Computing Jet Transport Coefficients On The Lattice

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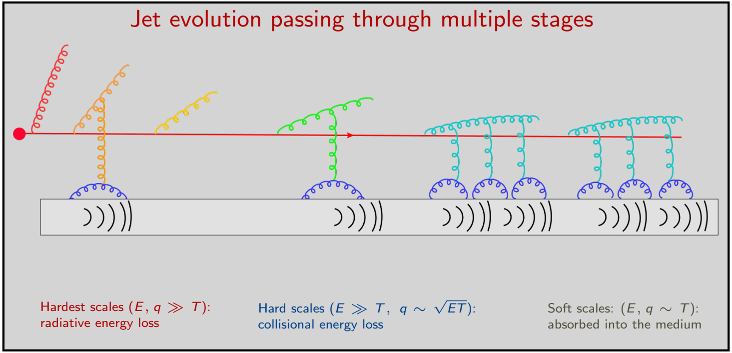
11th International Conference on Hard and Electromagnetic Probes of High-Energy Nuclear Collisions, **HARD PROBES 2023**

Aschaffenburg, Germany, March 28, 2023

Jets are complicated

- Originate in earliest stages of HIC
- Distinct stages of evolution as virtuality or respective hard scale is lowered



- Real-time dynamics encoded in **collision kernel** $\boxed{\frac{\mathrm{d}^4 W_{Q,F,T}(k)}{\mathrm{d}^4 k}}$, depends on
  - the hard scale $Q \sim E, \boldsymbol{q}, M$
  - the flavor $F \sim R, M$
  - the medium $T \sim T, m_D, N_f, \{\mu\}$

From Jet Transport Models to Model Independence

- Jet modification accumulated across (partially) independent processes and different stages ⇒ Gaussian process ⇒ moments of the **collision kernel**

Model-independent measure of momentum broadening

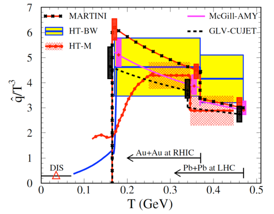
transverse

$$\hat{q} \equiv \frac{\langle \boldsymbol{k}_\perp^2 \rangle_L}{L} \propto C_R \alpha_s^2 T^3 \log\left(\frac{\text{hard}}{\text{soft}}\right) + \dots$$

longitudinal

$$\hat{e}_2 \equiv \frac{\langle \boldsymbol{k}_3^2 \rangle_L}{L} \propto C_R \alpha_s^2 T^3 \log\left(\frac{\text{hard}}{\text{soft}}\right) + \dots$$

Modeling transverse momentum broadening



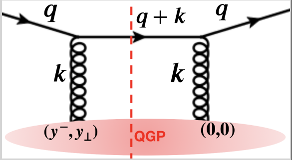
1. Many models are consistent, but **JUMPS** between RHIC and LHC
2. **Log-like rise** of $\hat{q}/T^3$ towards T_{pc} from above, sudden drop to zero.
3. **Strongly-coupled**, inviscid droplet at RHIC & LHC modeled as weakly-coupled HTL medium?

- JET collaboration[1] comparing and averaging many different models
- Model-independent study of strongly-coupled medium ⇒ **LATTICE**

[1] Burke:2013yra

A Hard Quark Scattering on QGP

- **Tree-level** scattering amplitude $\mathcal{M}$
- **Collision kernel** $\frac{\mathrm{d}^4 W(k)}{\mathrm{d}^4 k} = \frac{\mathrm{d}^4}{\mathrm{d}^4 k} \frac{|\mathcal{M}|^2}{2N_C}$
- **Transport coefficient**



$$\hat{q}_j = \sum_n \frac{e^{-\beta E_n}}{ZT_l} \int \mathrm{d}^4 k\, k_j^2 \frac{\mathrm{d}^4 W(k)}{\mathrm{d}^4 k}$$

$$\simeq c_0 \alpha_s \int \frac{\mathrm{d}y^- \mathrm{d}^2 y_\perp \mathrm{d}^2 k_\perp}{(2\pi)^3} \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp \cdot \boldsymbol{y}_\perp - \mathrm{i}\frac{\boldsymbol{k}_\perp^2}{q^-} y^-} \left\langle \mathrm{Tr}\left[F^{+j}(0) F_j^{+}(y^-, y_\perp)\right] \right\rangle_T$$

following standard steps ($A^- = 0$ gauge, promote $\partial_j \to D_j$, leading virtuality, ...) [2]

