Slow modes and momentum anisotropy in far-from-equilibrium QGP

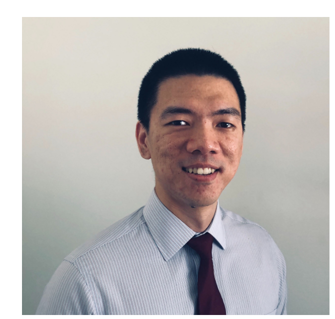
- Introduction
- Far-from-equilibrium slow modes from kinetic theory and their relations to hydro. modes.
  - Novel slow modes associated with momentum anisotropy.
- Implications for the origin of flow in small systems?

Yi Yin

Institute of Modern Physics, Lanzhou, CAS

Hard Probe, Mar. 27-31, 2023

Based on: J. Brewer (CERN), Weiyao Ke (LANL), Li Yan (Fudan), YY, 2212.00820



Weiyao Ke @ LANL

Brewer@ CERN

Far-from-equilibrium QGP

- Heavy-ion collision: a unique opportunity for studying off-equilibrium properties of quark-gluon matter.

- Significant progress in theory (attractor, KOMPOST etc) but important questions still open: e.g. what are the relevant d.o.fs at early time.

Works by many; Review: Burgers et al, Rev.Mod.Phys. 2020

- Near equilibrium: slow modes are conserved densities as described by hydro. → the evolution of azimuthal anisotropy is driven by hydro. modes.

- Far from equilibrium: are there slow modes? how does azimuthal anisotropy evolve (the origin of flow in small systems)? This talk: addressing the questions within kinetic theory models.

Multipole Moments

- Consider the phase space distribution $f(\vec{p};\tau)$ of gluons in an Bjorken expanding plasma

$$L_{l,m}(\tau) \equiv \int_{\vec{p}} p\, P_l^m(\theta)\cos(m\phi) f(\vec{p};\tau)$$

  - e.g. $L_{0,0} = T^{00}, L_{2,0} \propto \epsilon - p_L/3, L_{2,1} = T^{0x}, L_{2,2} \propto T^{xx} - T^{yy}$.

  - m: the shape of the distribution in transverse plane ($m \geq 2$ measures the azimuthal anisotropy)

  - l: the correlation strength at scale $\Delta v_z \sim \cos(\pi/l)$.

- Collecting $L_{l,m}$ with the same m and parity $(z \to -z)$ $s = \pm$ into "a bigger vector" $\Psi_{m,\pm}$,

  - e.g $\psi_{0+} = \left(L_{0,0}, L_{2,0}, \ldots\right), \psi_{2+} = \left(L_{2,2}, L_{4,2}, \ldots\right)$

RTA Kinetic theory

- Consider the angular distribution $F(\tau, \hat{p}) = \int dpp^2\, p f(\tau, \vec{p})$

  and its evolution eqn

$$\partial_\tau F - \frac{\hat{p}_z(1-\hat{p}_z^2)}{\tau}\partial_{\hat{p}_z} F + \frac{4\hat{p}_z^2}{\tau} F = -C[f]$$

- Using the generalized relaxation time approximation collision integral (conserving energy-momentum) :

$$C[F] = -\frac{F - T^{00} - 3(T^{0x}\hat{p}_x + T^{0y}\hat{p}_y + \tau T^{0\eta}\hat{p}_z)}{\tau_R},$$

- The evolution eqns for moments (different sectors (ms) do not mix).

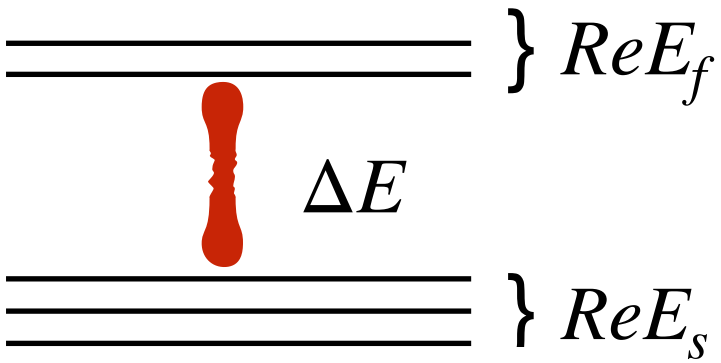
$$\partial_\tau \psi_{ms} = \mathscr{H}_{ms}(\tau)\, \psi_{ms}$$

