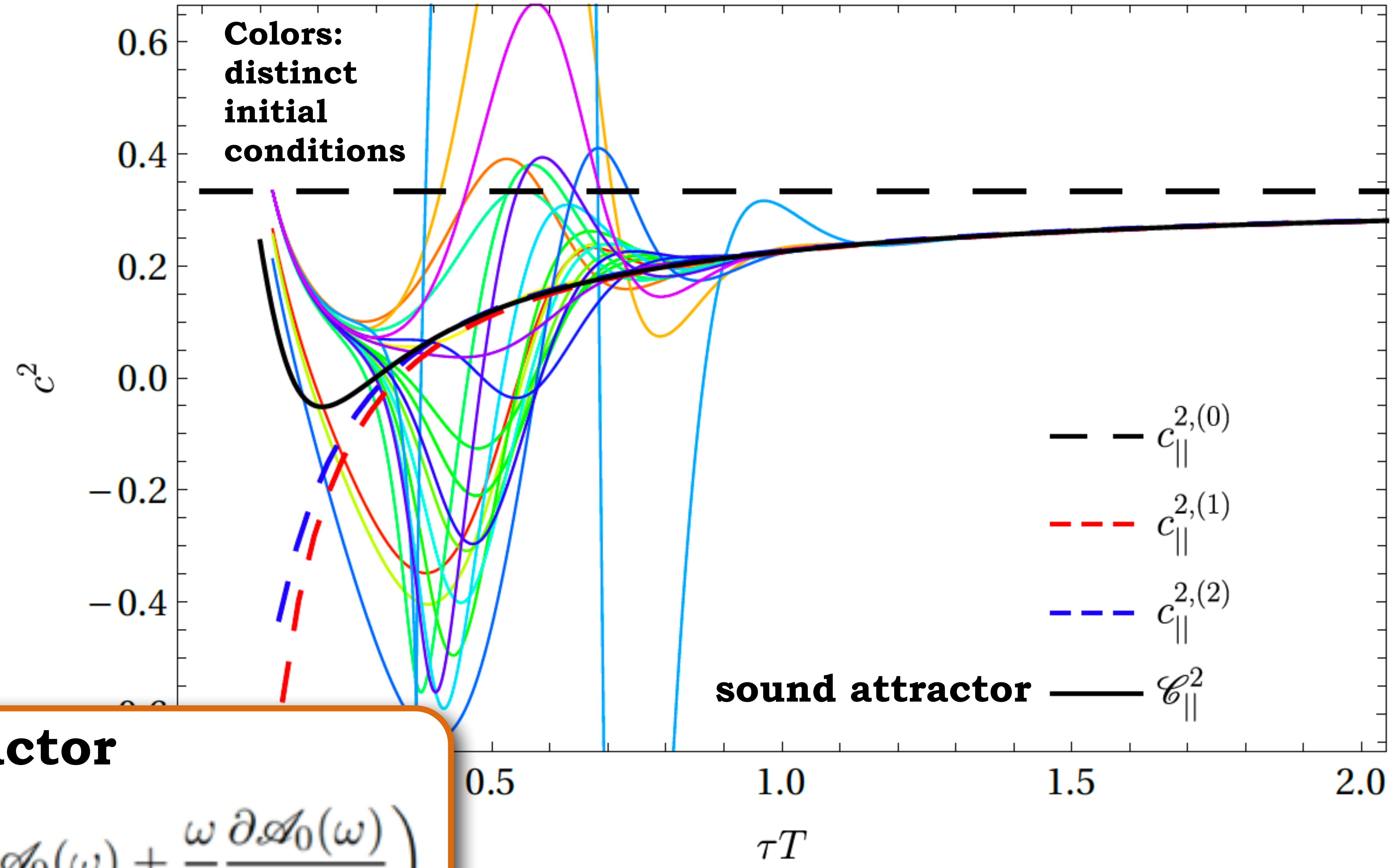

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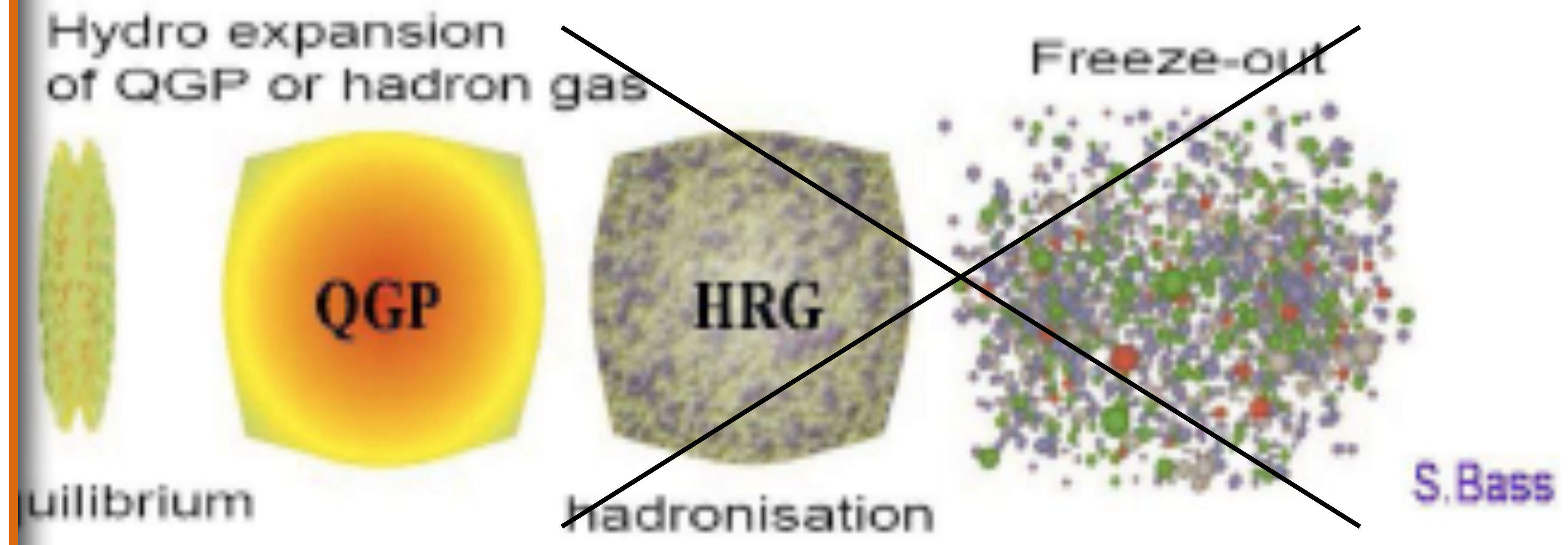
Sound attractor

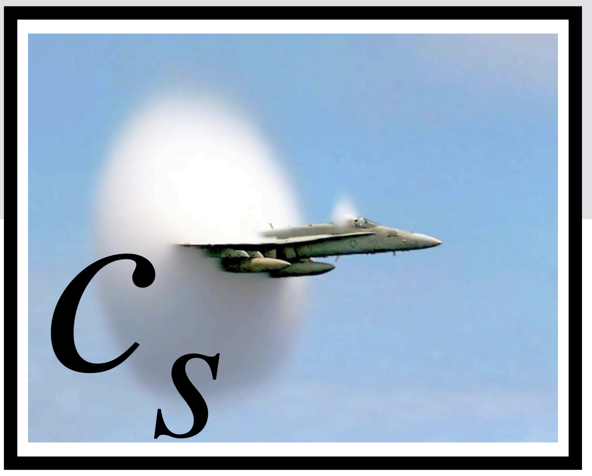
$$\mathscr{C}_{||}^2 = \frac{1}{3} - \frac{2}{9}\left(\mathscr{A}_0(\omega) + \frac{\omega}{4}\frac{\partial \mathscr{A}_0(\omega)}{\partial \omega}\right)$$

$$\mathscr{A}(\omega) = \frac{P_\perp - P_{||}}{\mathscr{P}}, \quad \mathscr{P} = \epsilon/3$$

$$\mathscr{A}_0(\omega) = \frac{2530\omega - 276}{3975\omega^2 - 570\omega + 120}$$

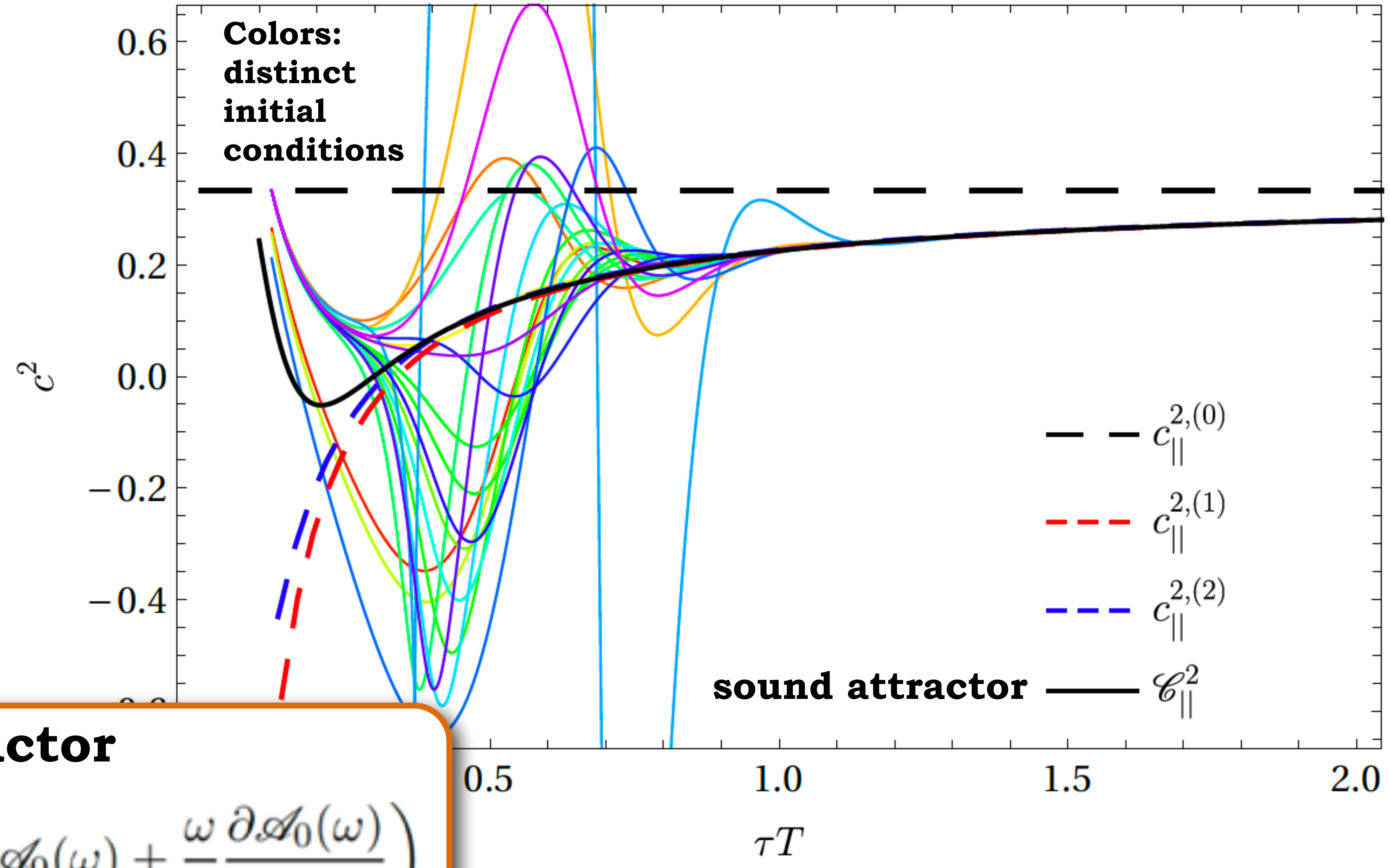
[Spalinski; PLB (2018)]





2. "Speed of sound" in Bjorken-expanding QGP

[Cartwright,Kaminski,Knipfer; (2022)]



Temperature from energy:

$$T = (\epsilon/\sigma_{SB})^{1/4}$$

Equilibrium speed of sound

$$c_s^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_s$$

Transverse/longitudinal speed of sound far from equilibrium

$$c_\perp^2 = -\frac{\partial \langle T^{x_1}_{x_1}\rangle}{\partial \langle T^0_0\rangle}, \qquad c_{||}^2 = -\frac{\partial \langle T^{\xi}_{\xi}\rangle}{\partial \langle T^0_0\rangle}$$

➡Verify with perturbative calculation

$$g_{\mu\nu}(\tau) + h_{\mu\nu}^{(\text{sound})}$$

Using technique from [Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]

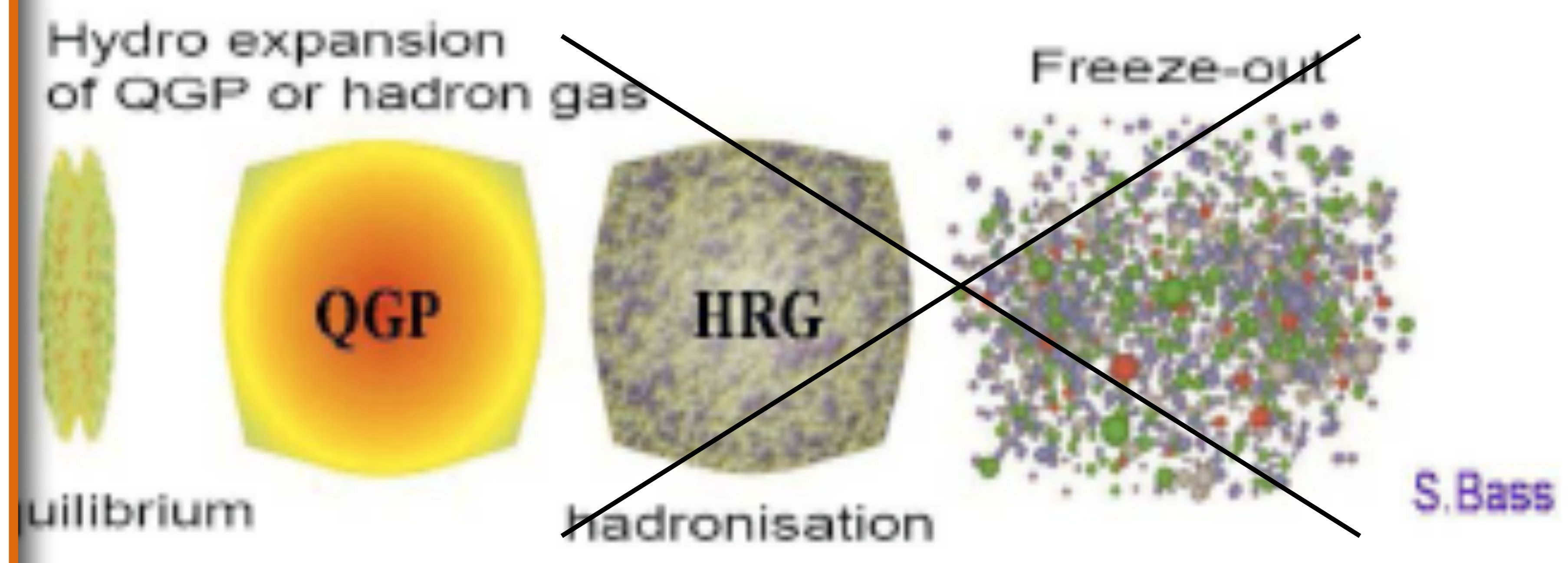
Sound attractor

$$\mathscr{C}_{||}^2 = \frac{1}{3} - \frac{2}{9}\left(\mathscr{A}_0(\omega) + \frac{\omega}{4}\frac{\partial \mathscr{A}_0(\omega)}{\partial \omega}\right)$$

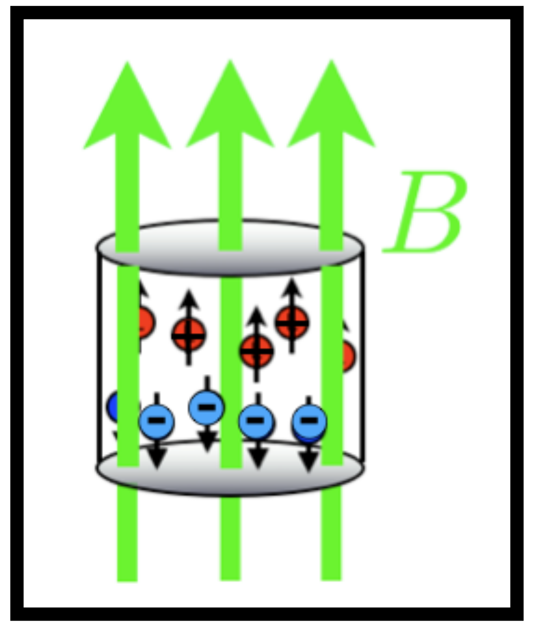
$$\mathscr{A}(\omega) = \frac{P_\perp - P_{||}}{\mathscr{P}}, \qquad \mathscr{P} = \epsilon/3$$

$$\mathscr{A}_0(\omega) = \frac{2530\omega - 276}{3975\omega^2 - 570\omega + 120}$$

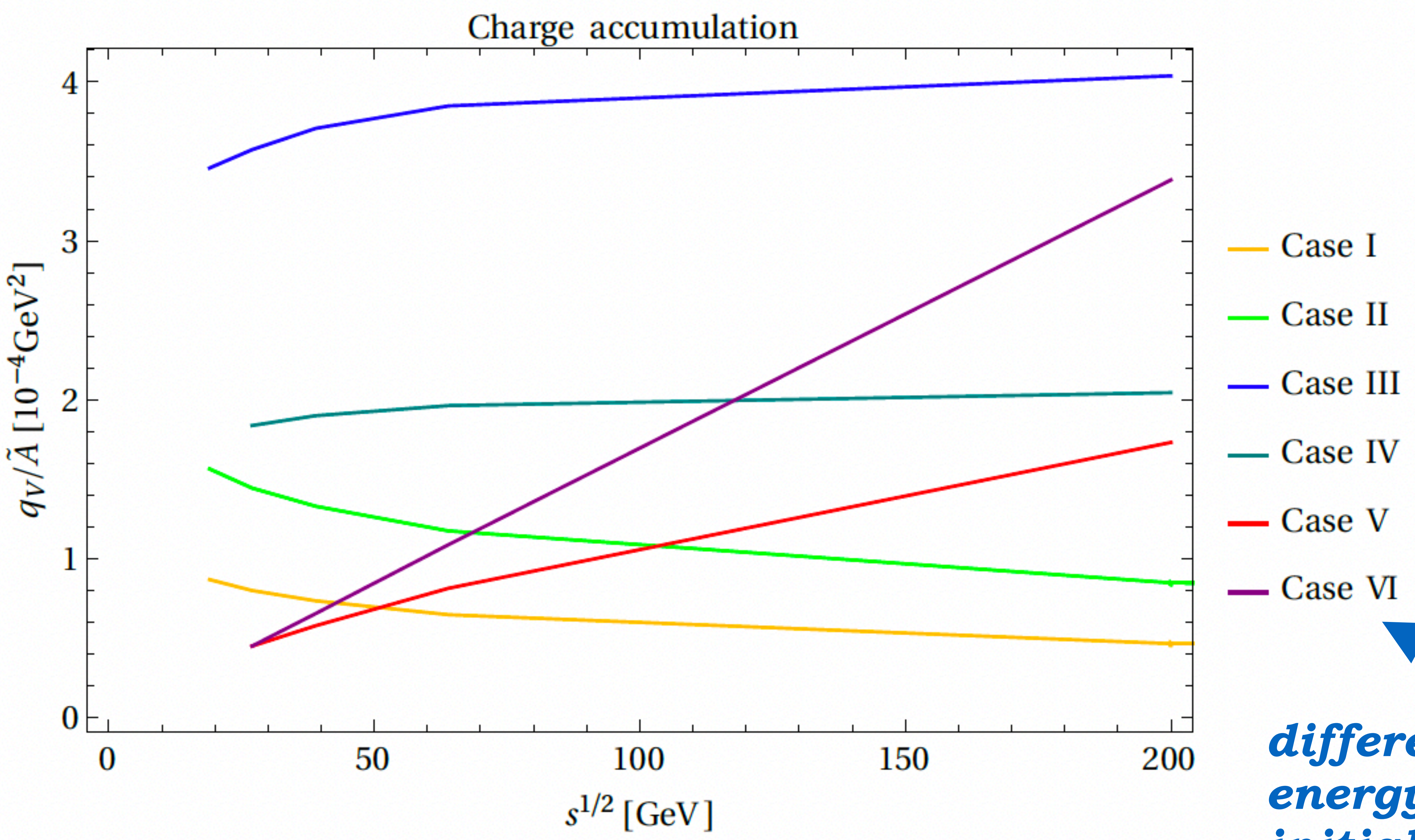
[Spalinski; PLB (2018)]



3. Chiral Magnetic Effect in Bjorken-expanding plasma



[Cartwright,Kaminski,Schenke; PRC (2022)]



Charge accumulated in detector over time $\Delta\tau = \tau_f - \tau_i$ due to Chiral Magnetic Effect (CME)

$$q_V/\tilde{A} = \int_{\tau_i}^{\tau_f} \mathrm{d}\tau\tau\,\langle J^1\rangle$$

Area: $\tilde{A} = \int \mathrm{d}x_2 \mathrm{d}\xi$

different combinations of initial energy, initial chiral imbalance, initial magnetic field

Equilibrium CME

$$J^\mu = \xi_\chi B \qquad \xi_\chi = C\,\mu_A$$

➡CME more likely to be seen at higher energies?

compare: [Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

Compare to experiments: *top-RHIC energy: [STAR Collaboration; (2021)]* no CME, only background

low-energy update: [STAR Collaboration; (2022)] no CME, only background

high energy update: [ALICE Collaboration; (2022)] no CME, only background?

Discussion

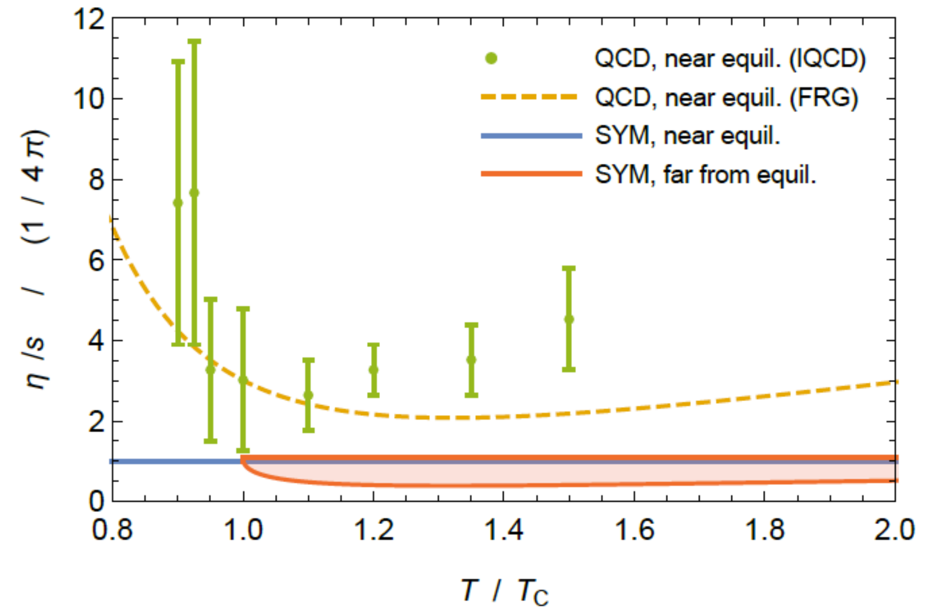
Summary

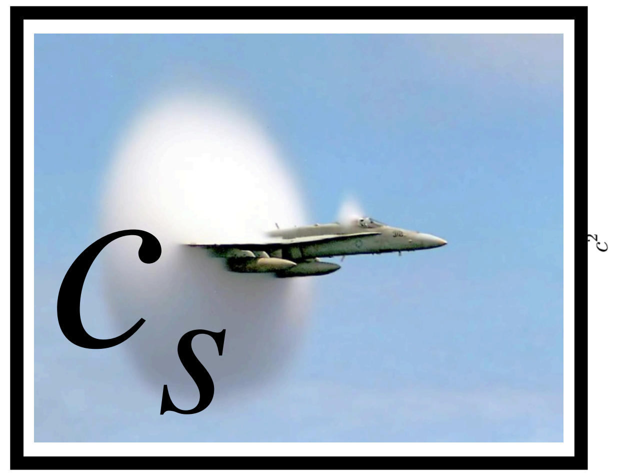
- proposed far from equilibrium definitions for "shear viscosity" and "speed of sound"
- "speed of sound" has *hydrodynamic attractor* *compare [Spalinski; PLB (2018)]* *[Heller, Spalinski; PRL (2015)]* *[Heller et al; PRL (2021)]*
- Chiral Magnetic Effect more likely to be seen at *high* energies?

