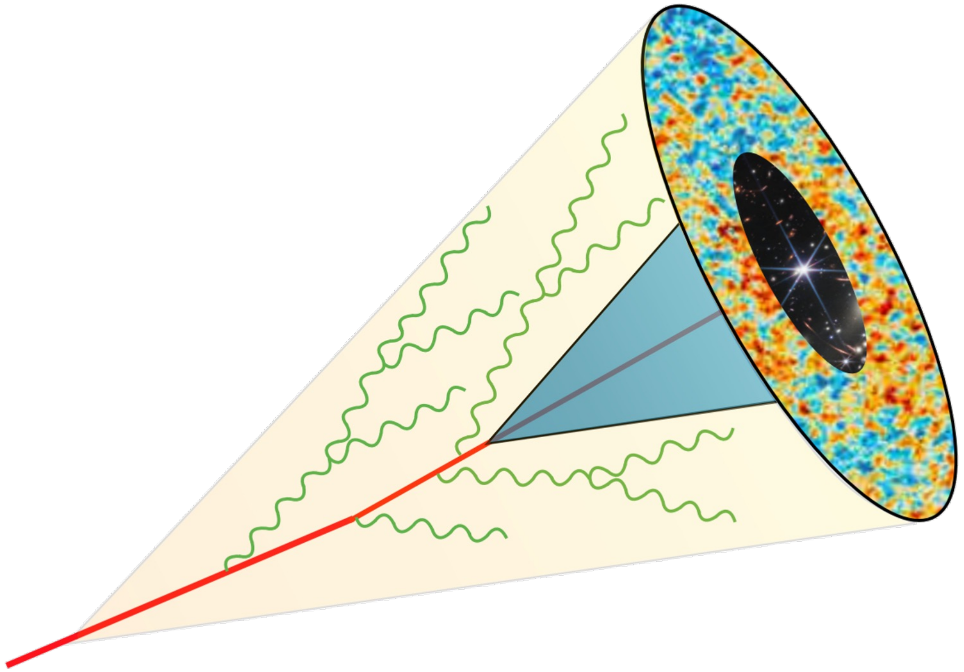
Exploring the deadcone effect in heavy ion collisions with energy correlators



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In collaboration with Carlota Andres, Fabio Dominguez, Cyrille Marquet, Ian Moult

29/03/2023

What are energy correlators?

Energy correlators are a unique class of collider observables.

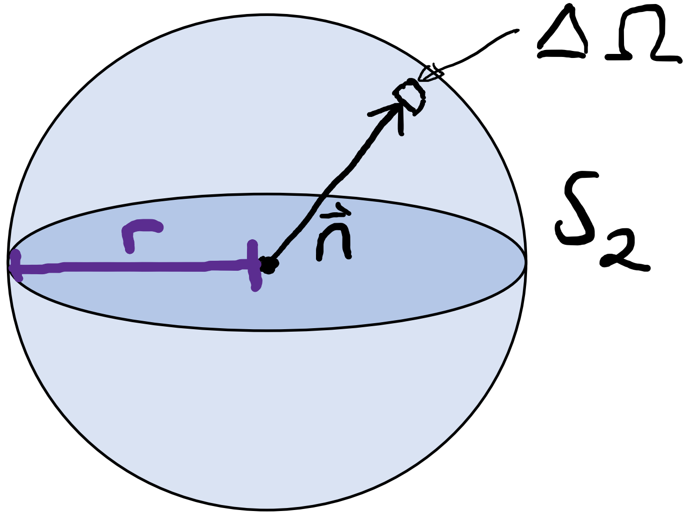
They are correlation functions of a fundemental QFT operator:

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

Thus, by measuring energy correlators we are directly measuring the spectrum of a theory.

However, they can also also be defined in terms of familiar inclusive cross-sections.

Intuition



What is $\mathcal{E}(\vec{n})$ phyiscally?

$\mathcal{E}(\vec{n})$ =Energy flux through ΔΩ

≡ Idealised calorimeter output at a calorimeter cell at position $\vec{n}$.

Therefore, when the S-matrix is $S_{fi} = \langle \text{final}|\text{initial}\rangle$,

$$\mathcal{E}(\vec{n})|\text{final}\rangle = \sum_i E_i\, \delta^{(2)}(\vec{n} - \vec{n}_i)\, |\text{final: i}\rangle.$$

Correlators as inclusive cross-sections

The correlation function we will look at in this talk is

$$\langle \text{final}|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)|\text{final}\rangle.$$

This is referred to as the energy-energy correlator (EEC).
Some simple manipulations give,

$$\langle \text{final}|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)|\text{final}\rangle = \langle \text{final}; i,j| \sum_{i,j} E_i E_j \delta^{(2)}(\vec{n}_1 - \vec{n}_i)\delta^{(2)}(\vec{n}_2 - \vec{n}_i)\,|\text{final}: i,j\rangle,$$

$$= \sum_{i,j} \sum_{\text{initial}} \langle \text{final}; i,j|\text{initial}\rangle\langle \text{initial}|\text{final}: i,j\rangle\, E_i E_j \delta^{(2)}(\vec{n}_1 - \vec{n}_i)\delta^{(2)}(\vec{n}_2 - \vec{n}_i)\,,$$

$$= \sum_{i,j} \sum_{\text{initial}} \left|S_{fi}(\text{initial} \to i,j + \text{anything else})\right|^2 E_i E_j \delta^{(2)}(\vec{n}_1 - \vec{n}_i)\delta^{(2)}(\vec{n}_2 - \vec{n}_i)\,,$$

Correlators as inclusive cross-sections

$$\frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\int \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}\boldsymbol{n}_i \mathrm{d}\boldsymbol{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\boldsymbol{n}_i - \boldsymbol{n}_1)\delta^{(2)}(\boldsymbol{n}_j - \boldsymbol{n}_2)$$

Where i, j are final state hadrons and σ_{ij} is the inclusive cross section to produce i, j with a hard scale Q.

We integrate out the global $SO(3)$ symmetry (ingoring the beam axis) to find the distribution we're interested in.

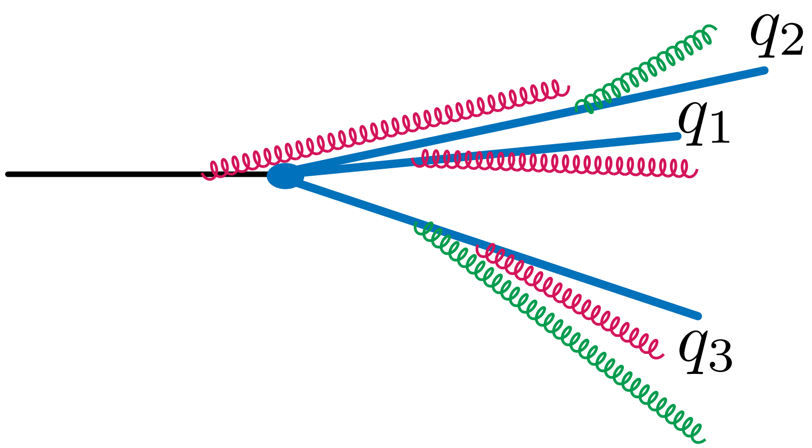
$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \int \mathrm{d}\boldsymbol{n}_{1,2}\frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}}\delta(\boldsymbol{n}_2 \cdot \boldsymbol{n}_1 - \cos\theta)$$

Correlators as inclusive cross-sections

These relations tell us how to find the EEC experimentally.

$$\frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\int \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}\boldsymbol{n}_i\mathrm{d}\boldsymbol{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\boldsymbol{n}_i-\boldsymbol{n}_1)\delta^{(2)}(\boldsymbol{n}_j-\boldsymbol{n}_2) \qquad \frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \int \mathrm{d}\boldsymbol{n}_{1,2}\frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}}\delta(\boldsymbol{n}_2\cdot\boldsymbol{n}_1-\cos\theta)$$

The correlators are ‘counting experiments’ just like cross-section measurements.



