Data-driven $\hat{q}$ in a hard-soft factorized parton energy loss approach

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Outline

- Hard-soft factorized parton energy loss
- Calculation using pQCD transport coefficients
- Data-driven analysis of parton transport properties

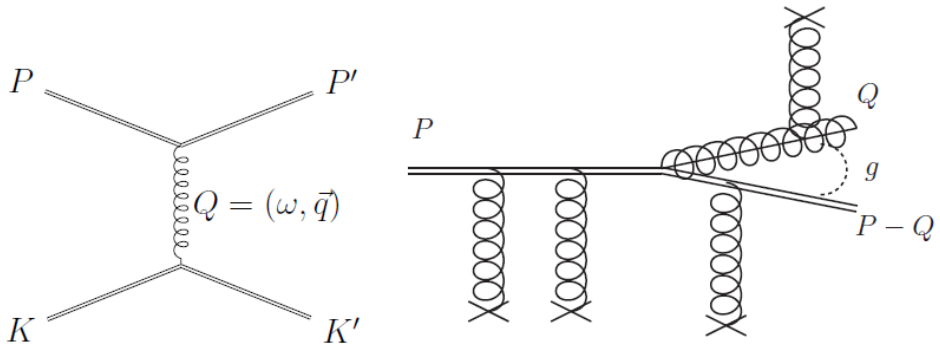
Weakly-coupled effective kinetic formalism

- Weakly-coupled: perturbative parton-medium interactions
- Effective: quarks and gluons are considered as quasi-particles
- Kinetic: dynamics of quasi-particles are described by transport equations

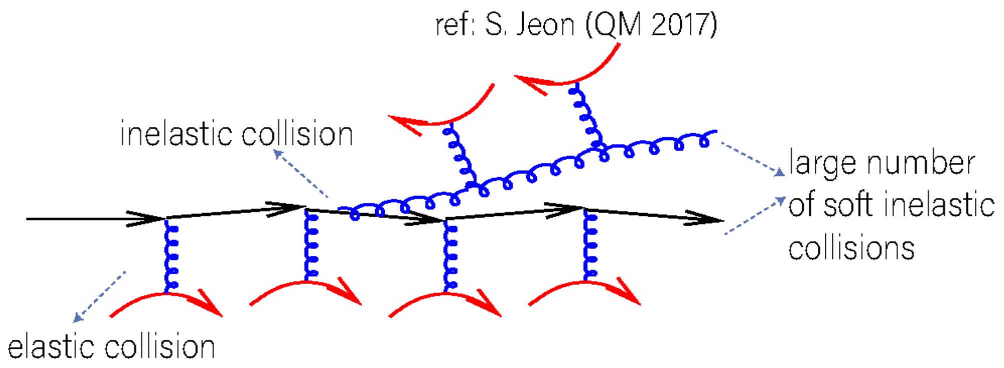
Leading-order realizations (e.g. MARTINI):

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^a(\mathbf{p}, \mathbf{x}, t) = -\mathcal{C}_a^{2\leftrightarrow 2}[f] - \mathcal{C}_a^{1\leftrightarrow 2}[f]$$

Assuming medium length ≫
radiation formation time ≫
scattering mean free path



Hard-soft factorization of parton energy loss



Interactions with the medium:

- Large number of soft interactions
- Rare hard scatterings

Frequent soft interactions can be treated stochastically as diffusion process.
Parton energy loss factorized as **hard interactions + diffusion process**.

Benefits of the hard-soft factorization

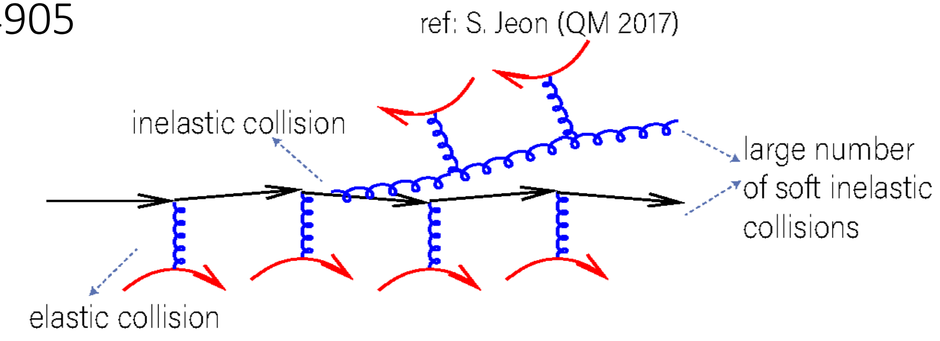
- Non-perturbative effects absorbed in soft transport coefficients.
- Soft transport coefficients can be constrained from measurements.
- Stochastic description is numerically more efficient.
- Can be extended to next-to-leading order of parton-medium interaction.

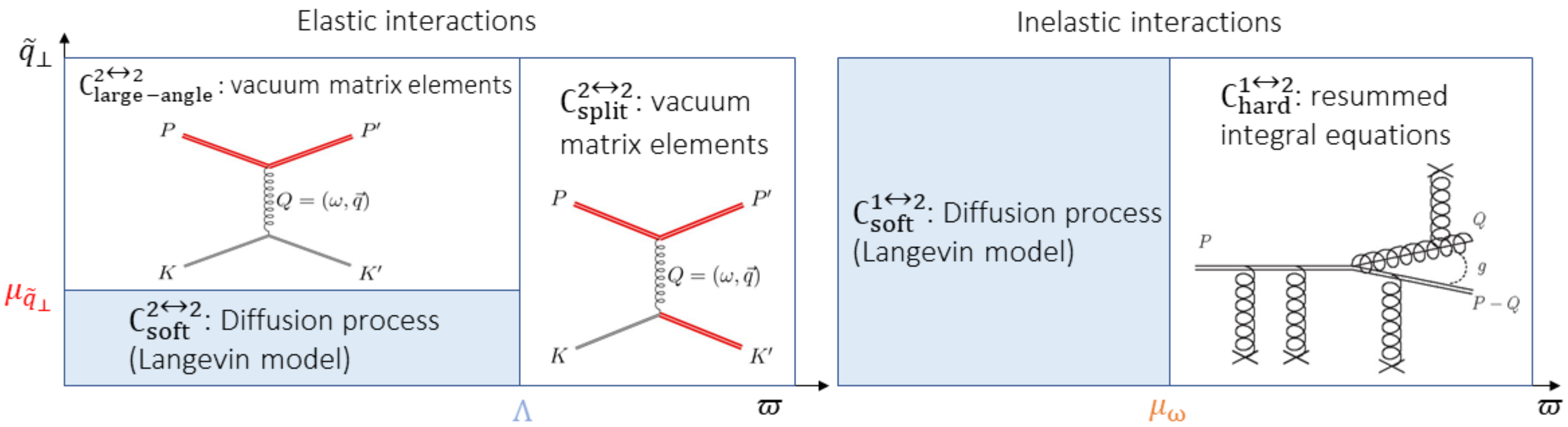
Hard-soft factorization of parton energy loss

Theory: J. Ghiglieri, G.D. Moore & D. Teaney: JHEP **03** (2016) 095
Implementation: T. Dai, J.F. Paquet, D. Teaney & S.A. Bass: PRC**105** (2022) 034905

Hard-soft factorization:

$$C_a^{2\leftrightarrow 2} + C_a^{1\leftrightarrow 2}$$
$$= C_a^{\mathrm{large-angle}}(\mu_{\tilde{q}_\perp}, \Lambda) + C_a^{\mathrm{split}}(\Lambda) + C_a^{\mathrm{large-}\varpi}(\mu_\varpi) + C_a^{\mathrm{diff}}(\mu_{\tilde{q}_\perp}, \mu_\varpi)$$





Soft interactions – drag and diffusion

Number and identity preserving soft interactions are described stochastically with drag and diffusion.

$$C^{\text{diff}}[f] = -\frac{\partial}{\partial p^i}\left[\eta_D(p)p^i f(p)\right] - \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i\hat{p}^j\hat{q}_L(p) + \frac{1}{2}(\delta^{ij} - \hat{p}^i\hat{p}^j)\hat{q}(p)\right)f(p)\right]$$