Slow modes

- The collection of low-lying eigenmodes which are gapped from the others.

$$\mathscr{H}(\tau)\phi(\tau) = E(\tau)\phi(\tau)$$

non-Hermitian complex

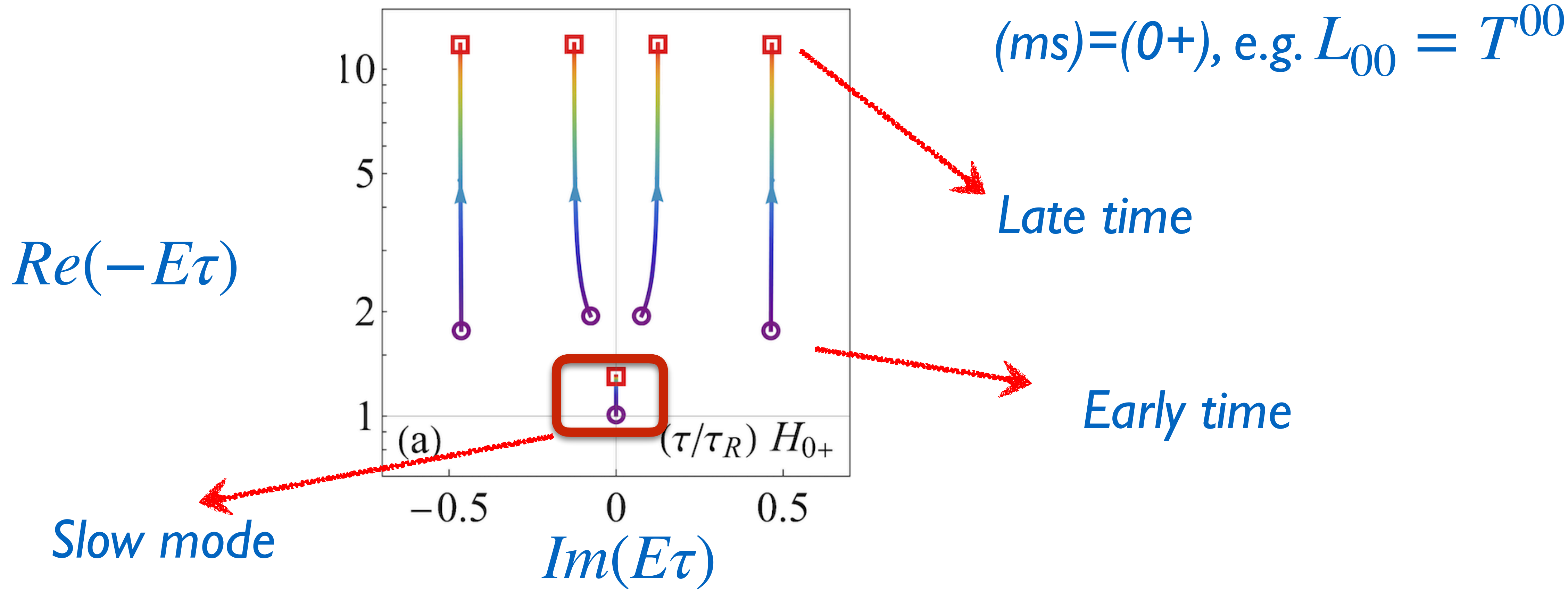


(decay rate $ReE \geq 0$ because of expansion and collision)

- Dominate the dynamics under adiabatic conditions.

Brewer-Li Yan-YY, PLB 21', Brewer-Scheihing-Hitschfeld-YY, JHEP 22', Mikheev-Mazeliauskas-Berges, PRD 22';