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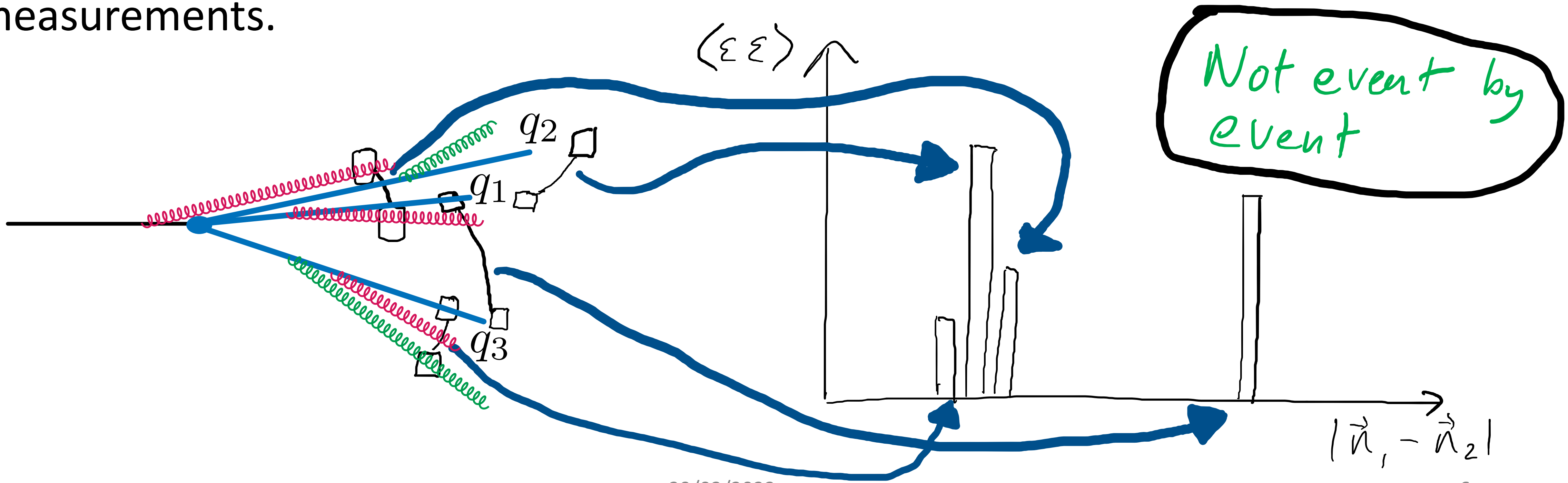

Correlators as inclusive cross-sections

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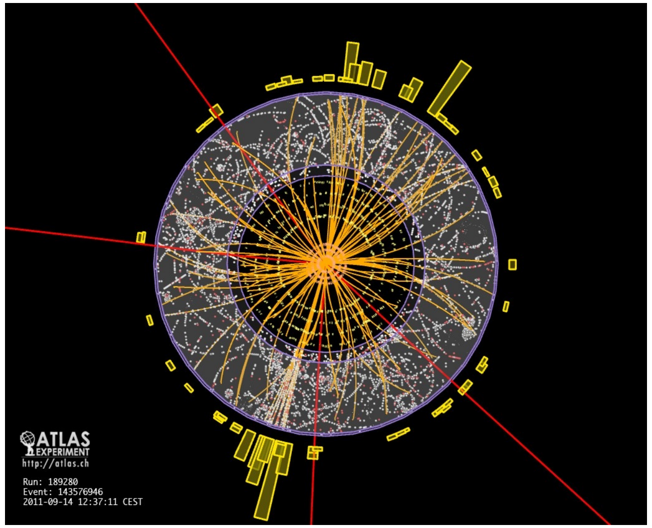

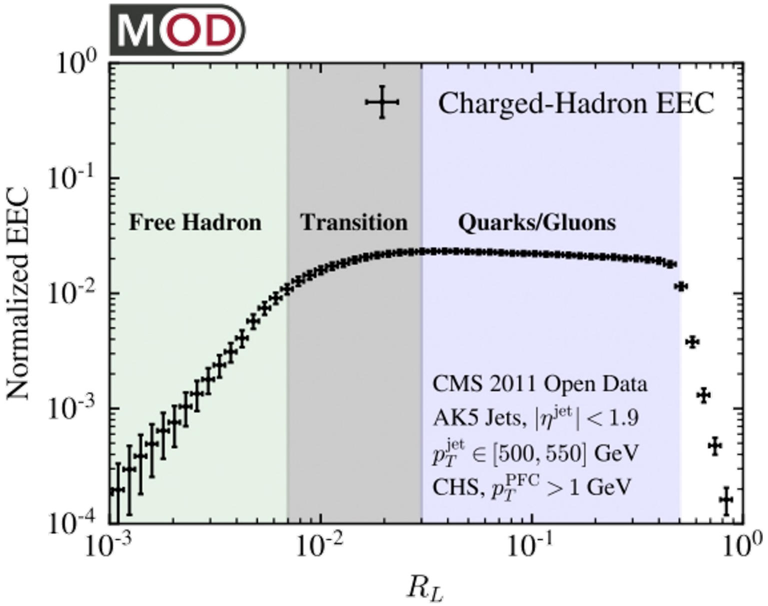

Komiske, Moult, Thaler, Zhu 2201.07800

What do we expect to see in the EEC?

Let's consider the definition of the EEC in terms of the S-matrix

$$\sum_{i,j} \sum_{\text{initial}} \left|S_{fi}(\text{initial} \to i, j + \text{anything else})\right|^2 E_i E_j \delta^{(2)}(\vec{n}_1 - \vec{n}_i)\delta^{(2)}(\vec{n}_2 - \vec{n}_i),$$

And consider measuring the correlator on a massless quark-jet.

Jets are dominated by soft and collinear radiation.

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And consider measuring the correlator on a massless quark-jet.

The EEC is dominated by collinear radiation.

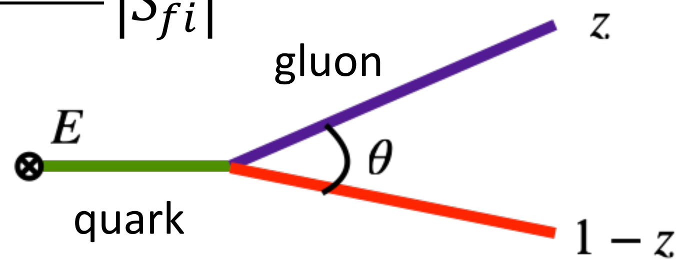
What do we expect to see in the EEC?