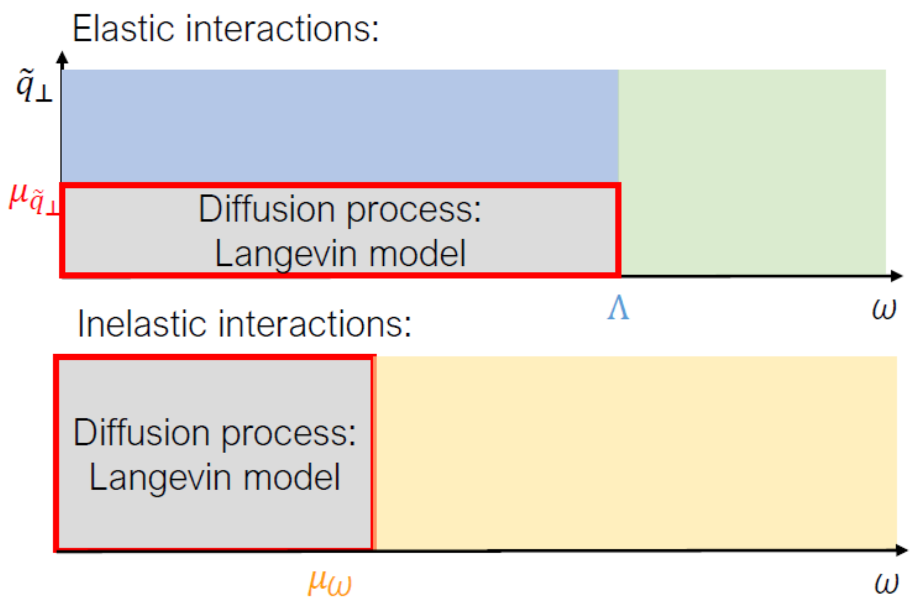
$$\hat{q}_{\text{pQCD}} \equiv \frac{d}{dt}\langle(\Delta p_\perp)^2\rangle = \frac{g^2 C_R T m_D^2}{4\pi} ln\left[1 + \left(\frac{\mu_{\tilde{q}_\perp}}{m_D}\right)^2\right]$$

$$\hat{q}^{\text{elas}}_{\text{L,pQCD}} \equiv \frac{d}{dt}\left\langle\left(\Delta p_L^{\text{elas}}\right)^2\right\rangle = \frac{g^2 C_R T M_\infty^2}{4\pi} \ln\left[1 + \left(\frac{\mu_{\tilde{q}_\perp}}{M_\infty}\right)^2\right]$$

$$\hat{q}^{\text{inel}}_{\text{L,pQCD}} \equiv \frac{d}{dt}\left\langle\left(\Delta p_L^{\text{inel}}\right)^2\right\rangle = \frac{(2 - ln2)g^4 C_R C_A T^2 \mu_\omega}{4\pi^3}$$

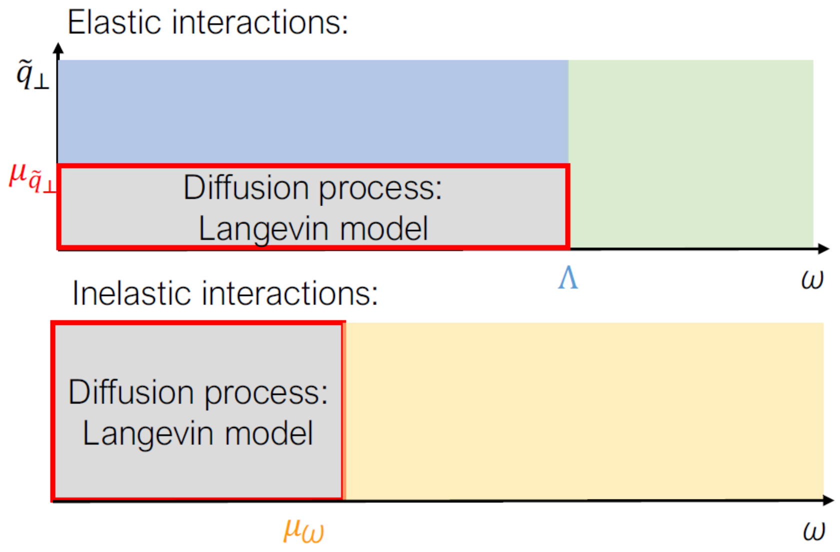
$$\eta_D(p) = \frac{\hat{q}_L}{2Tp} + \frac{1}{2p^2}(\hat{q} - 2\hat{q}_L)$$

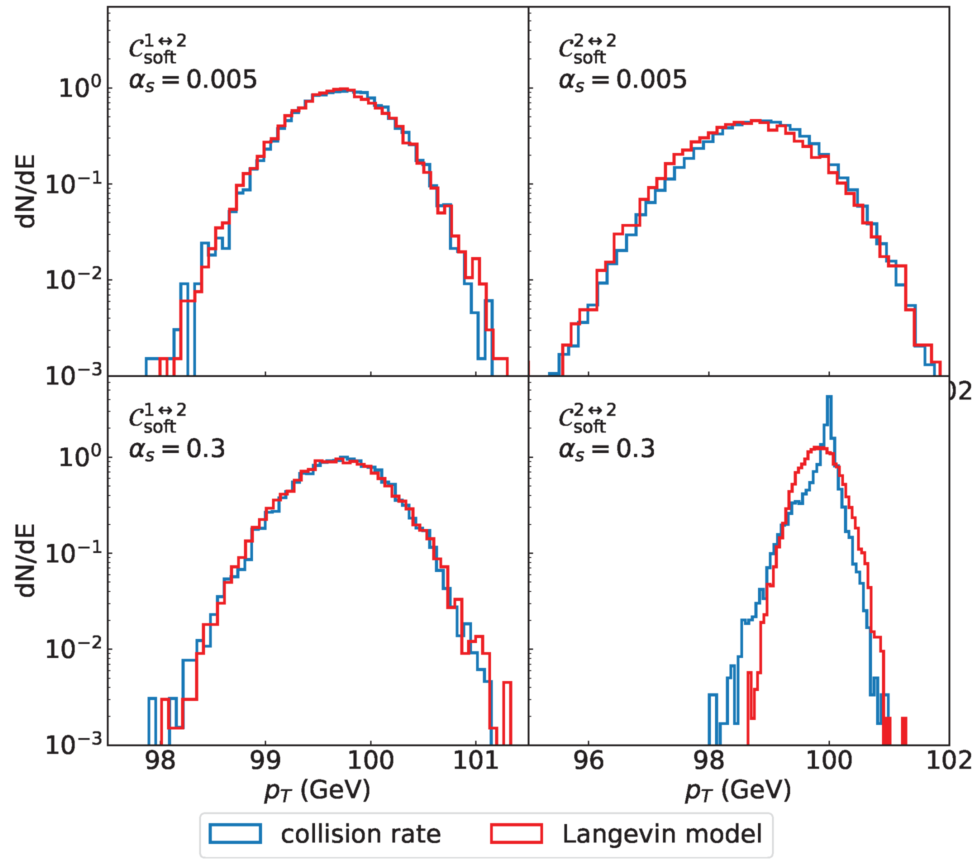
- Include both elastic and inelastic soft interactions.
- Treated with Langevin model.