Eigenvalues in hydro. sector



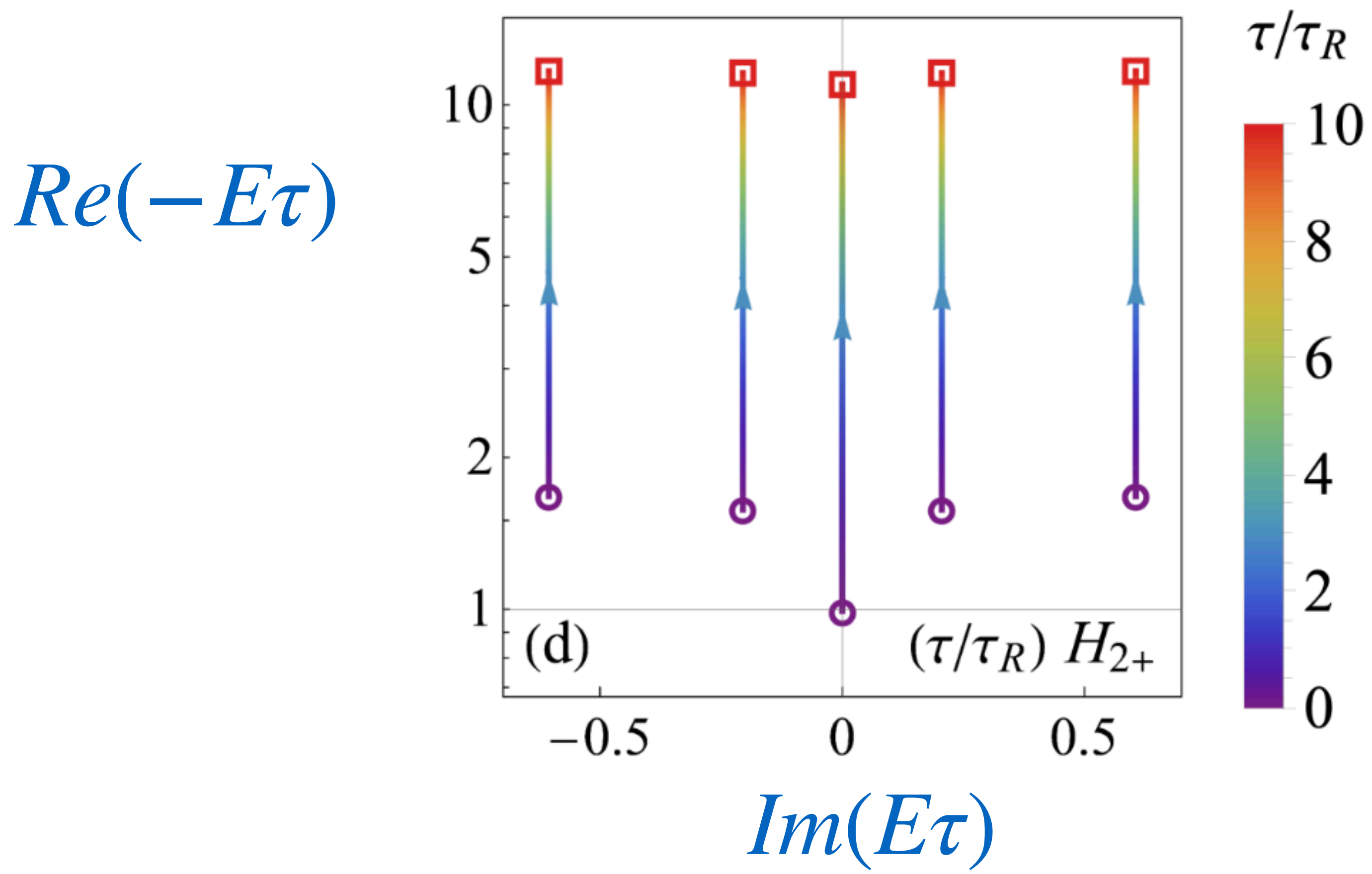
- There is an early-time slow mode associated with energy density.

$$\phi^G_{0+} = \left(T^{00}, L_{20}, \ldots\right) \longrightarrow \phi^G_{0+} = \left(T^{00}, 0, \ldots\right) \quad \tau$$

- Similar results for (1+) sector (containing T^{0x}); one-to-one correspondence between conserved density and early-time slow modes?

Anisotropy sector ($m \geq 2, s = +$)