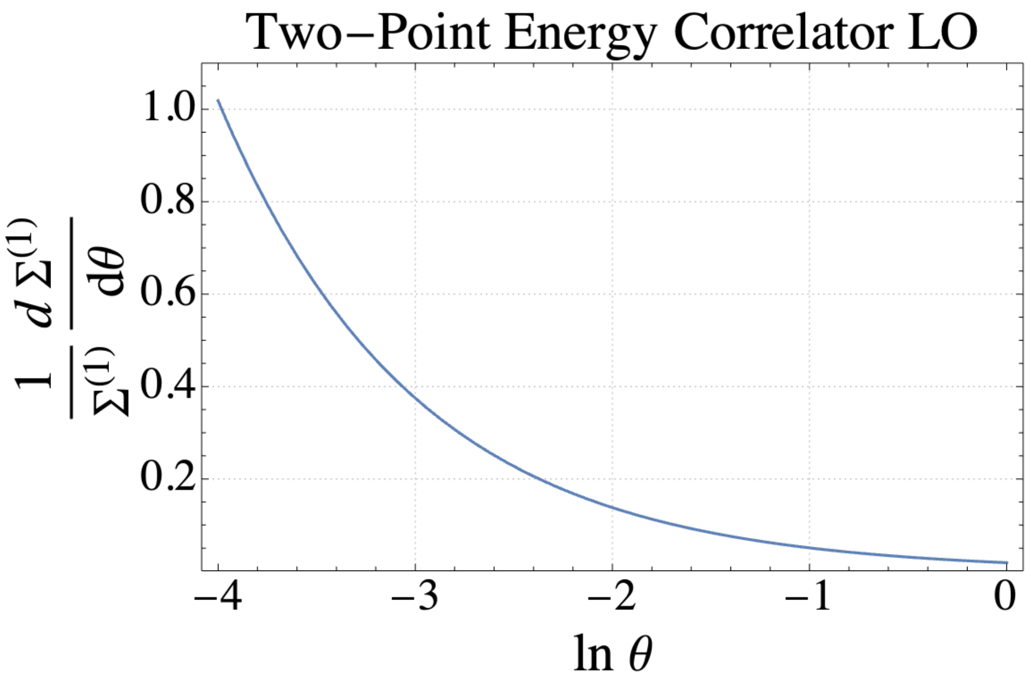
For now in *vacuum*…

What do we expect to see in the EEC?

At LO

$$\left|S_{fi}(\text{initial} \to i, j + \text{anything else})\right|^2 \sim \frac{1}{\theta_{ij}} \frac{1-(1-z)^2}{z} \left|S_{fi}\right|^2$$





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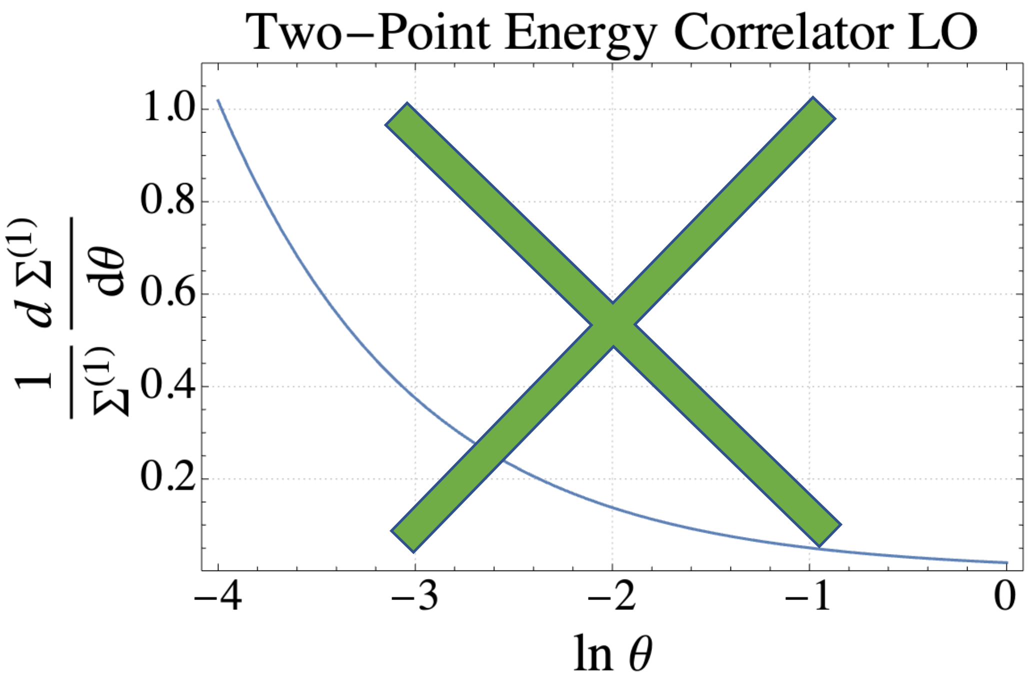
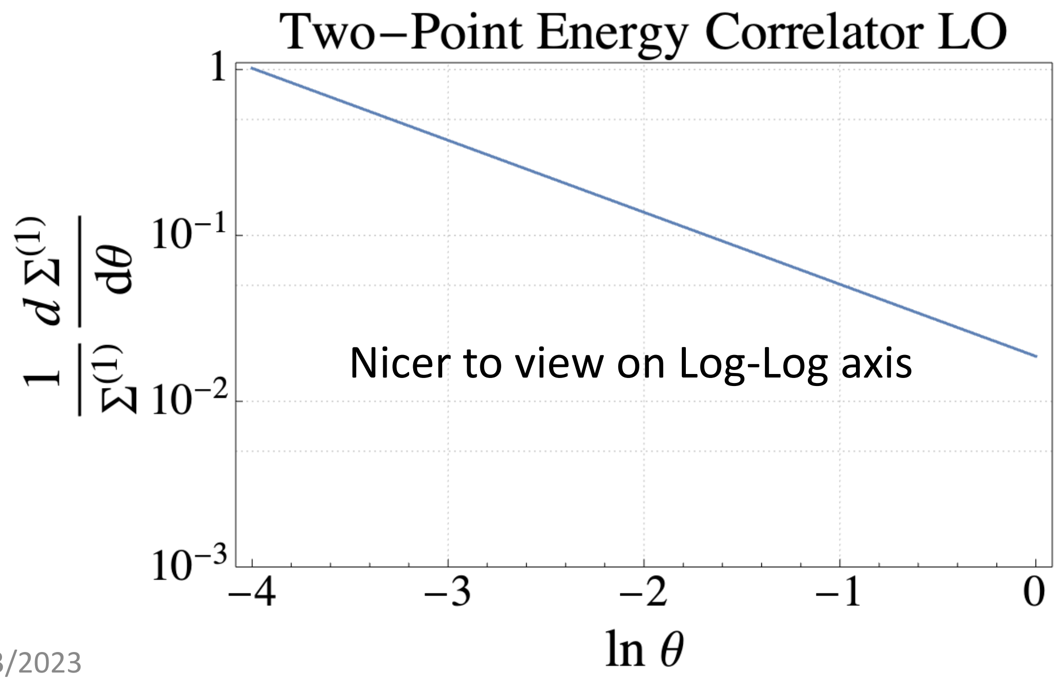




The deadcone

Now we look at a massive quark jet, still in *vacuum*…

At LO and in the soft limit of the emitted gluon:

$$\left|S_{fi}(\text{initial} \to i, j + \text{anything else})\right|^2 \sim \frac{{\theta_{ij}}^3}{\left({\theta_{ij}}^2 + {\Theta_0}^2\right)^2} \left|S_{fi}\right|^2$$

Where $\Theta_0 = m_Q/E$. This is the usual splitting kernel we think of when talking about the *deadcone*.