Brick test: weak coupling and beyond

We compare: collison rate treatment v.s. stochastic treatment
We use: pure glue medium; screened matrix elements for collision rates
We plot: energy distribution of a hard gluon propagating in a static medium





$\Rightarrow$ C_a^{soft} only

T = 300 MeV

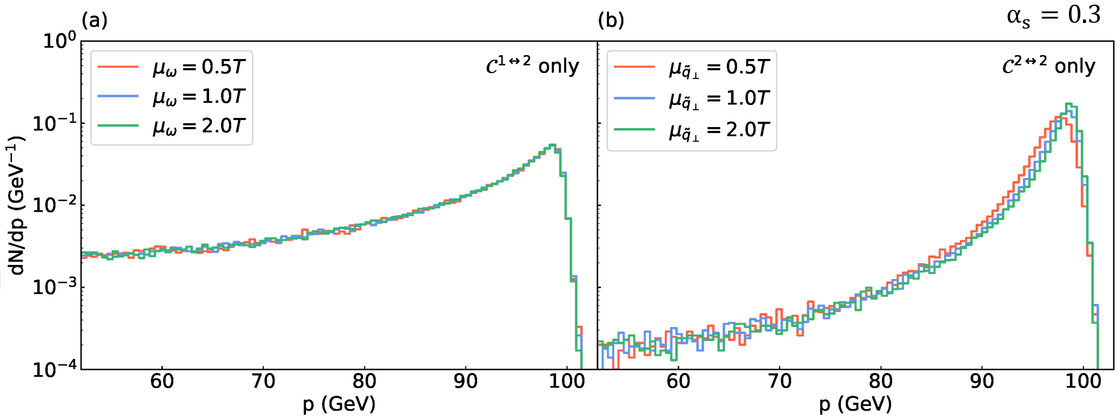
E_0 = 100GeV

Evolution time:

$t = (0.3/\alpha_s)^2 \text{fm/c}$

$C_a^{\text{soft}} + C_a^{\text{hard}}$ $\Leftarrow$

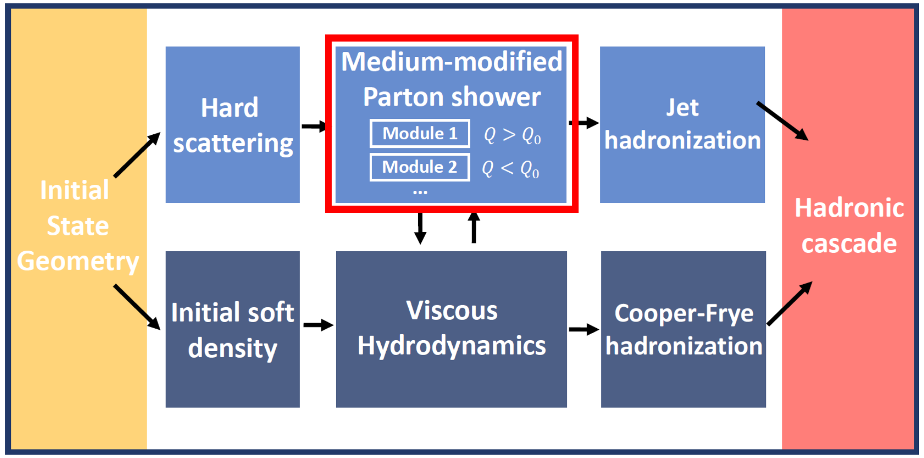
We validate: the dependence of the **single parton energy distribution** on the hard-soft cutoff



Energy loss is **weakly dependent on the cutoffs.**

Estimation of Q_0 in different collision systems

Q_0: switch between high virtuality stage to low virtuality stage.



k_T distribution

- Medium-induced emissions: integration of $\hat{q}$ along the path.
- Vacuum emission: k_T much larger than that accumulated in collisions during the formation.

Rough estimation of Q_0:
(assuming the medium evolves as conformal Bjorken expansion)

$$Q_0^2 \sim \langle k_T^2 \rangle(\tau_f, \tau_0) = \int_{\tau_0}^{\tau_f} d\tau \hat{q}(\tau)$$
$$\approx \frac{1}{3}\left[\tau T^3\right]\left[\frac{\hat{q}(T)}{T^3}\right]\int_{T_f}^{T_0}\frac{dT'}{T'}$$
$$= \frac{1}{3}\left[\tau T^3\right]\left[\frac{\hat{q}(T)}{T^3}\right]\ln(T_0/T_f).$$
$$\sim \tau T^3.$$

collision system	τ_{hydro} (fm/c)	T_0 (GeV)	$\tau_{hydro} T_0^3$ (GeV^2)
Au+Au, 200 GeV, 0-10%	0.5	0.41	0.17
Au+Au, 200 GeV, 20-30%	0.5	0.37	0.13
Au+Au, 200 GeV, 40-50%	0.5	0.35	0.11
Pb+Pb, 2760 GeV, 0-5%	1.2	0.38	0.33
Pb+Pb, 2760 GeV, 30-40%	1.2	0.34	0.24

- Introduce parameter: $Q_0^2 \sim \tilde{Q}_0 \cdot \tau_{\mathrm{hydro}} T_0^3$

Model output using perturbation theory

Experimental measurement:

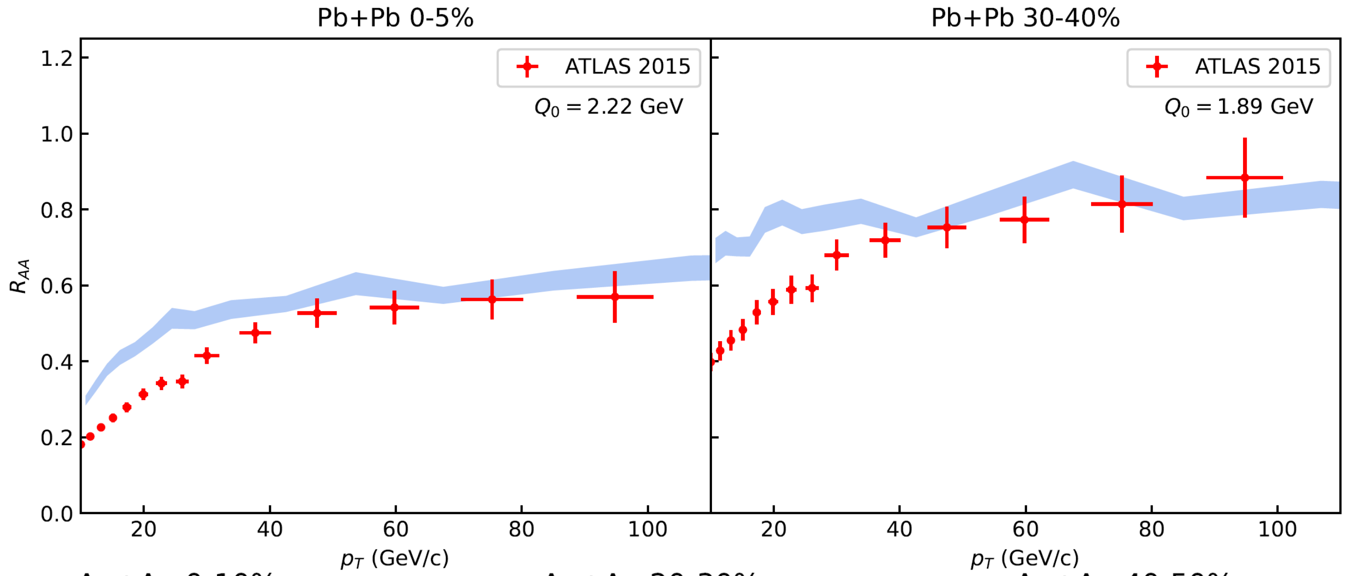
$$R_{AA}(\eta, p_T) = \frac{1}{\langle N_{coll} \rangle} \frac{dN_{AA}}{d\eta dp_T} \Big/ \frac{dN_{pp}}{d\eta dp_T}$$

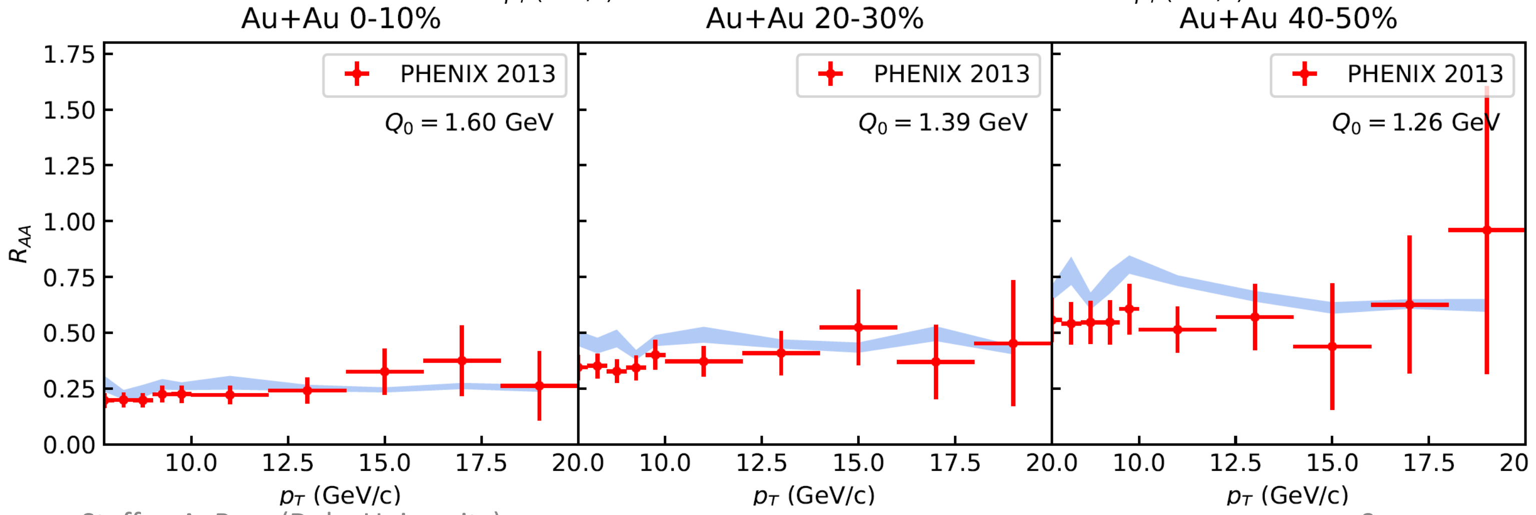
to quantify the effect of QGP.