- $c_0 = \frac{16\pi\sqrt{2}}{N_C^2 - 1} C_R$, $C_R = C_F = 4/3$
- $\alpha_s(\mu^2)$ at soft scale $(\# T)^2$, m_D^2
- *near light-cone separated* gauge field-strength tensors $F^{+j}(\cdot) \Rightarrow$ gauge covariant

- Transverse $\boxed{\hat{q} = \hat{q}_1 + \hat{q}_2}$ and longitudinal $\boxed{\hat{e}_2 = \hat{q}_3}$ broadening.

[2] Majumder:2012sh

Generalized Transport Coefficient

- **LATTICE** cannot handle a scale $q^- \gg 1/a$ (cf. HQET [Caswell:1985ui]) or *near light-cone separation* as in PDFs (cf. LAMET [Ji:2013dva])
- To **compute** $\hat{q}_j$ **non-perturbatively** use one of the following tools

- Make $\hat{q}_j$ gauge invariant via appropriate Wilson lines[3] or
- Define a generalized coeff.[2] $\widehat{Q}_j(q^+)$ with thermal discontinuity $\hat{q}_j$,

$$\widehat{Q}_j(q^+) \simeq c_0 \alpha_s \int \frac{\mathrm{d}^4 y \mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{e}^{ik\cdot y} 2q^-}{(q+k)^2 + i\varepsilon} \left\langle \mathrm{Tr}\left[F^{+j}(0) F^+_j(y)\right] \right\rangle_T$$

- Expand den. for space-like ($q^+ \simeq -q^-$), promote $\partial_3 \to D_3$, scale $\nu \lesssim 1/a \ll q^-$

$$\widehat{Q}_j(q^+ \simeq -q^-) \simeq \frac{c_0 \alpha_s}{q^-} \sum_{n=0}^{\infty} \left[\frac{\nu}{q^-}\right]^n \left\langle \mathrm{Tr}\left[F^{+j} \Delta^n F^+_j\right] \right\rangle_T , \quad \Delta \equiv \frac{i\sqrt{2} D_3}{\nu}$$

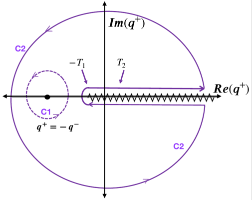
- Odd powers of Δ $\Leftrightarrow$ parity violation $\Rightarrow$ only even powers
- For medium at rest no spatial-temporal mixed terms á la $F^{3j} F^0_j$

[2] Majumder:2012sh
[3] GarciaEchevarria:2011md

Thermal vs Vacuum Discontinuities

- Contour integration along $C_1 \Rightarrow$

$$\mathcal{I} = \oint_{C_1} \frac{\mathrm{d}q^+}{2\pi \mathrm{i}} \frac{\widehat{Q}_j(q^+)}{(q^- + q^+)} \simeq \widehat{Q}_j(q^+ \simeq -q^-)$$



- Integrate after deformation to $C_2 \Rightarrow$

$$\mathcal{I} = \oint_{C_2} \frac{\mathrm{d}q^+}{2\pi \mathrm{i}} \frac{\widehat{Q}_j(q^+)}{(q^- + q^+)} \simeq \int_{-T_1}^{T_2} \frac{\mathrm{d}q^+}{2\pi \mathrm{i}} \frac{\mathrm{Disc}\left[\widehat{Q}_j(q^+)\right]}{(q^- + q^+)} + \int_0^{\infty} \frac{\mathrm{d}q^+}{2\pi \mathrm{i}} \frac{\mathrm{Disc}\left[\widehat{Q}_j(q^+)\right]}{(q^- + q^+)}$$

- thermal $\mathrm{Disc}\left[\widehat{Q}_j(q^+)\right]\big|_{q^+ \sim T} \simeq \hat{q}_j$, width $T_\delta \equiv T_1 + T_2 \simeq 2\sqrt{2}T$
- time-like hard quark undergoing vacuum-like (T-independent) splitting