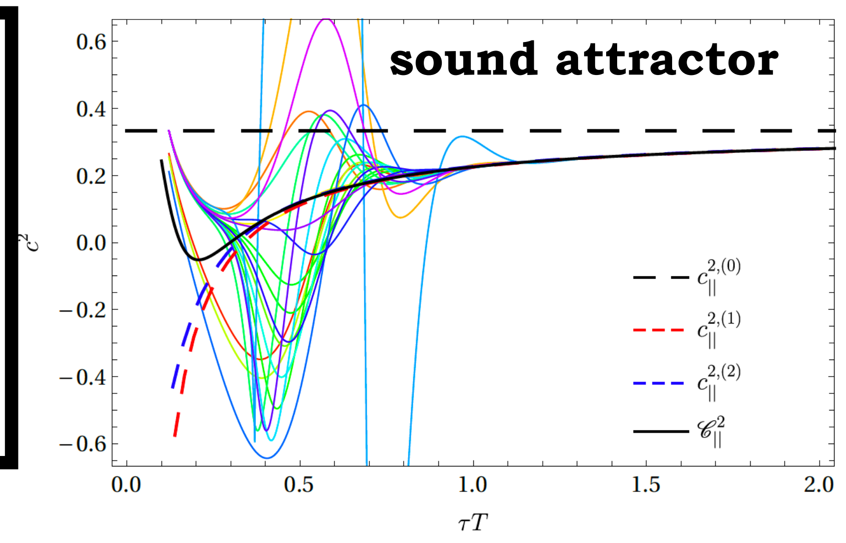
Outlook

- **compute speed of sound directly** from sound sector fluctuations around *Bjorken-expanding* holographic plasma
- include **dynamical magnetic field and dynamically created axial imbalance** to model QGP and CME *[AdS4CME Collaboration]*
- far from equilibrium "hydrodynamics" (effective field theory) *[Romatschke; PRL (2017)]*

$$\frac{\eta}{s}$$







Talk by Yi Yin

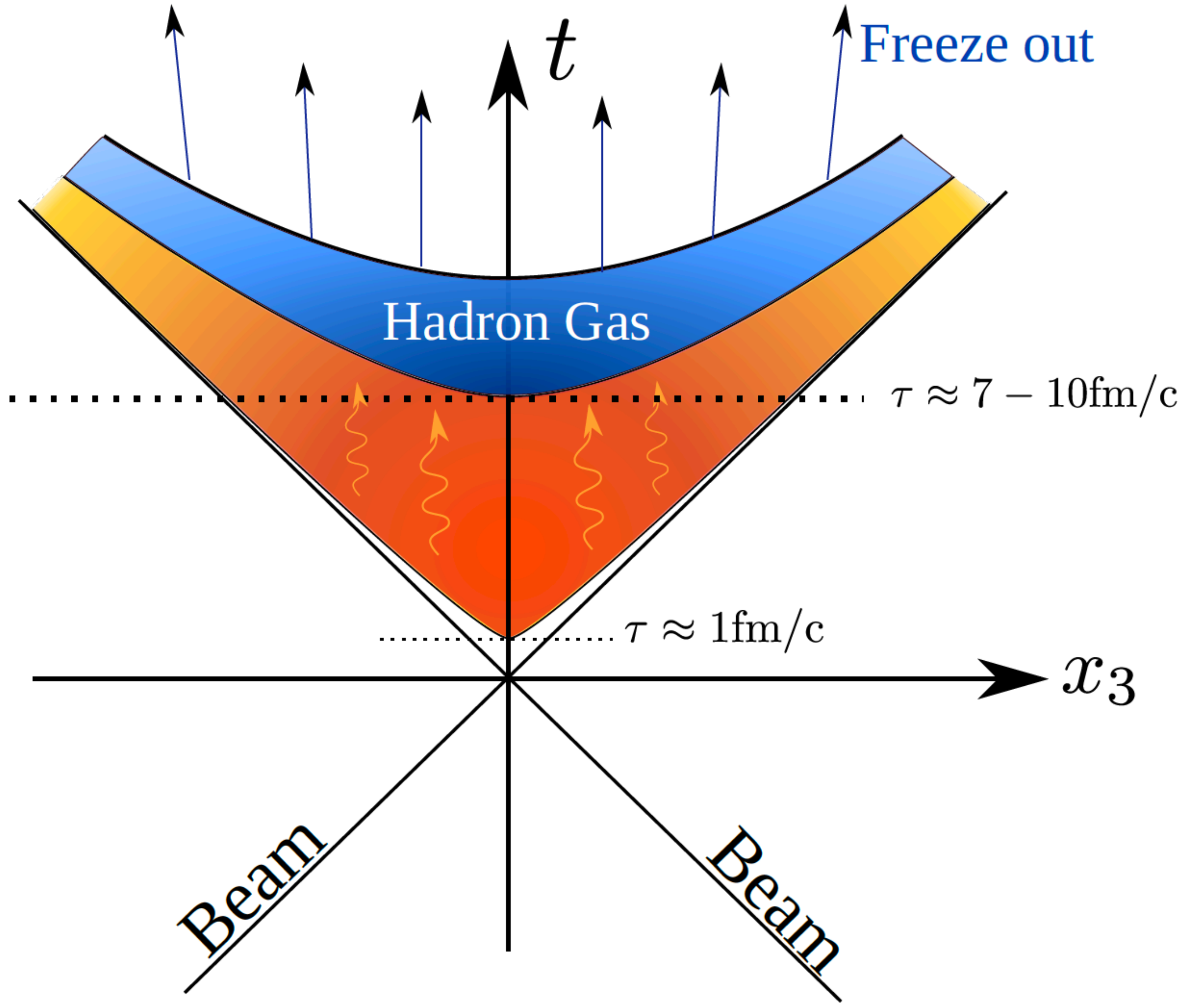
Talk by Clemens Werthmann

APPENDIX

2. Bjorken - expanding plasma

[Cartwright,Kaminski,Knipfer; (2022)]

- far away from equilibrium thermodynamic quantities are not well-defined
- plasma is approximately boost invariant along the beam-line
- initially large anisotropy between that direction and the transverse plane



proper time $$\tau = \sqrt{t^2 - x_3^2}$$

Ideal hydrodynamics:

$$u_\nu \partial_\mu T^{\mu\nu} = u_\nu \partial_\mu \left((\epsilon + P) u^\mu u^\nu - P q^{\mu\nu} \right)$$
$$= \partial_\tau \epsilon + \frac{4}{3\tau}\epsilon \,, \quad \epsilon = \epsilon_0 \left(\frac{\tau_0}{\tau} \right)^{4/3} ,$$

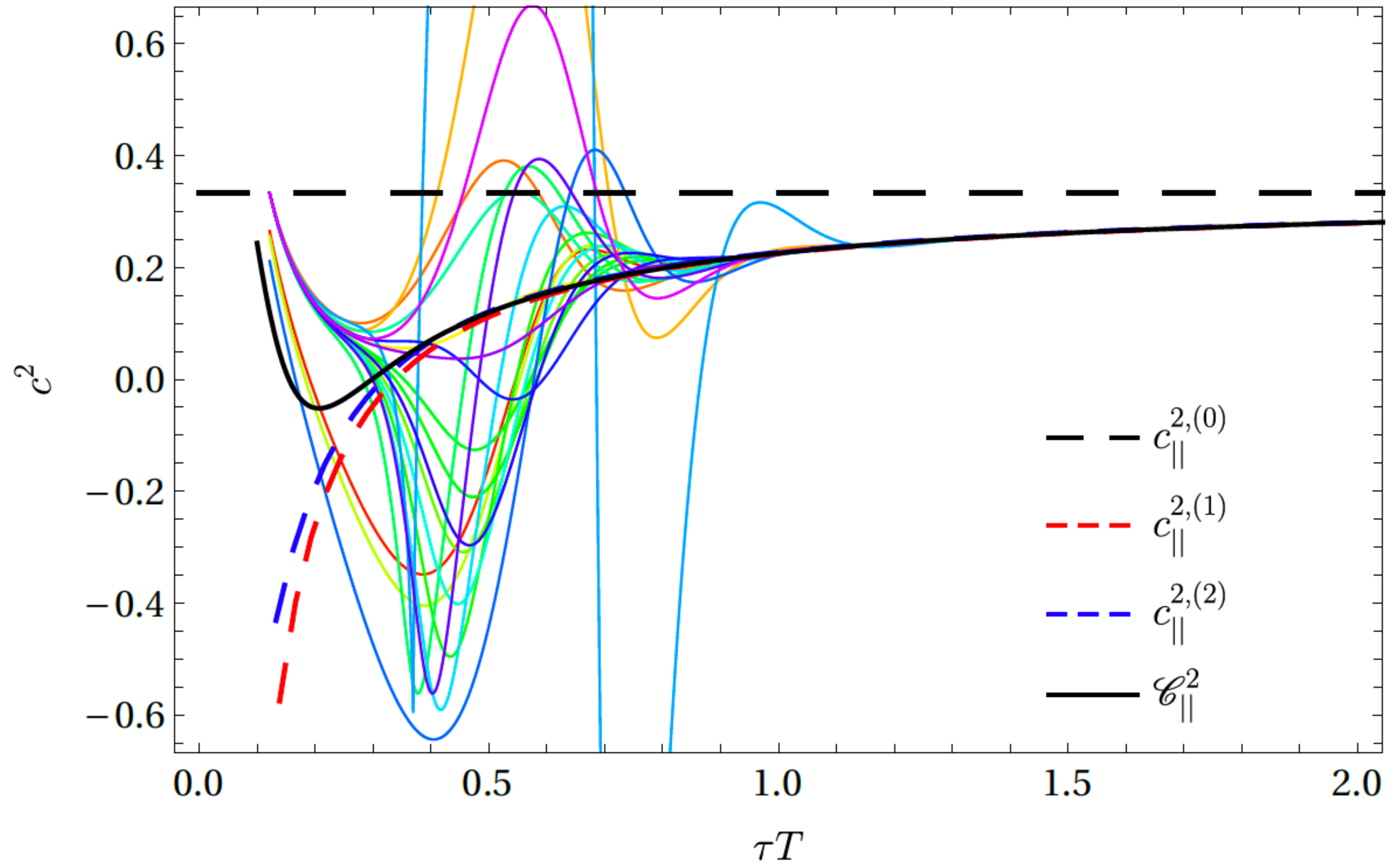
$$\tilde{T}^{\mu\nu} = \begin{pmatrix} \frac{P x_3^2 + t^2 \epsilon}{t^2 - x_3^2} & 0 & 0 & \frac{t x_3 (P+\epsilon)}{t^2 - x_3^2} \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ \frac{t x_3 (P+\epsilon)}{t^2 - x_3^2} & 0 & 0 & \frac{x_3^2 (P+\epsilon)}{t^2 - x_3^2} + P \end{pmatrix}$$

Viscous hydrodynamics (second order):

$$\partial_\tau \epsilon + \frac{4\epsilon}{3\tau} = \frac{4\eta}{3\tau^2} + \frac{8\eta\tau_\pi}{9\tau^3} - \frac{8\lambda_1}{9\tau^3}$$

➡ At late times, the system is still expanding and approximately isotropic.

2. “Speed of sound” attractor



[Cartwright,Kaminski,Knipfer; (2022)]

$$\omega = \tau T \qquad c_s^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_s$$

$$\mathscr{A}_0(\omega) = \frac{2530\omega - 276}{3975\omega^2 - 570\omega + 120}$$

[Spalinski; PLB (2018)]

$$\mathscr{A}(\omega) = \frac{P_\perp - P_{||}}{\mathscr{P}}, \quad \mathscr{P} = \epsilon/3$$

$$\partial_\omega \mathscr{A} = \frac{\partial_\omega \epsilon}{\epsilon}\left(3\Delta c^2 - \mathscr{A}(\omega)\right), \quad \Delta c^2 = c_{||}^2 - c_\perp^2 \qquad 2c_\perp^2 + c_{||}^2 = 1$$

$$\frac{\partial_\omega \epsilon}{\epsilon} = \frac{4}{\omega}$$

$$\Delta c^2 = -\frac{1}{3}\left(\frac{\omega}{4}\partial_\omega - 1\right)\mathscr{A}(\omega)$$

$$\Delta c^2 = \frac{-1}{2} + \frac{3}{2}c_{||}^2 = 1 - 3c_\perp^2$$

$$\mathscr{C}_\perp^2 = \frac{1}{3} + \frac{1}{9}\left(\mathscr{A}_0(\omega) + \frac{\omega}{4}\frac{\partial \mathscr{A}_0(\omega)}{\partial \omega}\right)$$

$$\mathscr{C}_{||}^2 = \frac{1}{3} - \frac{2}{9}\left(\mathscr{A}_0(\omega) + \frac{\omega}{4}\frac{\partial \mathscr{A}_0(\omega)}{\partial \omega}\right)$$

2. Holographic Bjorken - expanding plasma

[Cartwright,Kaminski,Knipfer; (2022)]

Metric Ansatz :

$$ds^2 = 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + e^{B(v,r)}S(v,r)^2(\mathrm{d}x_1^2 + \mathrm{d}x_2^2) + S(v,r)^2 e^{-2B(v,r)}\mathrm{d}\xi^2$$

$$\lim_{r\to\infty}\frac{1}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$$

Anisotropy function :

$$B = z^4 B_\mathrm{s} + \Delta_B$$

Initial conditions :

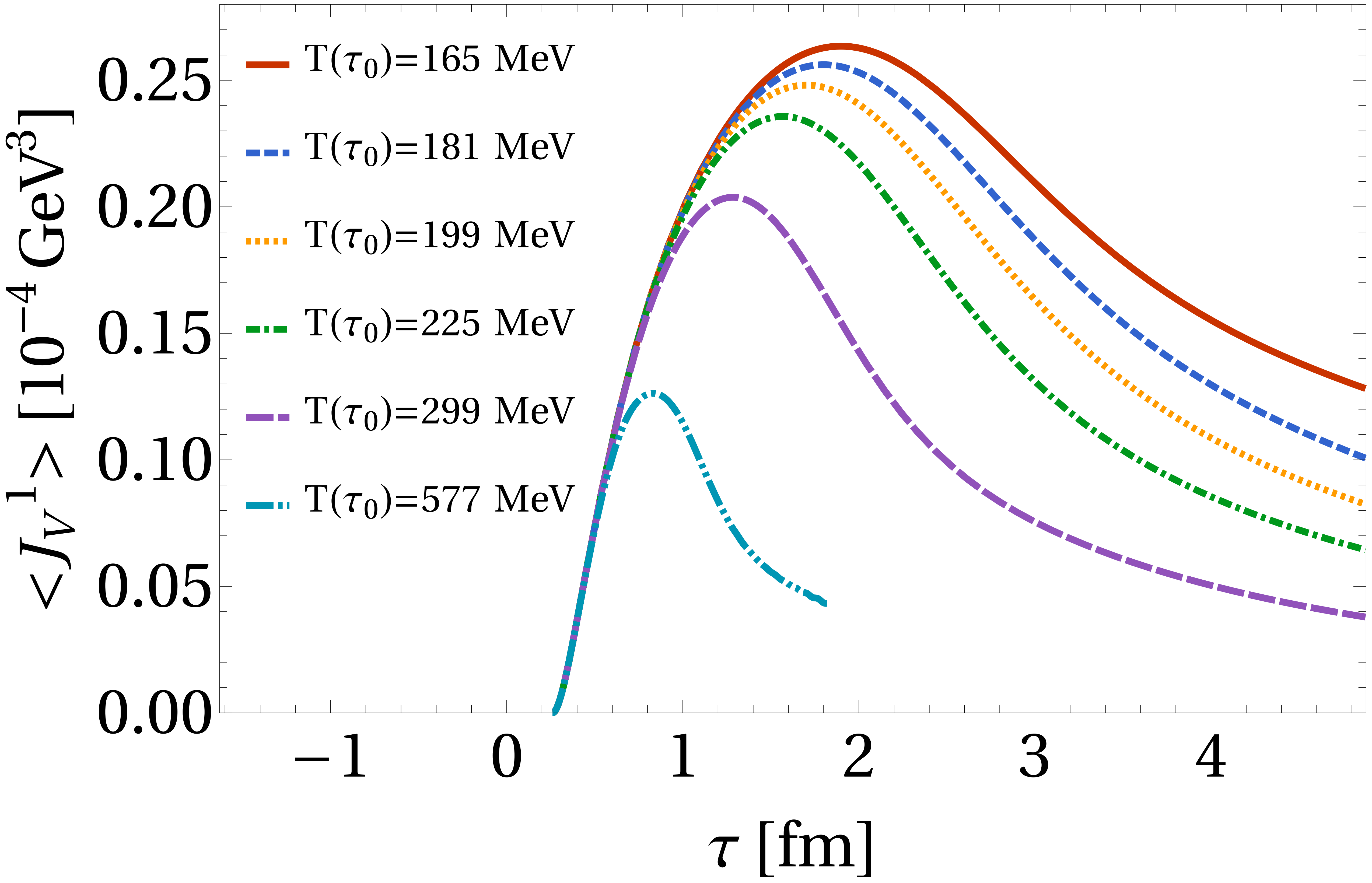
$$B_\mathrm{s}(z, v_0) = \Omega_1 \cos(\gamma_1 z) + \Omega_2 \tan(\gamma_2 z) + \Omega_3 \sin(\gamma_3 z) + \sum_{i=0}^{5}\beta_i z^i$$
$$+ \frac{\alpha}{z^4}\left[-\frac{2}{3}\ln\left(1 + \frac{z}{v_0}\right) + \frac{2z^3}{9v_0^3} - \frac{z^2}{3v_0^2} + \frac{2z}{3v_0}\right],$$

3. Bjorken - expanding plasma (case I)

Initial state:
constant B,
pressure anisotropy

time-dependent μ_5,
plasma expanding along beam line

Matching to QCD:
SUSY value for α
L=1fm fixes κ



Fixed initial $eB \approx m_\pi^2, \quad n_A = 0.00032\,\mathrm{GeV}^3$

➡**CME more likely to be seen at lower energies ???**

3. Bjorken - expanding plasma (case I)

[Cartwright,Kaminski,Schenke; PRC (2022)]

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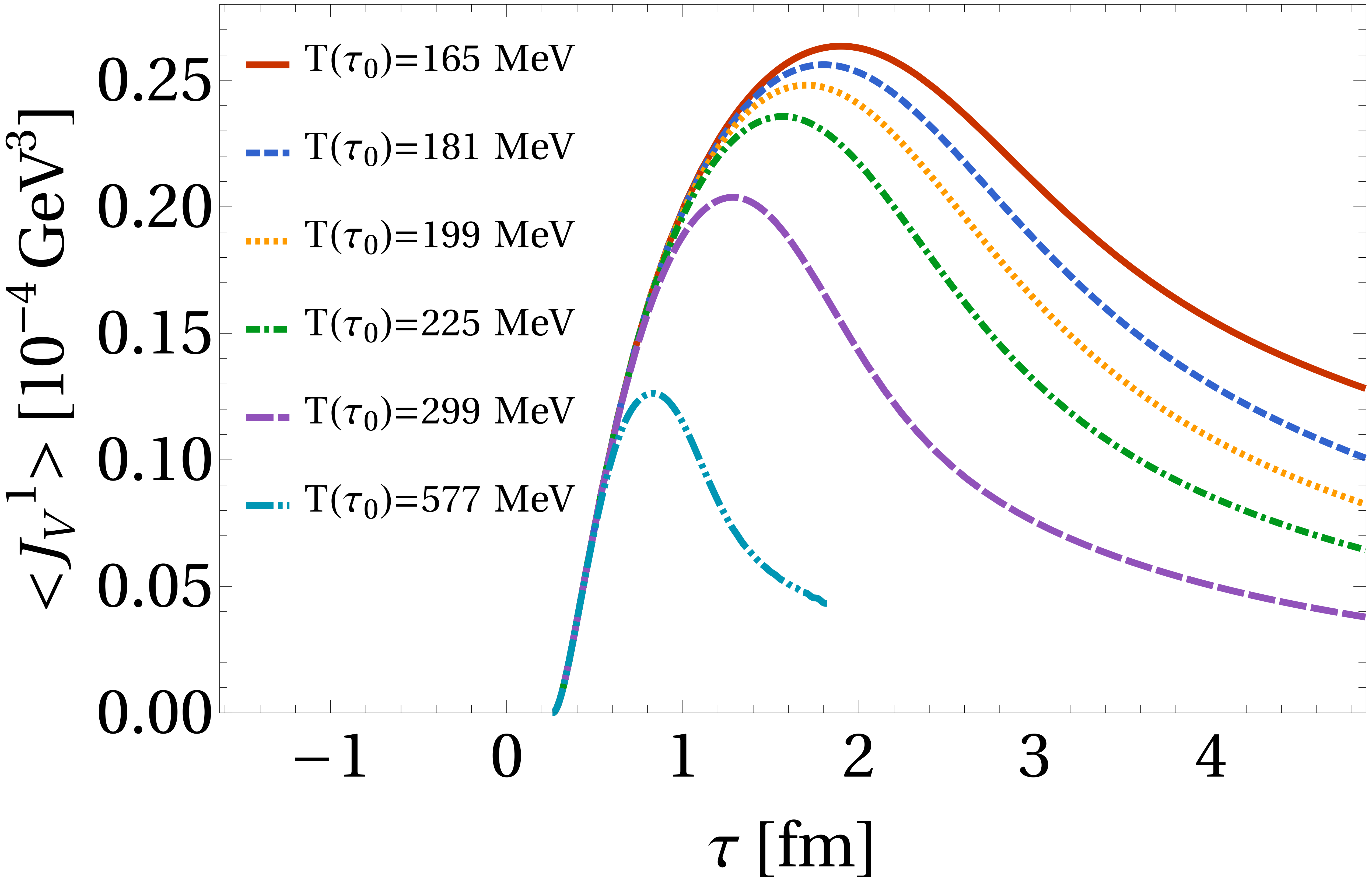

