Precise description of medium-induced emissions

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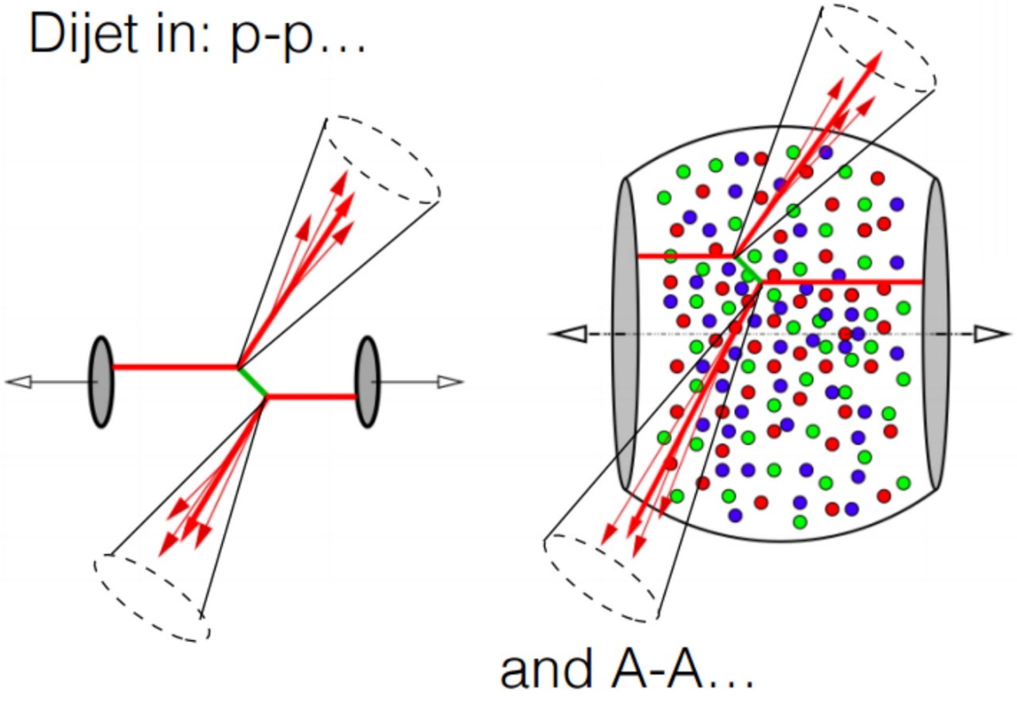
In collaboration with Konrad Tywoniuk
Based on 2303.12119

Jets in heavy-ion collisions

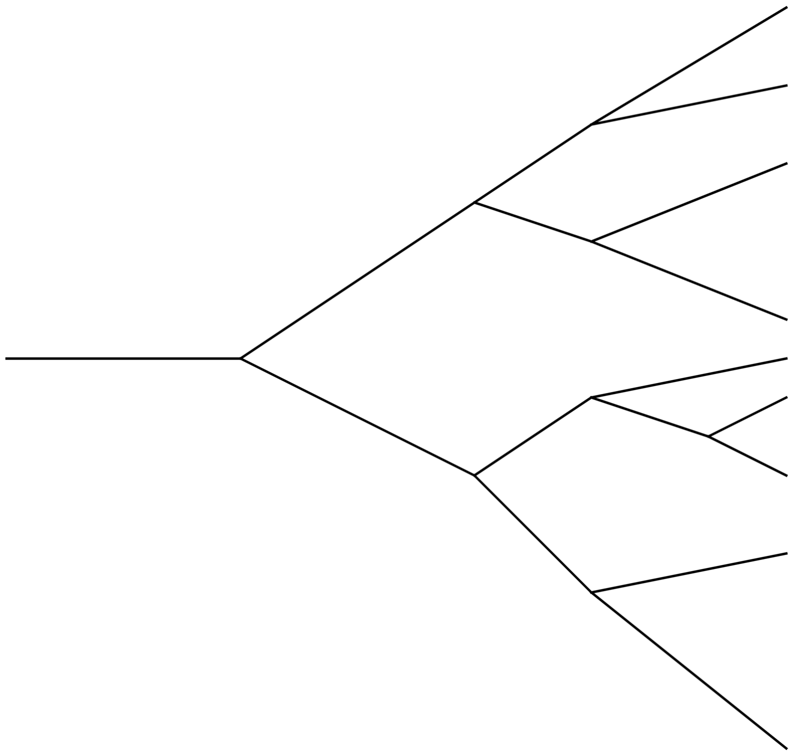
- Colliding two heavy nuclei creates quark-gluon plasma
- Jet must go through the medium (QGP) to reach the detector
- Medium interacts with jet and modifies it
- This is called jet quenching



[C. Andres (2022)]

Jets in heavy-ion collisions

- Partons going through the medium scatter with medium constituents
- Scatterings induce emissions
  - More emissions compared to vacuum jets



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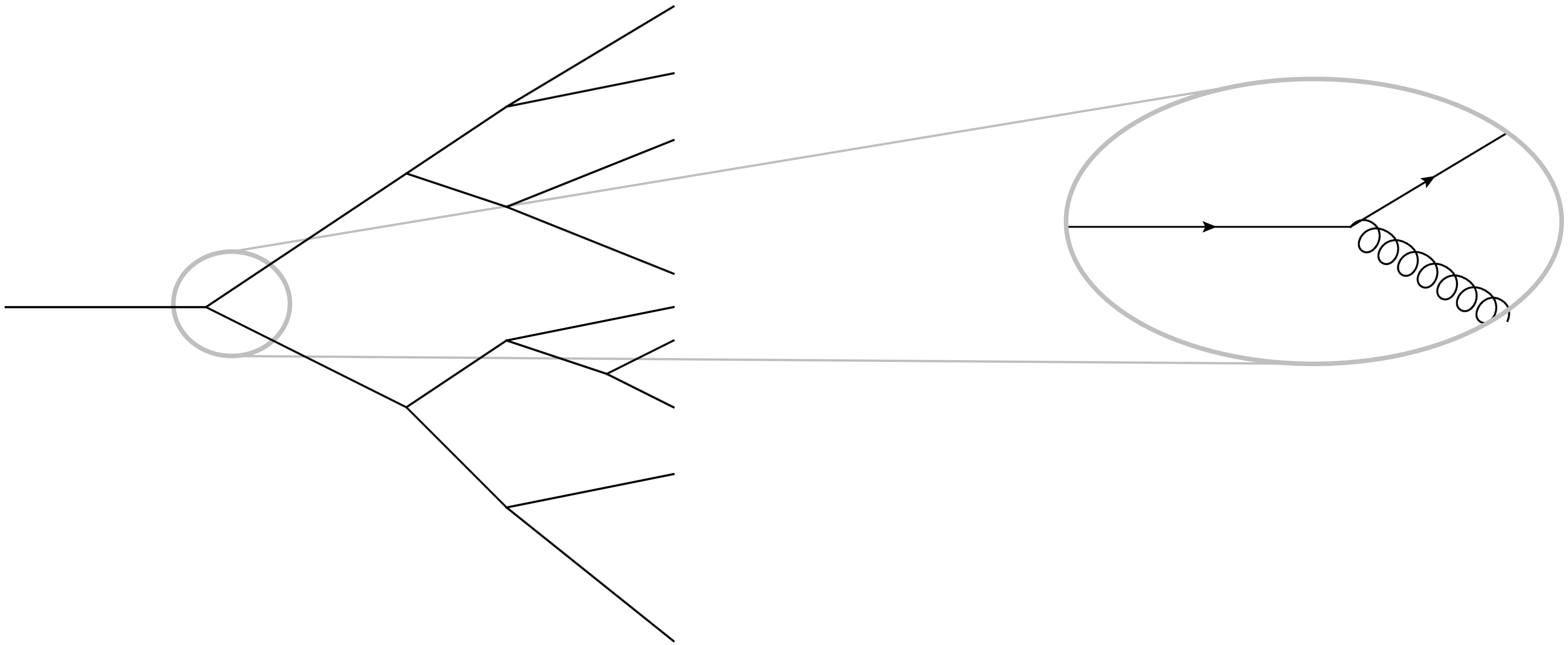
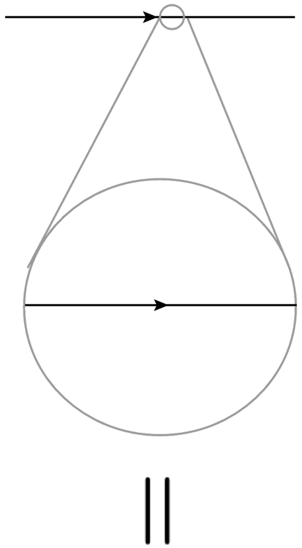


- To calculate many emissions we need to be able to calculate just one emission!

Calculating jet quenching

Vacuum propagator



$$\frac{i\delta^{ij}}{\not{p} - m + i\varepsilon}$$

Calculating jet quenching

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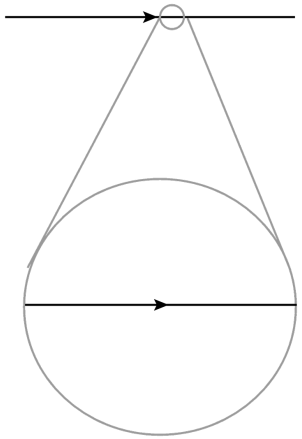

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NICE
EASY

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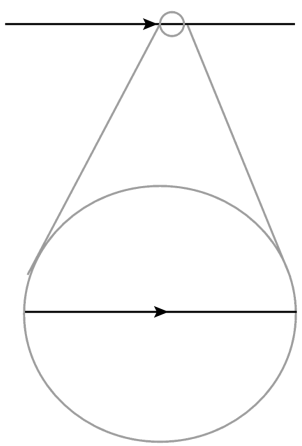

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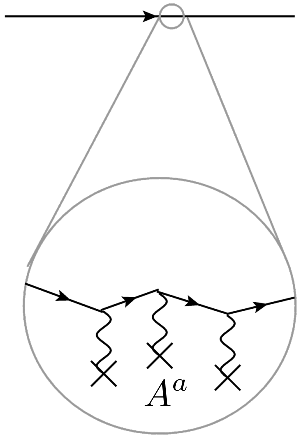
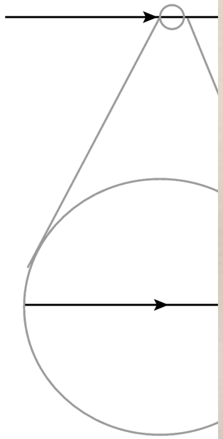
Medium propagator



$$\| \qquad \mathcal{G}(\boldsymbol{x}, t; \boldsymbol{x}_0, t_0) = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}} \mathcal{D}\boldsymbol{r} \exp\left[i\frac{E}{2} \int_{t_0}^{t} \mathrm{d}s \dot{\boldsymbol{r}}^2(s) \right] V_R\left(t, t_0; \boldsymbol{r}(t)\right)$$