The deadcone

Now we look at a massive quark jet, still in *vacuum*…

However, we are not interested in the soft limit, rather the collinear:

$$\frac{{\theta_{ij}}^3}{\left({\theta_{ij}}^2 + {\Theta_0}^2\right)^2} \Rightarrow \frac{2\mu^{2\epsilon} g_s^2}{(p_j + p_k)^2 - m_{(jk)}^2} T_R \left[1 - \frac{2}{d-2}\left(2z(1-z) - \mu_{q\bar{q}}^2\right)\right]$$

Catani, Dittmaier, Trocsanyi hep-ph/0011222

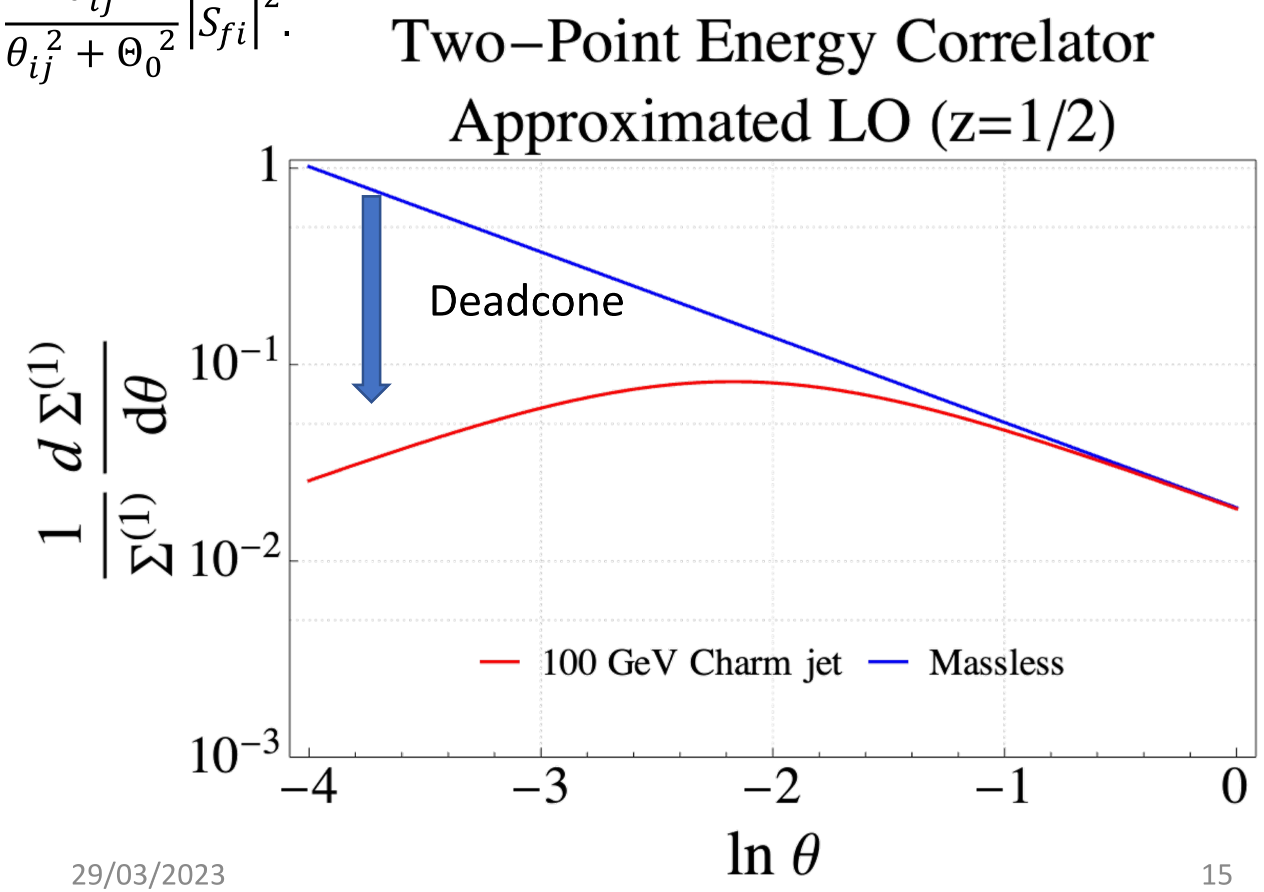
And so

$$\left|S_{fi}(\text{initial} \to i, j + \text{anything else})\right|^2 \sim \text{when } z = \frac{1}{2} \sim \frac{\theta_{ij}}{{\theta_{ij}}^2 + {\Theta_0}^2}\left|S_{fi}\right|^2.$$

The deadcone

Now we look at a massive quark jet, still in *vacuum*…

$|S_{fi}(\text{initial} \to i, j + \text{anything else})|^2 \sim \text{when } z = \frac{1}{2} \sim \frac{\theta_{ij}}{\theta_{ij}^2 + \Theta_0^2} |S_{fi}|^2.$



The EEC on a quenched jet

Now we consider the *medium* case…

The EEC on a quenched jet

This is an very abridged summary, see Fabio's talk "Determining the onset of color coherence with energy correlators" 15:20 for a detailed discussion.

In recent work (2209.11236, 2303.03413), we showed that the EEC can be computed on a medium quenched jet as

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \frac{1}{\sigma}\int \mathrm{d}z \left(g^{(n)}(\theta,\alpha_s) + F_{\mathrm{med}}(z,\theta)\right)\frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta\mathrm{d}z} z^n(1-z)^n \left(1+\mathcal{O}\left(\frac{\bar{\mu}_s}{Q}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\theta\, Q}\right)$$

The vacuum-like piece

The medium modification to the vacuum result

The EEC on a *massless* quenched jet

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma}\int dz \left(g^{(n)}(\theta,\alpha_s) + F_{\rm med}(z,\theta)\right)\frac{d\sigma_{qg}^{\rm vac}}{d\theta dz} z^n(1-z)^n\left(1+\mathcal{O}\left(\frac{\bar{\mu}_s}{Q}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{\theta\, Q}\right)$$