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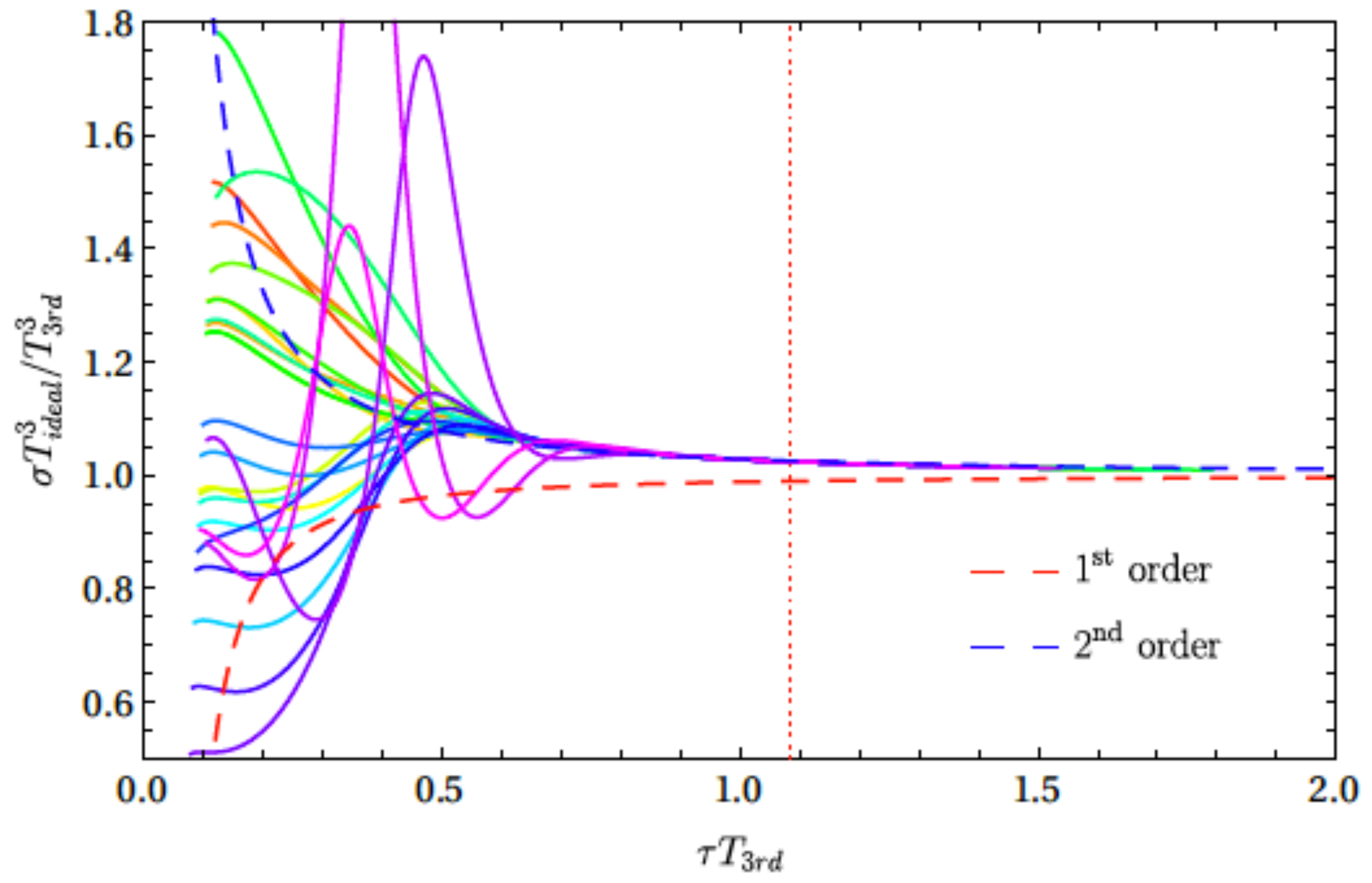
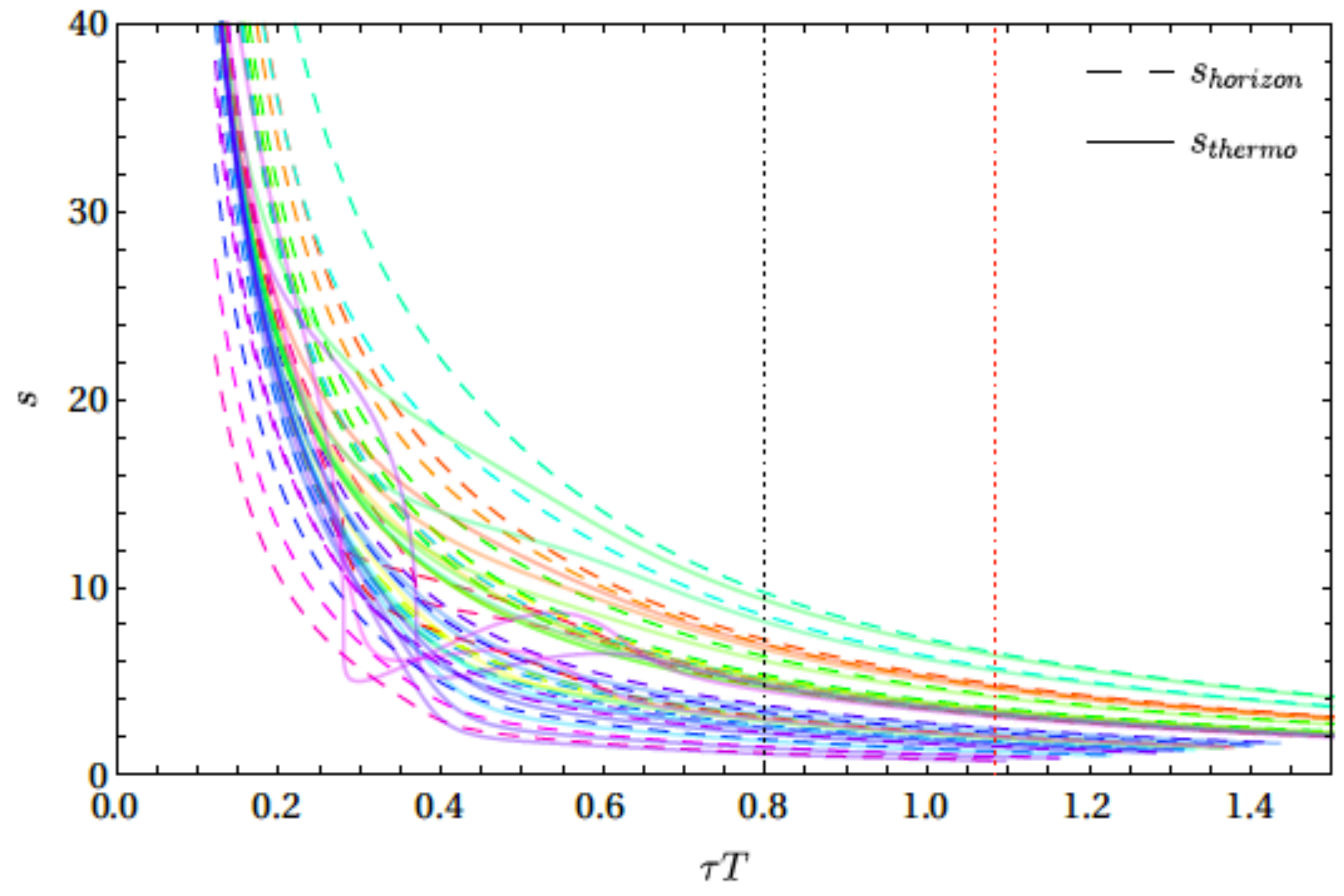
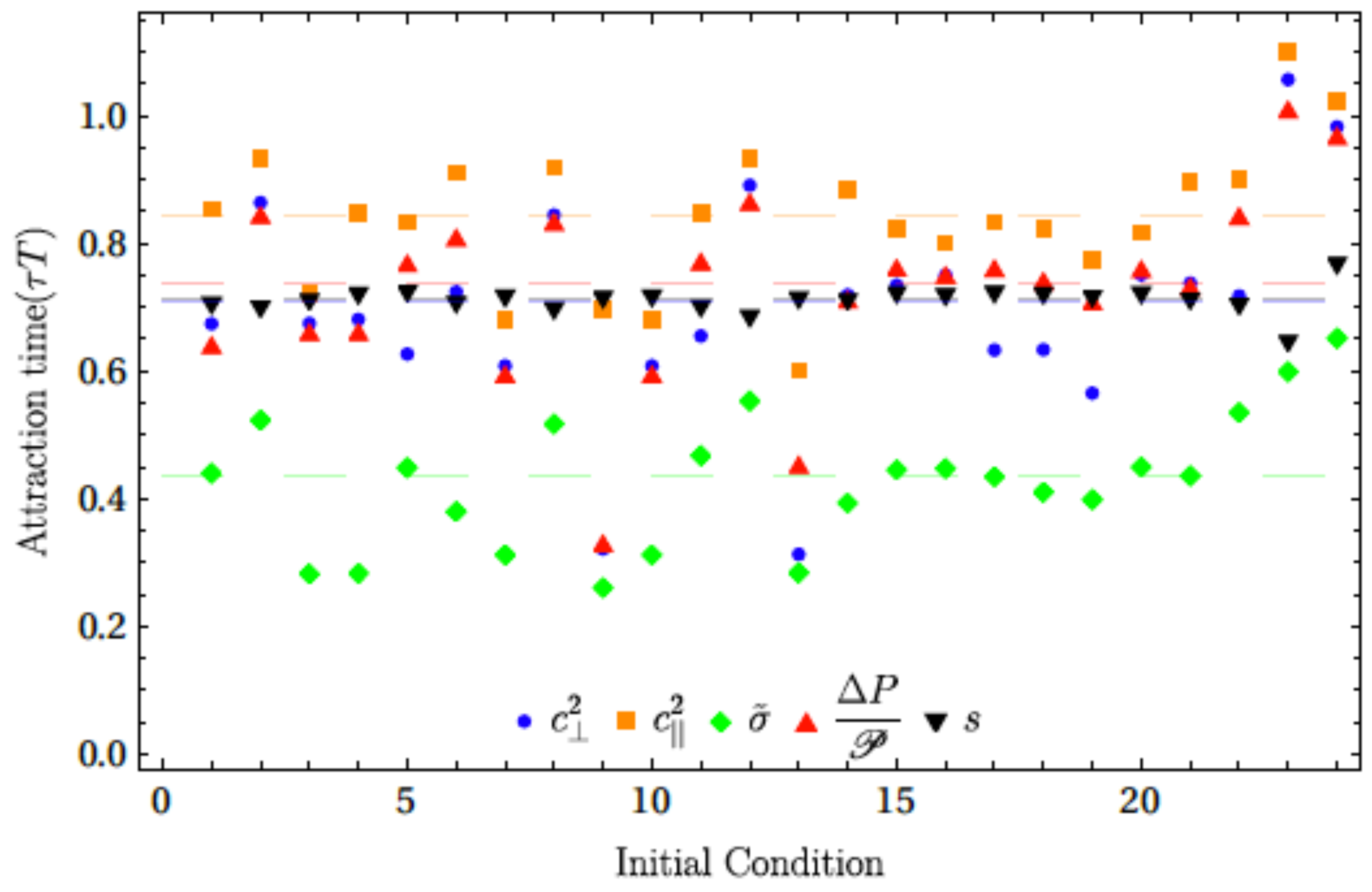
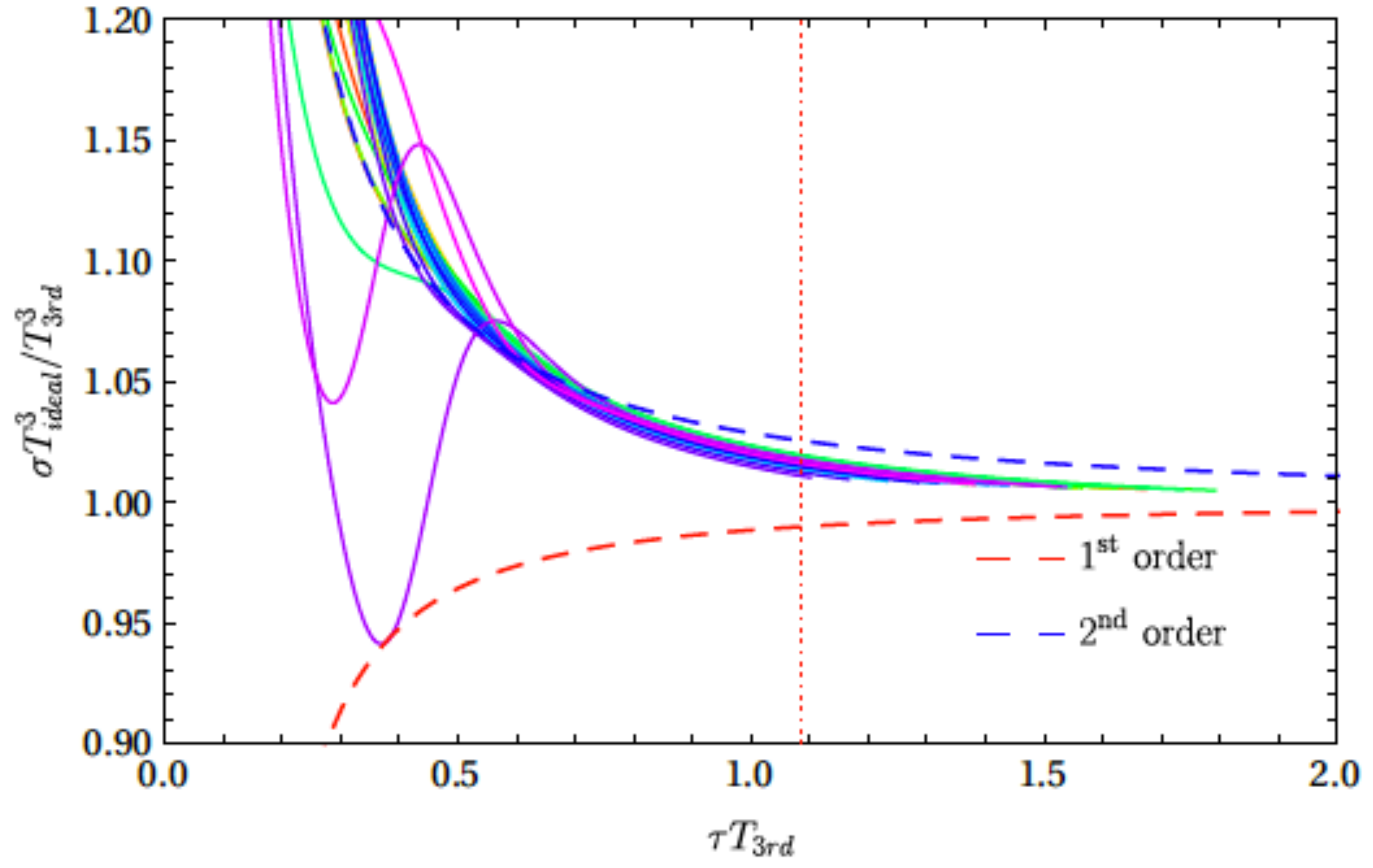
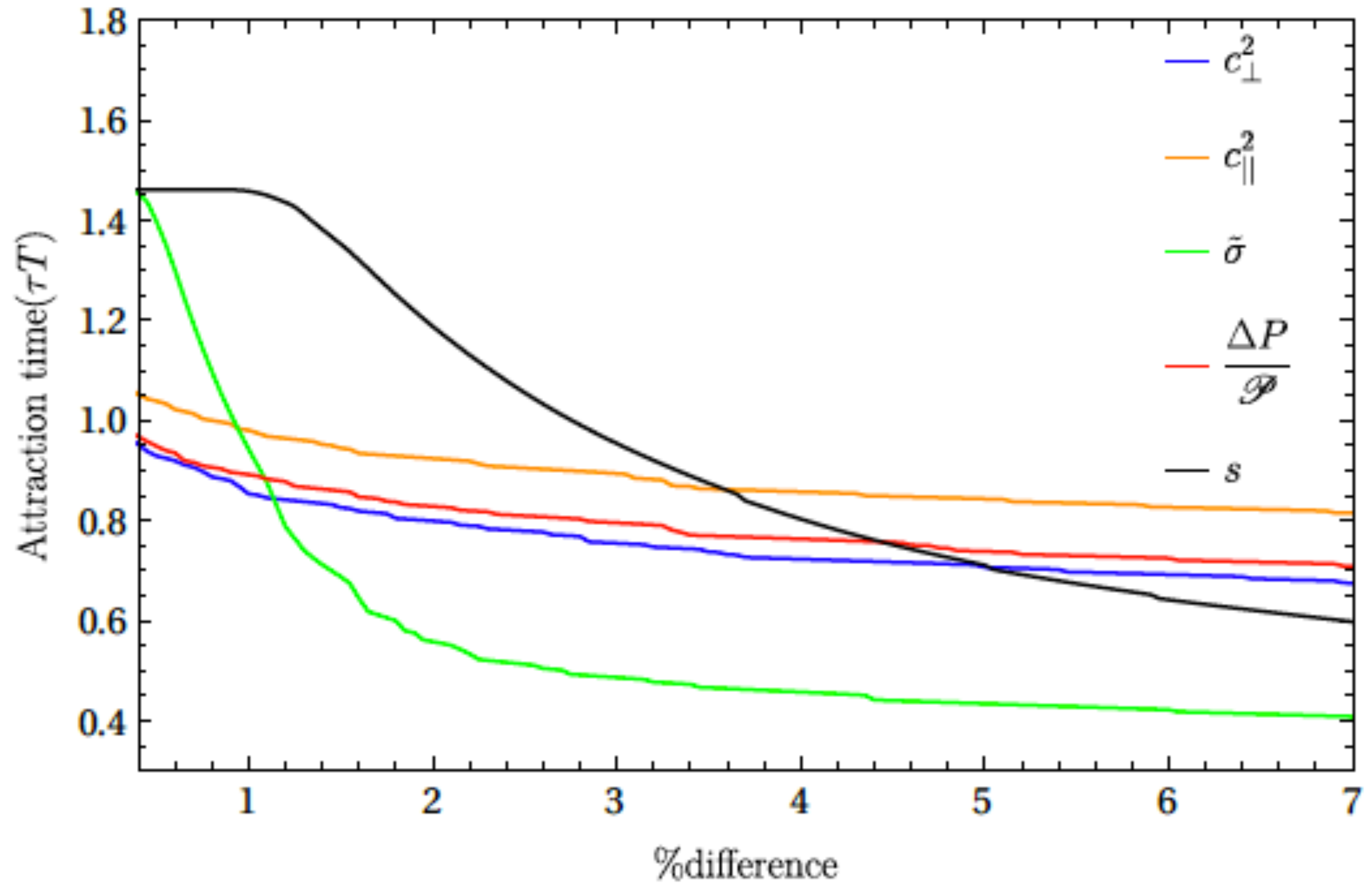






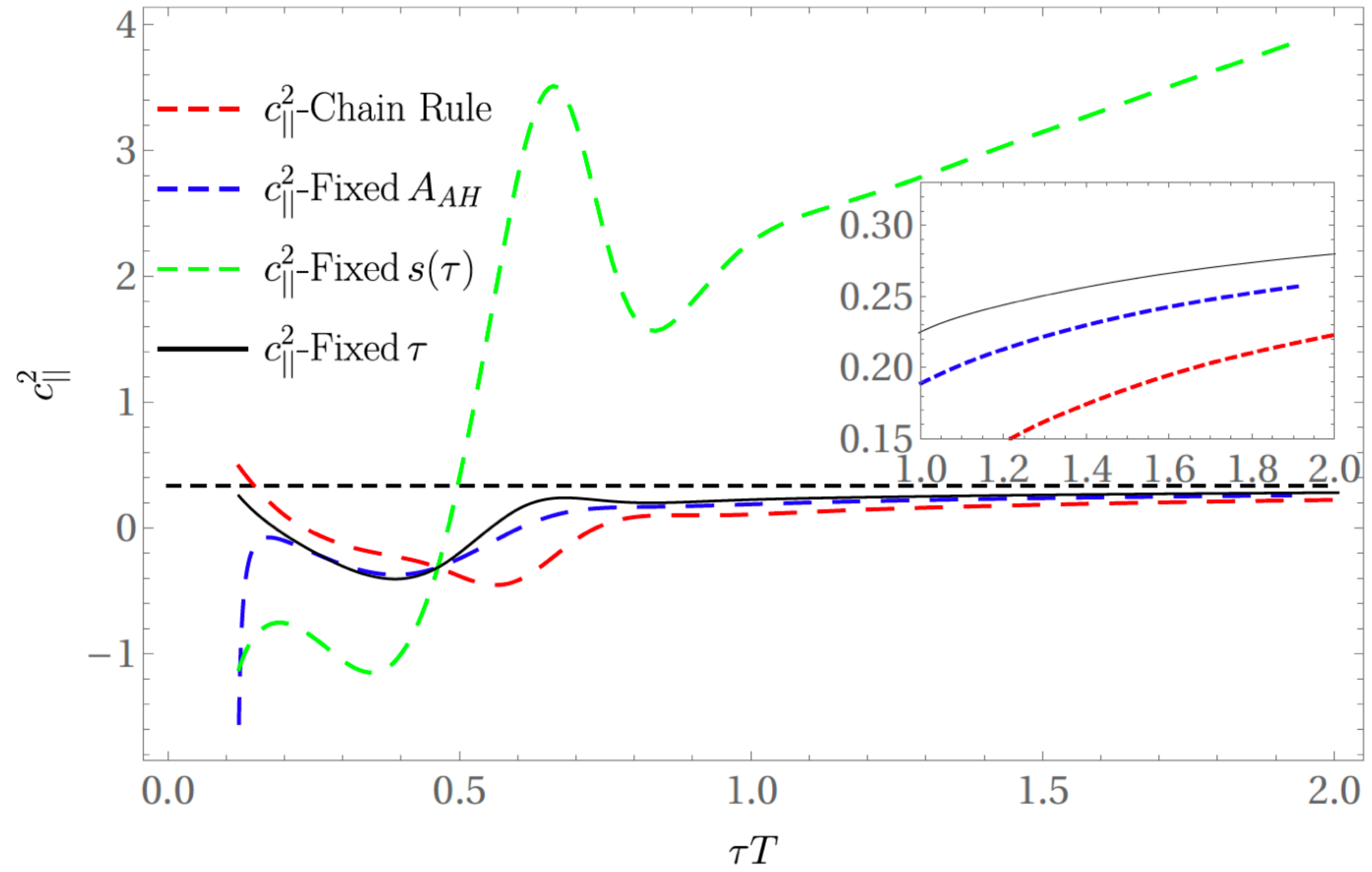


$$\epsilon(\tau) + P(\tau) = s(\tau)T(\tau)$$



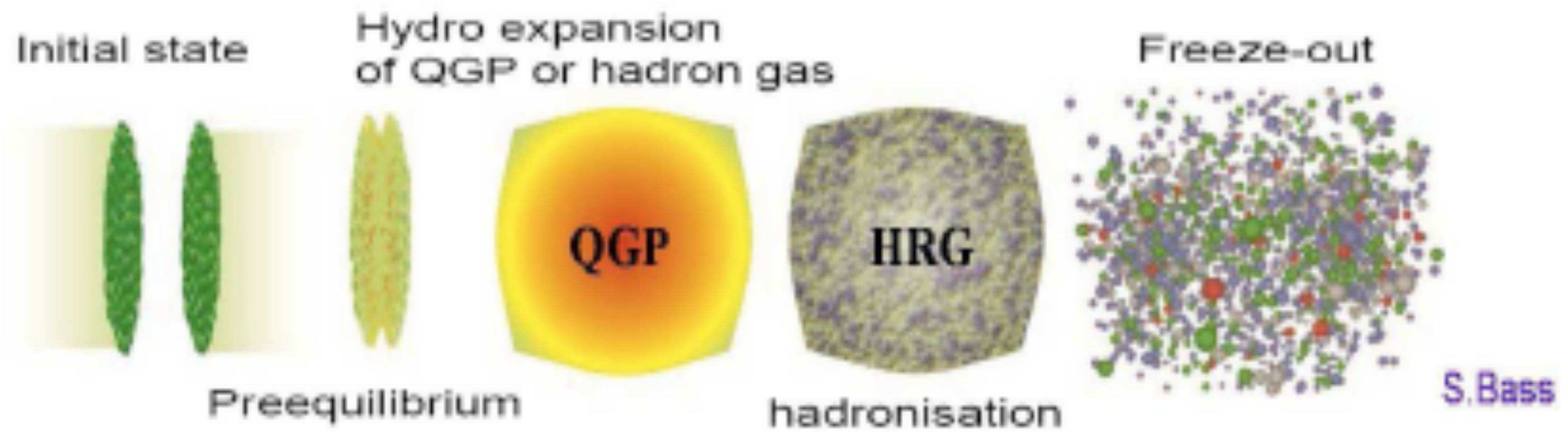
2. “Speed of sound” in Bjorken - expanding plasma

[Cartwright,Kaminski,Knipfer; (2022)]



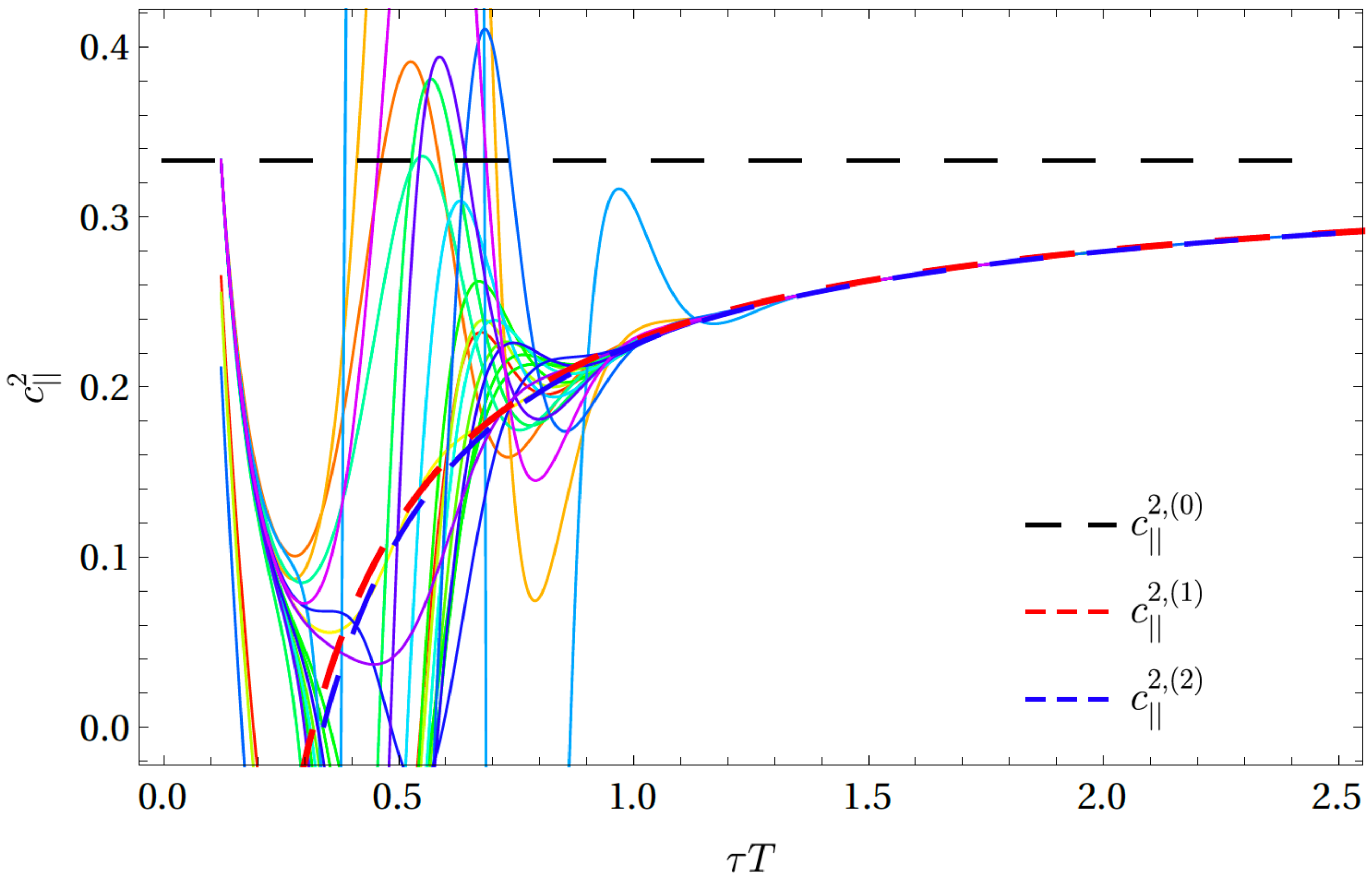
$$c_s^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_s$$

$$c_\perp^2 = -\frac{\partial \langle T^{x_1}_{x_1}\rangle}{\partial \langle T^0_0\rangle}, \qquad c_{||}^2 = -\frac{\partial \langle T^{\xi}_{\xi}\rangle}{\partial \langle T^0_0\rangle}$$