Medium evolution: hydrodynamic simulation
High virtuality parton: DGLAP
High energy parton: hard-soft factorized model

Hard-soft factorized model in JETSCAPE:

- Perturbative soft transport coefficients.
- Difficult to tune multiple parameters to match multiple measurements at the same time.

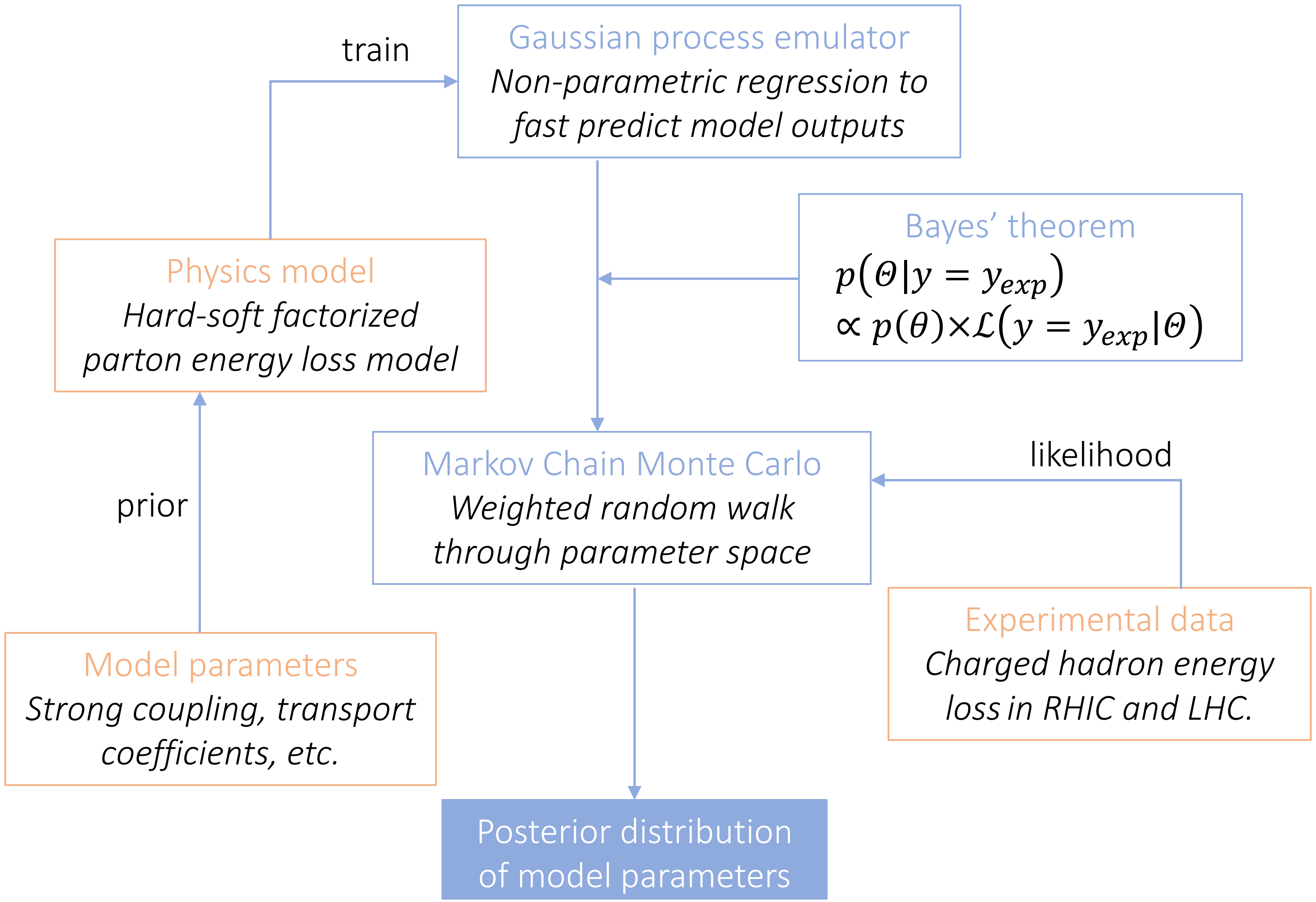




Bayesian model-to-data comparison

Goal: model-to-data comparison

- Simultaneously describe several sets of experimental data.
- Quantitatively estimate the model parameters.
- Quantify the non-perturbative effects of the soft interactions.



Parameterize the soft sector

Parameter	Min	Max
$\alpha_{s,\text{hard}}^{\text{inel}}$	0.1	0.4
T^*	0.16	0.5
$\beta_\perp$	-0.8	2
$\beta_\parallel$	-0.8	2
$\tilde{Q}_0$	6.8	20.6

soft sector transport coefficients parametrized in terms of $\beta_\perp$, $\beta_\parallel$:

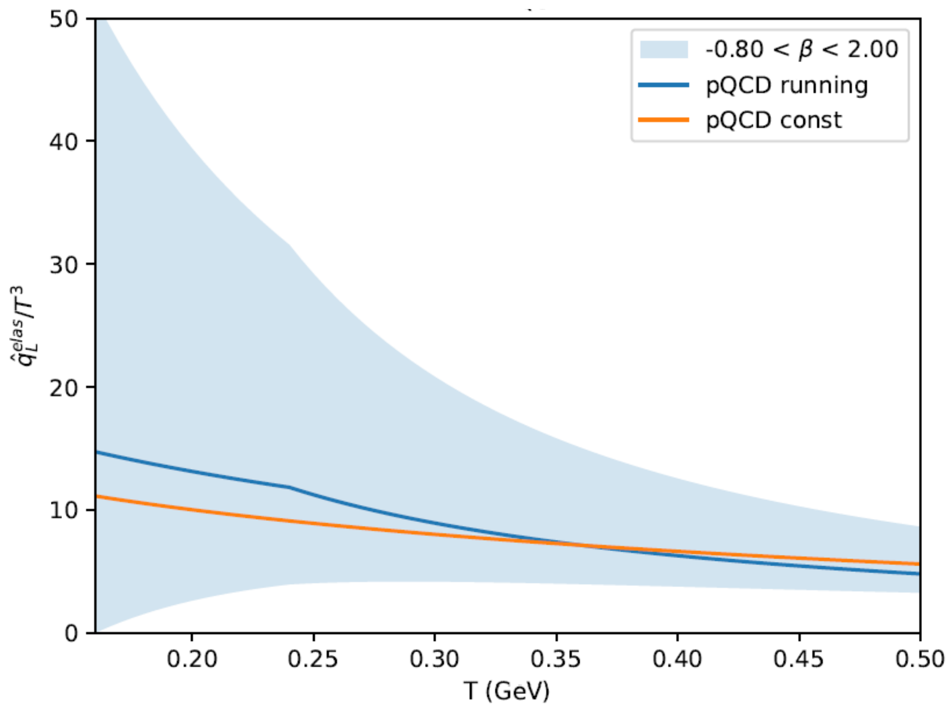
$$\hat{q}(\beta_\perp, T^*) = \hat{q}_{\text{pQCD}}(T^*)\left(1 + \beta_\perp \frac{\Lambda_{\text{QCD}}}{T}\right)$$

$$\hat{q}_L(\beta_\parallel, T^*) = \hat{q}_{L,\text{pQCD}}(T^*)\left(1 + \beta_\parallel \frac{\Lambda_{\text{QCD}}}{T}\right)$$

coupling constant used in the soft sector is regulated by T^*:

$$\alpha_{s,\text{soft}}(T^*) = \frac{g_{\text{soft}}^2(T^*)}{4\pi} = \frac{4\pi}{9} \frac{1}{\log\left[\left(\frac{2\pi \max(T, T^*)}{\Lambda_{\text{QCD}}}\right)^2\right]}$$