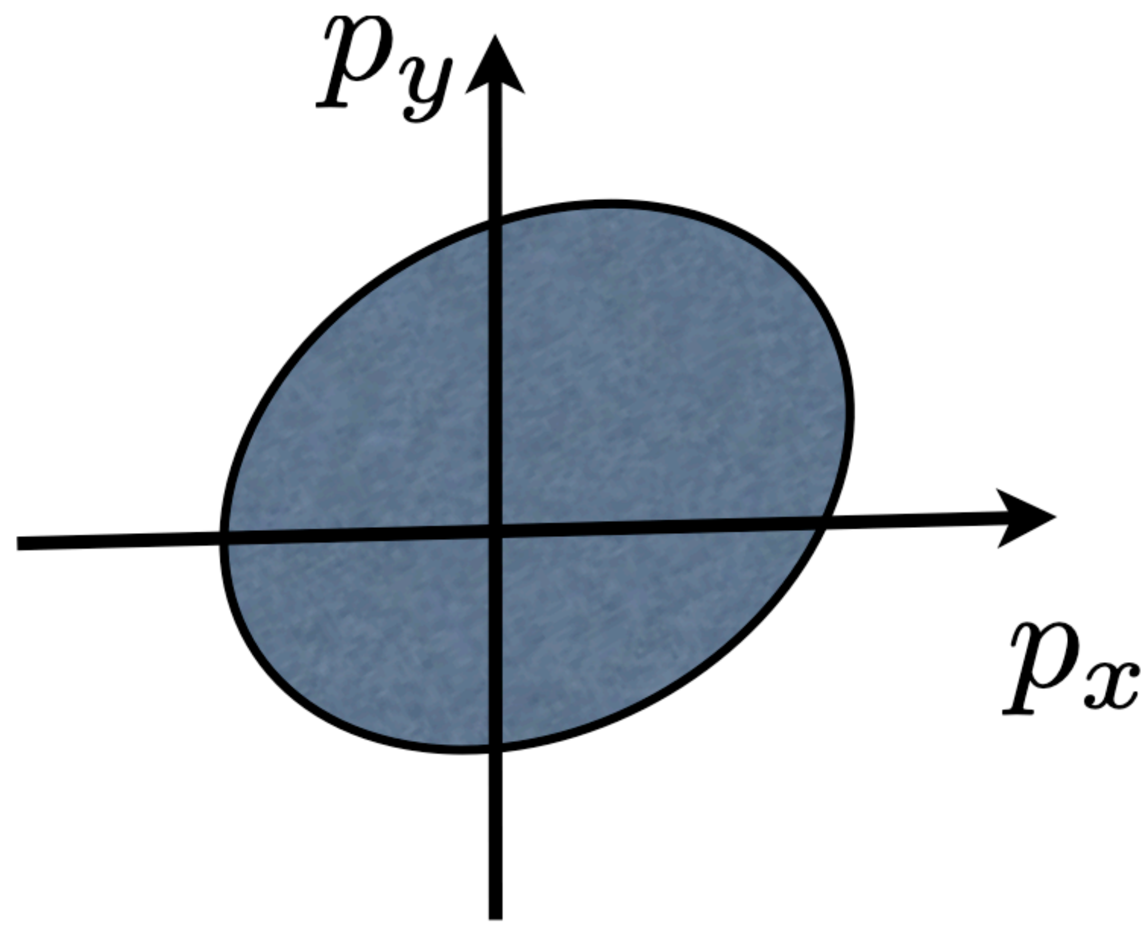
(ms)=(2+), e.g $L_{22} = T^{xx} - T^{yy}$



- Early-time slow mode exists for $m \geq 2$ (anisotropy) sectors.
- Democracy among different m(s) at early time:

$$\lim_{\tau \to 0} E^G_{0+} = E^G_{1+} = E^G_{2+} = E^G_{3+} = \ldots = \frac{1}{\tau}$$

Shapes in phase space as slow modes



- The phase space volume is preserved (Liouville theorem) in collisionless limit. (c.f. 2203.05004).

  - For Bjorken expansion at early time, the shape of transverse distribution evolves slowly.

Nonlinear Bosonization of Fermi Surfaces:
The Method of Coadjoint Orbits

Luca V. Delacrétaz,[1,2] Yi-Hsien Du,[1] Umang Mehta,[1,3] and Dam Thanh Son[1,2,4]