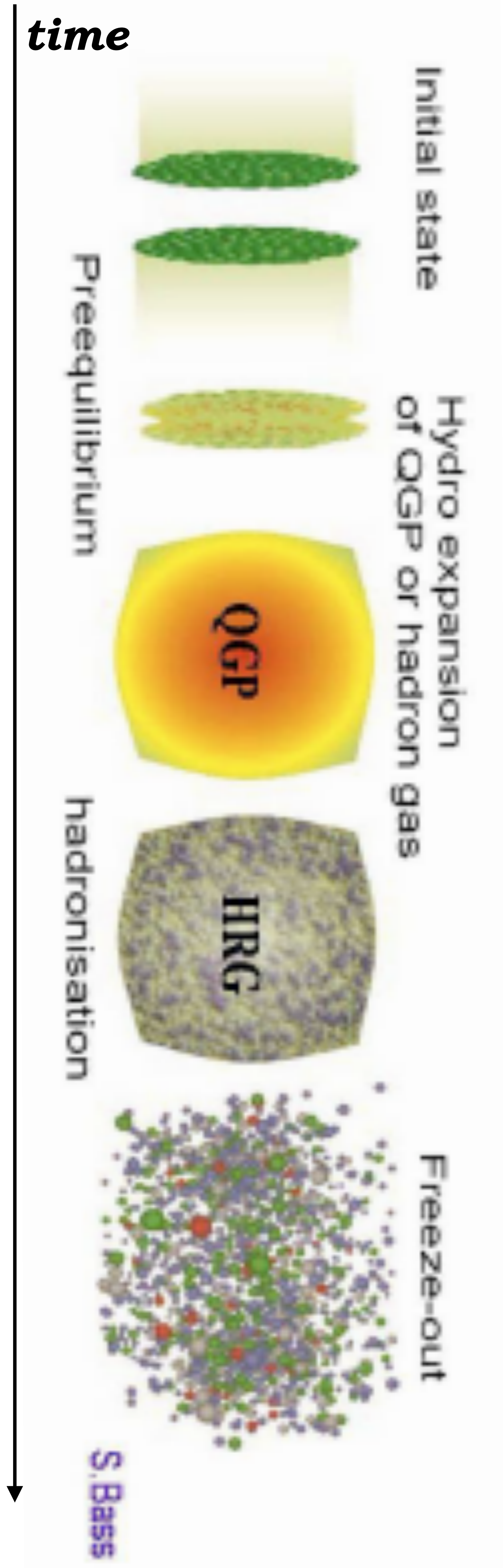
Hydrodynamic approximations :

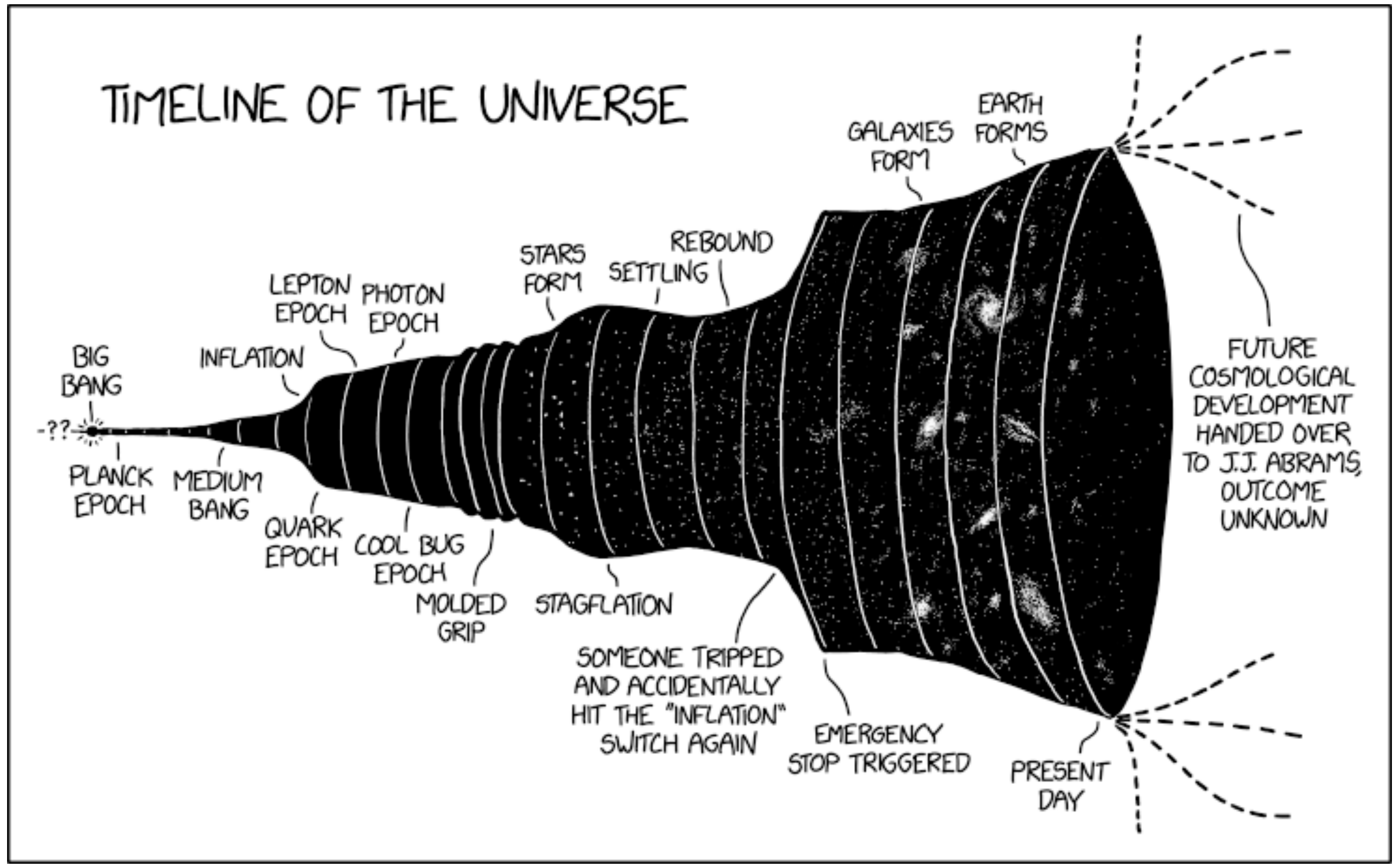
$$c_{||}^{2,(2)} = c_s^2 - \frac{4C_\eta}{3\tau T} - \frac{16C_\eta(1-C_\lambda)C_\pi}{27\tau^2T^2},$$



Invitation: hydrodynamics far from equilibrium

HYDRODYNAMICS





[https://www.xkcd.com/2240/]

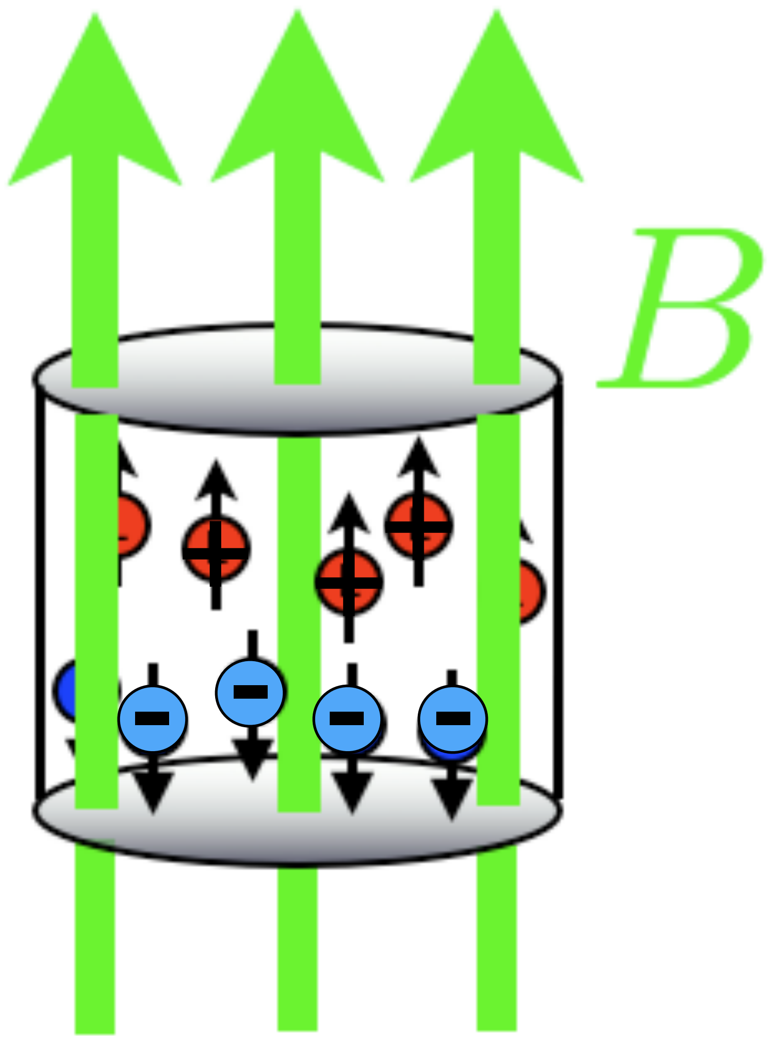
➡longitudinal expansion in HICs

➡universe expands

➡hydrodynamics around non-equilibrium states

The Chiral Magnetic Effect (CME) caused by chiral anomaly

[Kharzeev; PRC (2004)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]



Electric charge current: $$J^\mu = \xi_\chi B$$

Chiral magnetic conductivity: $$\xi_\chi = C\,\mu_A$$

Anomalous axial current divergence: $$\nabla_\mu J^\mu_A = C\,E \cdot B$$

axial charges are generated in aligned E- and B-fields

Needed:

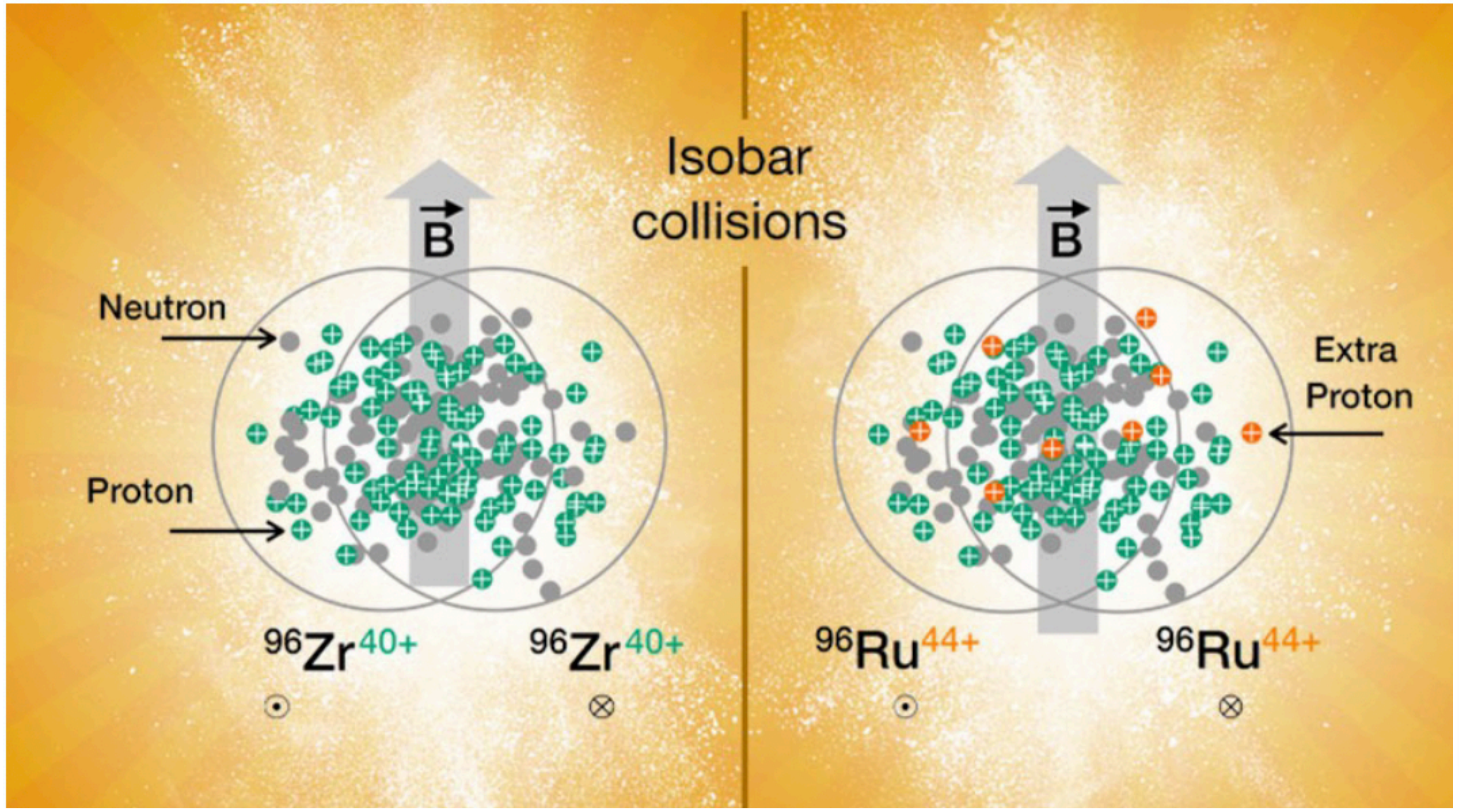
- ➡chiral anomaly
- ➡axial charge
- ➡magnetic field
- ➡sufficient life time

Invitation: CME in heavy ion collisions - RHIC isobar run

Magnetic field B is large in collision experiments:

RHIC	$B \approx 10^{19} G$
LHC	$B \approx 10^{20} G$

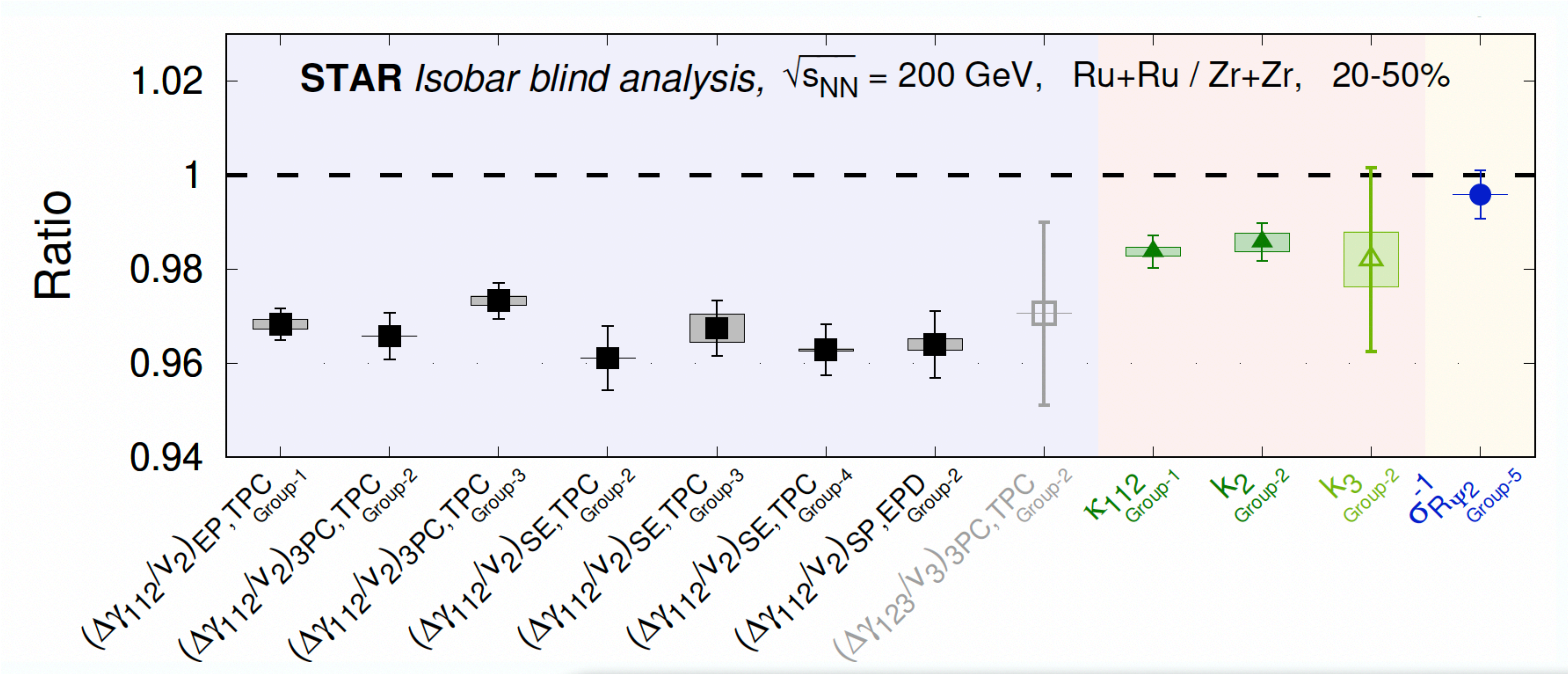
- early RHIC (2009, 2014) and LHC (2013) results hint at CME, but inconclusive; too dirty (cond-mat observed CME)
- isobar run approved at RHIC (2017)



taken from Helen Caines' talk at 6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions (Nov 1-5, 2021)

➡Larger charge creates larger magnetic field, so larger CME in Ru
➡otherwise identical (?)

Invitation: CME in heavy ion collisions - RHIC isobar analysis



If CME present, we expect:

$$\frac{\text{Measure}(\text{Ru}+\text{Ru})}{\text{Measure}(\text{Zr}+\text{Zr})} > 1$$

But plot shows all ratios <1 !

➡**No CME according to pre-blind criteria**

➡**Ru and Zr not as identical as expected: multiplicities and initial geometries differ**

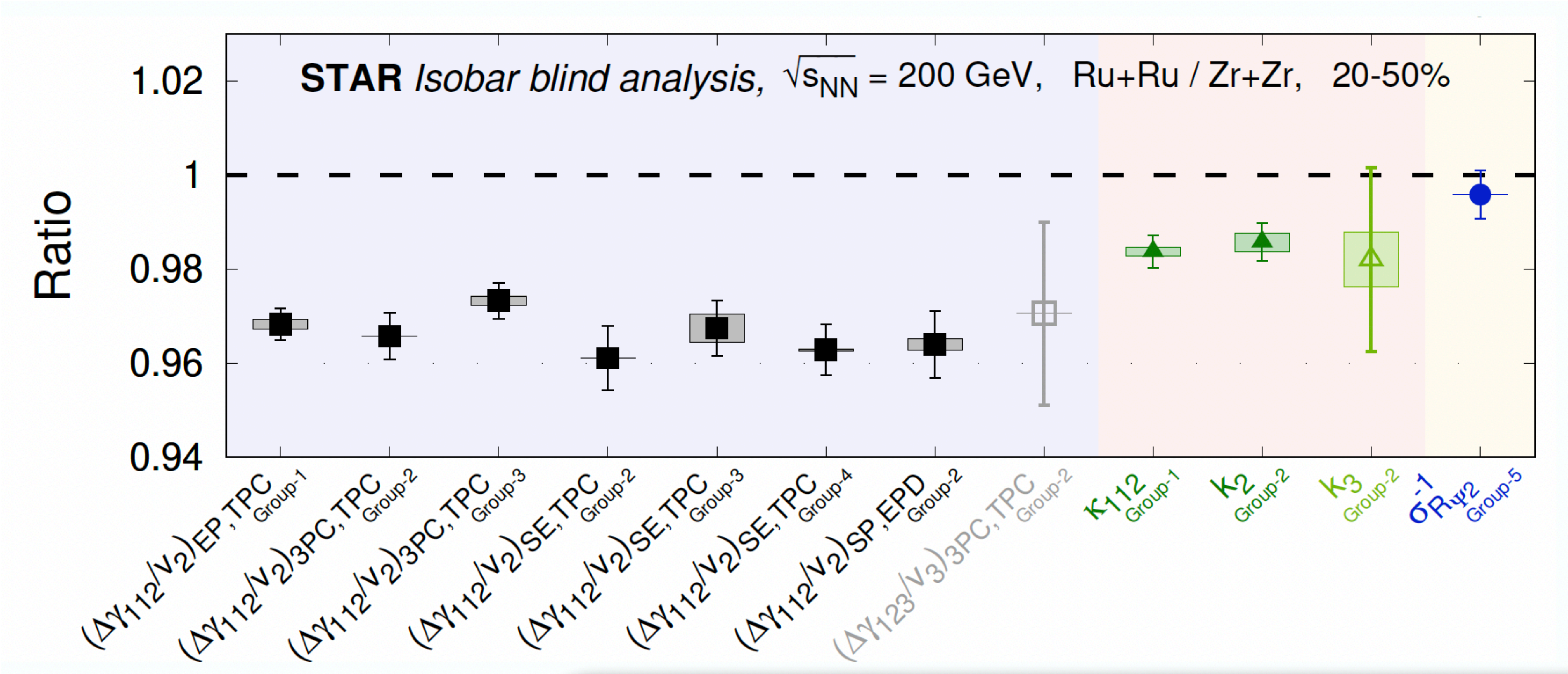
➡**don't know axial charge or magnetic field**

➡**signal-to-background ratio unclear**

➡**more runs? need theoretical understanding**

Invitation: CME in heavy ion collisions - RHIC isobar analysis

top-RHIC energy: [STAR Collaboration; (2021)]
low-energy update: [STAR Collaboration; (2022)]
high energy update: [ALICE Collaboration; (2022)]