- Vacuum subtraction isolates thermal momentum broadening

$$\frac{\hat{q}_j}{T^3} \simeq c_0 \alpha_s \frac{T}{T_\delta} \sum_{n=0}^{\infty} \left[\frac{\nu}{q^-}\right]^{2n} \frac{1}{T^4} \left\langle \mathrm{Tr}\left[F^{+j} \Delta^{2n} F^{+}_{j}\right]\right\rangle_{(T-V)},$$

- Can be evaluated on the **LATTICE** and the continuum limit be taken

Connection to the Equation of State

- For an infinitely hard quark $q^- \to \infty$ the sum terminates at $n = 0$
- The transverse operator sum reduces to the triplet comp. of the EMT (i.e. entropy density of pure gauge plasma at rest in temperature units)

$$\frac{s}{T^3} = \sum_{j=1}^{2} \frac{1}{T^4} \left\langle \mathrm{Tr}\left[F^{+j} F^{+}_{j}\right] \right\rangle_{(T-V)}$$

- Transport coefficient related as $\boxed{\hat{q} \simeq \frac{2\pi\alpha_s(\mu^2)}{N_C}\; s}$ for a pure gauge plasma[4]
- Since s is a physical observable, $\hat{q}$ inherits scheme dependence of $\alpha_s(\mu^2)$!
- Estimate **approximation error** for a hard quark $E \gg T \sim m_D$ in LO HTL[5]

$$\frac{\hat{q}}{T^3} \simeq \frac{N_C^2 - 1}{N_C} \alpha_s(m_D^2)\left[\zeta(3)\frac{48}{11}\right] \simeq \left\{\frac{21}{22}\right\} \frac{2\pi\alpha_s(m_D^2)}{N_C} \frac{s_{\mathrm{SB}}}{T^3} \;\Rightarrow\; \frac{\delta\hat{q}}{\hat{q}} \gtrsim \frac{1}{22} \approx 4.5\%$$

[4] Kumar:2020wvb
[5] Arnold:2008vd

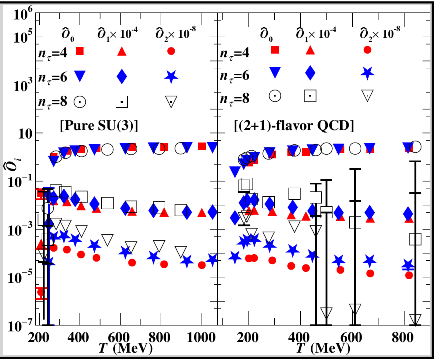
Lattice calculation

- Factorize off hard scale $q^- \gg 1/a$, evaluate *THE REST* on **LATTICE**
- Rephrase intermediate scale $\nu = T = 1/aN_\tau$ after vacuum subtraction
- Wick rotation: $x^0 \to -\mathrm{i}x_4,\ A^0 \to +\mathrm{i}A_4,\ F^{0j} \to +\mathrm{i}F_{4j}$

$$\frac{\hat{q}_j}{T^3} \simeq c_0 \frac{T}{T_\delta} \sum_{n=0}^{\infty} \left[\frac{T}{q^-}\right]^{2n} \left[\alpha_s \widehat{O}_n\right]^{(R)},\ \widehat{O}_n = \frac{1}{T^4} \left\langle \mathrm{Tr}\left[F_{3j}\Delta^{2n}F_{3j} - F_{4j}\Delta^{2n}F_{4j}\right]\right\rangle_{(T-V)}$$

- $\widehat{O}_n$ on **LATTICE**, extrapolate scale-independent form to continuum limit

- Lattice setup: aspect ratio 4 for $T > 0$ @ $N_\tau = 4, 6, 8, 10$
- Wilson action in quenched, LW/HISQ in $(2+1)$ QCD[6]
- Clover lattice operator $\widehat{\mathcal{F}}_{\mu\nu}$, projected to $\mathfrak{su}(2)$ algebra
- Bare results for $10^{-2n}\widehat{O}_{0,1,2}$, weights mimic suppression for a hard, 100 GeV quark