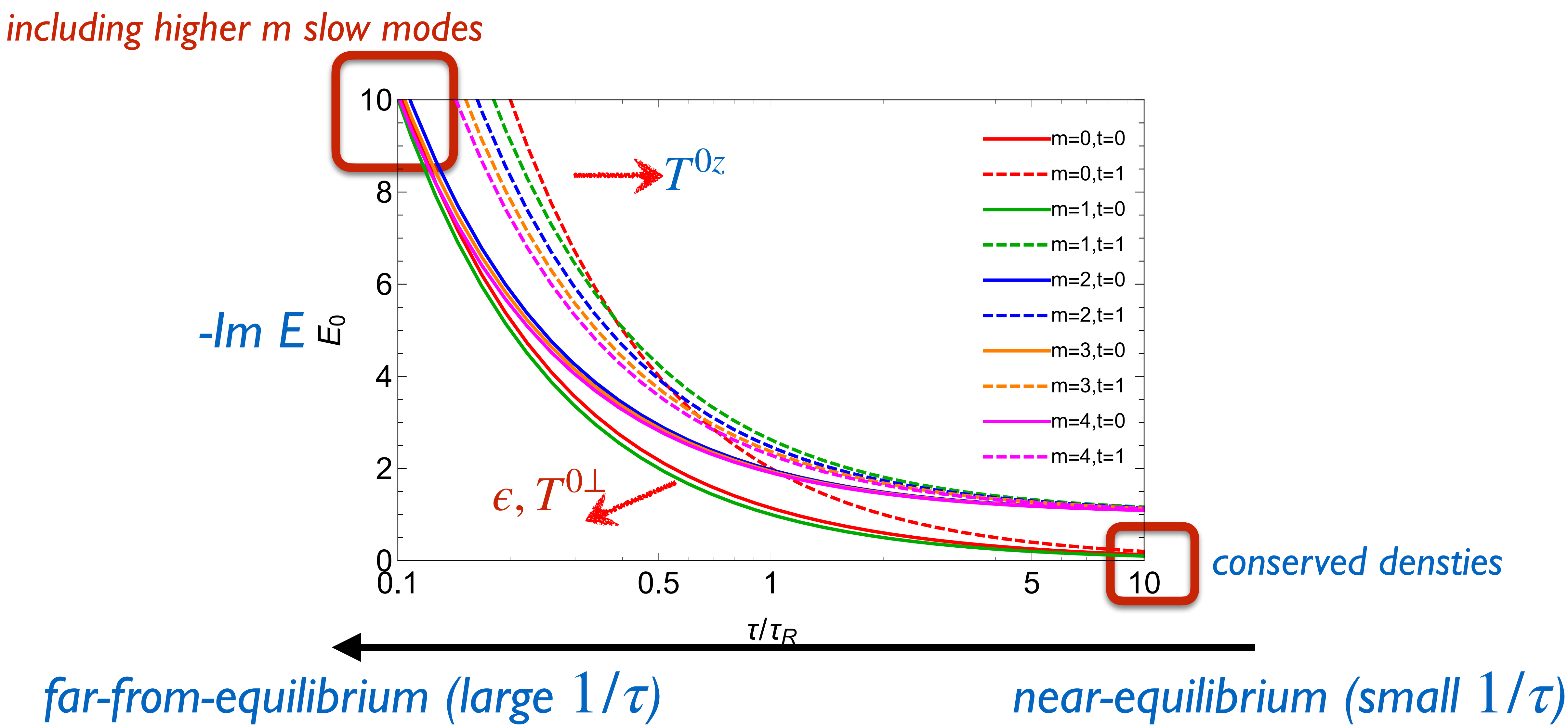
[1] *Kadanoff Center for Theoretical Physics,*
University of Chicago, Chicago, Illinois 60637, USA

Collisionless dynamics of general non-Fermi liquids from hydrodynamics of emergent conserved quantities

Dominic V. Else
Perimeter Institute for Theoretical Physics, Waterloo, ON N2L 2Y5, Canada

Many recent developments on emergent symmetries (slow modes), dynamics of shape in collisionless kinetic theory.

Slow modes in and out of equilibrium



- Showing the diverse relation to conserved densities.
- Implications for the memory of initial azimuthal an-isotropy?