Calcula

Vacuum pro



$$||$$

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Calculating jet quenching

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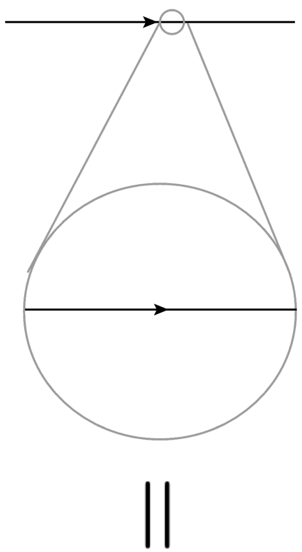

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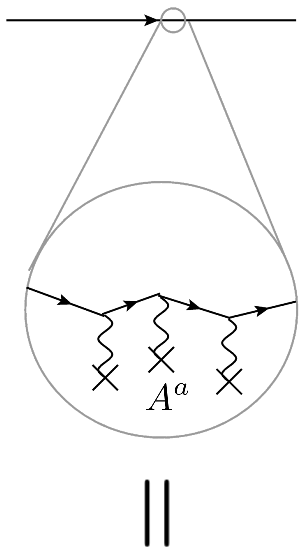

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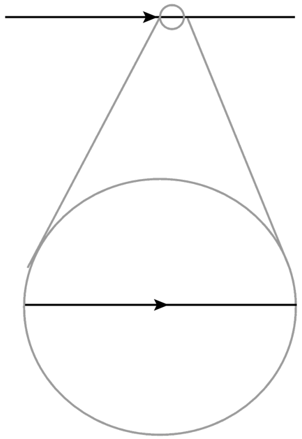

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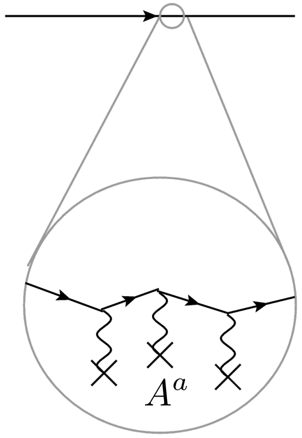

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Varying path leads to
path integral

Color rotation leads to:
Wilson line

$$V_R\left(t,t_0;\mathbf{r}(t)\right) = \mathcal{P}\exp\left[ig\int_{t_0}^{t} \mathrm{d}s\,A^a(s,\mathbf{r}(s))T_R^a\right]$$

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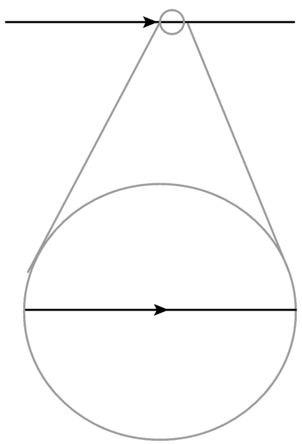

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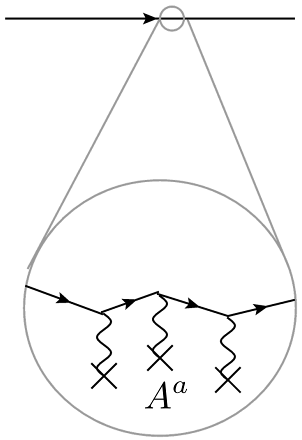

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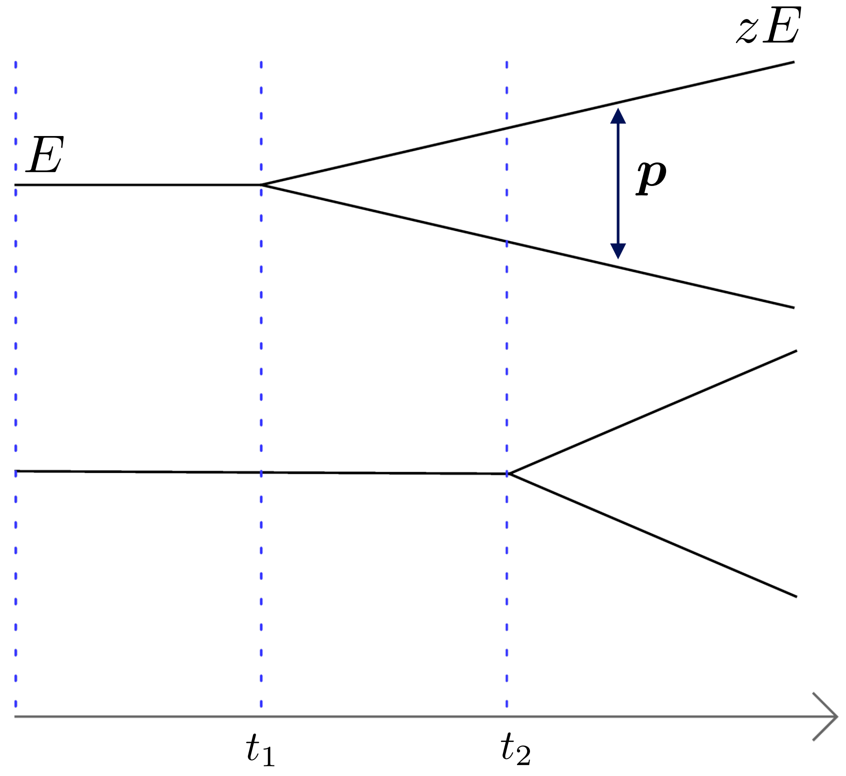
NOT NICE
NOT EASY

Medium-induced emissions

The spectrum of induced emissions:

$$(2\pi)^2 \frac{\mathrm{d}I}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}} = \frac{\alpha_s}{\omega^2} P_{a\to bc}(z)\mathrm{Re}\int_0^\infty \mathrm{d}t_1 \int_{t_1}^\infty \mathrm{d}t_2 \int_{\boldsymbol{l}_1\boldsymbol{l}_2\bar{\boldsymbol{l}}_2} \boldsymbol{l}_1 \cdot \bar{\boldsymbol{l}}_2\, \mathcal{Q}(\boldsymbol{p},\boldsymbol{l}_2,\bar{\boldsymbol{l}}_2|t_\infty,t_2)\mathcal{K}(\boldsymbol{l}_2,\boldsymbol{l}_1|t_2,t_1)$$

- This is not nice. Let's break it down



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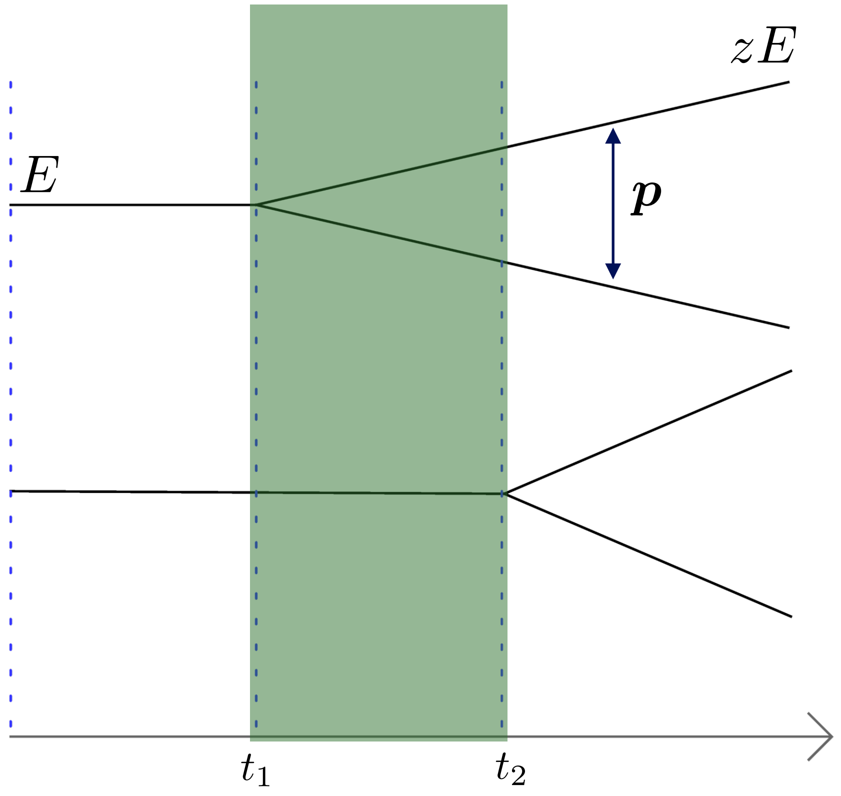
- Splitting kernel: $\mathcal{K}(\boldsymbol{l}_2,\boldsymbol{l}_1|t_2,t_1) \sim \langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger \rangle$
  - The splitting process itself

✔ Can solve analytically ☺



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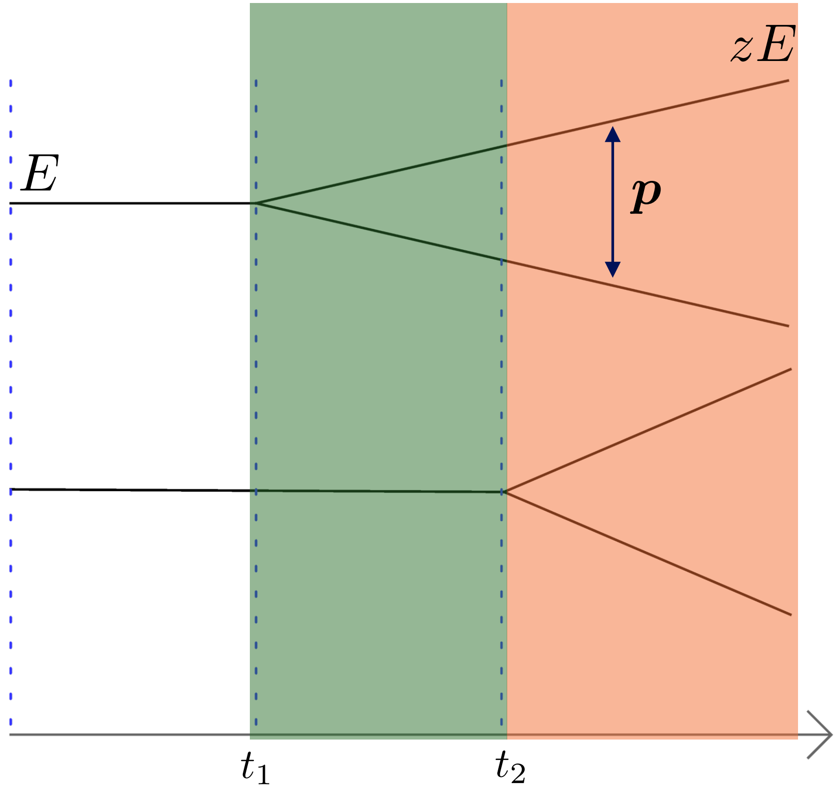
  - The evolution of the two partons after the splitting
  - Coherence → decoherence