$\tilde{Q}_0$: fixes the virtuality separation scale $Q_0^2 \sim \tilde{Q}_0 \cdot \tau_{\text{hydro}} T_0^3$

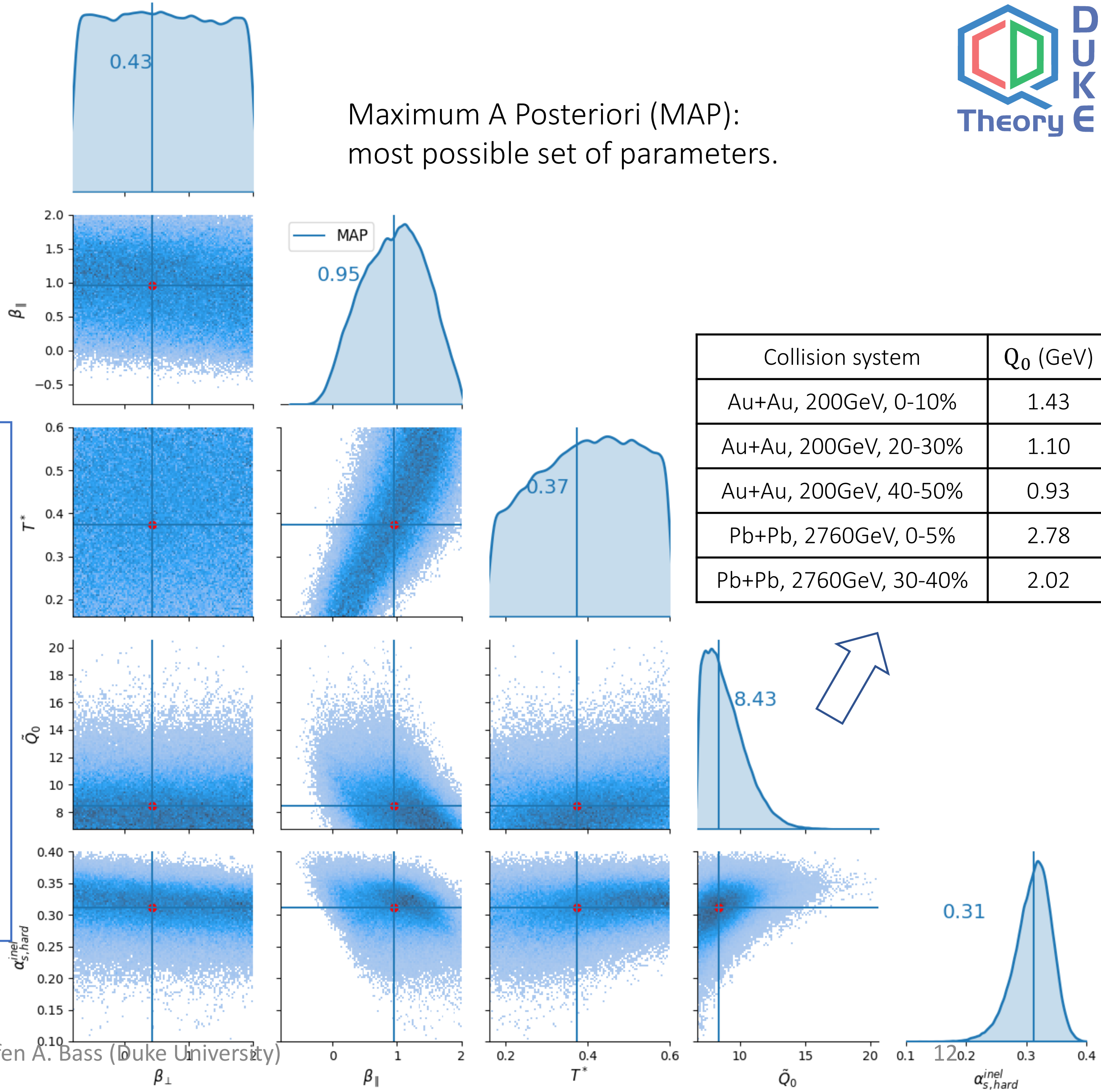


Parameterized transport coefficients reproduce the pQCD calculation at large energy scale.

Experimental data - posterior

Constrain model parameters using experimental data: Au+Au ($\sqrt{s_{NN}} = 200$ GeV) and Pb+Pb ($\sqrt{s_{NN}} = 2760$ GeV) at different centralities.

Maximum A Posteriori (MAP): most possible set of parameters.

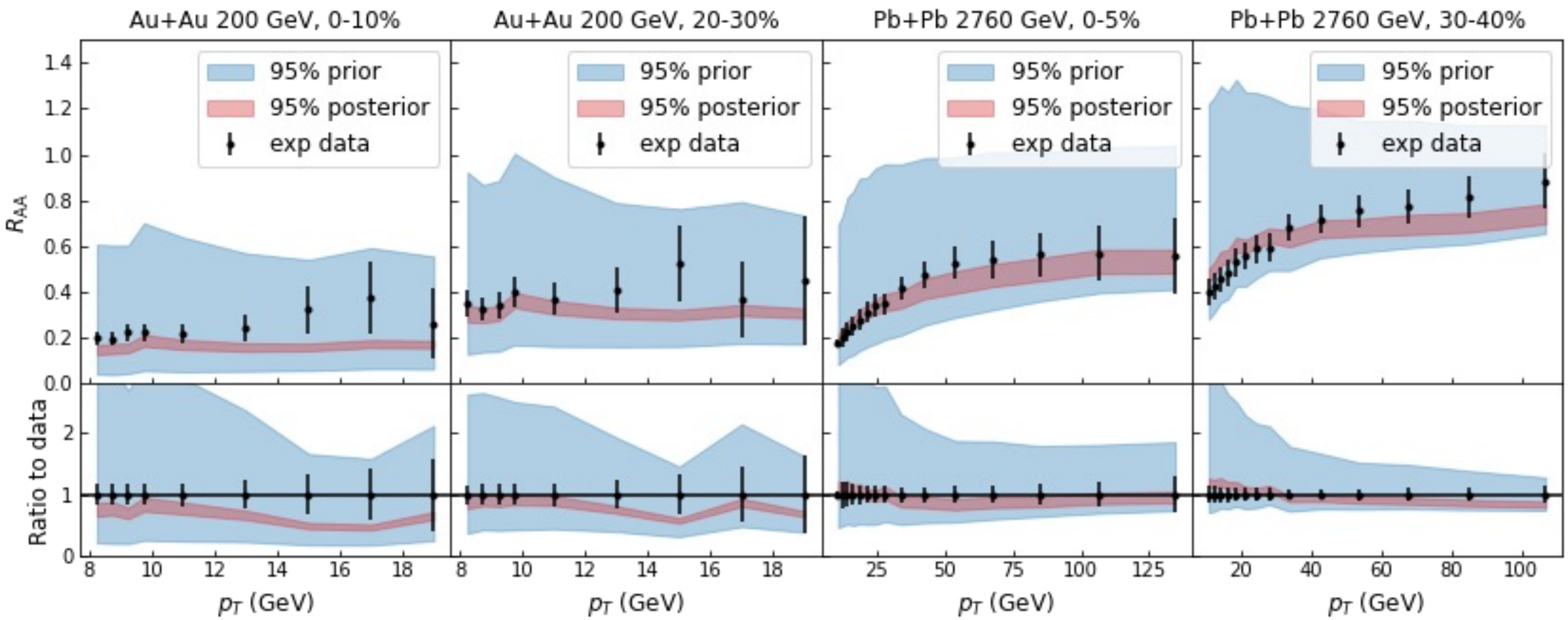


Collision system	Q_0 (GeV)
Au+Au, 200GeV, 0-10%	1.43
Au+Au, 200GeV, 20-30%	1.10
Au+Au, 200GeV, 40-50%	0.93
Pb+Pb, 2760GeV, 0-5%	2.78
Pb+Pb, 2760GeV, 30-40%	2.02

The posterior distribution constrained using Bayesian model-to-data comparison:

- R_{AA} is not sensitive to $\beta_\perp$: no constrain on $\beta_\perp$.
- MAP is $\beta_\parallel = 0.95$.
- Differs from pQCD assumption: $\beta_\parallel = 0$.
- Non-perturbative effects in soft interactions.
- $\beta_\parallel$ correlated to T^* : need additional observables to disambiguate.