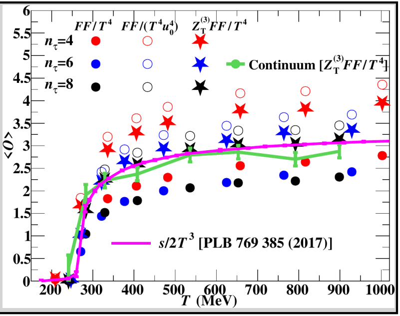


[6] HotQCD:2014kol

$\hat{q}$ in pure gauge theory

- $\widehat{O}_{n>0}$: additive mixing with T-dependent lower-dim. operators, coefficients not known yet $\Rightarrow$ postpone by restricting to $q^- \to \infty$ case
- $\widehat{O}_0 = T_G^{(3)}$ is proportional to triplet comp. of gluon contrib. to EMT

- Transverse sum: $\hat{q} = \hat{q}_1 + \hat{q}_2$
- Renormalize[7] $\mathcal{Z}_T^{(3)}\widehat{O}_0^{(B)} = \widehat{O}_0^{(R)}$
- Tadpole improvement $(1/u_0^4)$ overestimates $\widehat{O}_0^{(R)}$ by $\approx 10\%$
- Take continuum limit of $\widehat{O}_0^{(R)}$,
- cf. QCD in a **moving frame**[7]
- Multiply continuum $\widehat{O}_0^{(R)}$ by $\alpha_s^{(R)}(\mu^2)$ in $\overline{\text{MS}}$ scheme



- Full QCD: $T_G^{(3),(B)}$ does not renormalize multiplicatively!

$$\begin{pmatrix} T_G^{(3),(R)} \\ T_Q^{(3),(R)} \end{pmatrix} = \begin{pmatrix} \mathcal{Z}_{GG}^{(3)} & \mathcal{Z}_{GQ}^{(3)} \\ \mathcal{Z}_{QG}^{(3)} & \mathcal{Z}_{QQ}^{(3)} \end{pmatrix} \begin{pmatrix} T_G^{(3),(B)} \\ T_Q^{(3),(B)} \end{pmatrix}$$

- Mixing matrix for LW/HISQ action and quark contrib. $T_Q^{(3),(B)}$ not known

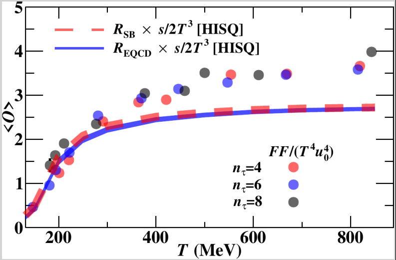
[7] Giusti:2015daa,Giusti:2016iqr

Estimating the effects of Unquenching

- Renormalization/mixing not under control: no continuum limit!
- Estimate gluon contrib. to entropy density in SB limit

$$R_{\rm SB} = \frac{s_{\rm SB}(N_f=0)}{s_{\rm SB}(N_f=3)} = \frac{32}{95} \Rightarrow T_G^{(3),(R)}(N_f=3) \simeq R_{\rm SB}\, T_{QCD}^{(3),(R)}(N_f=3)$$

- Tadpole improved $O_0^{(B)}/u_0^4 T^4$, within 30% of $R_{\rm SB}\, T_{QCD}^{(3),(R)}$
- Similar result as $\mathcal{O}(g^6)$ EQCD[8]
- Cutoff effects in pure gauge $O_0^{(B)}/u_0^4 T^4 \lesssim 10\%$ @ $N_\tau \geq 6$
- NLO renormalization factors of Wilson/Wilson action $\lesssim N_C \times 10\%$



- Full QCD: Tadpole improved $O_0^{(B)}/u_0^4 T^4$ @ $N_\tau = 6$, assigned 30% uncertainty band, sufficient to **estimate the effects of Unquenching**

[8]Laine:2006cp

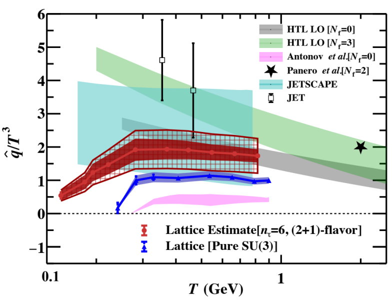
Transverse momentum broadening

- Consider only $q^- \to \infty$ on the lattice **(continuum limit in pure gauge)**
- For QCD: $N_\tau = 6$, tadpole improved, 30% uncertainty band (hashed)
- Multiply with $\overline{\text{MS}}$ $\alpha_s(\mu^2)$ @ NLO for $N_f = 0$ or 3, scale $\mu = (2\ldots4)\pi T$