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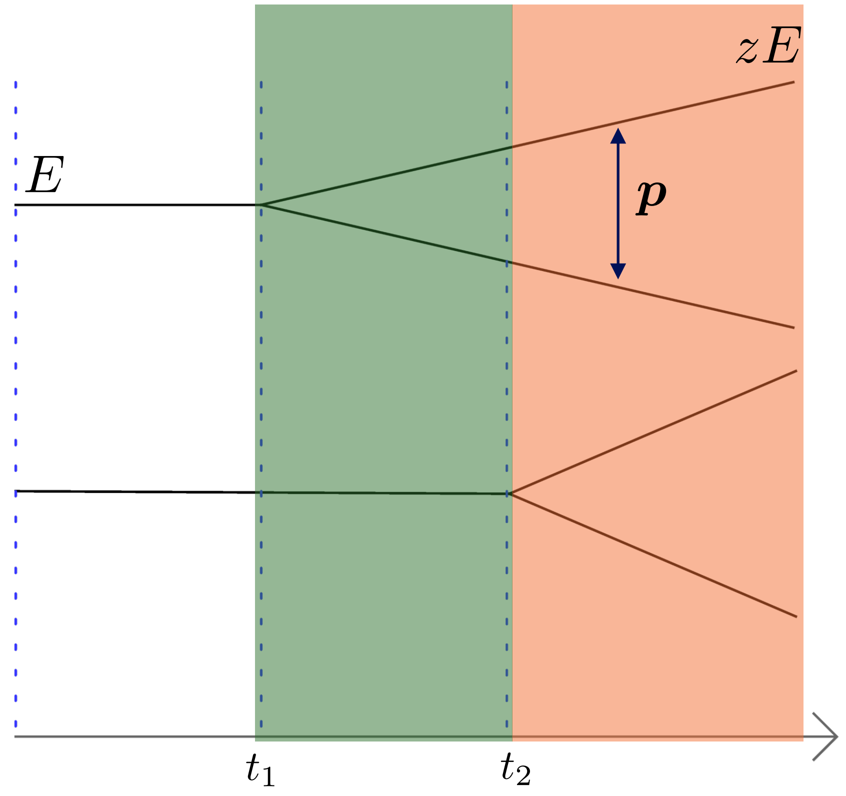

Plan

1. Calculate full emission spectrum numerically
2. Calculate emission spectrum using large-Nc approximation
3. Calculate emission spectrum using eikonal approximation

The quadrupole

- The quadrupole is a path integral of four Wilson lines

$$\mathcal{Q}(\boldsymbol{u}_{\mathrm{f}}, \bar{\boldsymbol{u}}_{\mathrm{f}}, \boldsymbol{u}_2, \bar{\boldsymbol{u}}_2 | t_f, t_2) = \int_{\boldsymbol{u}_2}^{\boldsymbol{u}_{\mathrm{f}}} \mathcal{D}\boldsymbol{u} \int_{\bar{\boldsymbol{u}}_2}^{\bar{\boldsymbol{u}}_{\mathrm{f}}} \mathcal{D}\bar{\boldsymbol{u}}\, \mathrm{e}^{i\frac{\omega}{2}\int_{t_2}^{t_f} \mathrm{d}s\, (\dot{\boldsymbol{u}}^2 - \dot{\bar{\boldsymbol{u}}}^2)} \langle V V^\dagger V V^\dagger \rangle$$

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- Can also calculate through a system of Schrödinger equations

$$\left[i\frac{\partial}{\partial t} + \frac{\partial_{\boldsymbol{u}}^2 - \partial_{\bar{\boldsymbol{u}}}^2}{2\omega} \right] \mathcal{Q}_i(\boldsymbol{u}, \bar{\boldsymbol{u}}, \boldsymbol{u}_2, \bar{\boldsymbol{u}}_2 | t, t_2) = i\mathbb{M}_{ij}(\boldsymbol{u}, \bar{\boldsymbol{u}}) \mathcal{Q}_j(\boldsymbol{u}, \bar{\boldsymbol{u}}, \boldsymbol{u}_2, \bar{\boldsymbol{u}}_2 | t, t_2)$$

- Sum over all the different color states the partons can be in
  - $\gamma \to q\bar{q}$: 2 states, $g \to gg$: 24 states

$\mathbb{M}_{ij}$ = Potential matrix that connects the different color states

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- We solved the Schrodinger equation numerically for $\gamma \to q\bar{q}$
- Want to compare to approximate analytical solutions

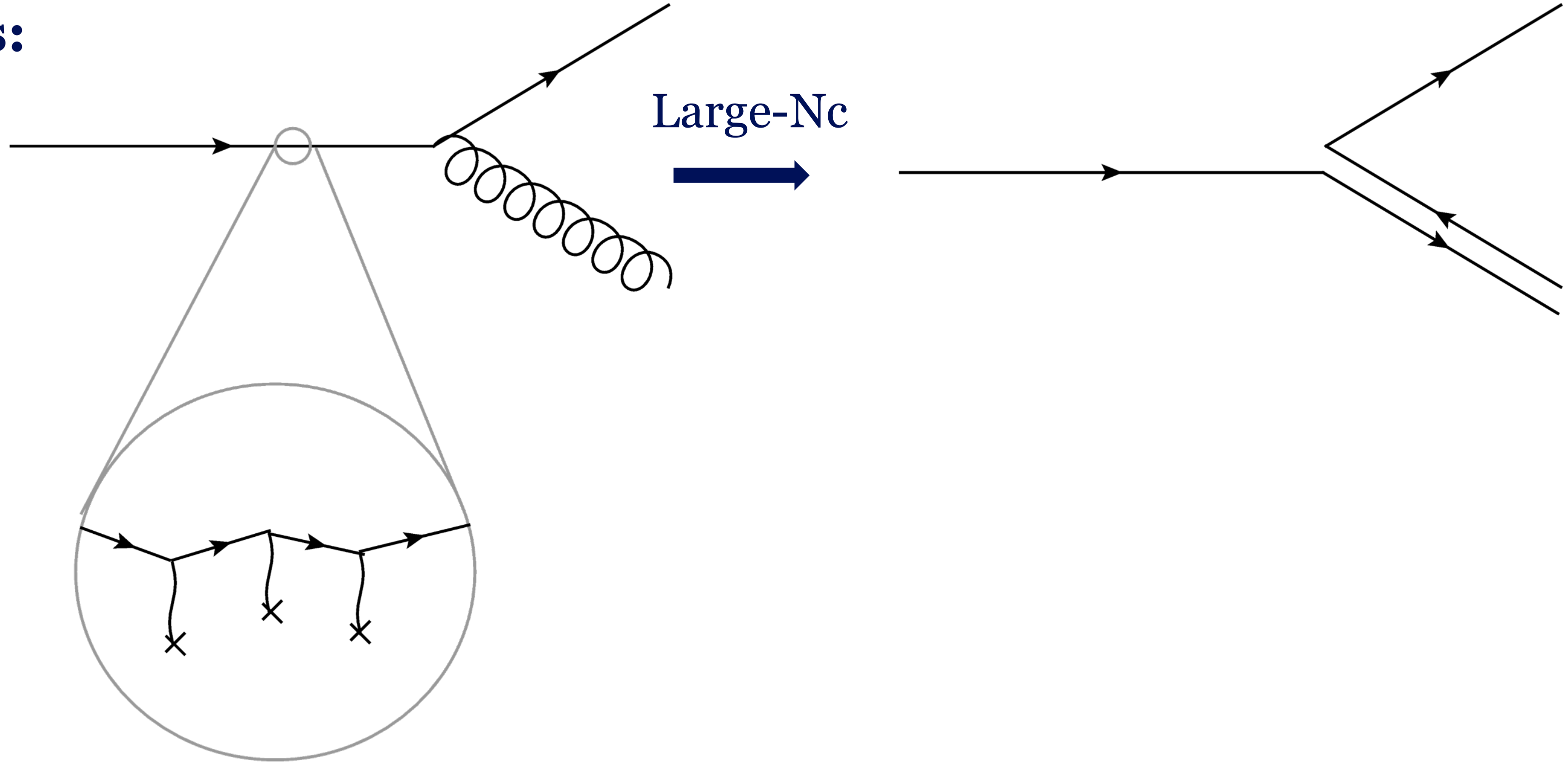
Plan

1. Calculate full emission spectrum numerically ✓
2. Calculate emission spectrum using large-N_c approximation
3. Calculate emission spectrum using eikonal approximation

Analytical solutions

Two simplifying approximations:

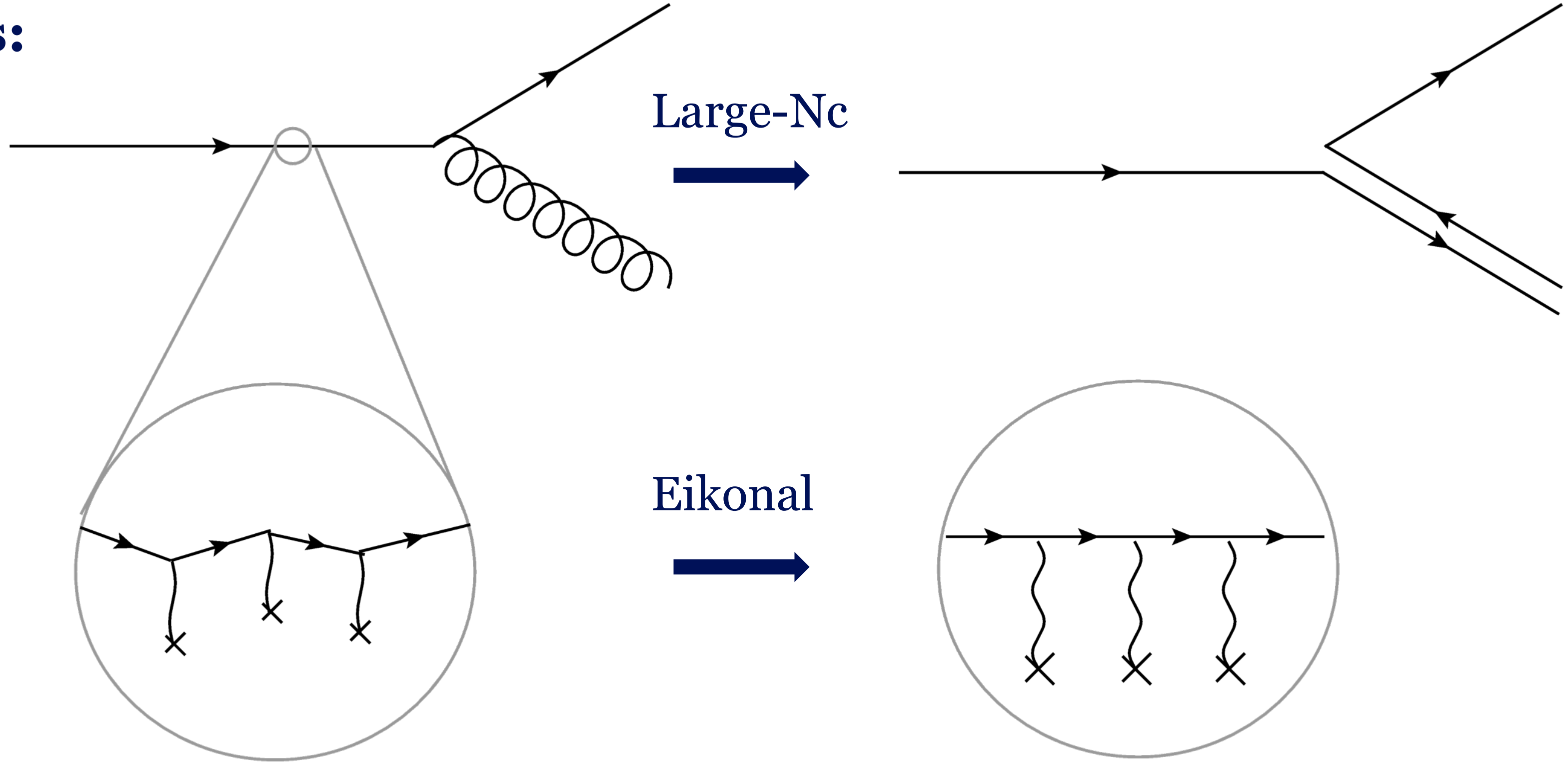
1. Large-Nc approximation
   - Take number of colors (Nc) to infinity
   - Used all the time