Real data – observable emulation

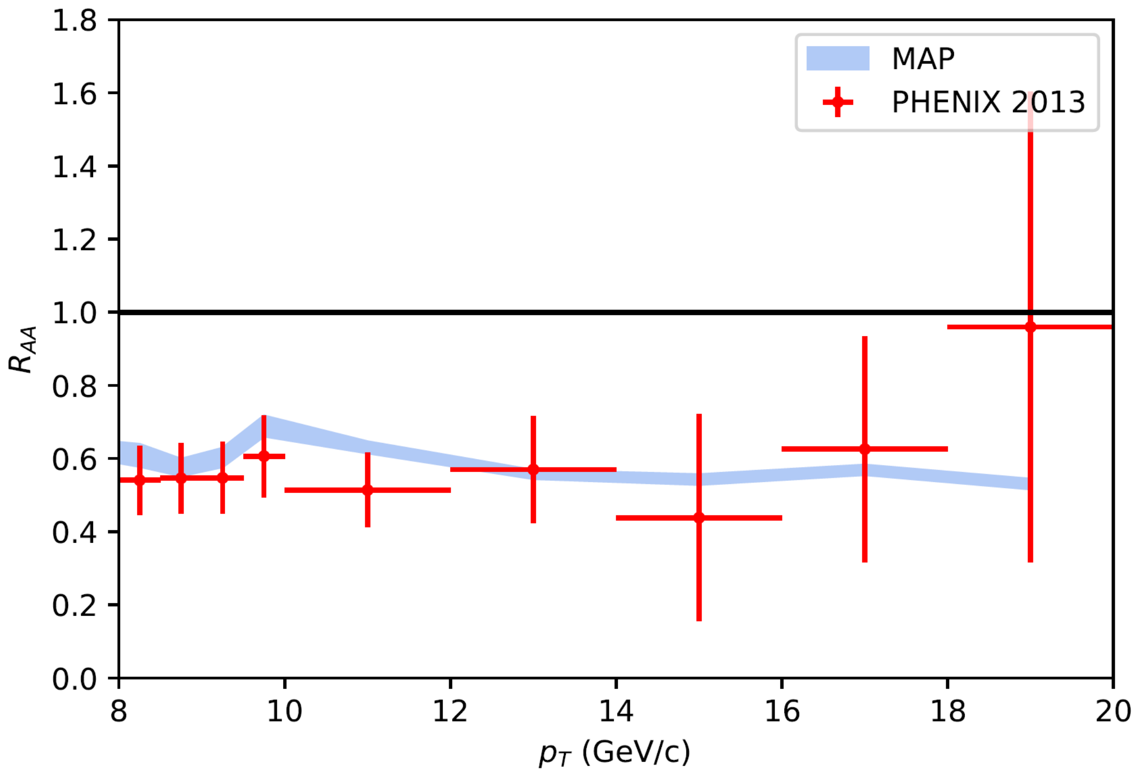


Model-to-data comparison constrain the large prior range to a small posterior range close to the experimental observation.

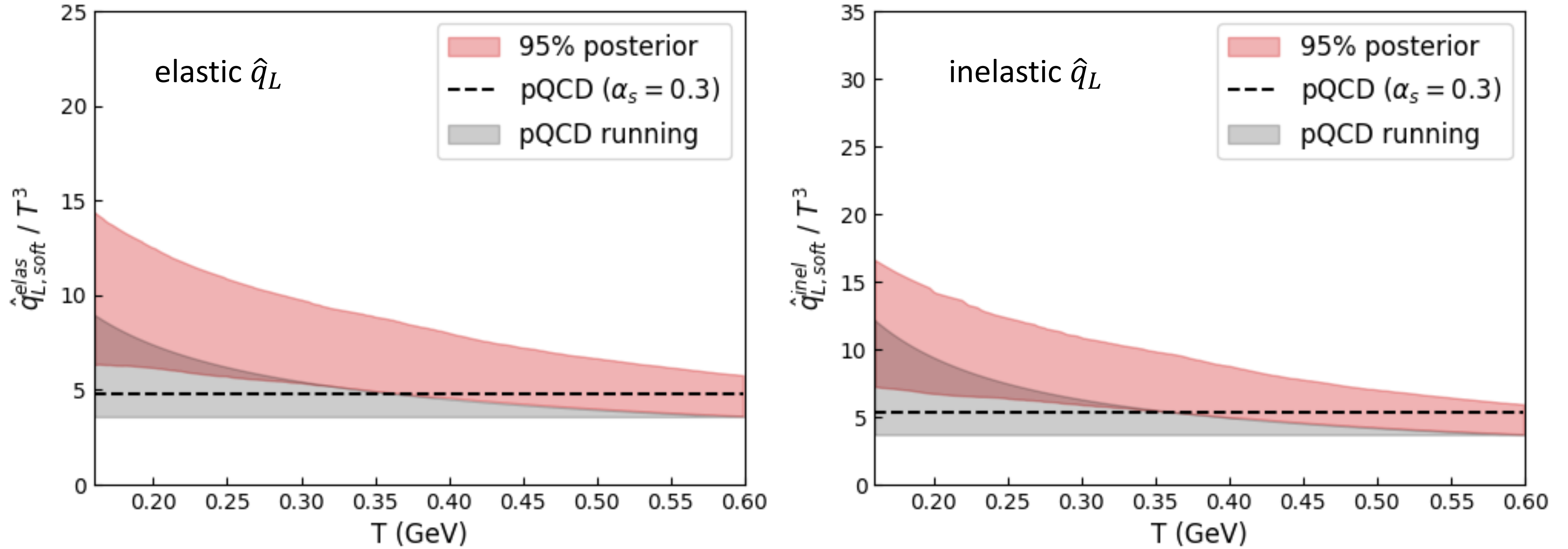
Predict experimental measurement

Apply the posterior model parameters to calculate new observable:
Au + Au collisions, 40-50% centrality.

Constrained model parameters can describe data not used in the analysis.