The vacuum-like piece. When the bracket is expanded this term at LO gives $1/\theta$ divergence already discussed. In general known at NLO+NNLL. We use LO+NLL here:

$$\frac{1}{\sigma}\frac{d\sigma_{qg}^{\rm vac}}{d\theta dz} = \frac{\alpha_s(\theta Q)}{\pi} C_F \frac{1+(1-z)^2}{z\,\theta} + \mathcal{O}(\alpha_s^2,\theta^0)$$

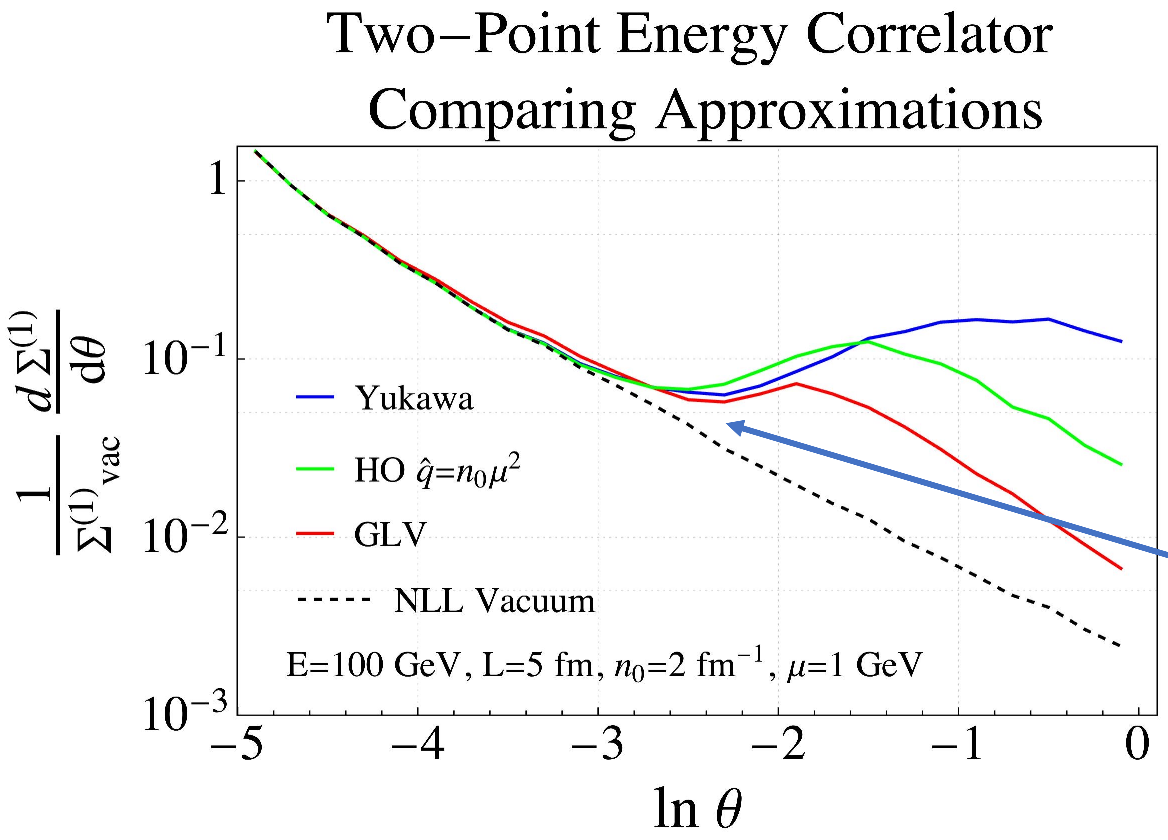
$$g^{(1)} = \left(\left[\left(\frac{\alpha_s(Q)}{\alpha_s(\theta Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\right]_{qq} + \frac{2n_f(\gamma_{qg}(2)-\gamma_{qg}(3)) + \gamma_{gg}(2)-\gamma_{gg}(3)}{\gamma_{qq}(2)-\gamma_{qq}(3)+\gamma_{gq}(2)-\gamma_{gq}(3)}\left[\left(\frac{\alpha_s(Q)}{\alpha_s(\theta Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\right]_{gq}\right) + \mathcal{O}\left(\alpha_s(Q)^n \ln(\theta)^{n-1}\big|_{n\geq 1}\right) + \mathcal{O}(\theta), \quad \text{(A.23)}$$

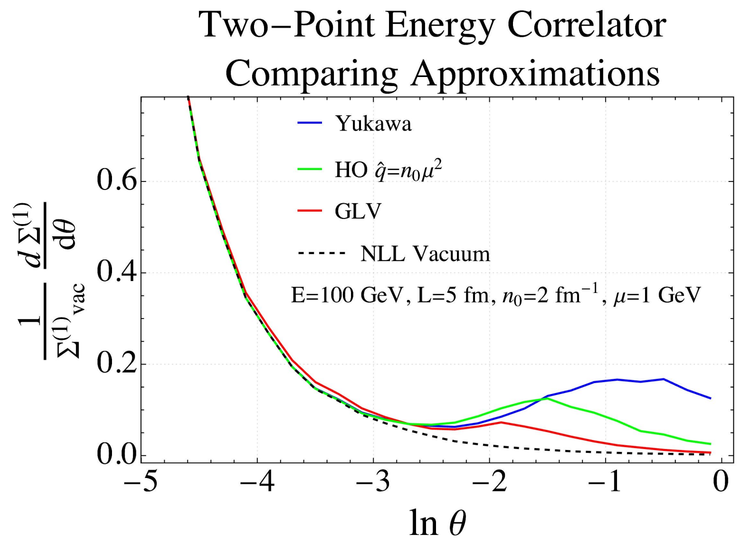
The medium modification to the vacuum result is computed with in the BDMPS-Z framework:

- Large "plus" component, decoupling between transverse and longitudinal dynamics.
- propagators in a background field averaged at cross-section level
- Resulting path integral solved in various approximations: semi-hard Wilson lines, GLV.

Where $\hat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J), & 2n_f\gamma_{qg}(J), & 0 \\ \gamma_{gq}(J), & \gamma_{gg}(J), & 0 \\ 0, & 0, & \gamma_{g\tilde{g}}(J) \end{pmatrix}$ is the spin-J twist-2 QCD anomalous dimension matrix.

The EEC on a *massless* quenched jet





Universal feature of all the models is an onset angle, at angles smaller than the onset the spectrum is vacuum-like.

$$t_f = \frac{2}{z(1-z)E\theta^2} \qquad \theta_L \sim (EL)^{-1/2}$$

Andres, Dominguez, JH, Marquet, Moult 2303.03413

The EEC on a *massive* quenched jet

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \frac{1}{\sigma}\int \mathrm{d}z \left(g^{(n)}(\theta,\alpha_{\mathrm{s}}) + F_{\mathrm{med}}(z,\theta)\right)\frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta\mathrm{d}z} z^n(1-z)^n\left(1+\mathcal{O}\left(\frac{\bar{\mu}_{\mathrm{s}}}{Q}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\theta\, Q}\right)$$

The vacuum-like piece.
When the bracket is expanded this term at LO gives $1/\theta$ divergence already discussed. In general known at NLO+NLL. We use LO combined with an approximated NLL which lies within the bands from scale variation of the full result, achieved by resuming collinear radiation into a modified coupling.