- HTL@LO[9] for $q^- = 100$ GeV for $N_f = 0$ or 3 $\left(m_D^2 = \frac{C_A + T_F N_f}{3} g^2 T^2\right)$

$$\frac{\hat{q}}{T^3} = \frac{C_F \zeta(3)}{4\pi^3}\left[2C_A + 3T_F N_f\right] g^4 \ln\left[\frac{2ET}{m_D^2}\right]$$

- HTL@NNLO soft contr., $T \approx 2$ GeV for $N_f = 2$ [Panero:2013pla]
- Stochastic vacuum model for $N_f = 0$ [Antonov:2007sh]
- **Jetscape** (Pheno) [Soltz:2019aea]
- **JET** collaboration[1] (Models)



[9] He:2015pra
[1] Burke:2013yra

Longitudinal momentum broadening

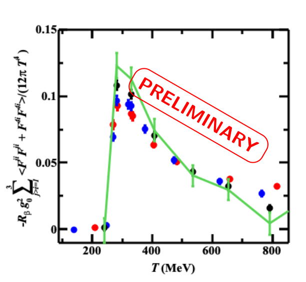
- So far little attention to $\hat{e}_2$, as power-suppressed for light flavors vs $\hat{q}$

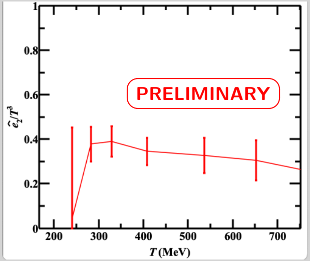
- Similar issues with $\widehat{O}_{n>0}$: postpone, consider $q^- \to \infty$
- Similar issues with Unquenching: postpone, consider pure gauge
- First term in $\hat{e}_2 = \hat{q}_3$ vanishes!

$$\frac{\hat{e}_2}{T^3} \simeq \frac{c_0}{2\sqrt{2}}\frac{1}{2}\left[\alpha_s^{(R)} \frac{\langle \mathrm{Tr}[F_{j3}^2 - F_{j4}^2]\rangle_{(T-V)}^{(R)}}{T^4} - \underbrace{\left(\frac{\alpha_s \langle \mathrm{Tr}[F_{j3}^2 + F_{j4}^2]\rangle_{(T-V)}}{T^4}\right)^{(R)}}_{=-\frac{2\pi}{3b_0}T_G^{\mu\mu} = -\frac{2\pi}{3b_0}T_G^{(1)}}\right]$$

- $\hat{e}_2$ related to the gluonic **trace anomaly**

$$\frac{\hat{e}_2}{T^3} = \frac{1}{4}\frac{\hat{q}}{T^3} + \frac{2\pi^2}{3N_C b_0}\frac{T_G^{(1)}}{T^4}$$





Summary

- Jet transport coefficients $\hat{q}$ and $\hat{e}_2$ computed on the **LATTICE**
- **Tree-level** hard quark scattering on **non-perturbative medium**
- **OPE** in T/q^- yields a series of gauge-invariant **local operators**
- **Leading-twist ops** $(q^- \to \infty)$ tied to **SU(3) Equation of State:**

$$\frac{\hat{q}}{T^3} \simeq \frac{2\pi\alpha_s(\mu^2)}{N_C} \frac{T_G^{(3)}}{T^4} \quad \text{and} \quad \frac{\hat{e}_2}{T^3} = \frac{1}{4}\frac{\hat{q}}{T^3} + \frac{2\pi^2}{3N_C b_0}\frac{T_G^{(1)}}{T^4}$$

weak T dependence, smooth decrease to zero in scaling region

- Higher-twist operators mixing w. lower-d ops: need more work!
- Unquenching to Full QCD w. further mixing: needs more work!
- Evaluate **NLO scattering** through OPE on the **LATTICE**

- **Experimental sensitivity** to $\hat{e}_2$ larger for **heavy-quark jets**
- Extension of framework to **heavy-quark jets** straightforward. . . ?

Thank you for your attention!