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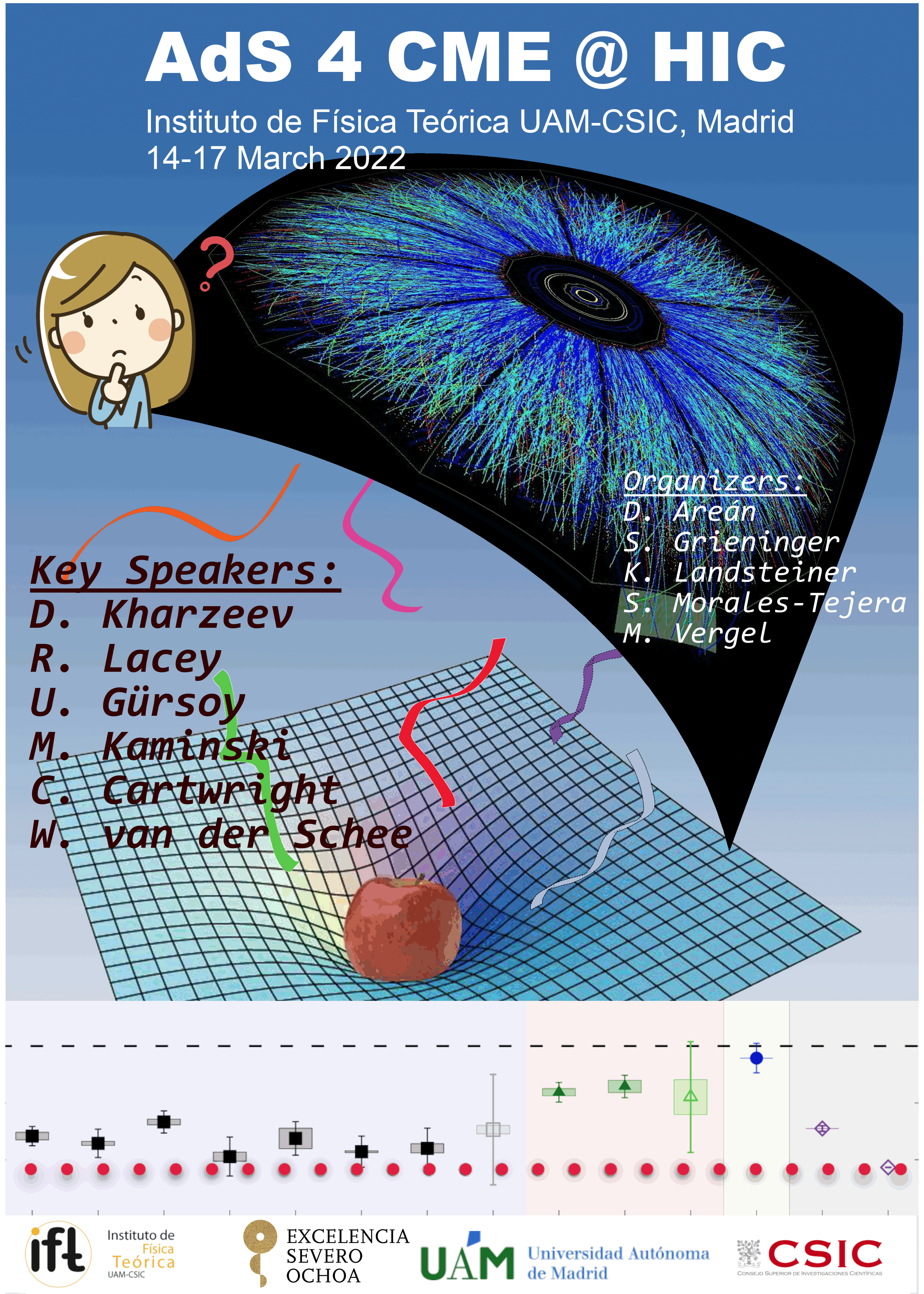
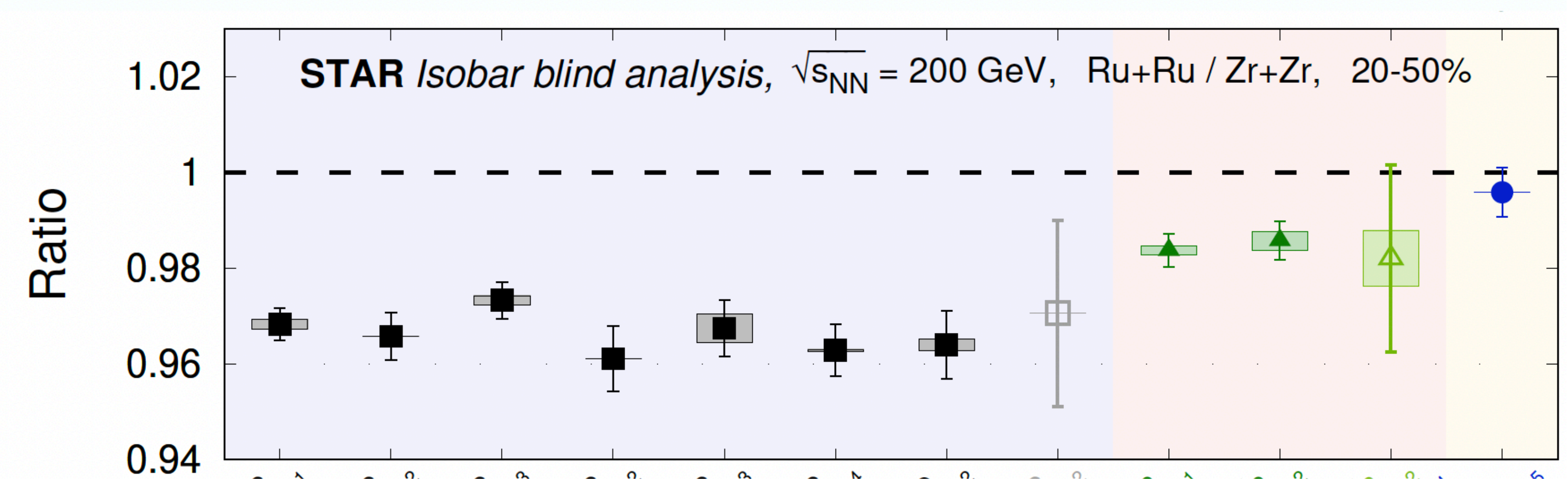
AdS4CME Collaboration

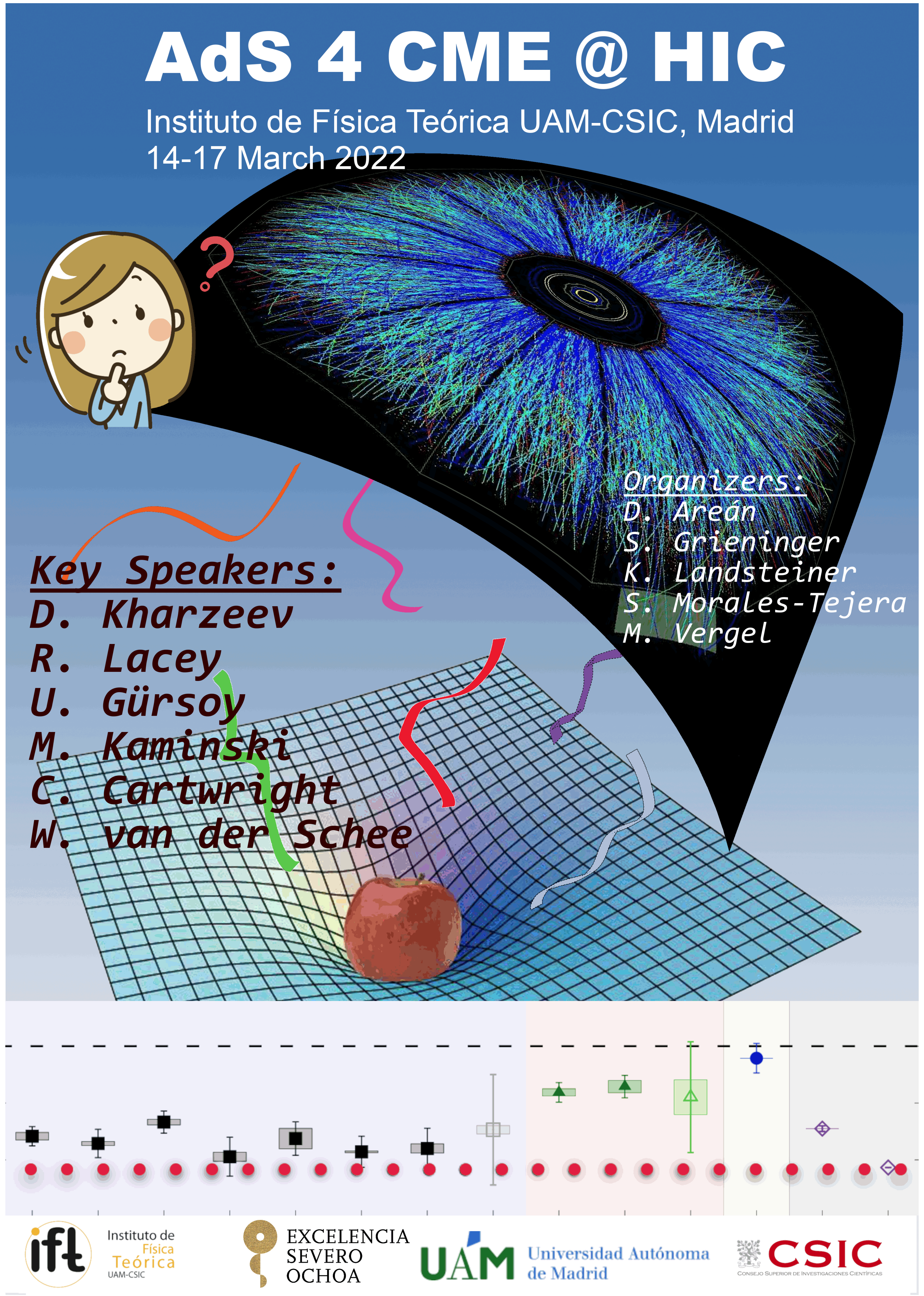


Participants: Dmitri Kharzeev, Karl Landsteiner, Umut Gürsoy, MK

To be invited: Wilke van der Schee, Daniel Arean, Björn Schenke, Sebastian Grieninger, Casey Cartwright, Sergio Morales Tejera, Pablo Saura Bastida, Nabil Iqbal, Nick Poovuttikul, Martin Ammon, Matti Jarvinen, Ho Ung Yee, Misha Stephanov, Jenfing Liao, Saso Grozdanov, Ruth Gregory, Arpit Das, Helen Caines, Andrea Danu, Mei Huang, Jacquelyn Noronha-Hostler, …



AdS4CME Collaboration

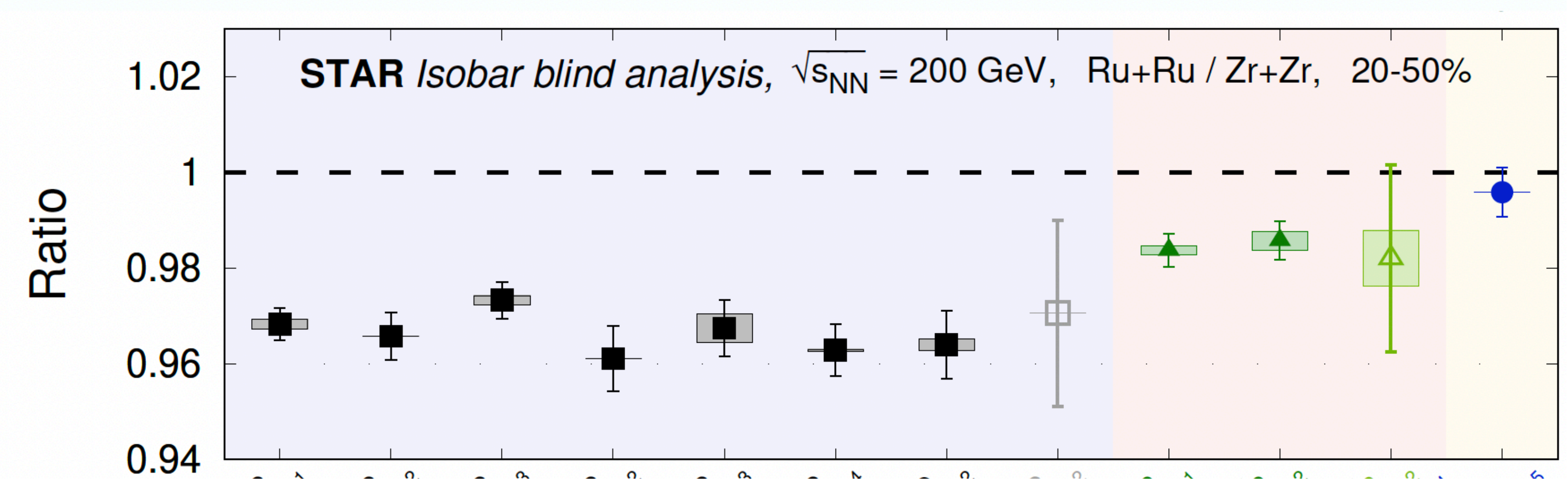


https://ads4cme.wixsite.com/ads4cme

Upcoming Workshop at ECT*, Trento, Italy
March 13-17, 2023

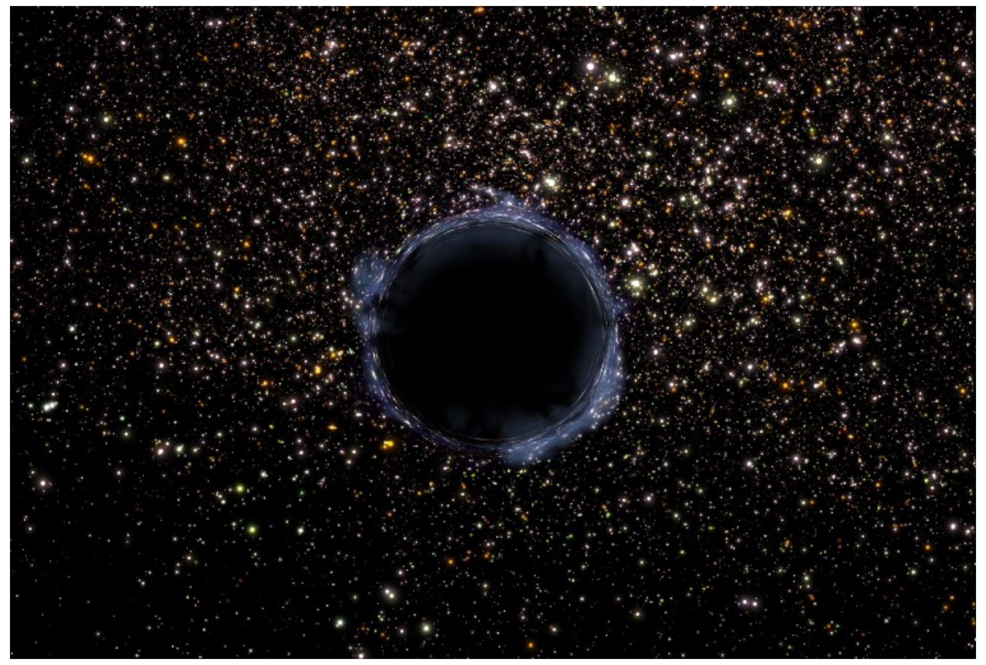
Participants: Dmitri Kharzeev, Karl Landsteiner, Umut Gürsoy, MK

To be invited: Wilke van der Schee, Daniel Arean, Björn Schenke, Sebastian Grieninger, Casey Cartwright, Sergio Morales Tejera, Pablo Saura Bastida, Nabil Iqbal, Nick Poovuttikul, Martin Ammon, Matti Jarvinen, Ho Ung Yee, Misha Stephanov, Jenfing Liao, Saso Grozdanov, Ruth Gregory, Arpit Das, Helen Caines, Andrea Danu, Mei Huang, Jacquelyn Noronha-Hostler, …



Preliminary re-analysis of isobar data suggested lower baseline, implying a CME-signal (with 1 to 5 sigma)

Choose a holographic model to compute CME current



Recall: needed for CME is

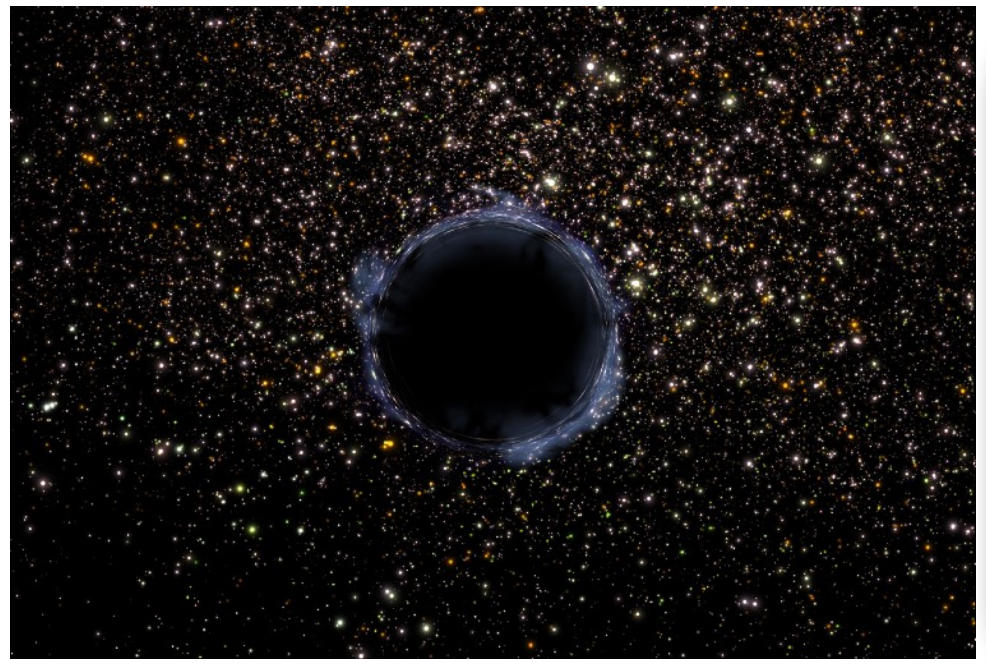
➡chiral anomaly

➡axial charge

➡magnetic field

➡sufficient life time

Holographic model with axial current only



➡ **use as holographic dual to charged state in strong *B***

➡ ***N*=4 Super-Yang-Mills coupled to external (E,B)-fields**

[Ammon, Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; arXiv:2012.09183]

Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

5-dimensional Einstein-Maxwell action encodes N=4 Super-Yang-Mills theory with axial U(1) gauge symmetry

5-dimensional Chern-Simons term encodes chiral anomaly

Charged magnetic black branes dual to charged thermal state with *B*

[D'Hoker, Kraus; JHEP (2010)]

- charged magnetic analog of Reissner-Nordstrom black brane
- asymptotically AdS_5

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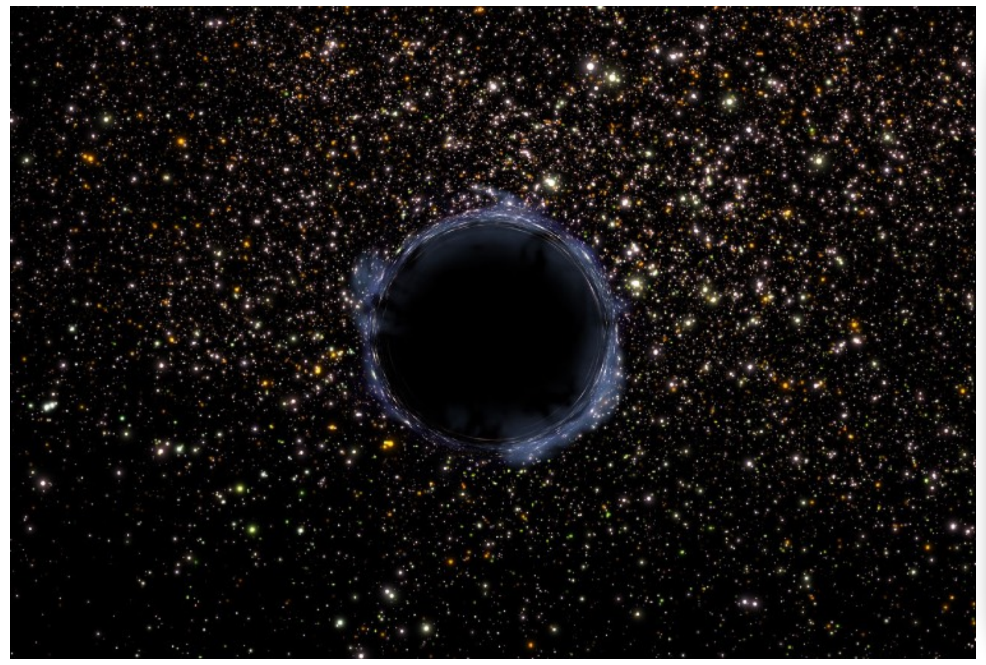

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Charged magnetic black branes dual to charged thermal state with *B*

[D'Hoker, Kraus; JHEP (2010)]

- charged magnetic analog of Reissner-Nordstrom black brane
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➡axial *B*
➡axial charge
➡axial current only

Chiral effects in vector and axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD electromagnetic *U(1)*)

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_\chi \; B^\mu + \xi_{VA} B_A^\mu$$

chiral magnetic effect

Axial current (e.g. QCD axial *U(1)*)

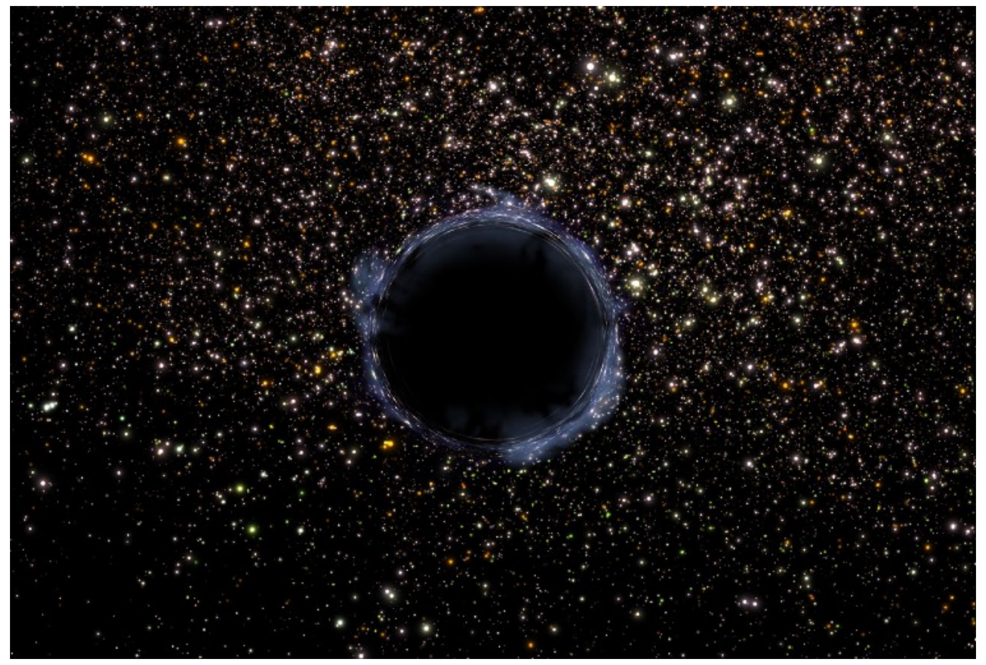
$$J_A^\mu = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu$$

chiral vortical effect

chiral separation effect

➡**phenomenology needs *both* currents**

Holographic model with two currents



Einstein-Maxwell-Chern-Simons action with two gauge fields A_μ and V_μ

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - 2\Lambda - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{L^2}{4}F^{(5)}_{\mu\nu}F^{\mu\nu}_{(5)} + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\left(3F_{\nu\rho}F_{\sigma\tau} + F^{(5)}_{\nu\rho}F^{(5)}_{\sigma\tau}\right)\right)$$

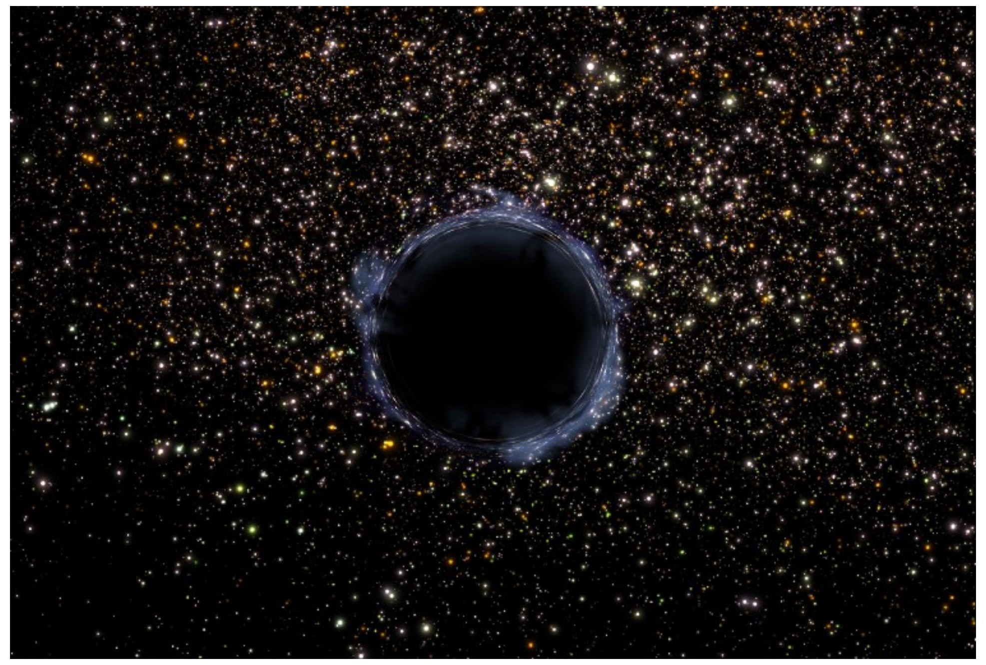
gravitational coupling κ — *Einstein-Hilbert* — *Maxwell* — *"axial Maxwell"* — *Chern-Simons coupling α* — *Chern-Simons term encoding chiral anomaly*

5D vector gauge field $V_\mu \longleftrightarrow$ *4D conserved vector current*

$$J^\mu_V = \cdots + \xi_V\omega^\mu + \xi_{VV}B^\mu + \xi_{VA}B^\mu_A$$

5D axial gauge field $A_\mu \longleftrightarrow$ *4D anomalous axial current*

$$J^\mu_A = \cdots + \xi\omega^\mu + \xi_B B^\mu + \xi_{AA}B^\mu_A$$



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[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