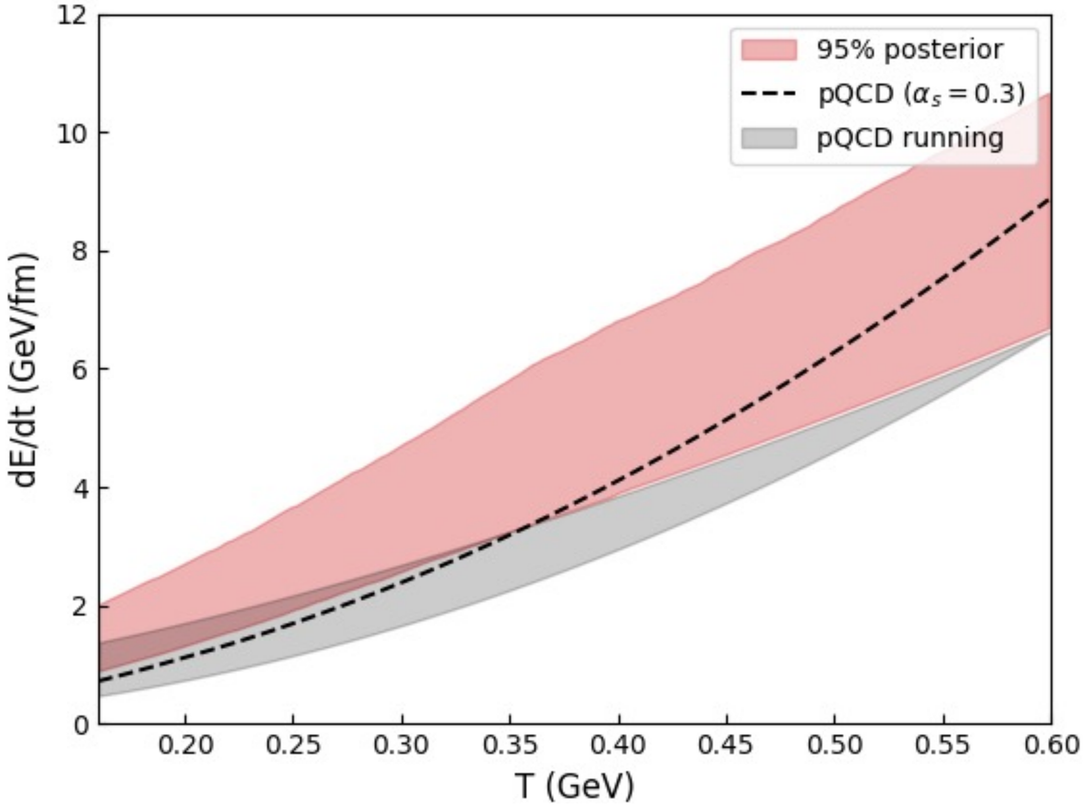
Properties of parton transport



Estimate the temperature dependence of the soft transport coefficients using the posterior of model parameters:

- Large prior range is constrained to small posterior range.
- Constrained soft $\hat{q}_L/T^3$ decreases slowly as the temperature increase.
- Above pQCD value: non-perturbative effects.

Properties of parton transport



Interactions with energy transfer $\omega < 4T$.

average energy loss per unit length of a parton traversing the QGP:

- calibration results show similar trends as the pQCD calculation

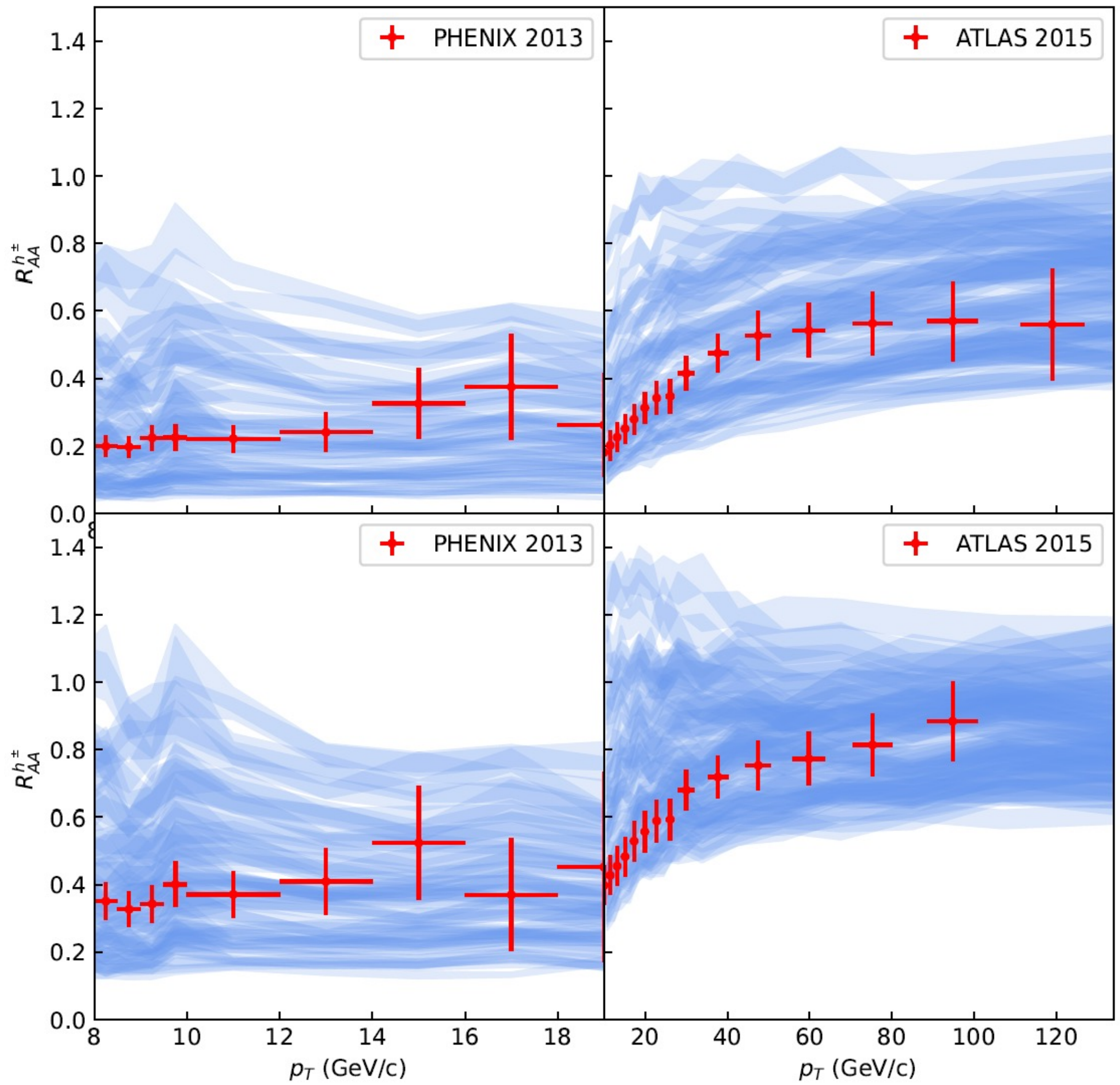
Conclusion & Outlook

- Factorize the soft interactions out as a drag and diffusion process.
- Bayesian model-to-data comparison to constrain drag and diffusion coefficients.
- Quantify the non-perturbative effects in soft interactions.

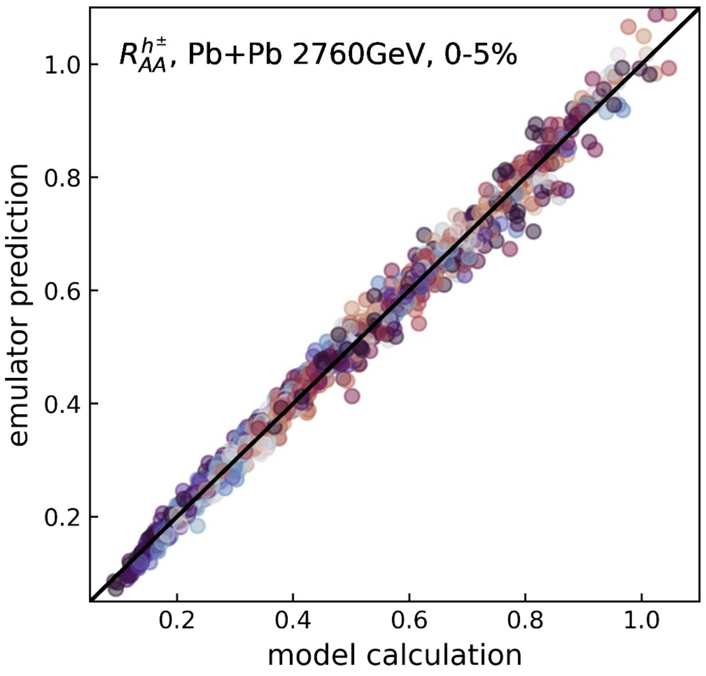
- Add more features to the hard-soft factorized model (e.g. include finite-size effect).
- More flexible parameterization of soft transport coefficients.
- Compare with more observables.