Analytical solutions

Two simplifying approximations:

1. Large-Nc approximation
   - Take number of colors (Nc) to infinity
   - Used all the time
2. Eikonal approximation
   - Partons travel on straight lines
   - Better for high energy
   - Used sometimes



The quadrupole at large-Nc

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- The factorizable part is

$$\mathcal{Q}_{\text{fac}}(\boldsymbol{p}, \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2 | t_\infty, t_2) = (2\pi)^2 \delta(\boldsymbol{l}_2 - \bar{\boldsymbol{l}}_2) \int_{\boldsymbol{u}} \mathrm{e}^{-i(\boldsymbol{p}-\boldsymbol{l}_2)\cdot \boldsymbol{u}} \mathcal{P}(z\boldsymbol{u}|t_\infty, t_2)\mathcal{P}((1-z)\boldsymbol{u}|t_\infty, t_2)$$

  - The two partons decohere immediately and broaden independently
  - Leads to analytical solution of emission spectrum $\dfrac{\mathrm{d}I}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}$

$\mathcal{P}(\boldsymbol{u}|t)$
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Transverse momentum broadening

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- The non-factorizable part is

$$\mathcal{Q}_{\text{non-fac}}(\boldsymbol{p}, \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2 | t_\infty, t_2) = \int_{t_2}^{\infty} \mathrm{d}t_3 \int_{\boldsymbol{l}_3} \int_{\boldsymbol{u}} \mathrm{e}^{-i(\boldsymbol{p}-\boldsymbol{l}_3)\cdot \boldsymbol{u}} \mathcal{P}(z\boldsymbol{u}|t_\infty, t_3) \mathcal{P}((1-z)\boldsymbol{u}|t_\infty, t_3) T(\boldsymbol{u}|t_3) \mathcal{Q}_{\text{initial}}(\boldsymbol{l}_3, \boldsymbol{l}_3, \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2 | t_3, t_2)$$

  - Two partons start out in some initial configuration $\mathcal{Q}_{\text{initial}}$
  - Decohere and start broadening independently at intermediate time t_3
  - Transition function $T(\boldsymbol{u}) \to 0$ in the soft limit $z \to 0$

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$\mathcal{P}(\boldsymbol{u}|t)$ = Transverse momentum broadening

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Usually only the factorizable term is kept in calculations

This is only safe in the soft limit

- The non-factorizable p

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1. Calculate full emission spectrum numerically ✓
2. Calculate emission spectrum using large-Nc approximation ✓
   - Sum of factorizable and non-factorizable terms
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Plan

Let's compare these

1. Calculate full emission spectrum numerically ✓
2. Calculate emission spectrum using large-Nc approximation ✓
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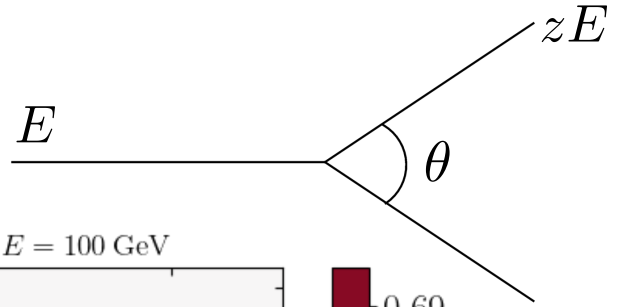
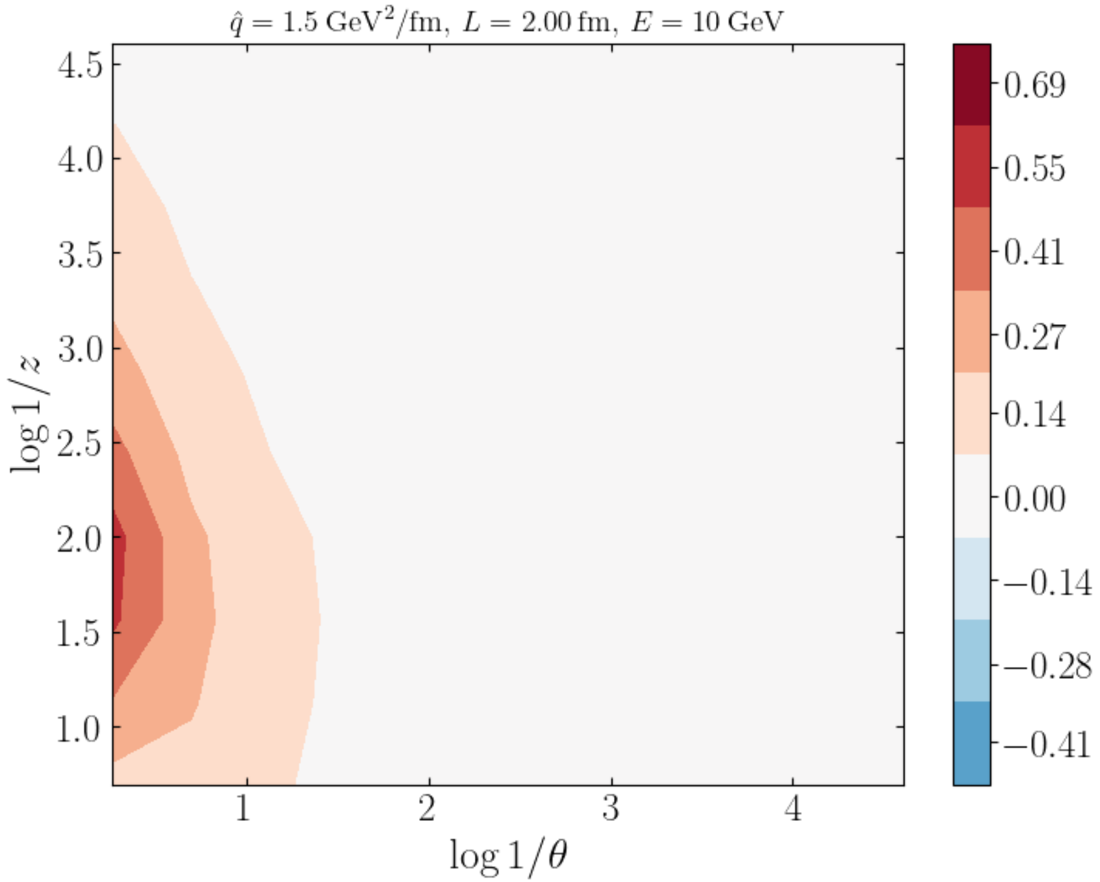
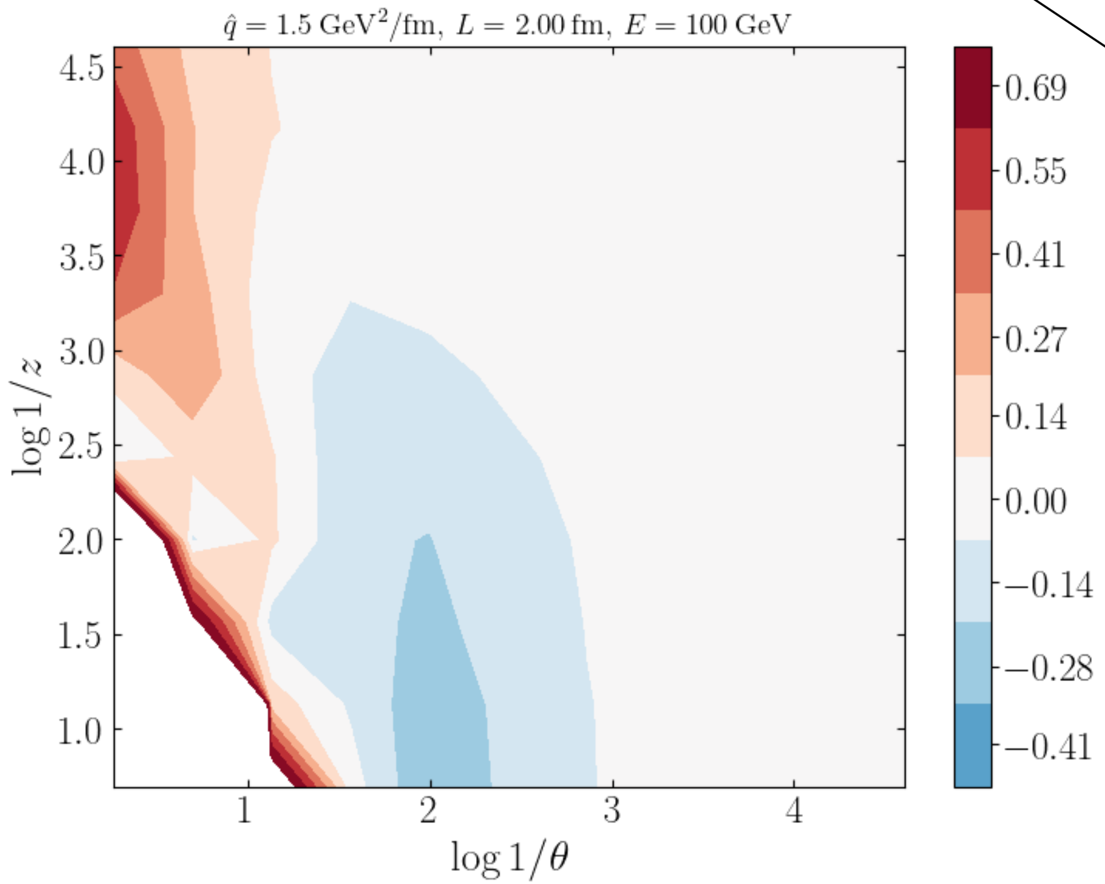
Finite Nc