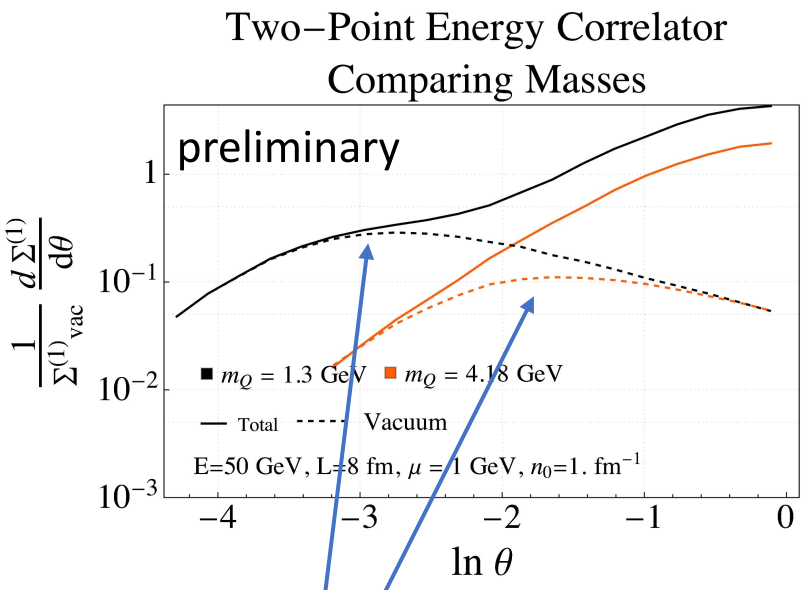
The medium modification to the vacuum result is computed with in the BDMPS-Z framework now with quark masses present in the propagators. The rest of the computation remains the same.
We here focus on the "Tilted" Wilson lines + Yukawa potential approximation.

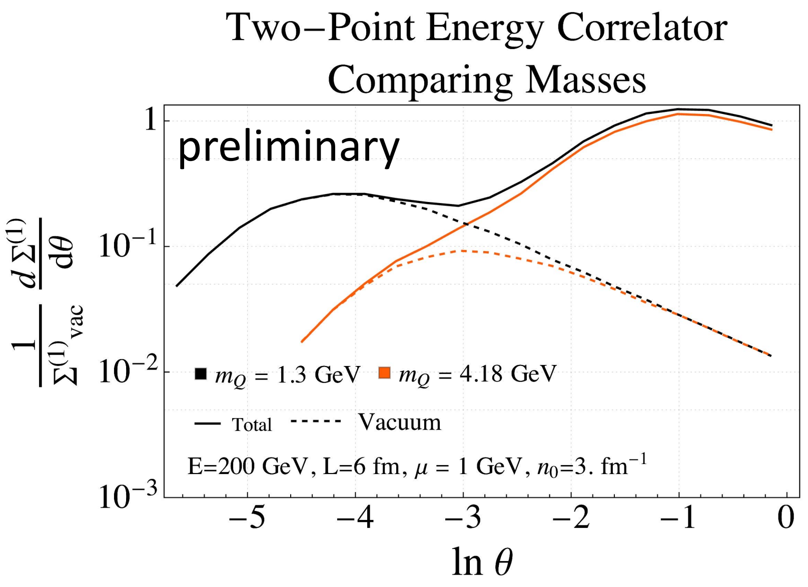
$$\frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta\mathrm{d}z} = \frac{2\mu^{2\epsilon}g_{\mathrm{s}}^2}{(p_j+p_k)^2 - m_{(jk)}^2}\, T_R\left[1 - \frac{2}{d-2}\left(2z(1-z) - \mu_{q\bar{q}}^2\right)\right]$$

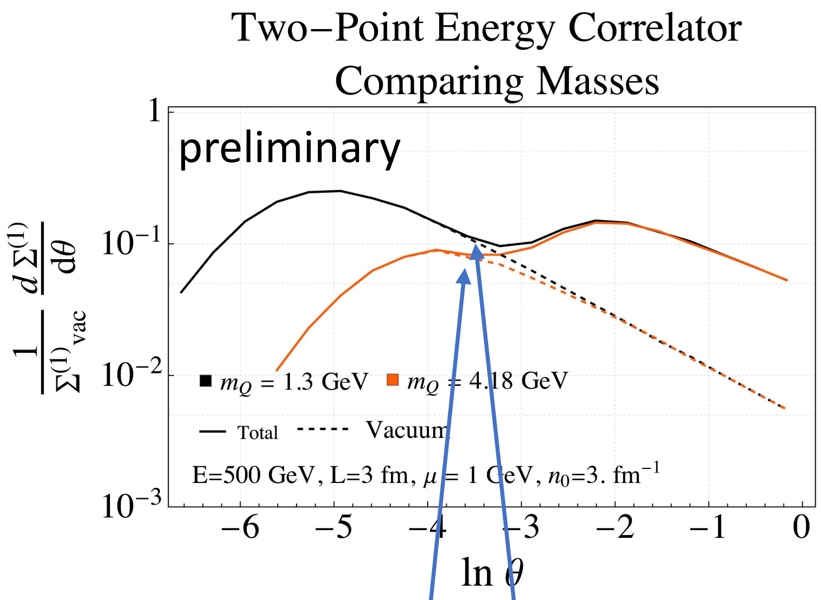
$$g^{(1)} = \left(\left[\left(\frac{\alpha_{\mathrm{s}}(Q)}{\alpha_{\mathrm{s}}(\theta Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\right]_{qq} + \frac{2n_f(\gamma_{qg}(2)-\gamma_{qg}(3)) + \gamma_{gg}(2)-\gamma_{gg}(3)}{\gamma_{qq}(2)-\gamma_{qq}(3)+\gamma_{gq}(2)-\gamma_{gq}(3)}\left[\left(\frac{\alpha_{\mathrm{s}}(Q)}{\alpha_{\mathrm{s}}(\theta Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\right]_{gq}\right) + \mathcal{O}\left(\alpha_{\mathrm{s}}(Q)^n \ln(\theta)^{n-1}\Big|_{n\geq 1}\right) + \mathcal{O}(\theta), \qquad \text{(A.23)}$$