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Isotropization (non-expanding plasma)

Initial state:

- Energy and axial charge corresponding to (T,μ_5) in final state
- Magnetic field is uniform and constant in time
- Dynamical pressure anisotropy vanishes
- CME current is absent

	"RHIC"	"LHC"
T	300MeV	1000MeV
μ_5	10 (100) MeV	10 (100) MeV
B	1 (0.1) m_π^2	15 (1.5) m_π^2

Matching couplings to QCD:

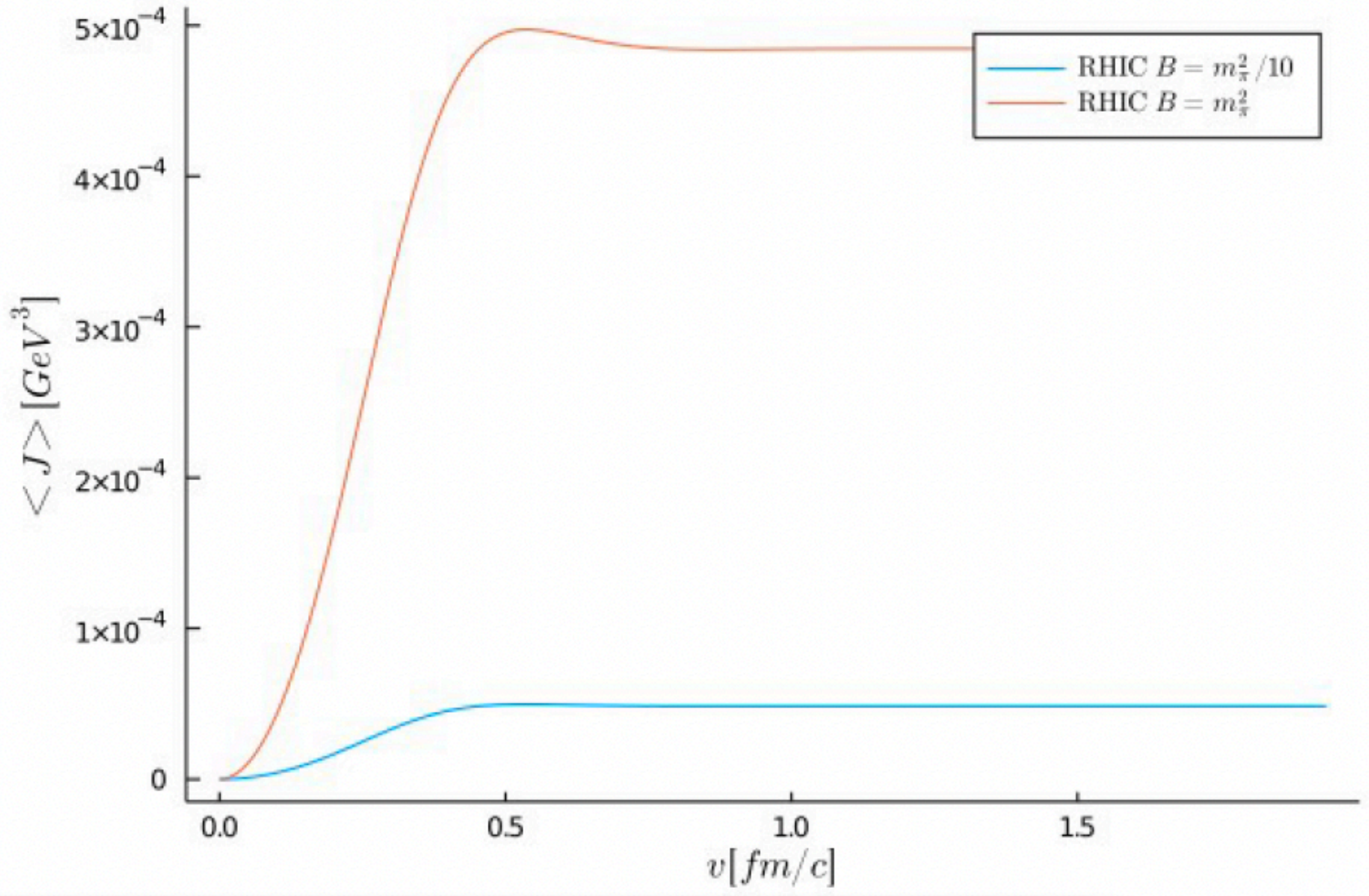
➔ Gravitational coupling: match to entropy

$$s_{BH} = \frac{4\pi^2 T^3}{2\kappa^2} \qquad s_{SB} = 4\left(\nu_b + \frac{7}{4}\nu_f\right)\frac{\pi^2 T^3}{90}$$

$$s_{BH} = \frac{3}{4} s_{SB} \qquad \Longrightarrow \quad \kappa^2 \approx 12.5$$

➔ Chern Simons coupling: match to anomaly

$$\frac{\alpha}{2\kappa^2} = A_{QCD} = \frac{1}{8\pi^2} \qquad \Longrightarrow \alpha \approx 0.316$$



➡CME more likely to be seen at RHIC than at LHC

➡lifetime of *B* crucial

Isotropization (non-expanding plasma)

[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

taken from Karl Landsteiner's talk at 6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

	"RHIC"	"LHC"
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$$J^\mu = \xi_\chi B$$

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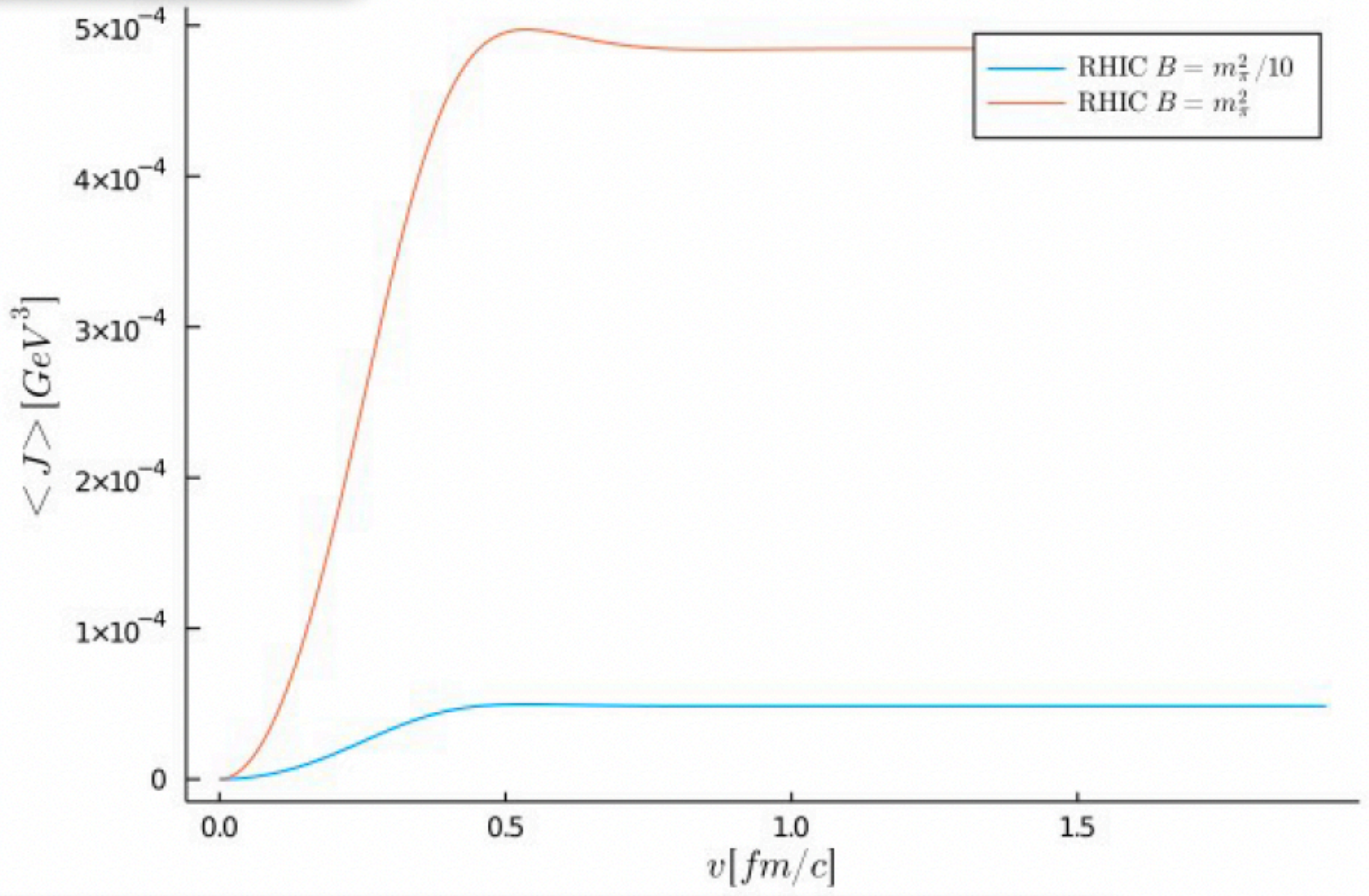

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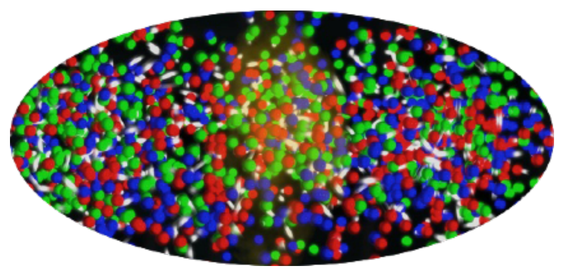
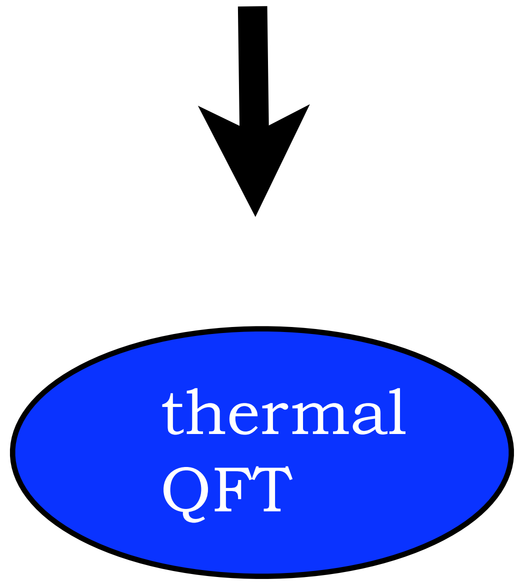
Far from equilibrium

Thermalization in
field theory:



T=0 particle "soup"



nonzero T plasma

Far from equilibrium

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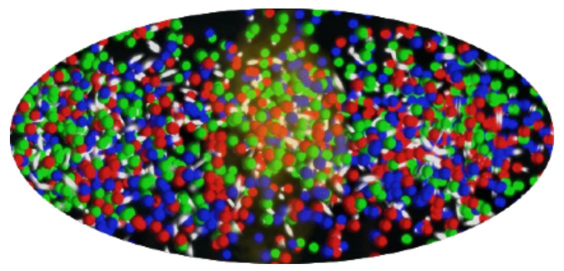
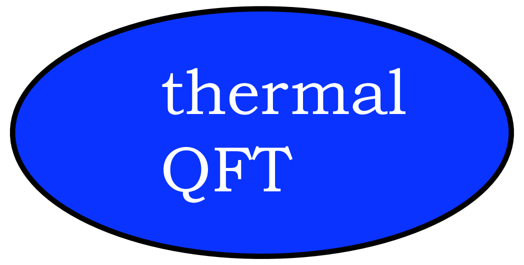
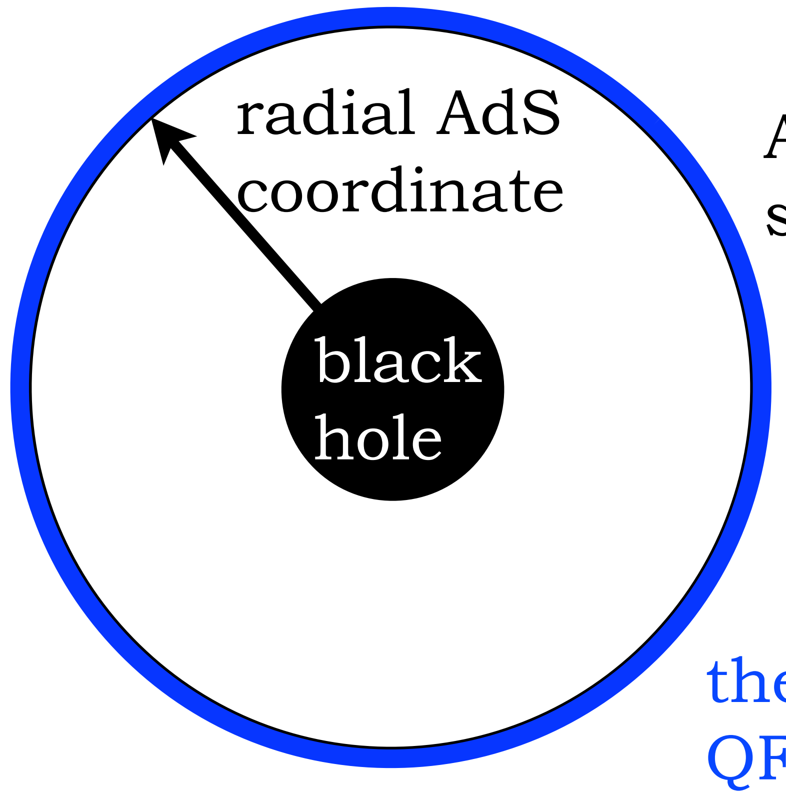


T=0 particle “soup”

correspondence



nonzero T plasma



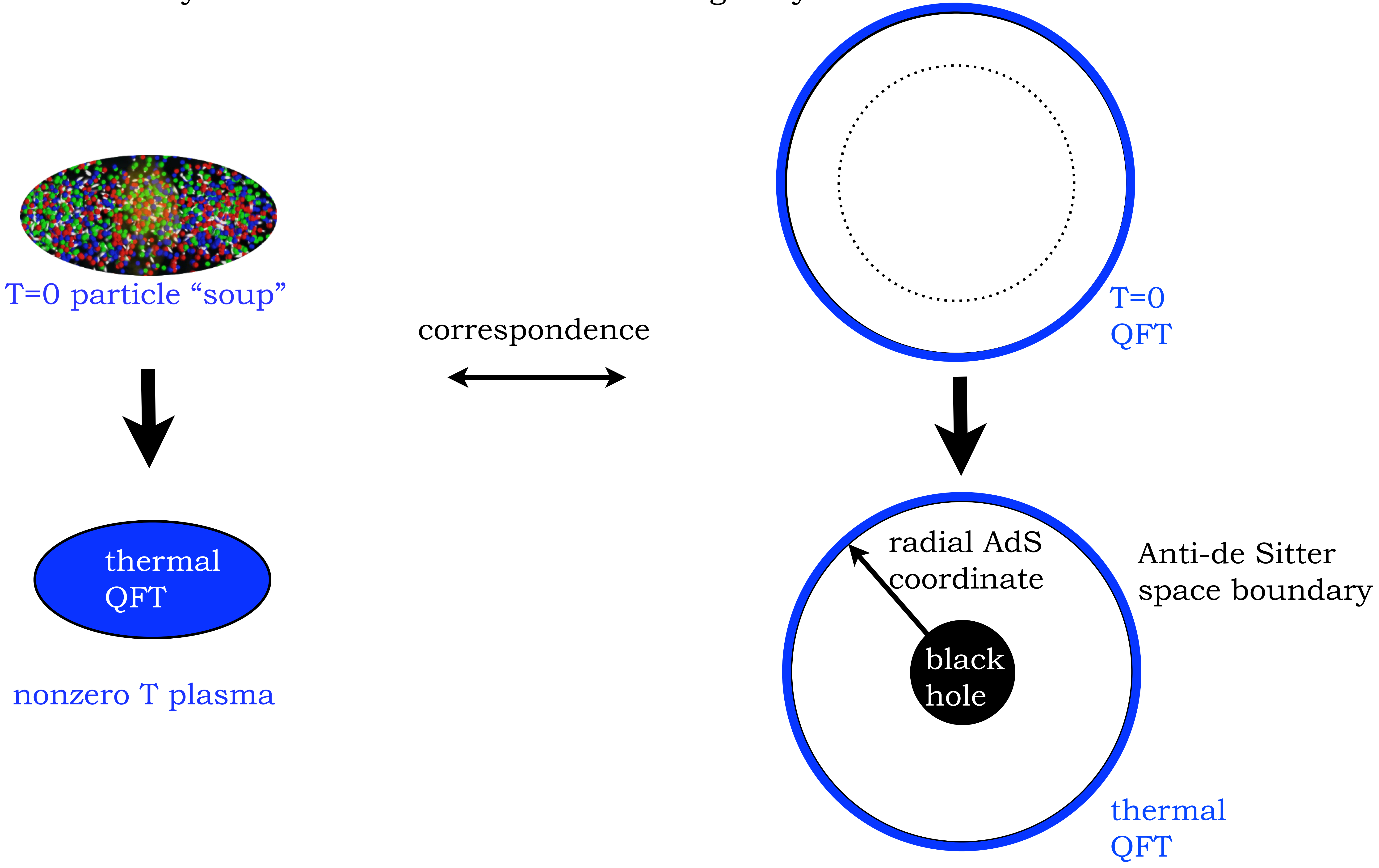
Anti-de Sitter
space boundary

thermal
QFT

Far from equilibrium

Thermalization in field theory:

Horizon formation in gravity:



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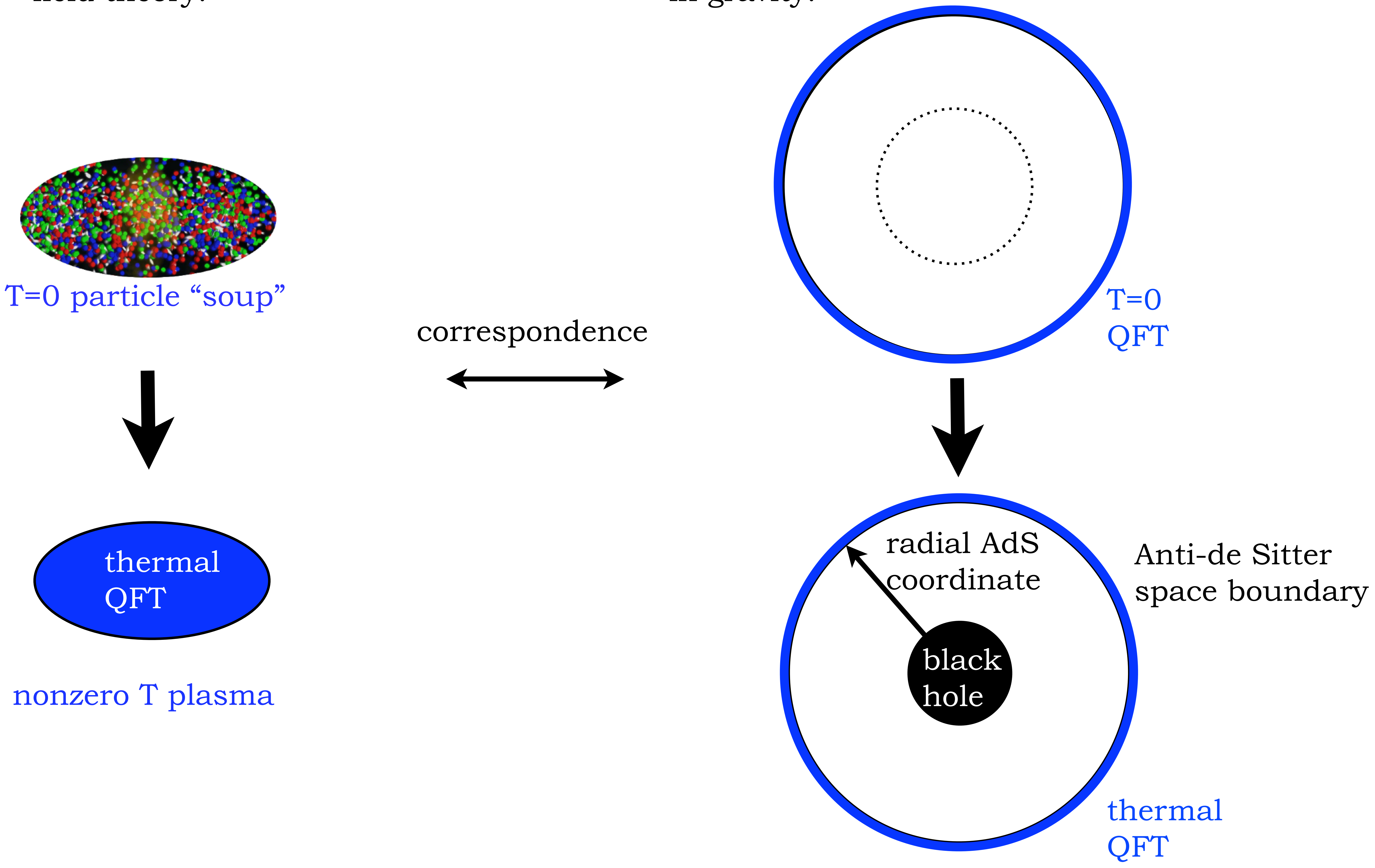