- Divide the spectrum into $\frac{\mathrm{d}I^{\mathrm{full}}}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}} = \frac{\mathrm{d}I^{\mathrm{vac}}}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}(1 + F_{\mathrm{med}})$
- The medium modification is given by F_{med}





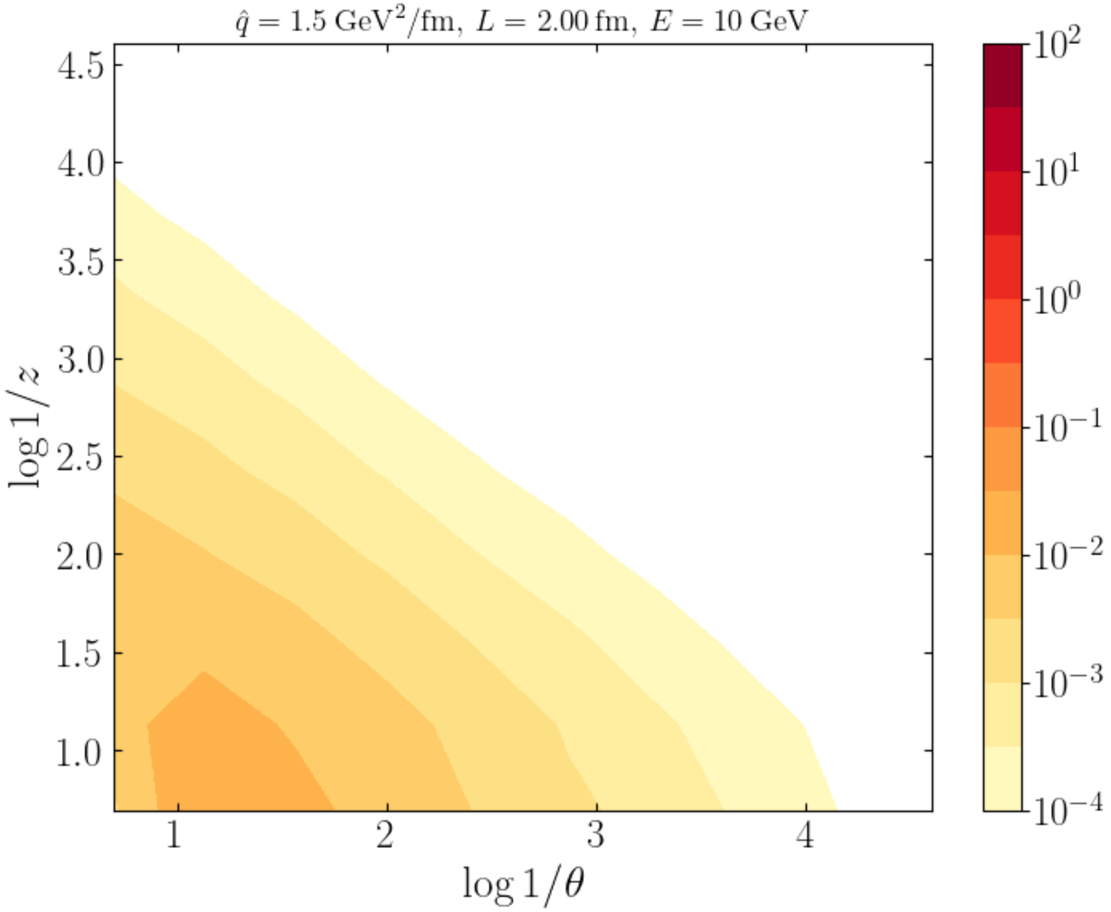


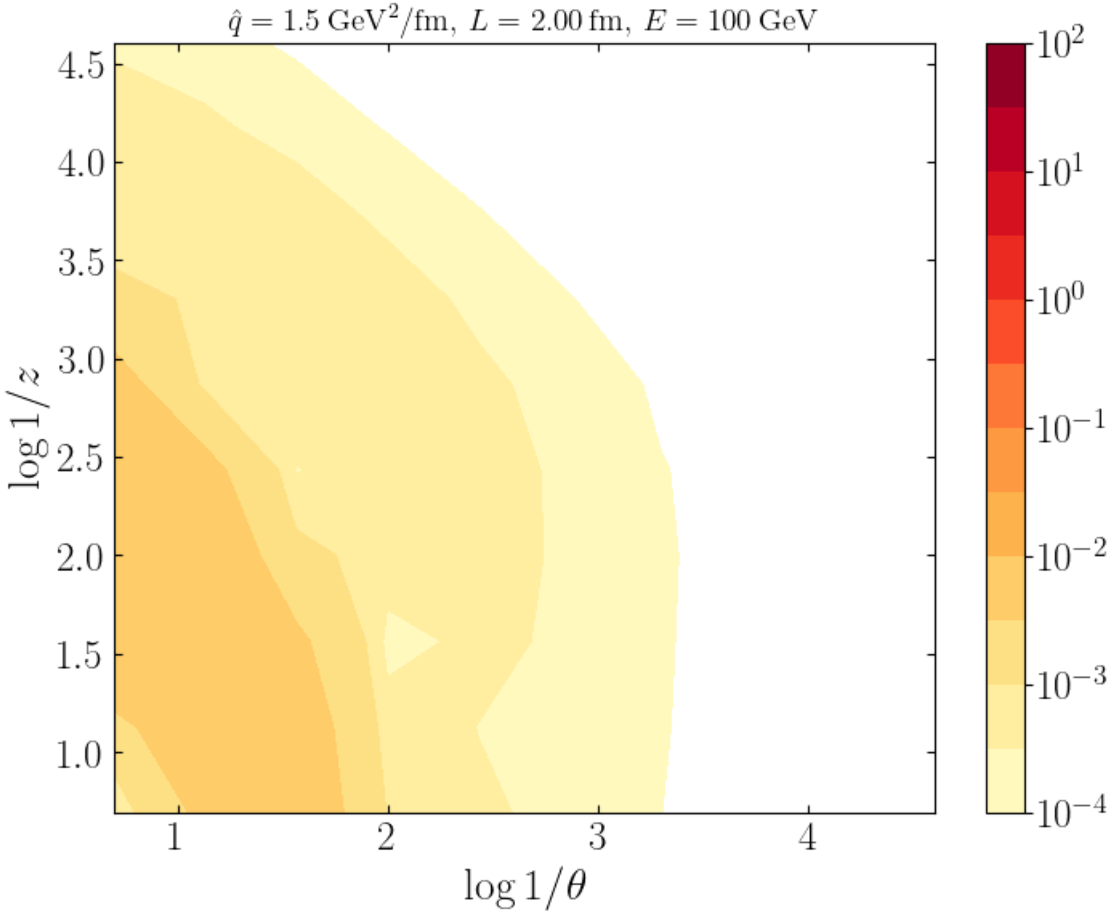
Difference finite Nc and large-Nc

$$\mathcal{Q} \simeq \mathcal{Q}_{\text{factorizable}} + \mathcal{Q}_{\text{non-factorizable}}$$

- Very small error, ~ 1 %

$$\frac{|F_{\text{med,finite}-\text{N}_\text{c}} - F_{\text{med,large}-\text{N}_\text{c}}|}{1 + |F_{\text{med,finite}-\text{N}_\text{c}}|}$$