The EEC on a *massive* quenched jet

Increasing energy







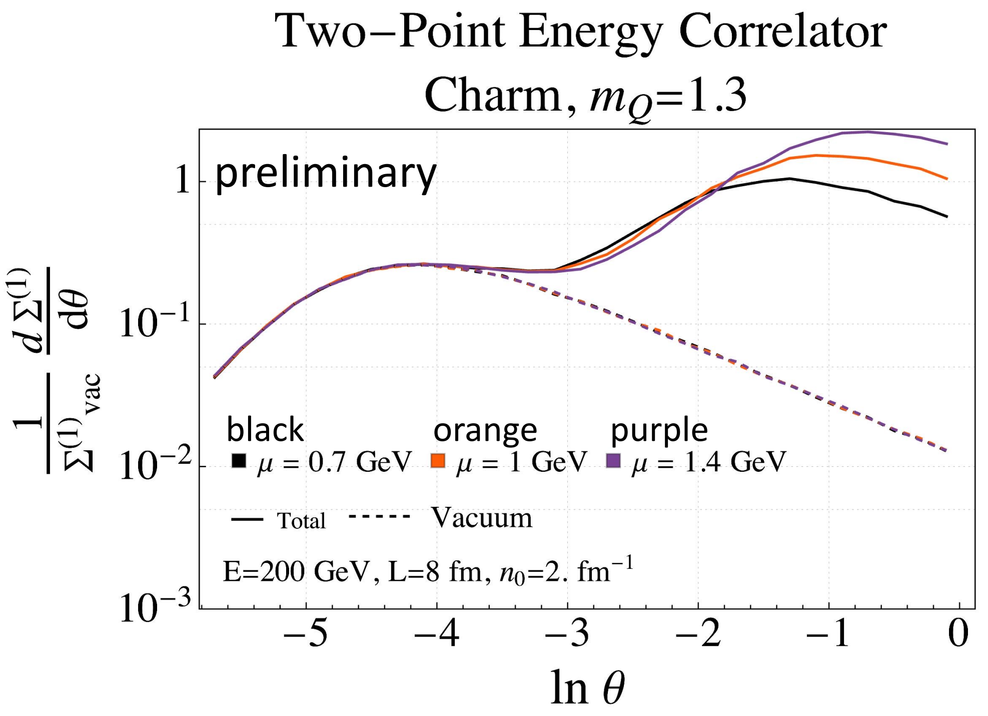
'Deadcone angle'
$\Theta_0 \sim m_Q/E$

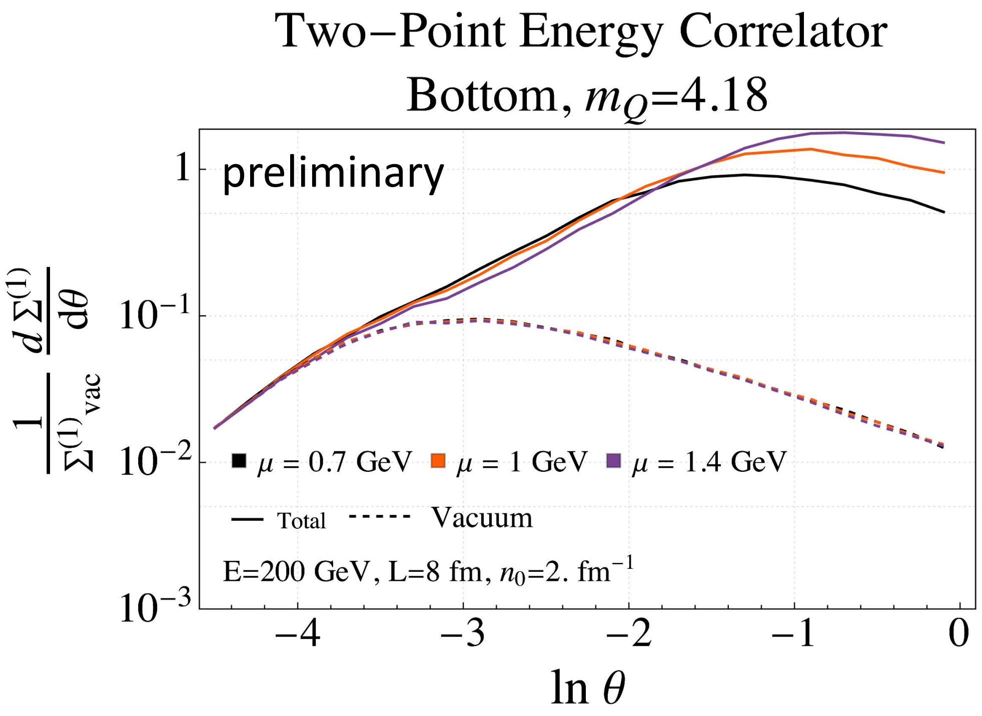
Increasing filling of the deadcone

'Onset angle'
$\theta_L \sim \sqrt{1/(LE)}$

Threshold scales with $\sqrt{E/L} \sim m_Q$.