The evolution rate

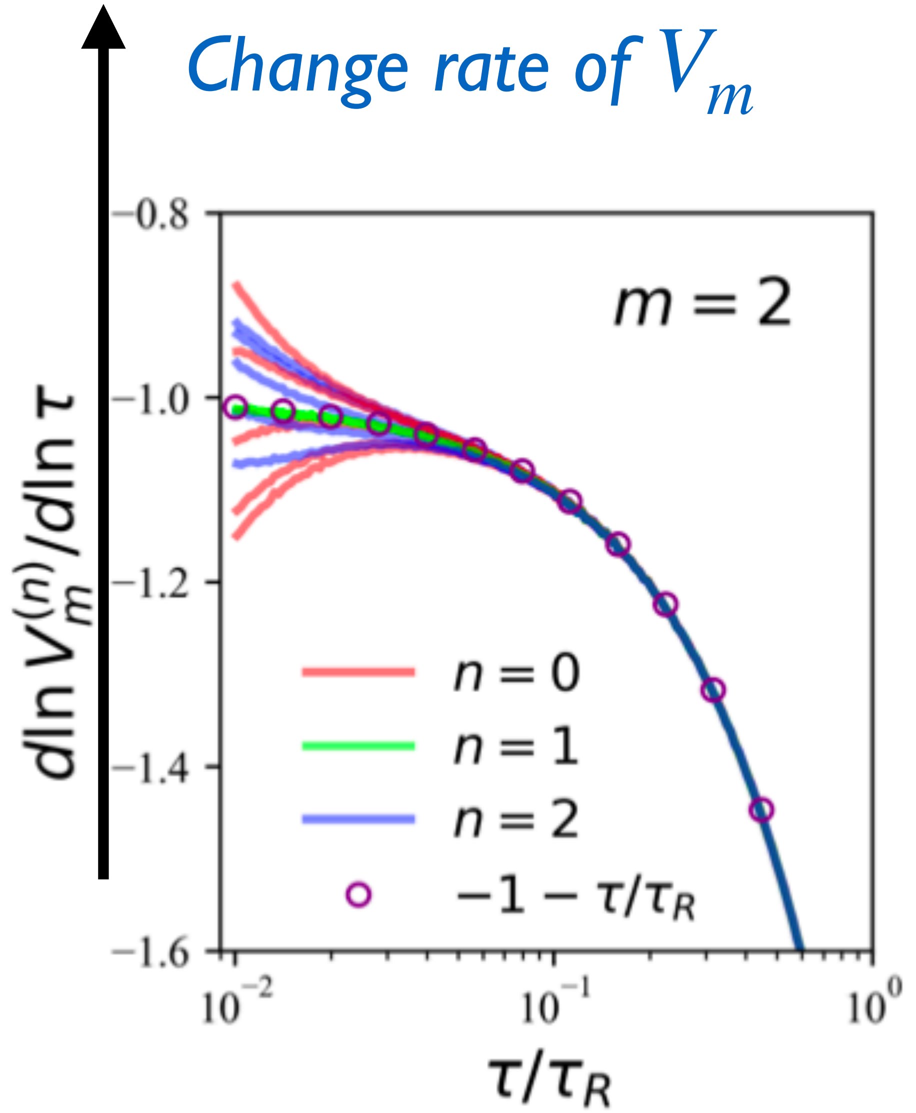
$$V_m^{(n)} = \int_{\hat{p}} (\frac{p_T}{p})^n F(\hat{p}) \, cos(m\phi)$$

Harmonics *distribution*

- Given the dominance of slow modes, the percentage change rate of $V_m^{(n)}$ is insensitive to many details of initial conditions (attractor behavior).