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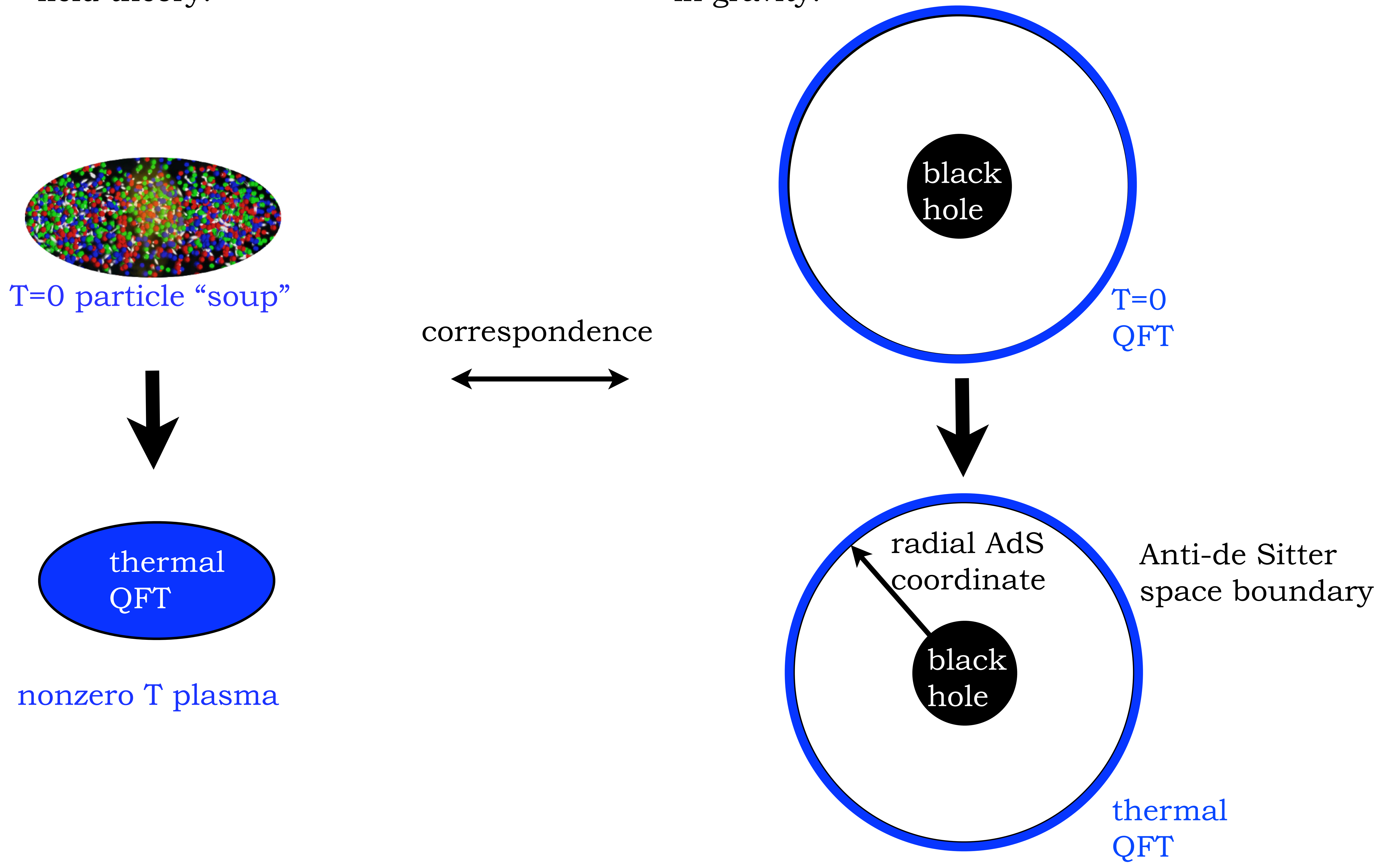
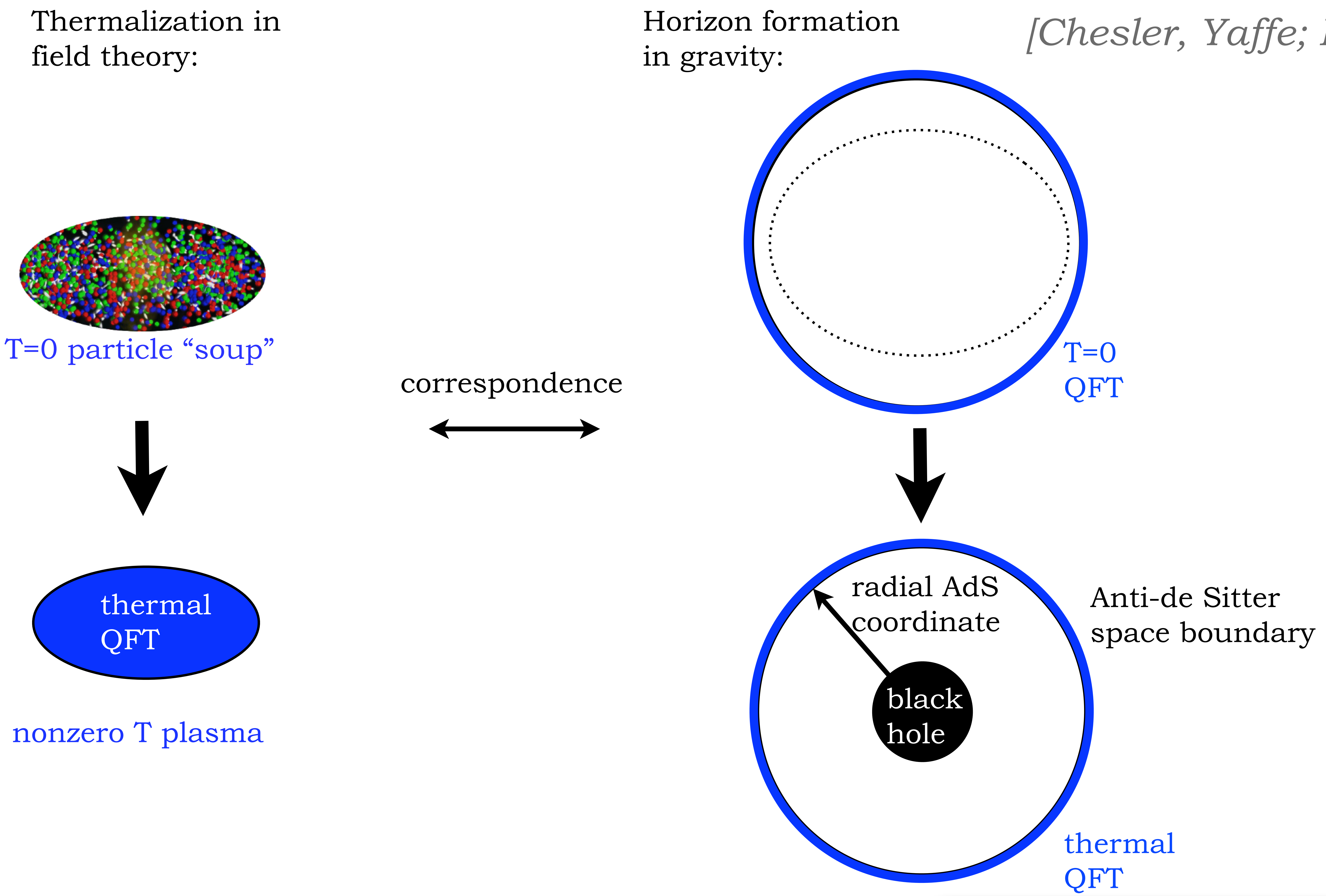


Far from equilibrium

[Janik,Peschanski; (2006)]

[Chesler, Yaffe; PRL (2009)]



➡solve time-dependent Einstein equations

Bjorken - expanding plasma

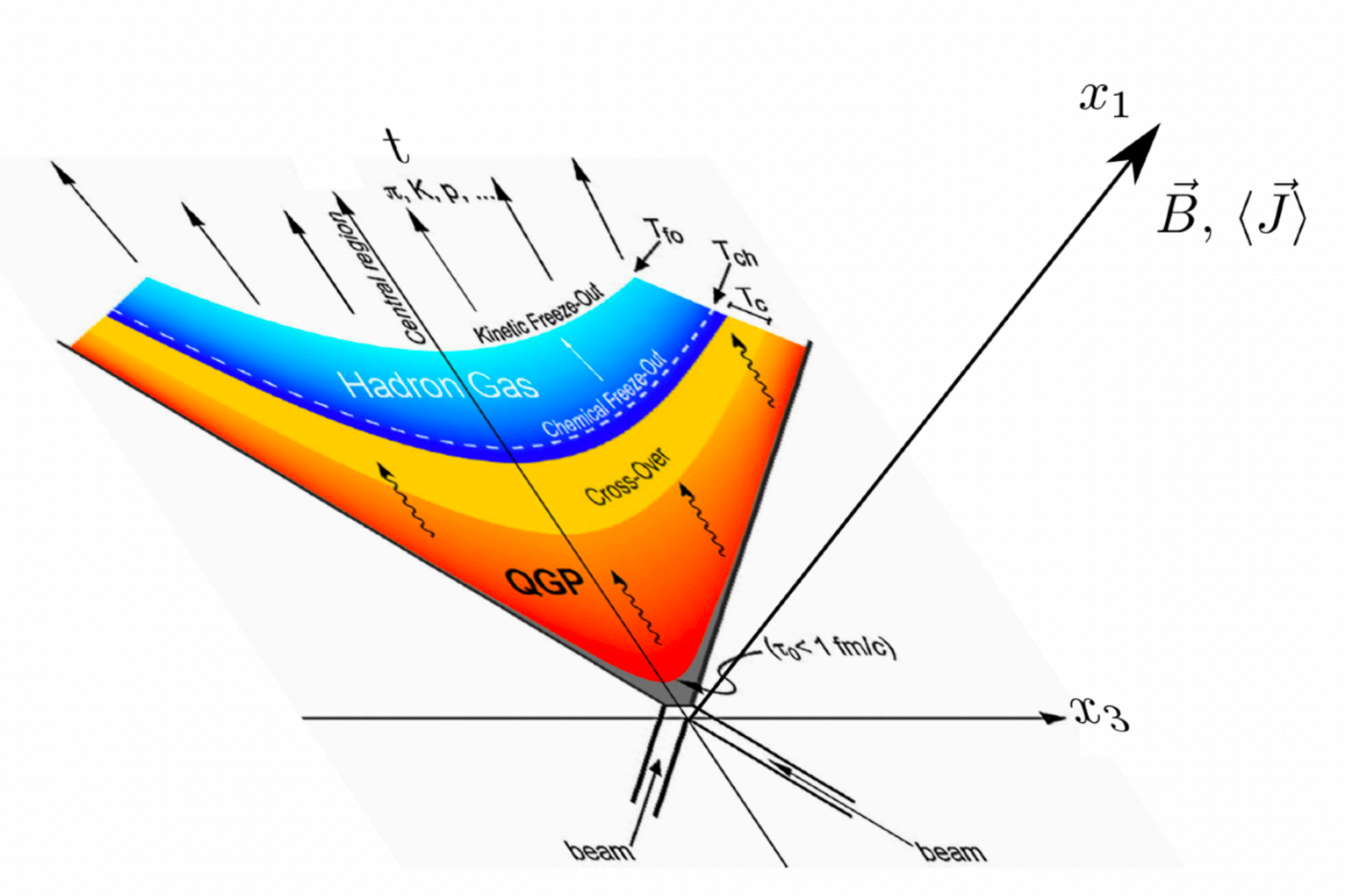
Milne coordinates $(\tau, x_1, x_2, \xi; r)$

$$\xi = \frac{1}{2}\ln[(t+x_3)/(t-x_3)]$$

$$\tau = \sqrt{t^2 - x_3^2}$$

Bjorken flow

$$\partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} - \frac{4}{3}\frac{\eta}{\tau^2} = 0$$



Metric Ansatz

$$\begin{aligned} ds^2 &= 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + F_1(v,r)\mathrm{d}v\mathrm{d}x_1 \\ &+ S(v,r)^2 e^{H_1(v,r)}\mathrm{d}x_1^2 + S(v,r)^2 e^{H_2(v,r)}\mathrm{d}x_2^2 \\ &+ L^2 S(v,r)^2 e^{-H_1(v,r)-H_2(v,r)}\mathrm{d}\xi^2, \end{aligned}$$

$$A_\mu = \frac{1}{L}(0, -\phi(v,r), 0, 0, 0),$$

$$V_\mu = \frac{1}{L}(0, 0, -V(v,r), b\,\xi, 0)\,,$$

Boost invariant metric at the boundary

$$\lim_{r\to\infty} \frac{L^2}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$$

$$\lim_{r\to\infty} V_a = V_a^{\mathrm{ext}} = \frac{1}{L}(0, 0, b\,\xi, 0)$$

$$q_5/L = L^4 S(v,r)^3 \phi'(v,r) + 8\alpha b V(v,r)\,,$$

$$\mathcal{E}_5 \equiv -\phi'(v,r) = \frac{q_5 L^{-1} - 8\alpha b V(v,r) L^{-4}}{S(v,r)^3}$$

Bjorken - expanding plasma

[Cartwright,Kaminski,Schenke; PRC (2022)]

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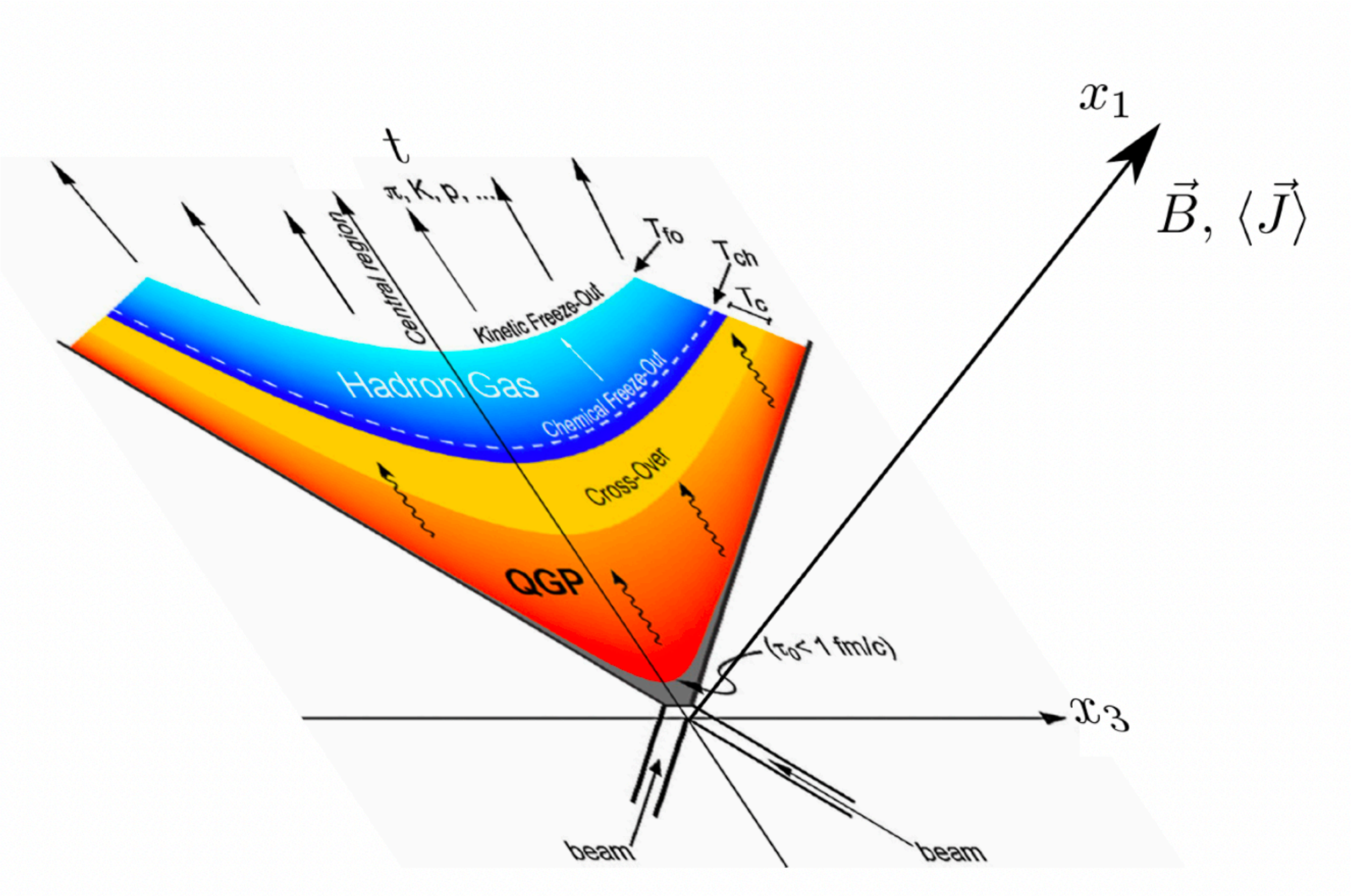

taken from Casey Cartwright's talk

Metric Ansatz

$$ds^2 = 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + F_1(v,r)\mathrm{d}v\mathrm{d}x_1 + S(v,r)^2 e^{H_1(v,r)}\mathrm{d}x_1^2 + S(v,r)^2 e^{H_2(v,r)}\mathrm{d}x_2^2 + L^2 S(v,r)^2 e^{-H_1(v,r)-H_2(v,r)}\mathrm{d}\xi^2,$$

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Bjorken - expanding plasma

[Cartwright,Kaminski,Schenke; PRC (2022)]

Bjorken flow equation

$$\partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} - \frac{4}{3}\frac{\eta}{\tau^2} = 0$$

Holographic Bjorken flow equation

$$-\frac{P_1(\tau)}{\tau} - \frac{P_2(\tau)}{\tau} - \frac{B_1(\tau)^2}{8\tau} + \partial_\tau \epsilon(\tau) + \frac{2\epsilon(\tau)}{\tau} = 0$$

Energy and pressures

$$\epsilon = \langle T_{00}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{3a_4(\tau)}{4L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2}\right),$$

$$P_1 = \langle T_{11}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} + \frac{h_4^{(1)}(\tau)}{L^4} + \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{1}{6\tau^4}\right),$$

$$P_2 = \langle T_{22}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} + \frac{h_4^{(2)}(\tau)}{L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{b^2}{16L^2\tau^2} - \frac{1}{6\tau^4}\right),$$

$$\tau^2 P_\xi = \langle T_{\xi\xi}\rangle = \frac{2L^3\tau^2}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} - \frac{h_4^{(1)}(\tau)}{L^4} - \frac{h_4^{(2)}(\tau)}{L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{b^2}{16L^2\tau^2} + \frac{1}{3\tau^4}\right)$$

$$\langle J^a_{(5)}\rangle = \frac{1}{2\kappa^2}\left(\frac{q_5 L}{\tau}, 0, 0, 0\right),$$

$$\langle J^a\rangle = \frac{1}{2\kappa^2}\left(0, 2V_2(\tau), 0, 0\right),$$

➡**CME current**

➡**time-dependent axial charge and *B***

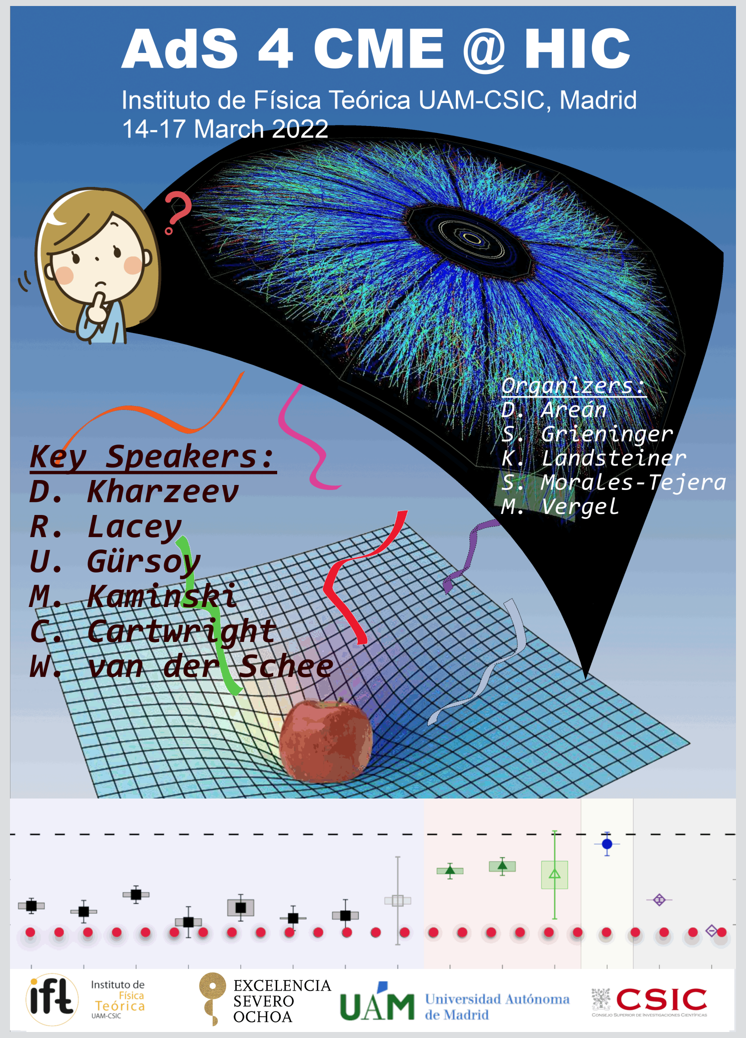
$$B^a = \frac{1}{2}\epsilon^{abcd}u_b F_{cd} \quad \Rightarrow \quad B^1 = \frac{b}{L\tau}$$

Recall the metric:

$$ds^2 = 2 dr dv - A(v,r) dv^2 + F_1(v,r) dv dx_1 + S(v,r)^2 e^{H_1(v,r)} dx_1^2 + S(v,r)^2 e^{H_2(v,r)} dx_2^2 + L^2 S(v,r)^2 e^{-H_1(v,r)-H_2(v,r)} d\xi^2,$$

Burning questions

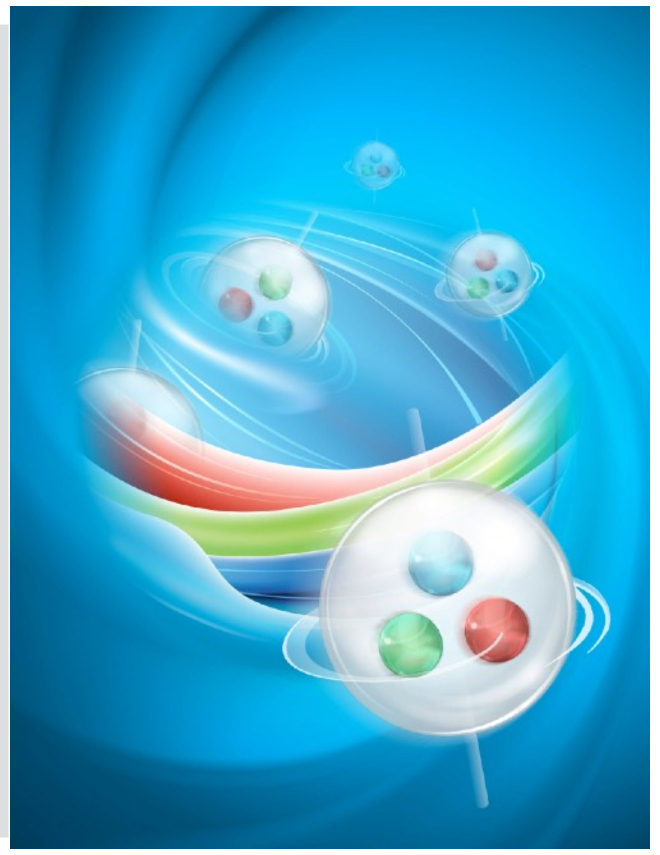
(Far from equilibrium) holography



- Compute initial values for the
  - ✳ axial charge / chemical potential
  - ✳ magnetic field
- How do they **depend on time**?
- How big is signal-to-background ratio?
- Is our holographic model a **good description of the time-dependence** (and energy-dependence) of the CME in HICs?

More background to spoil the signal

- Consider relevance of the 25 magnetic transport effects
  [Ammon, Grieninger, Hernandez, Kaminski, et al.; JHEP (2021)]
- Rotation leads to similar transport effects as magnetic field
  [Gallegos,Gursoy,Yarom; SciPost (2021)] *[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]*
  [Gallegos,Gursoy,Yarom; arXiv:2203.05044] *[Hongo,Huang,Kaminski,Stephanov,Yee; (2022)]*
  [Gallegos,Gursoy; JHEP (2020)] *[STAR; 2108.00044]*



[STAR; Nature (2017)]

Potential Discussion Topics

[Topological confinement in Skyrme holography](#)
Cartwright, Harms, Kaminski, Thomale
Class.Quant.Grav. 39 (2022) 13, 135002
2201.00105 [hep-th]

Topological or rotational non-Abelian gauge fields from Einstein-Skyrme holography
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Characteristic momentum of Hydro+and a bound on the enhancement of the speed of sound near the QCD critical point
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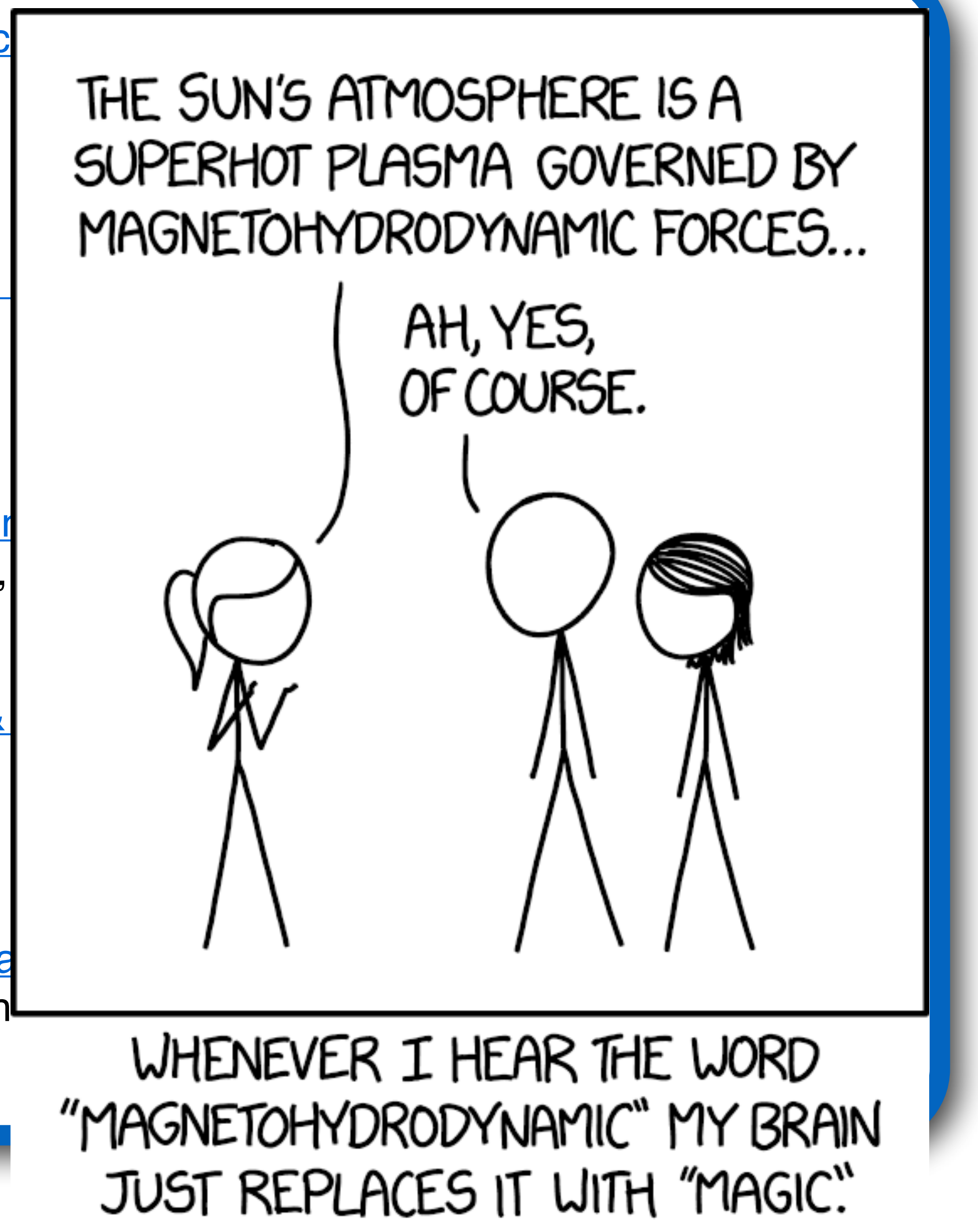