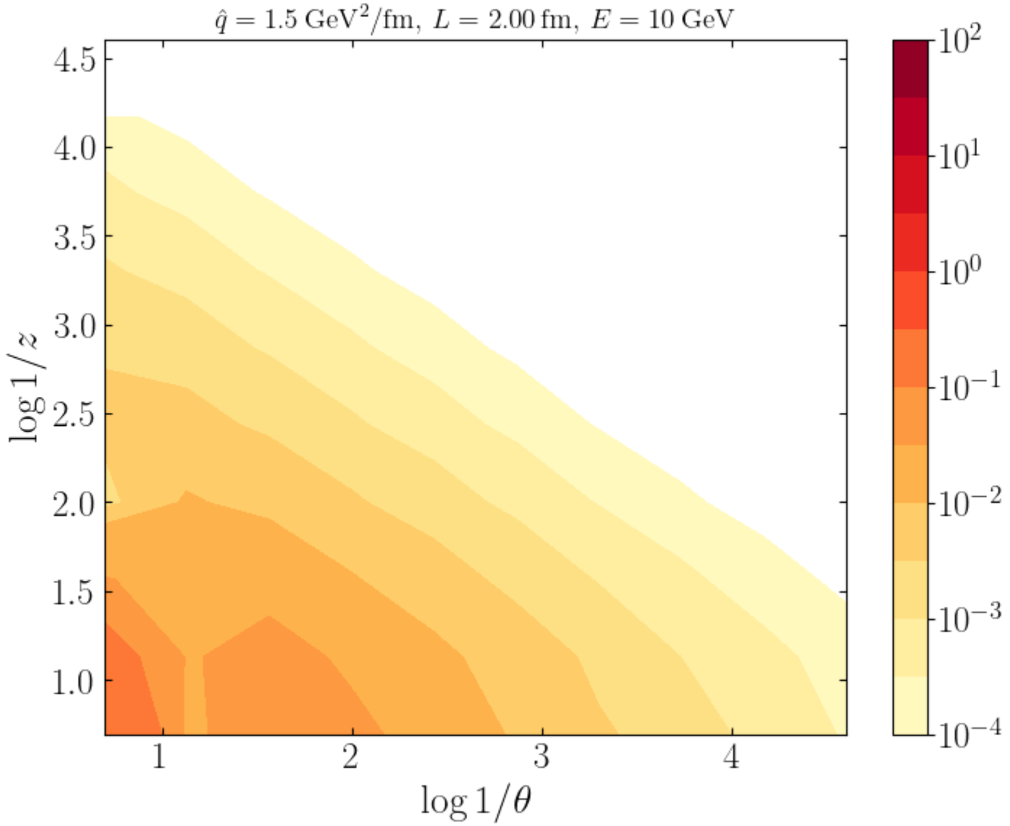


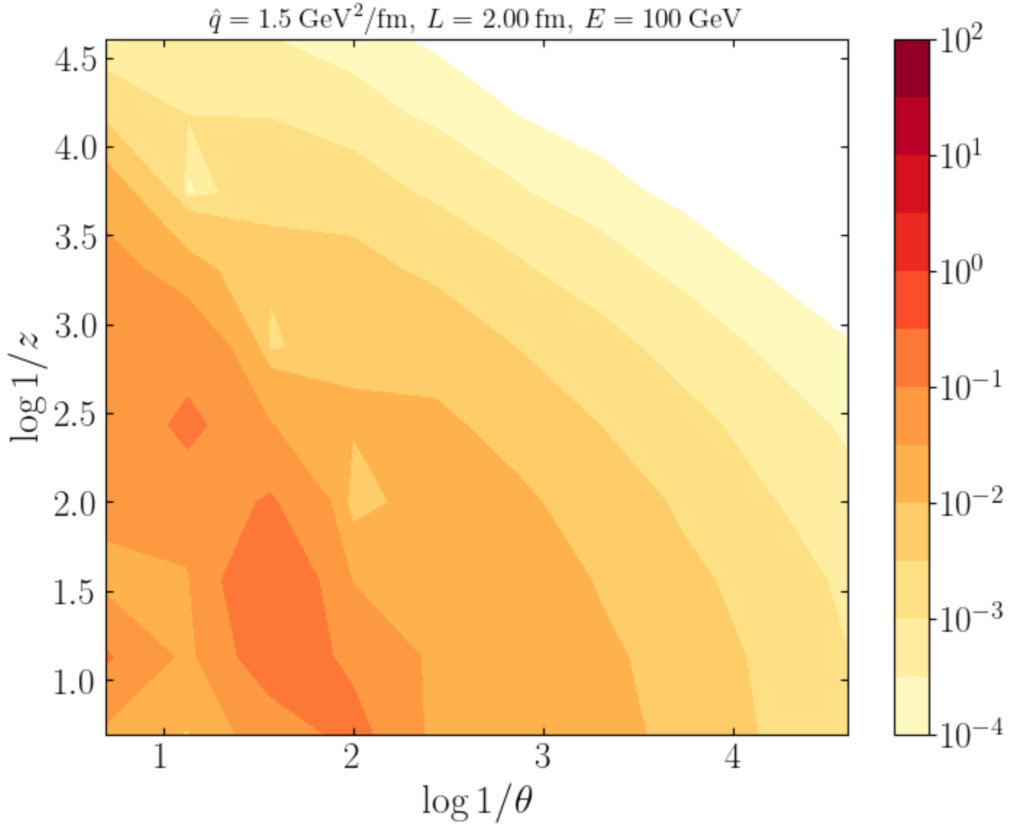
Difference finite Nc and factorizable

$\mathcal{Q} \simeq \mathcal{Q}_{\text{factorizable}}$

- Bigger error, up to > 10 %
- Small error for soft limit $z \to 0$, as expected

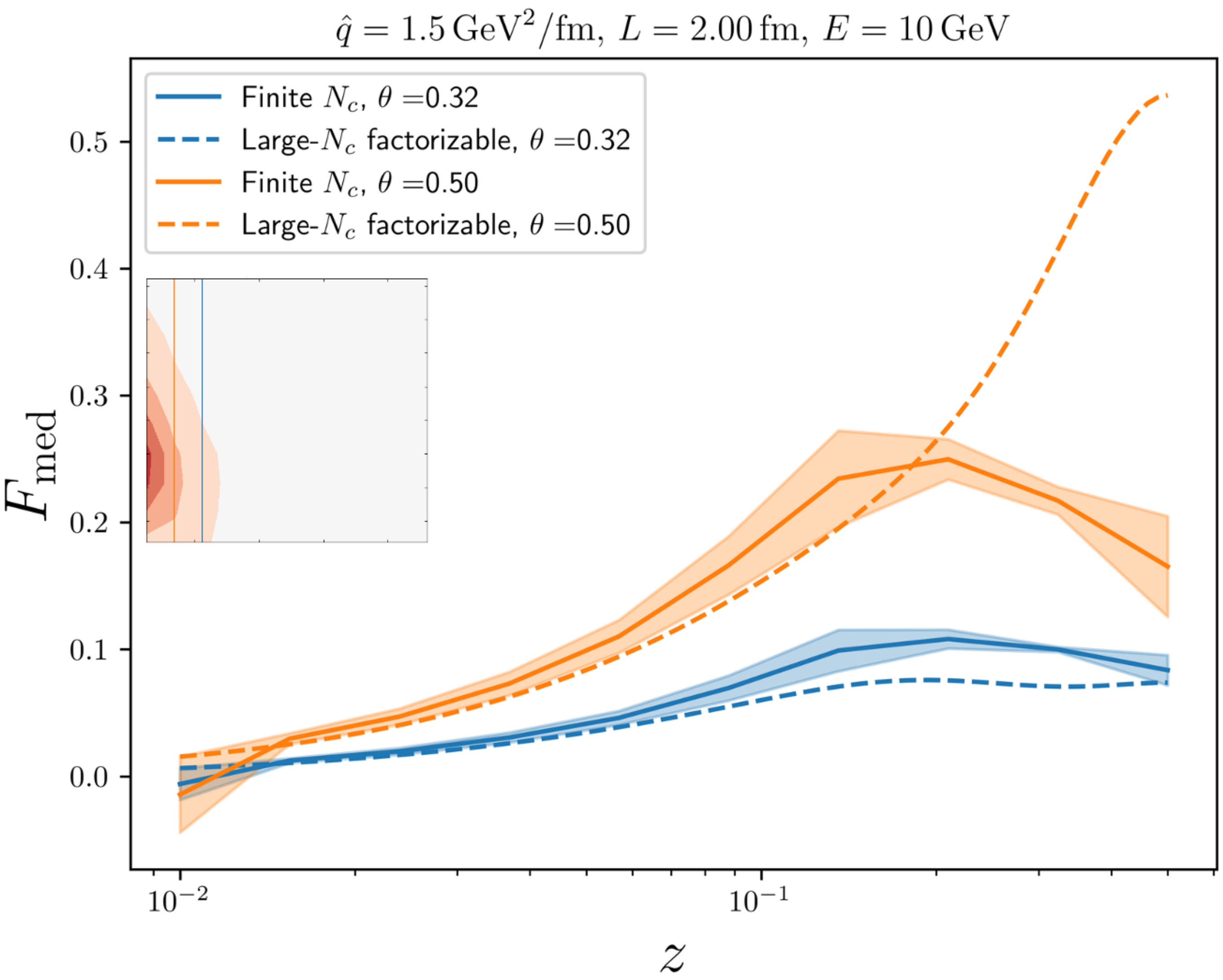
$$\frac{|F_{\text{med,finite}-\text{N}_\text{c}} - F_{\text{med,factorizable}}|}{1 + |F_{\text{med,finite}-\text{N}_\text{c}}|}$$





Difference finite Nc and factorizable

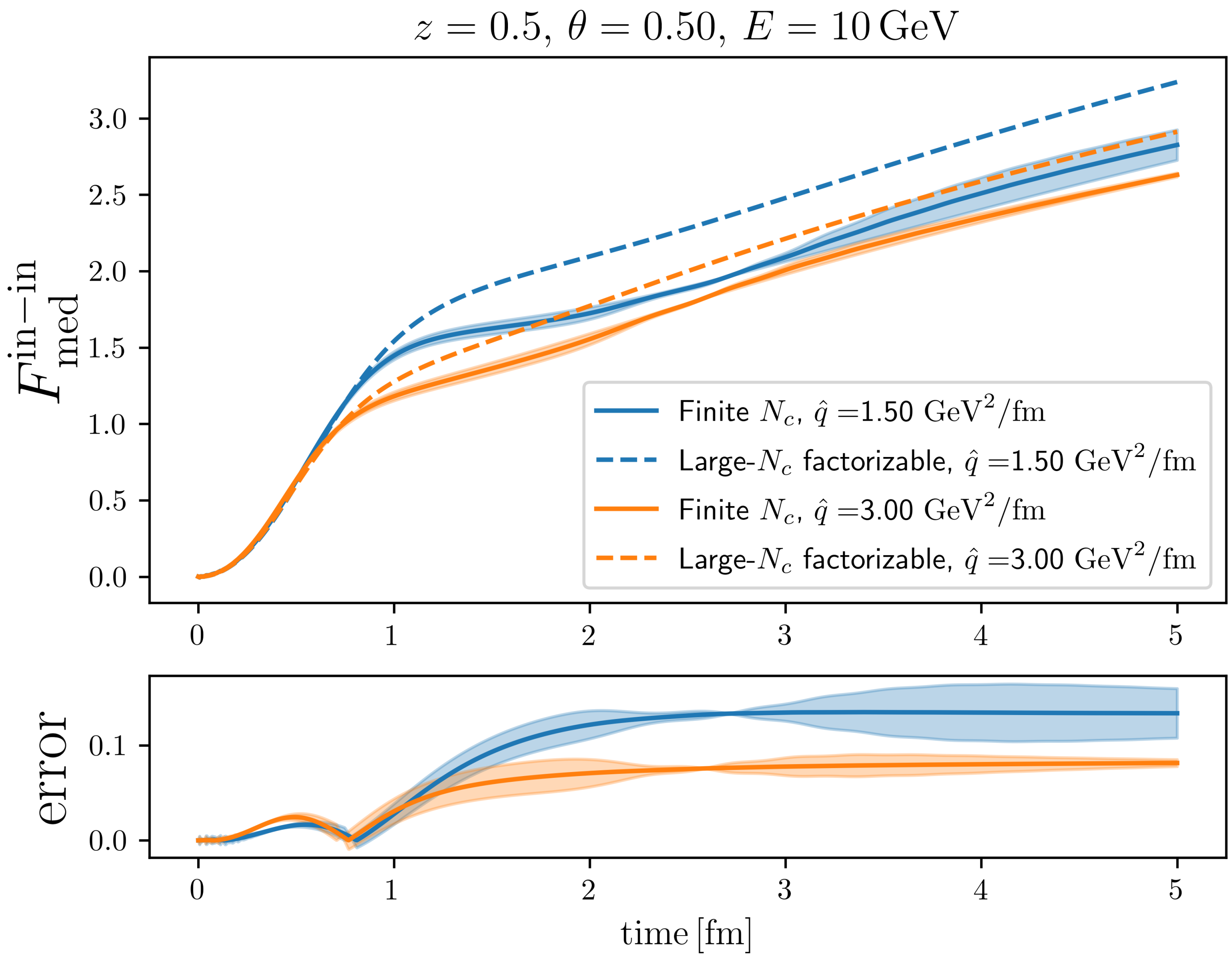
- Accurate for low z, worse for z ~ 0.5
- Peaks at different value of z



Difference finite Nc and factorizable

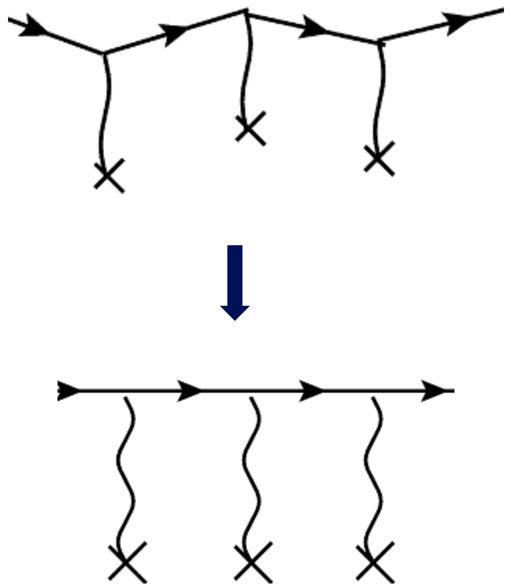
Time evolution of F_{med}

- Error becomes constant after some time
- Increasing $\hat{q}$ leads to smaller error