$$\tau\partial_\tau \log V_m^{(n)} = -E_{m+}^G$$

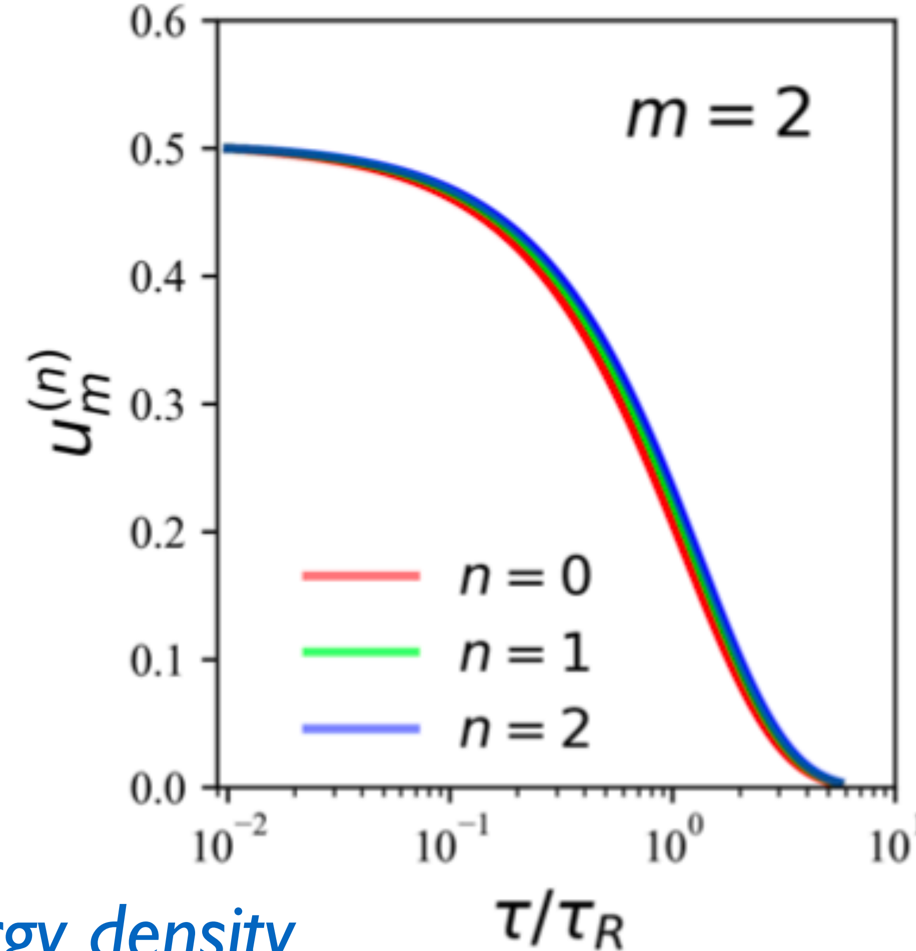
Ground state eigenvalue



Initial azimuthal anisotropy decay slowly

$$u_m^{(n)} = \frac{V_m^{(n)}}{V_0^{(n)}} \, (m \geq 2)$$

NB: u_2, u_3 is the proxy of elliptic, triangle flow etc. $u_2^{(2)} \propto (T^{xx} - T^{yy})/\epsilon$.



$$\tau \partial_\tau u_m = -(E_m^G - E_0^G) \sim -\frac{1}{\tau_R}$$

change rate of shape — *change rate of energy density*

- The memory of initial momentum space eccentricity will survive up to τ_R

- Without $m \geq 2$ slow modes, u_m would quickly decay as $(\tau/\tau_I)^{-\#}$.

Summary and outlook

- We study the slow modes of the far-from-equilibrium QGP using kinetic theory.
  - The relation between slow modes and conserved density is rather diverse.
  - Momentum space an-isotropy (the shape of distribution) evolve slowly.
- Implication: accounting for the prehydro. evolution of azimuthal an-isotropy in terms of slow modes. (Further check is needed, e.g. including spatial gradient effects, more general microscopic model).

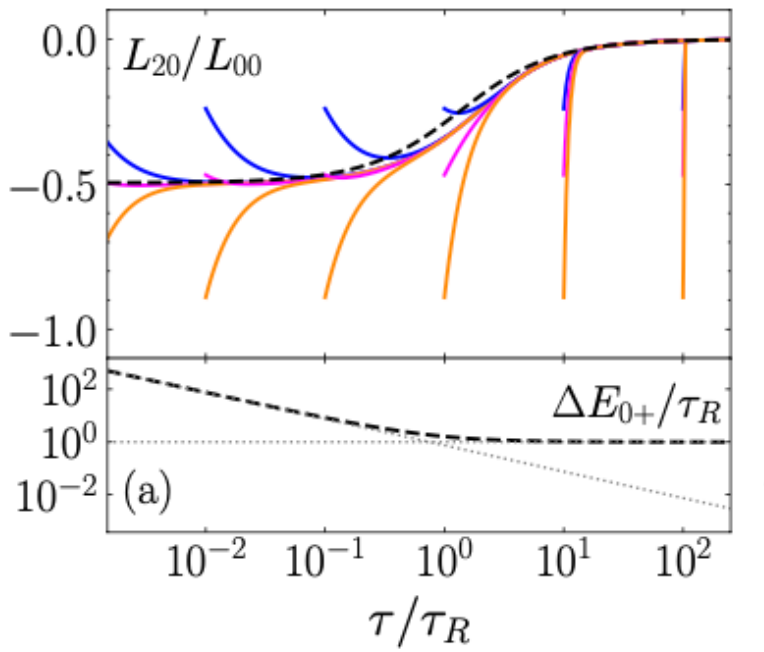
A broader view: the evolution of QGP vs scale