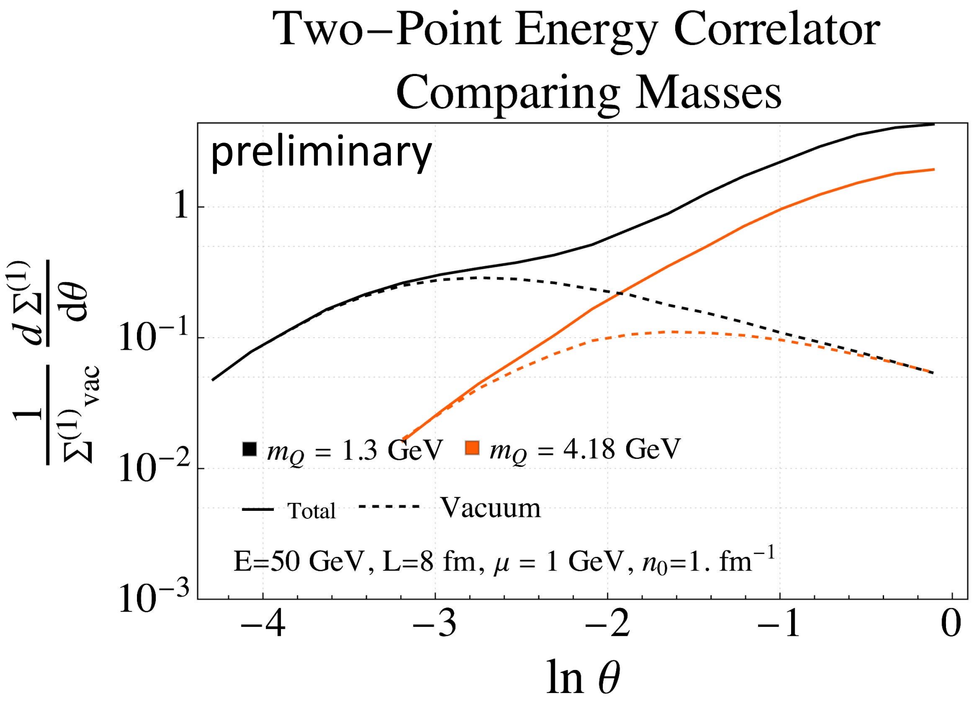
The EEC on a *massive* quenched jet

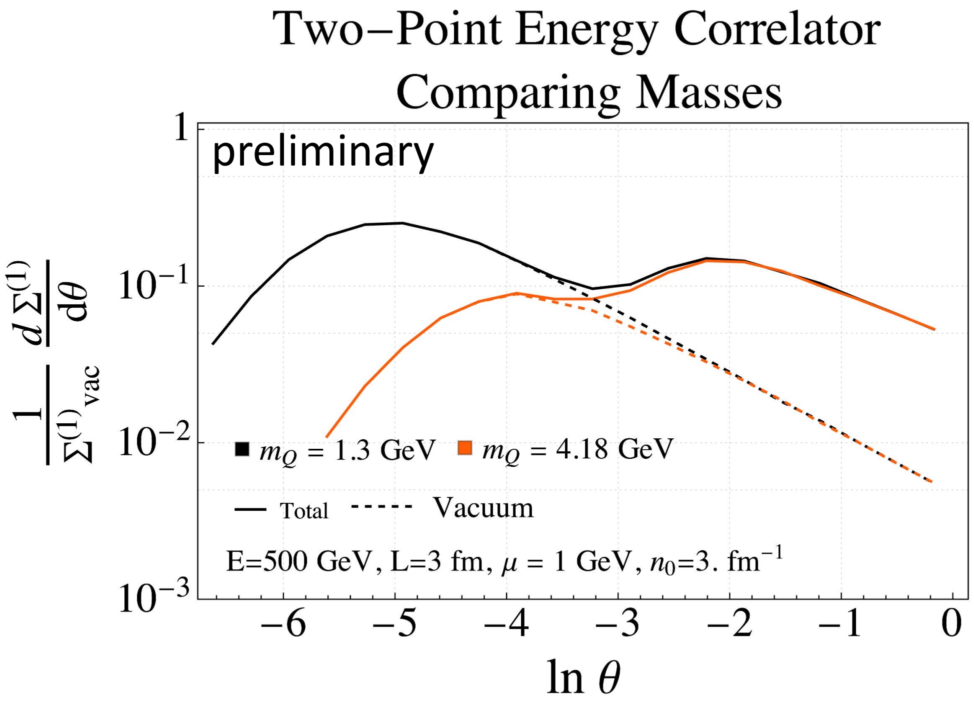




Sensitivity to medium parameters mostly found outside the deadcone

The EEC on a *massive* quenched jet





Wide angle mass sensitivity in the medium is only present when the deadcone is modified by the medium.
Implies a universality of the approach to the massless limit.

Conclusions

We have now initiated studies of *massless* and *massive* jets in HI collisions with Energy Correlators.

The simple and inclusive nature of energy correlators perhaps provides a window into degrees of theoretical control than can be very hard to otherwise achieve in jet substructure within HI physics. In this talk, we could directly read the behaviour of the correlators off of the QCD S-matrix element.

Our model is still simple, a static brick medium, but it does concretely demonstrate an impressive sensitivity to the deadcone can be found with the EEC.

We find that the deadcone is affected by the presence of a thermal medium in a nontrivial way.

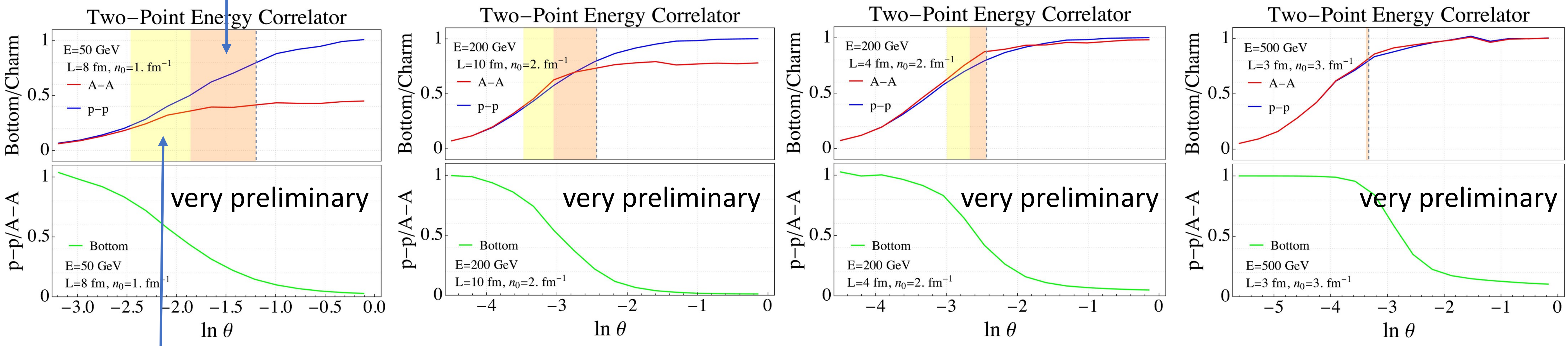
A more complete analysis will appear on arXiv soon…

Supplemental material

The EEC on a *massive* quenched jet

Fully filled cone, effectively like a modified b mass

Increasing E/L



Increasing filling of the deadcone

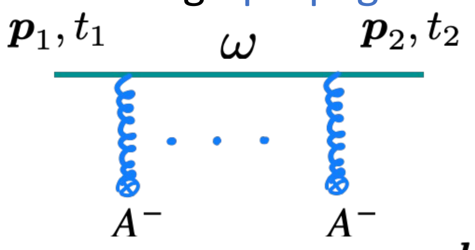
Partially filled cone,
Modified shape relative to vacuum (steeper suppression)

As requested by Mateusz…

Computing *massless* F_{med}

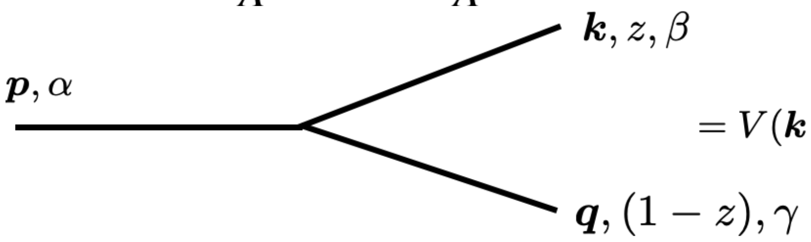
The formalism we use, based in BDMPS-Z:

- All particles have a large longitudinal momentum compared to their transverse momenta and therefore there is a decoupling between transverse and longitudinal dynamics.
- We work in a mixed representation with momentum coordinates in the transverse direction and "time" (+ coordinate) in the longitudinal direction.
- Multiple scatterings resumed through propagators in a background field



$$= \mathcal{G}_R(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1; \omega)$$

- Vacuum vertices



$$= V(\boldsymbol{k} - z\boldsymbol{p}, z) T^{\alpha\beta\gamma} (2\pi)^2 \delta^{(2)}(\boldsymbol{p} - \boldsymbol{k} - \boldsymbol{q})$$

- Background field averaged at the level of the cross section

$$\left\langle A^{a-}(\boldsymbol{q}_1, t_1) A^{b-\dagger}(\boldsymbol{q}_2, t_2) \right\rangle = \delta^{ab} \delta(t_2 - t_1) \delta^{(2)}(\boldsymbol{q}_1 - \boldsymbol{q}_2) v(\boldsymbol{q}_1)$$

Computing *massless* F_{med}

Full evaluation differential in z and θ not yet achieved for arbitrary medium parameters.

A recent impressive effort has set up a general numerical evaluation but only stable for short mediums arXiv:2303.12119

Two available approximation schemes:

- Opacity expansion ($N = 1$) arXiv:1807.03799
  - Unitarity problems can lead to negative cross sections.
  - Recursive formulas to generate all orders (not yet implemented numerically).

- "Tilted" Wilson lines
  - Resums multiple scatterings in the eikonal approximation.
  - Assumes semi-hard splittings (z not too small).
  - We implement this using both a Yukawa and HO potential for medium scatterings and for now using the leading colour limit.

arXiv:1907.03653

arXiv:2107.02542