Eikonal approximation

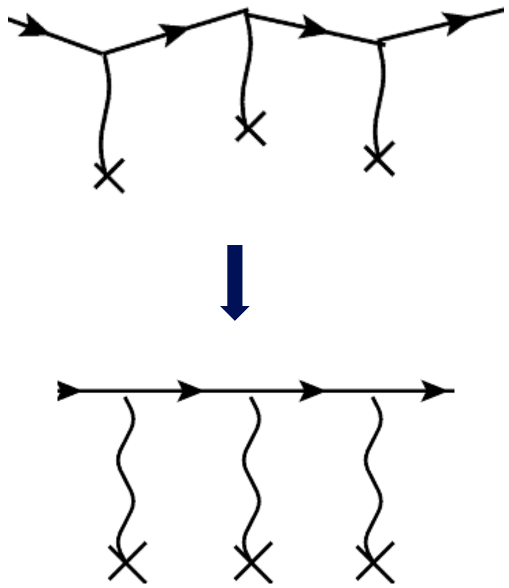
- Eikonal approximation: particles go on straight paths
  - Could be accurate at high energies



Eikonal approximation

- Eikonal approximation: particles go on straight paths
  - Could be accurate at high energies
- Path integral becomes easy

$$\mathcal{G}(\boldsymbol{x}, t; \boldsymbol{x}_0, t_0) = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}} \mathcal{D}\boldsymbol{r} \exp\left[i\frac{E}{2}\int_{t_0}^{t} \mathrm{d}s\, \dot{\boldsymbol{r}}^2(s)\right] V_R\left(t, t_0; \boldsymbol{r}(t)\right)$$
$$\simeq \frac{E}{2\pi i\,(t - t_0)} \exp\left[i\frac{E}{2}\frac{(\boldsymbol{x} - \boldsymbol{x}_0)^2}{t - t_0}\right] V_R\left(t, t_0; \boldsymbol{x}(t)\right)$$



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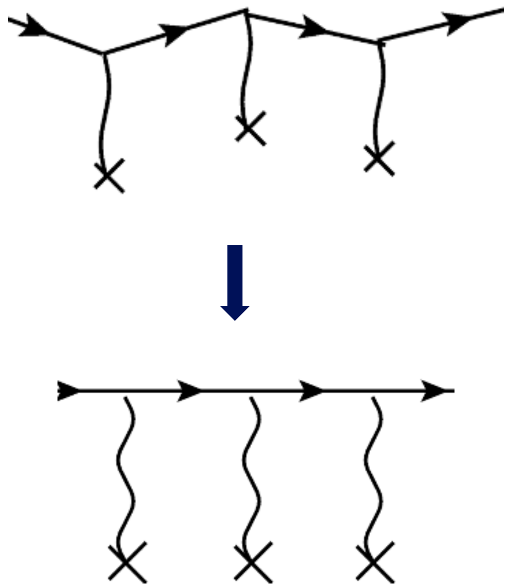
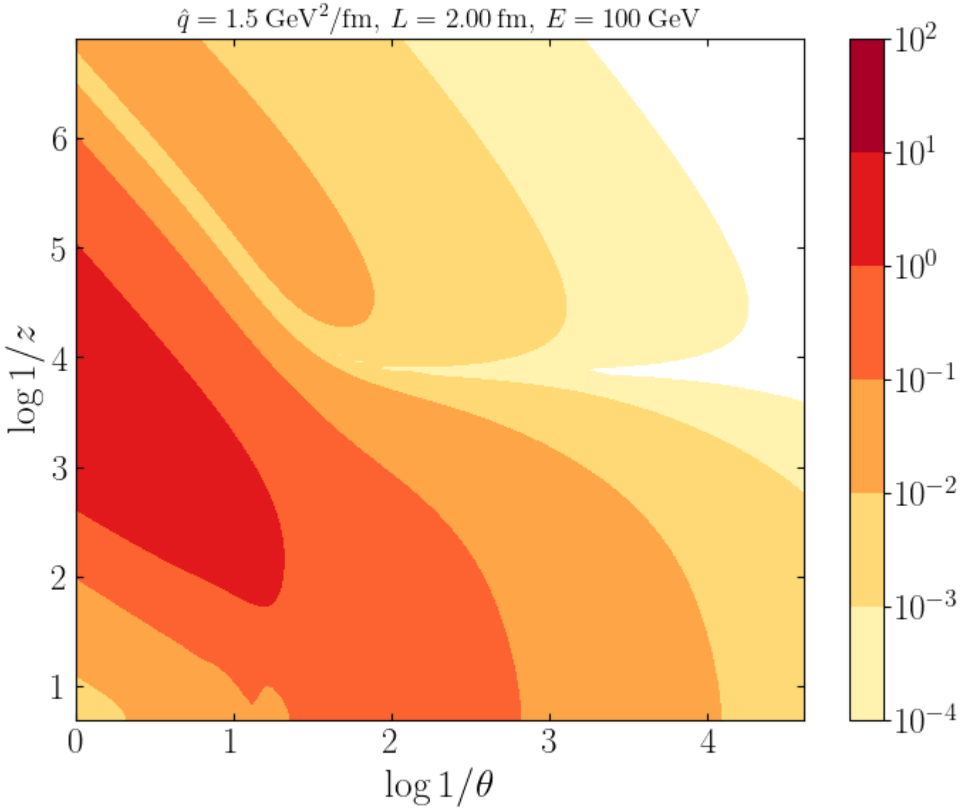


- The quadrupole now simplifies to

$$\mathcal{Q}(\boldsymbol{u}, \bar{\boldsymbol{u}}, \boldsymbol{u}_2, \bar{\boldsymbol{u}}_2 | t, t_2) \sim \langle \mathcal{G}\mathcal{G}^\dagger \mathcal{G}\mathcal{G}^\dagger \rangle$$
$$\sim \# \, \mathrm{e}^{(\cdots)} \, \mathrm{tr}[V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger]$$

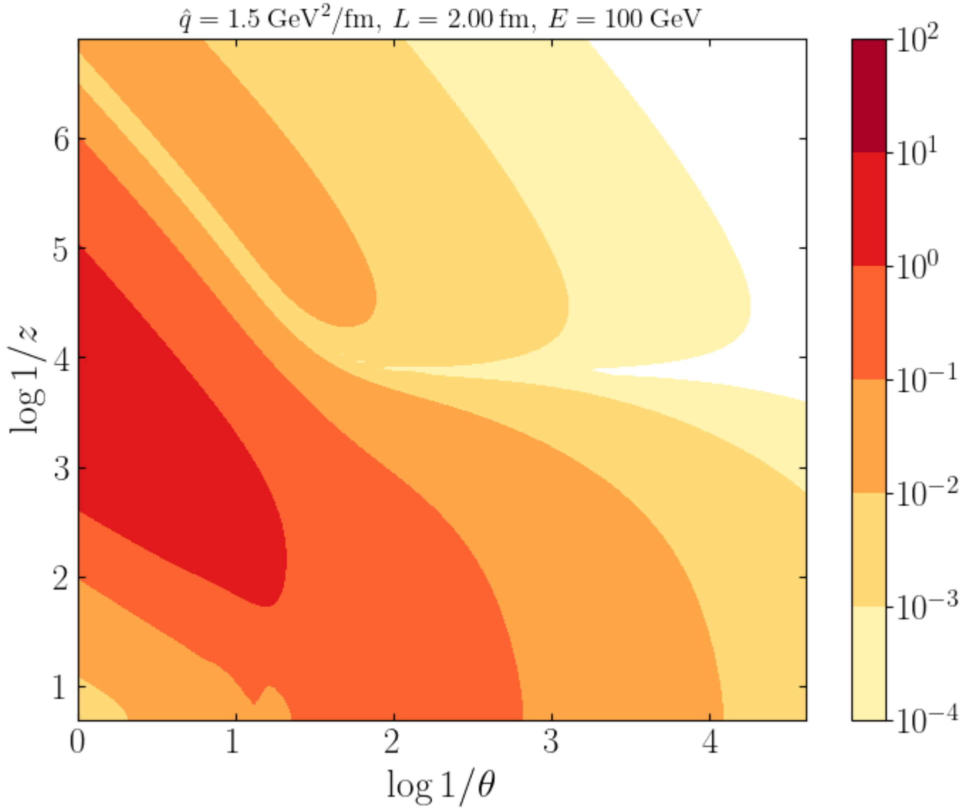
Eikonal and non-eikonal

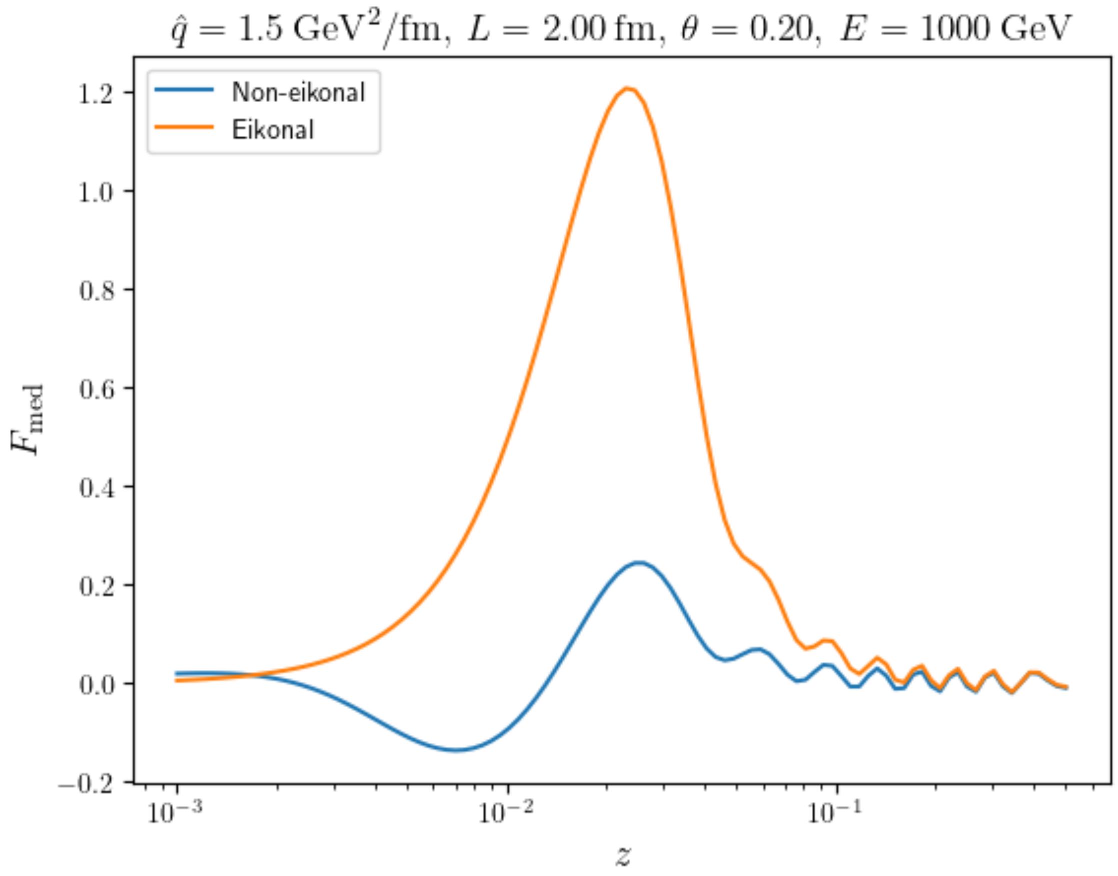
- Large error



Eikonal and non-eikonal

- Large error
- Better at higher energy
  - Still does not capture main contribution to spectrum





Plan

1. Calculate full emission spectrum numerically ✓
2. Calculate emission spectrum using large-Nc approximation ✓
3. Calculate emission spectrum using eikonal approximation ✓

Conclusion

Goal: calculate the medium-induced emission spectrum without commonly used approximations

- At finite Nc it can be done numerically through system of Schrödinger equations

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Conclusion

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In our numerics we found that

- Finite Nc → large-Nc: very small error
- Finite Nc → the factorizable part: small error at low z, larger error at finite z

Conclusion

Goal: calculate the medium-induced emission spectrum without commonly used approximations

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In our numerics we found that

- Finite Nc → large-Nc: very small error
- Finite Nc → the factorizable part: small error at low z, larger error at finite z

- Also studied the eikonal approximation
  - Associated with big error
  - Could be more accurate at higher energy

Thank you for your attention!