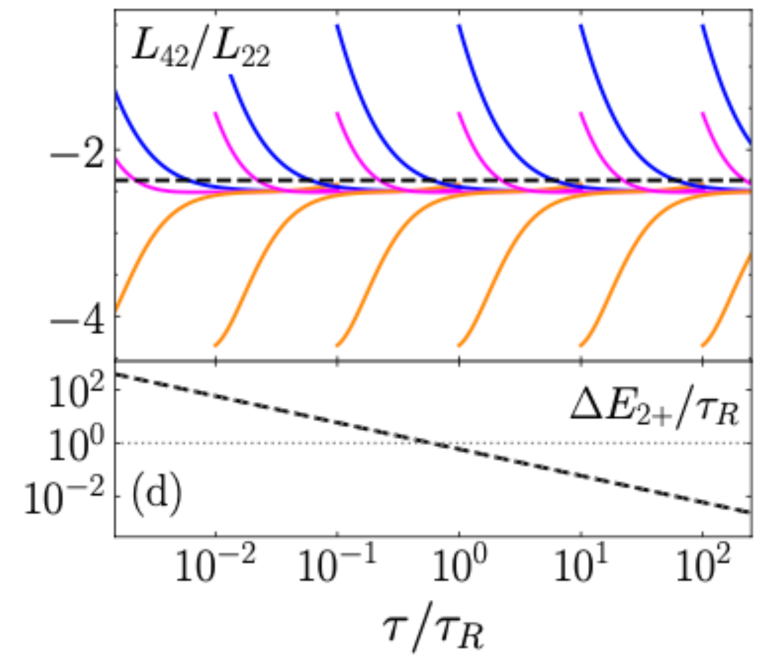
- This talk: QGP properties from equilibrium to far-from-equilibrium (QGP with varying expansion rate). Other perspectives:
  - Jet observables as a function of scattering angle.
  - Medium response with varying gradient (Jet-medium responses).

Ultimate goal: a comprehensive picture of QGP evolution!

Back-up

Attractor





- After the decay of fast modes, the ratio of moment is determined by that of slow mode and is insensitive to the initial condition (attractor behavior).

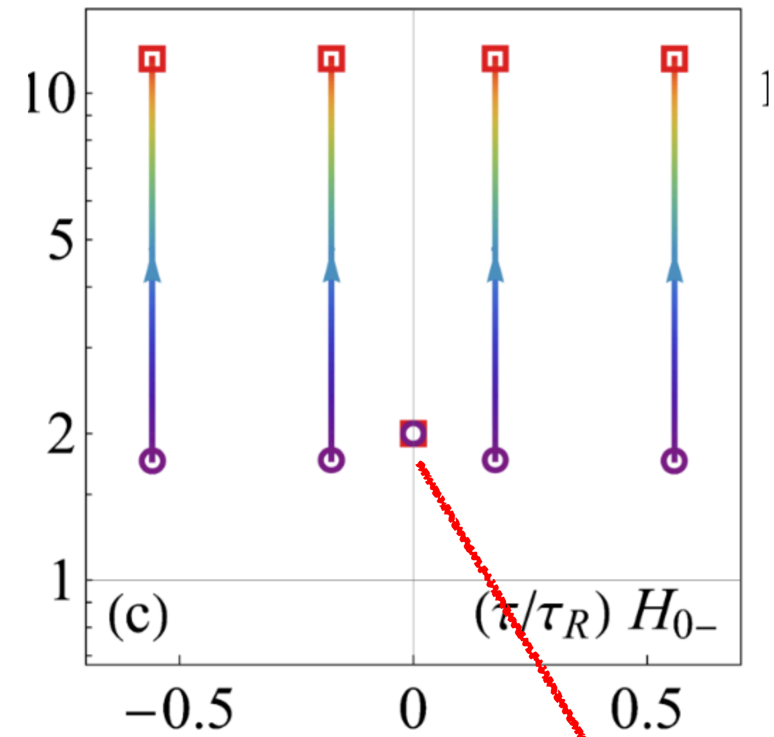
- Early-time attractor behavior for higher spin sectors.

- Implications for the memory of momentum space an-isotropy?

Longitudinal momentum density sector

(ms)=(0-), e.g. $L_{01} = T^{0z}$

$Im(-E\tau)$



$Re(E\tau)$

$\left(T^{0z}, \ldots\right)$

- T^{0z} is not related to the early-time slow modes.