Confinement & other phase transitions

Rotation & spin

QCD critical point & phase diagram

(Magneto)hydrodynamics

Neutron stars

Far-from-equilibrium dynamics

Chiral magnetic effect & anomalies

Anomalous hydrodynamics kicks neutron stars
Kaminski, Uhlemann, Bleicher, Schaffner-Bielich
Phys.Lett.B 760 (2016) 170-174, 1410.3833 [nucl-th]

Chiral hydrodynamics in strong external magnetic fields
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Shear transport far from equilibrium via holography
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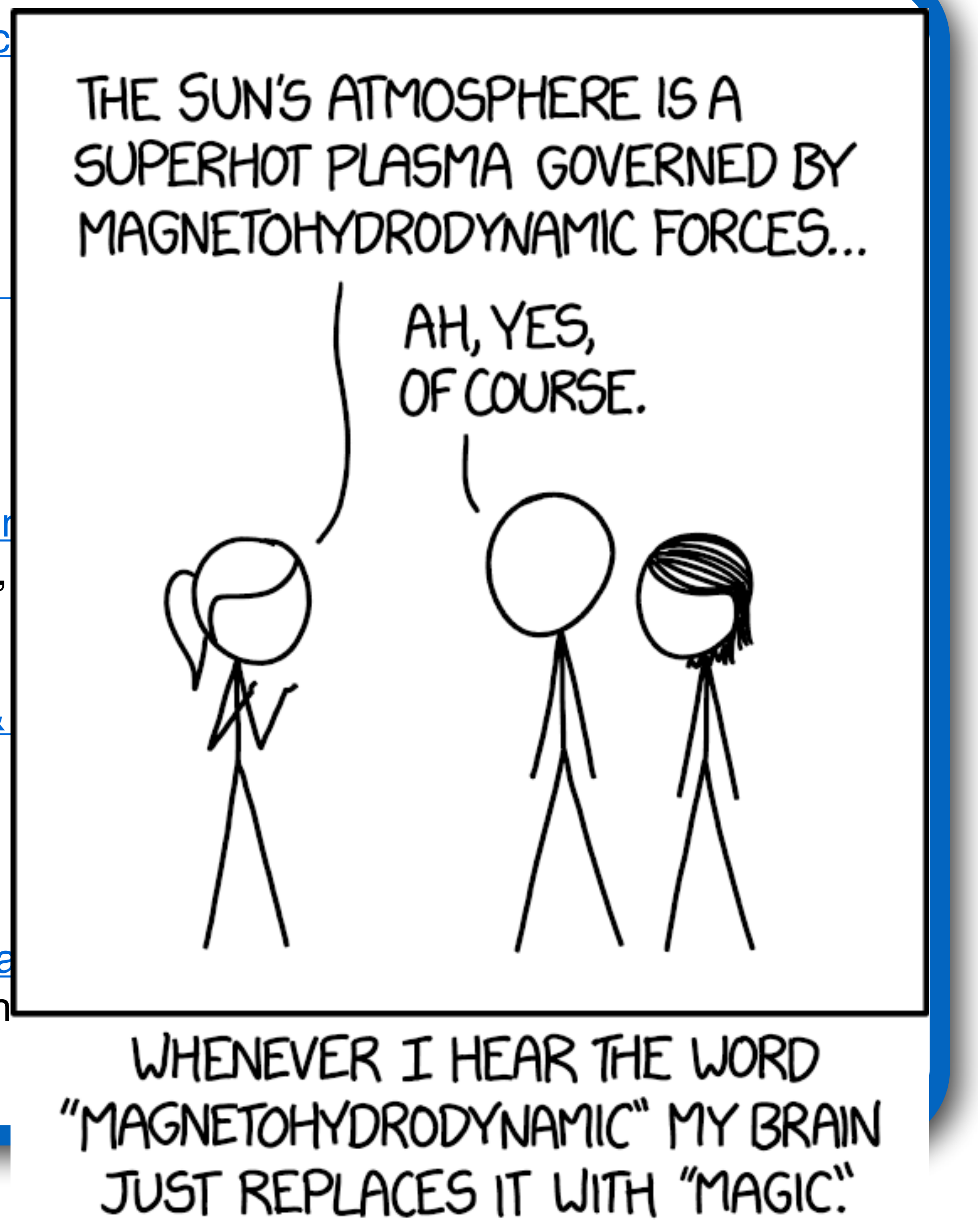

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see Matteo Baggioli's talk today

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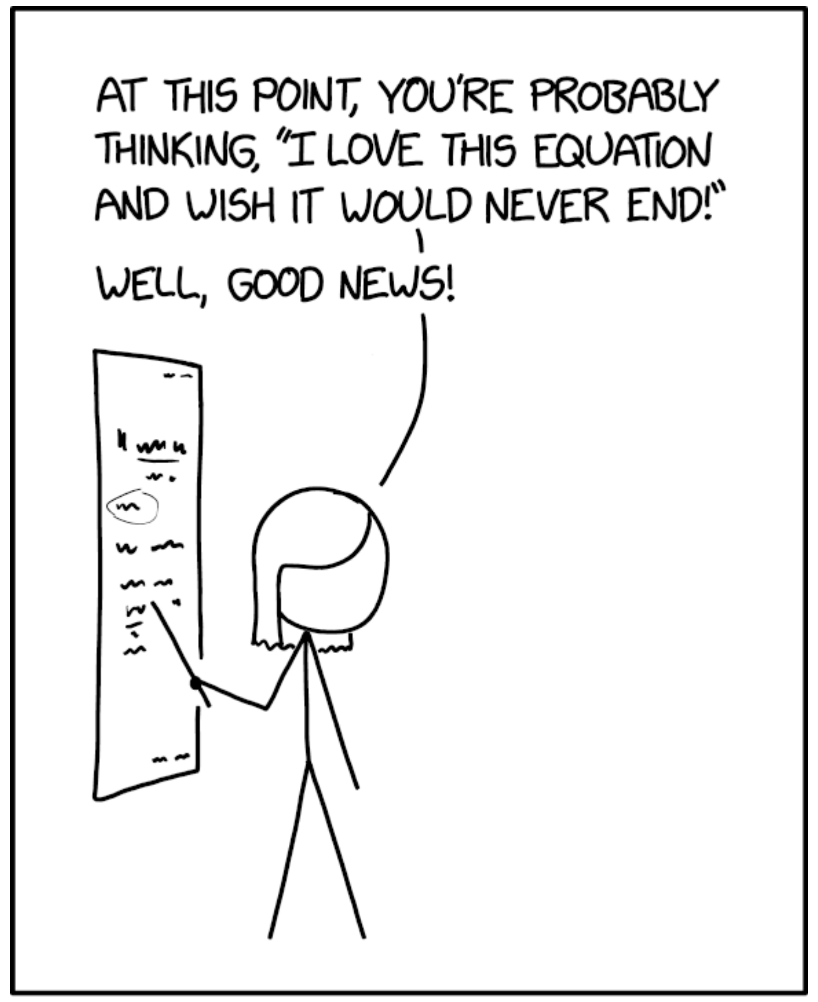
Invitation: Hydrodynamic expansion is asymptotic

Hydrodynamic expansion of dispersion relations
around far-from-equilibrium state

- asymptotic expansion: *coefficients* $\sim n!$
- attractors *[Heller, Spalinski; PRL (2015)]* *[Heller et al; PRL (2021)]*
- resurgence
- far-from-equilibrium holography *[Kurkela et al; PRL (2019)]* *[Janik, Jankowski, Soltanpanahi; PRL (2017)]*
- far-from-equilibrium fluid dynamics *[Romatschke; PRL (2017)]*

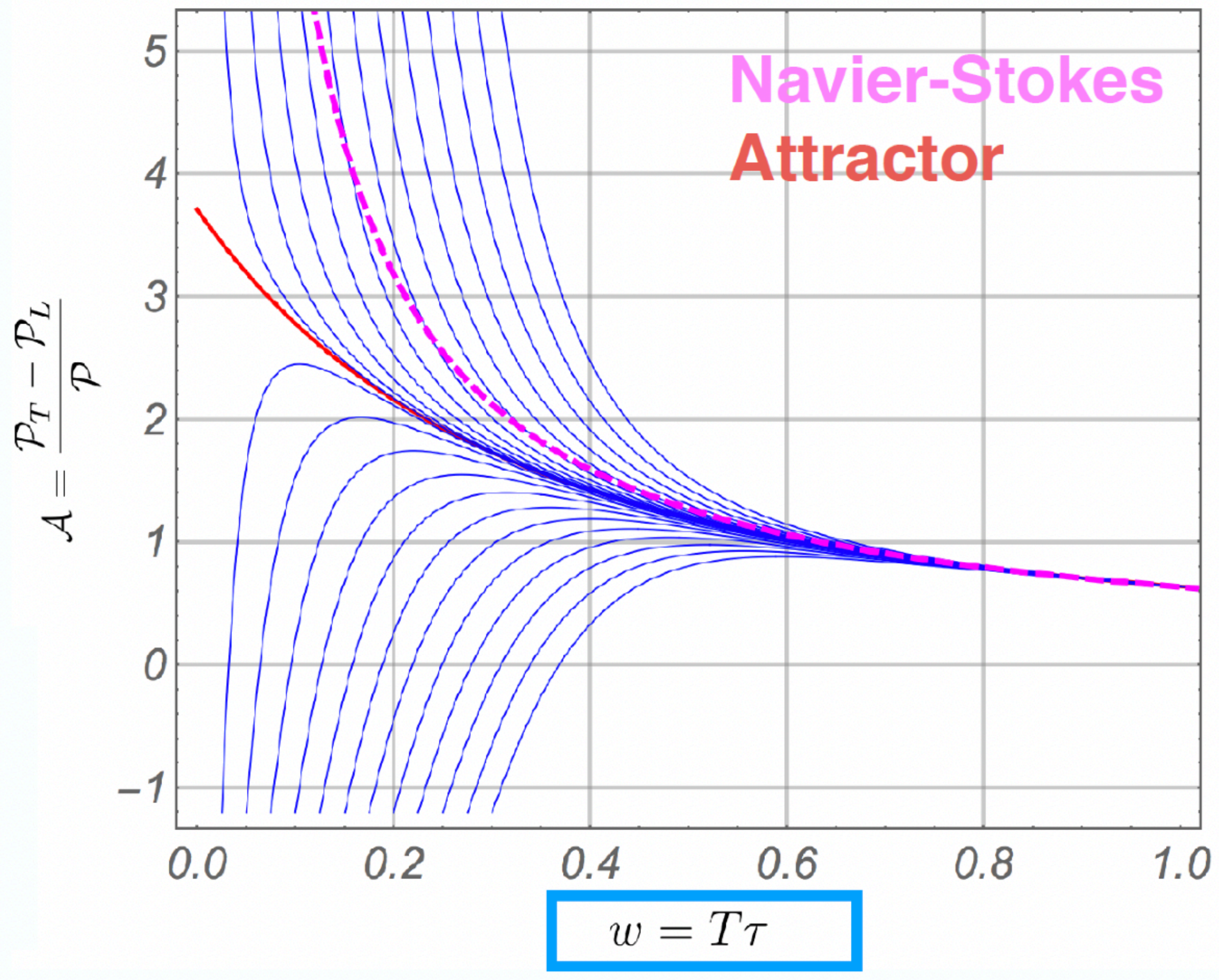


TAYLOR SERIES EXPANSION IS THE WORST.

➡asymptotic is worse

Pressure anisotropy in *N*=4 SYM:

$$\mathscr{A} = \underbrace{\frac{8C_\eta}{w}}_{\text{Navier–Stokes}} + \underbrace{\frac{16C_\eta C_\tau}{3w^2}}_{\text{2nd order}} + \ldots = \underbrace{\sum_{n>0} \frac{a_n^{(0)}}{w^n}}_{\text{gradient expansion}} + \underbrace{\left(\sigma\, w^{\frac{C_\eta}{C_\tau}} e^{-\frac{3}{2C_\tau}w}\right) \sum_{n\geq 0} \frac{a_n^{(1)}}{w^n}}_{\text{transseries sectors}} + \ldots$$



[from Talk by Spalinski at QuarkMatter22]

Bjorken - expanding plasma: C^3-code

[Cartwright,Kaminski,Schenke; PRC (2022)]

Holographic Model

Numerical routine

[Cartwright,Kaminski,Schenke, 2021]

1. Provide initial data $H_i(v_0,r), V(v_0,r), \epsilon(v_0), \xi(v_0)$

2. Solve line by line

$$
\begin{aligned}
0 &= zS(v,z)^2\left(H_1'(v,z)H_2'(v,z)+H_1'(v,z)^2+H_2'(v,z)^2\right)+ze^{-H_1(v,z)}V'(v,z)^2\\
&\quad+6\left(2S'(v,z)+zS''(v,z)\right)S(v,z)\,,\\
0 &= L^6b^2e^{H_1(v,z)}S(v,z)^2+\left(L^3q_5-8\alpha bV(v,z)\right)^2-24L^6z^2S(v,z)^4S'(v,z)\dot S(v,z)\\
&\quad-12L^6z^2S(v,z)^5\dot S'(v,z)-24L^6S(v,z)^6\,,\\
0 &= -64\alpha^2b^2e^{H_1(v,z)}V(v,z)+8\alpha bL^3q_5e^{H_1(v,z)}-L^6z^2S(v,z)^3\left(S'(v,z)\dot V(v,z)+\dot S(v,z)V'(v,z)\right)\\
&\quad+L^6z^2S(v,z)^4\left(H_1'(v,z)\dot V(v,z)+\dot H_1(v,z)V'(v,z)-2\dot V'(v,z)\right)\,,\\
0 &= -9z^2S(v,z)^3\left(H_1'(v,z)\dot S(v,z)+\dot H_1(v,z)S'(v,z)\right)-4z^2e^{-H_1(v,z)}S(v,z)^2V'(v,z)\dot V(v,z)\\
&\quad-6z^2{\dot H_1}'(v,z)S(v,z)^4-2b^2e^{H_1(v,z)}\,,\\
0 &= -6z^2{\dot H_2}'(v,z)S(v,z)^4+b^2e^{H_1(v,z)}+2z^2e^{-H_1(v,z)}S(v,z)^2V'(v,z)\dot V(v,z)\\
&\quad-9z^2S(v,z)^3\left(H_2'(v,z)\dot S(v,z)+\dot H_2(v,z)S'(v,z)\right)\,,\\
0 &= 3L^4S(v,z)^6\left(2L^2z^4A''(v,z)+4z^3A'(v,z)-L^2z^2\dot H_1(v,z)\left(2H_1'(v,z)+H_2'(v,z)\right)\right.\\
&\quad\left.-L^2z^2H_1'(v,z)\dot H_2(v,z)-2L^2z^2H_2'(v,z)\dot H_2(v,z)+8L^2\right)-5b^2L^6e^{H_1(v,z)}S(v,z)^2\\
&\quad+2L^6z^2e^{-H_1(v,z)}S(v,z)^4\left(36e^{H_1(v,z)}S'(v,z)\dot S(v,z)-V'(v,z)\dot V(v,z)\right)\\
&\quad-7\left(L^3q_5-8\alpha bV(v,z)\right)^2\,,\\
0 &= 3z^2A'(v,z)S(v,z)\dot S(v,z)+L^2e^{-H_1(v,z)}\dot V(v,z)^2+L^2\dot H_1(v,z)\dot H_2(v,z)S(v,z)^2\\
&\quad+L^2\dot H_1(v,z)^2S(v,z)^2+L^2\dot H_2(v,z)^2S(v,z)^2+6L^2S(v,z)\ddot S(v,z)\,.
\end{aligned}
$$

3. Obtain time derivative

$$\partial_v H_i(r,v) = \dot H_i - \frac{1}{2}A(r,v)\partial_r H_i(r,v)$$

$$\partial_v V(r,v) = \dot V - \frac{1}{2}A(r,v)\partial_r V(r,v)$$

4. Step forward in time

5. Provide new initial condition

$H_i(v_0+n\Delta v,r), V(v_0+n\Delta v,r), \epsilon(v_0+n\Delta v), \xi(v_0+n\Delta v)$

6. Repeat steps 2-5 until final time is reached

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Bjorken - expanding plasma: C^3-code

[Cartwright,Kaminski,Schenke; PRC (2022)]

Holographic Model

Numerical routine

[Cartwright,Kaminski,Schenke, 2021]

1. Provide initial data $H_i(v_0,r), V(v_0,r), \epsilon(v_0), \xi(v_0)$

2. Solve line by line

$$
\begin{aligned}
0 &= zS(v,z)^2\left(H_1'(v,z)H_2'(v,z)+H_1'(v,z)^2+H_2'(v,z)^2\right)+ze^{-H_1(v,z)}V'(v,z)^2\\
&\quad+6\left(2S'(v,z)+zS''(v,z)\right)S(v,z)\,,\\
0 &= L^6b^2e^{H_1(v,z)}S(v,z)^2+\left(L^3q_5-8\alpha bV(v,z)\right)^2-24L^6z^2S(v,z)^4S'(v,z)\dot{S}(v,z)\\
&\quad-12L^6z^2S(v,z)^5\dot{S}'(v,z)-24L^6S(v,z)^6\,,\\
0 &= -64\alpha^2b^2e^{H_1(v,z)}V(v,z)+8\alpha bL^3q_5e^{H_1(v,z)}-L^6z^2S(v,z)^3\left(S'(v,z)\dot{V}(v,z)+\dot{S}(v,z)V'(v,z)\right)\\
&\quad+L^6z^2S(v,z)^4\left(H_1'(v,z)\dot{V}(v,z)+\dot{H}_1(v,z)V'(v,z)-2\dot{V}'(v,z)\right)\,,\\
0 &= -9z^2S(v,z)^3\left(H_1'(v,z)\dot{S}(v,z)+\dot{H}_1(v,z)S'(v,z)\right)-4z^2e^{-H_1(v,z)}S(v,z)^2V'(v,z)\dot{V}(v,z)\\
&\quad-6z^2\dot{H}_1{}'(v,z)S(v,z)^4-2b^2e^{H_1(v,z)}\,,\\
0 &= -6z^2\dot{H}_2{}'(v,z)S(v,z)^4+b^2e^{H_1(v,z)}+2z^2e^{-H_1(v,z)}S(v,z)^2V'(v,z)\dot{V}(v,z)\\
&\quad-9z^2S(v,z)^3\left(H_2'(v,z)\dot{S}(v,z)+\dot{H}_2(v,z)S'(v,z)\right)\,,\\
0 &= 3L^4S(v,z)^6\Big(2L^2z^4A''(v,z)+4z^3A'(v,z)-L^2z^2\dot{H}_1(v,z)\left(2H_1'(v,z)+H_2'(v,z)\right)\\
&\quad-L^2z^2H_1'(v,z)\dot{H}_2(v,z)-2L^2z^2H_2'(v,z)\dot{H}_2(v,z)+8L^2\Big)-5b^2L^6e^{H_1(v,z)}S(v,z)^2\\
&\quad+2L^6z^2e^{-H_1(v,z)}S(v,z)^4\left(36e^{H_1(v,z)}S'(v,z)\dot{S}(v,z)-V'(v,z)\dot{V}(v,z)\right)\\
&\quad-7\left(L^3q_5-8\alpha bV(v,z)\right)^2\,,\\
0 &= 3z^2A'(v,z)S(v,z)\dot{S}(v,z)+L^2e^{-H_1(v,z)}\dot{V}(v,z)^2+L^2\dot{H}_1(v,z)\dot{H}_2(v,z)S(v,z)^2\\
&\quad+L^2\dot{H}_1(v,z)^2S(v,z)^2+L^2\dot{H}_2(v,z)^2S(v,z)^2+6L^2S(v,z)\ddot{S}(v,z)\,.
\end{aligned}
$$

3. Obtain time derivative

$$\partial_v H_i(r,v) = \dot{H}_i - \frac{1}{2}A(r,v)\partial_r H_i(r,v)$$

$$\partial_v V(r,v) = \dot{V} - \frac{1}{2}A(r,v)\partial_r V(r,v)$$

4. Step forward in time

5. Provide new initial condition

$H_i(v_0+n\Delta v,r), V(v_0+n\Delta v,r), \epsilon(v_0+n\Delta v), \xi(v_0+n\Delta v)$

6. Repeat steps 2-5 until final time is reached

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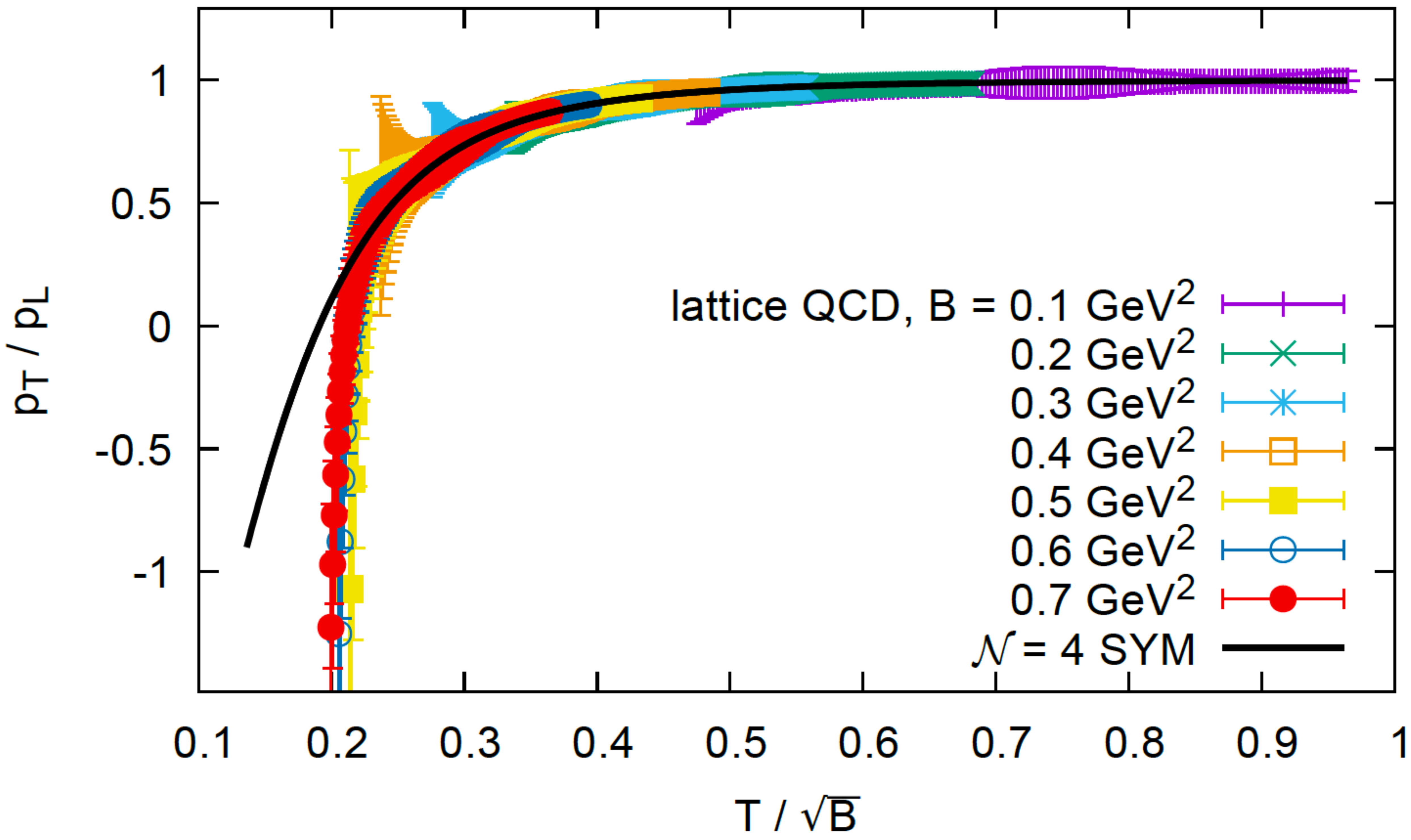
taken from Casey Cartwright's talk

Same magneto response in LQCD and N=4 SYM with magnetic field

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; JHEP (2018)]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $$p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$$

longitudinal pressure: $$p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$$

F_{QCD} ... *free energy*

L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*