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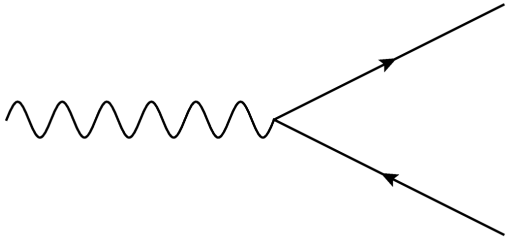
Large-Nc approximation

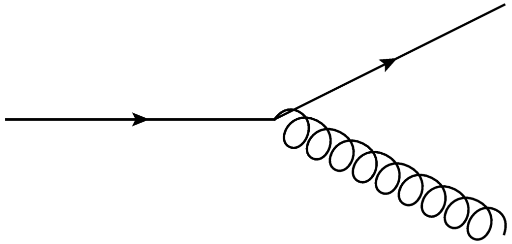
Three processes

Pair production



$$\frac{1}{N_c}\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar 2} V_{\bar 1}^\dagger]\rangle$$

Quark emitting gluon

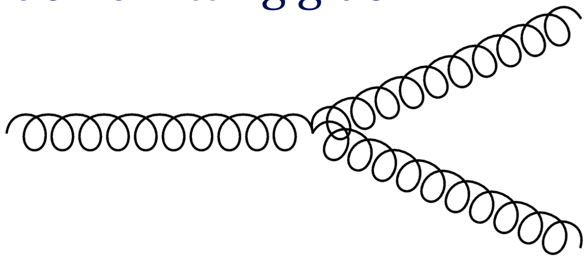


$$\frac{1}{N_c^2-1}\langle \mathrm{tr}[V_{\bar 1}^\dagger V_1 V_2^\dagger V_{\bar 2}]\,\mathrm{tr}[V_{\bar 2}^\dagger V_2] - \frac{1}{N_c}\mathrm{tr}[V_{\bar 1}^\dagger V_1]\rangle$$

↓ Large-Nc

$$\frac{1}{N_c^2}\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar 2} V_{\bar 1}^\dagger]\rangle\langle \mathrm{tr}[V_{\bar 2}^\dagger V_2]\rangle$$

Gluon emitting gluon



$$\frac{1}{N_c(N_c^2-1)}\langle \mathrm{tr}[V_1^\dagger V_{\bar 1}]\,\mathrm{tr}[V_2^\dagger V_{\bar 2} V_{\bar 1}^\dagger V_1]\,\mathrm{tr}[V_{\bar 2}^\dagger V_2] - \mathrm{tr}[V_1^\dagger V_{\bar 1} V_2^\dagger V_{\bar 2} V_{\bar 1}^\dagger V_1 V_{\bar 2}^\dagger V_2]\rangle$$

↓ Large-Nc

$$\frac{1}{N_c^3}\langle \mathrm{tr}[V_1^\dagger V_{\bar 1}]\rangle\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar 2} V_{\bar 1}^\dagger]\rangle\langle \mathrm{tr}[V_{\bar 2}^\dagger V_2]\rangle$$

- The same correlator of four Wilson lines present in all three processes
- Study $\gamma \to q\bar{q}$ as a proxy for all of them

The quadrupole at large-Nc

- Want to calculate the quadrupole analytically

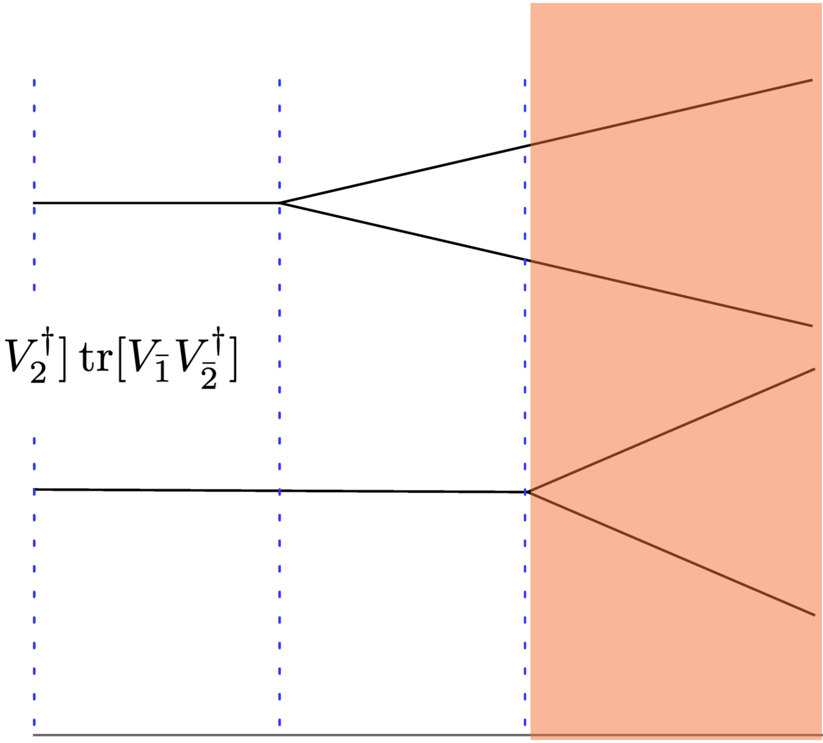
$$\mathcal{Q}(\boldsymbol{u}_\mathrm{f}, \bar{\boldsymbol{u}}_\mathrm{f}, \boldsymbol{u}_2, \bar{\boldsymbol{u}}_2 | t_f, t_2) = \int_{\boldsymbol{u}_2}^{\boldsymbol{u}_\mathrm{f}} \mathcal{D}\boldsymbol{u} \int_{\bar{\boldsymbol{u}}_2}^{\bar{\boldsymbol{u}}_\mathrm{f}} \mathcal{D}\bar{\boldsymbol{u}}\, \mathrm{e}^{i\frac{\omega}{2}\int_{t_2}^{t_f} \mathrm{d}s\, (\dot{\boldsymbol{u}}^2 - \dot{\bar{\boldsymbol{u}}}^2)} \frac{1}{N_c} \langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger] \rangle$$

- At large-Nc the four Wilson lines separate into two parts:

$$\frac{1}{N_c}\mathrm{tr}[V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger] \simeq \frac{1}{N_c^2}\mathrm{tr}[V_1 V_{\bar{1}}^\dagger]\,\mathrm{tr}[V_2 V_{\bar{2}}^\dagger] + \frac{1}{N_c^4}\int_{t_2}^{\infty} \mathrm{d}s\; \mathrm{tr}[V_1 V_{\bar{1}}^\dagger]\,\mathrm{tr}[V_2 V_{\bar{2}}^\dagger] T(s)\,\mathrm{tr}[V_1 V_2^\dagger]\,\mathrm{tr}[V_{\bar{1}} V_{\bar{2}}^\dagger]$$

- Correlator of two Wilson lines describes transverse momentum broadening

$$\mathcal{P}(\boldsymbol{r}_1 - \boldsymbol{r}_2 | t_\infty, t_2) = \frac{1}{N_c}\langle \mathrm{tr}[V_1 V_2^\dagger] \rangle$$
$$\simeq \mathrm{e}^{-C_F \int_{t_2}^{\infty} \mathrm{d}s\, n(s) \sigma(\boldsymbol{r}_1 - \boldsymbol{r}_2)}$$



- The quadrupole has two parts:

$$\mathcal{Q} = \mathcal{Q}_{\mathrm{factorizable}} + \mathcal{Q}_{\mathrm{non-factorizable}}$$

The potential matrix

$$\mathbb{M} = -\frac{1}{2}n(t)\begin{bmatrix} 2C_F(\sigma_{12}+\sigma_{\bar{2}\bar{1}}) + \frac{1}{N_c}\Sigma_1 & -\frac{1}{N_c}\Sigma_1 \\ -N_c\Sigma_2 & 2C_F(\sigma_{1\bar{1}}+\sigma_{\bar{2}2}) + \frac{1}{N_c}\Sigma_2 \end{bmatrix}$$

Here we have introduced

$$\Sigma_1 \equiv \sigma_{1\bar{2}} + \sigma_{2\bar{1}} - \sigma_{1\bar{1}} - \sigma_{2\bar{2}}$$
$$\Sigma_2 \equiv \sigma_{1\bar{2}} + \sigma_{\bar{1}2} - \sigma_{12} - \sigma_{\bar{1}\bar{2}}\,.$$

- Harmonic oscillator:

$$\mathbb{M} = -\frac{\hat{q}}{4C_F}\begin{bmatrix} C_F[\boldsymbol{u}^2+\bar{\boldsymbol{u}}^2] + \frac{1}{N_c}\boldsymbol{u}\cdot\bar{\boldsymbol{u}} & -\frac{1}{N_c}\boldsymbol{u}\cdot\bar{\boldsymbol{u}} \\ N_c z(1-z)(\boldsymbol{u}-\bar{\boldsymbol{u}})^2 & [C_F - N_c z(1-z)](\boldsymbol{u}-\bar{\boldsymbol{u}})^2 \end{bmatrix}$$

- Large-Nc:

$$\mathbb{M} = -\frac{\hat{q}}{4}\begin{bmatrix} \boldsymbol{u}^2+\bar{\boldsymbol{u}}^2 & 0 \\ 2z(1-z)(\boldsymbol{u}-\bar{\boldsymbol{u}})^2 & [z^2+(1-z)^2](\boldsymbol{u}-\bar{\boldsymbol{u}})^2 \end{bmatrix}$$