Backup Slides

Emulate the Monte Carlo model

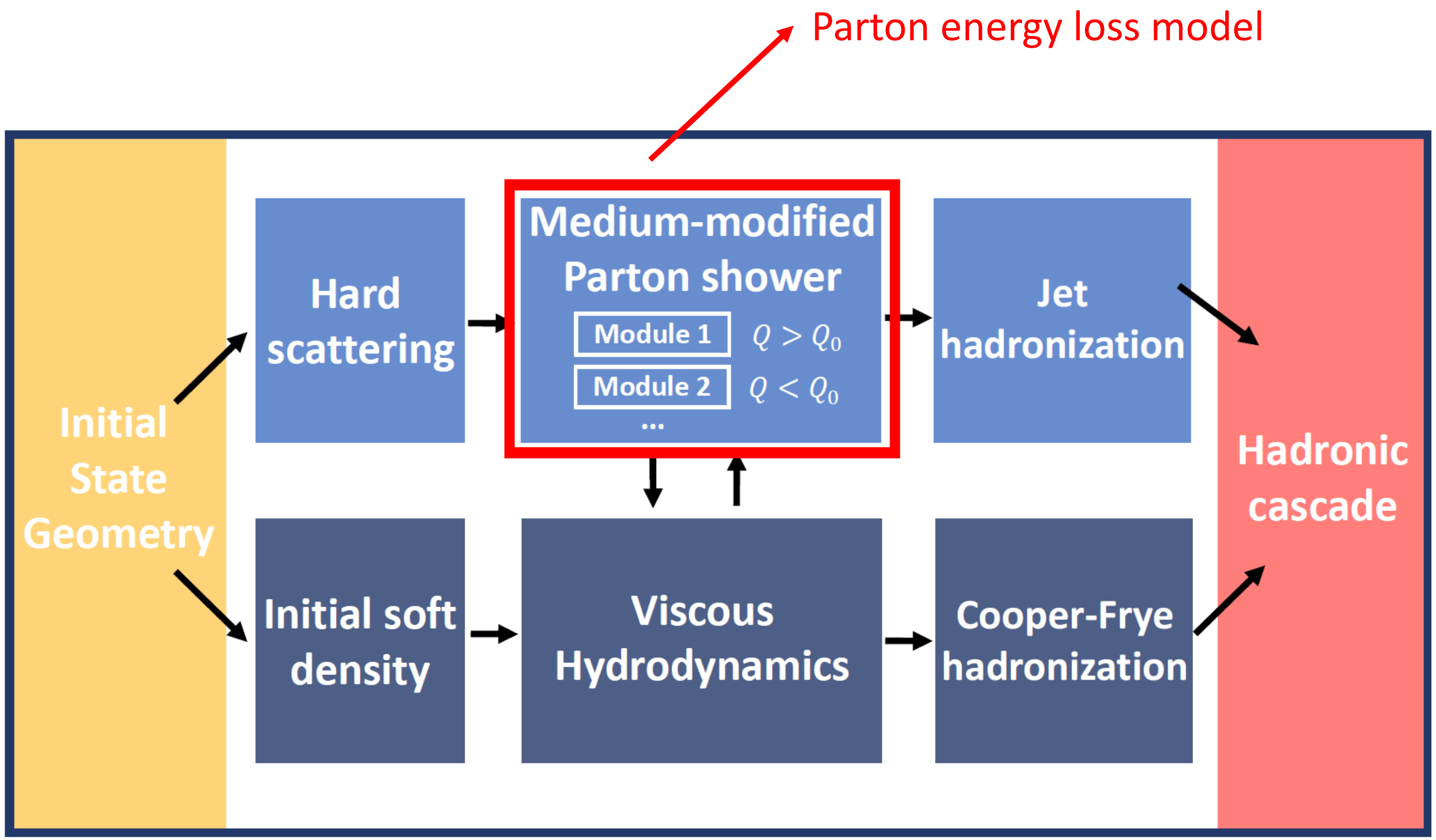


- Run hard-soft factorized model in JETSCAPE framework on the design points.
- The computationally expensive model requires a fast surrogate.
- The results on design points cover the data points.
- Design points should be enough for the emulator.



Validation of the model emulator: the Gaussian process emulator works well.

JETSCAPE framework



- A software framework to simulate the whole process of heavy ion collisions.
- A modular-based Monte Carlo event generator.
- An open-source package: https://github.com/JETSCAPE/JETSCAPE

Parton energy loss formalisms

Interference between neighboring scattering centers

- single hard scattering approximation
- multiple soft scattering approximation

Treatment of the underlying plasma

- static scattering centers
- dynamical entities

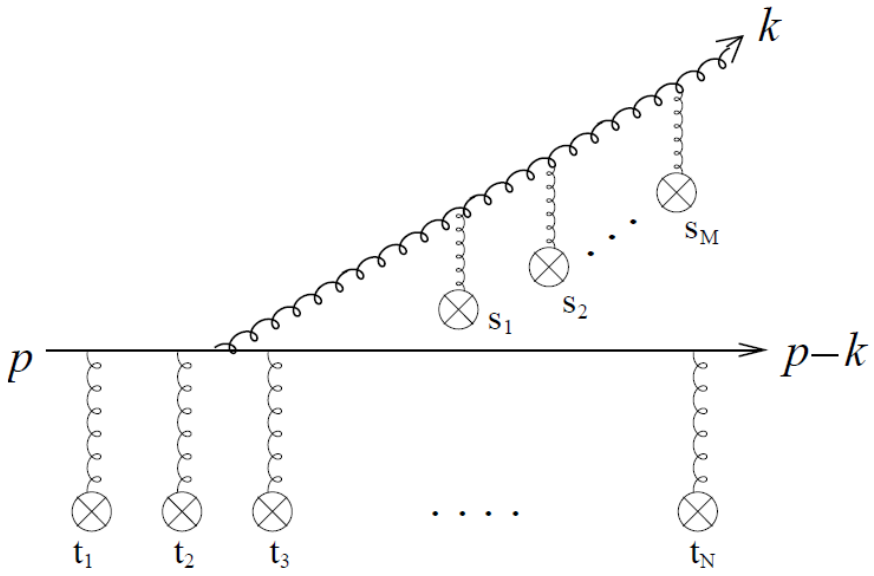


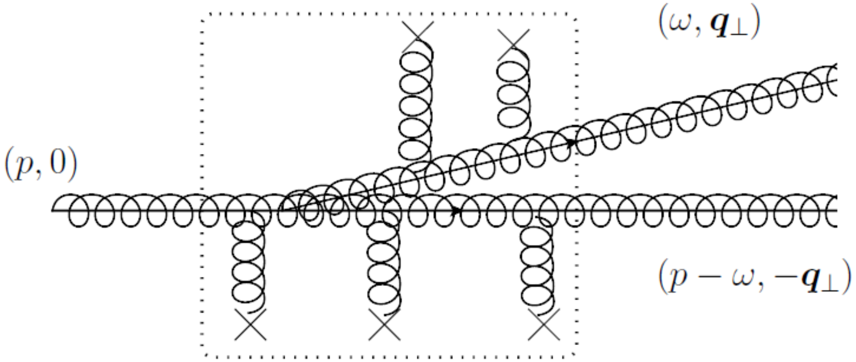
Figure: A typical diagram calculated using multiple soft scattering approximation.

- Strongly-coupled QGP: a quasi-particle description may not be justified
- Hard interactions: smaller non-perturbative effects
- Soft interactions: largest non-perturbative effects

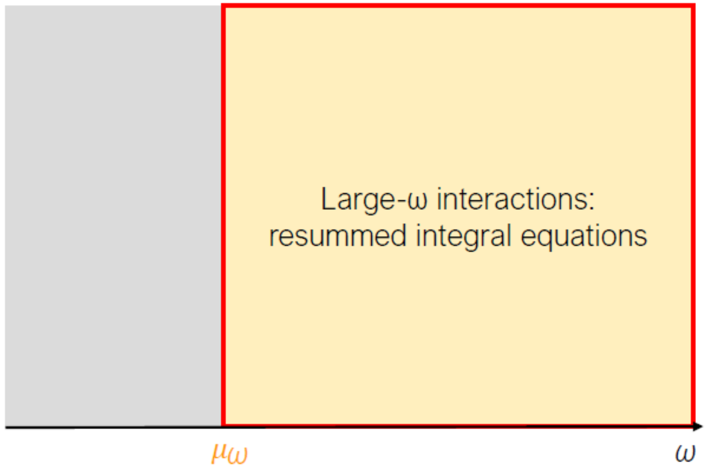
Hard $1 \leftrightarrow 2$ Interactions - Large ω Interactions

Multiple soft interactions with the plasma induce the collinear radiation of a parton of energy ω.

Soft scatterings are resummed and LPM effect is taken into account.

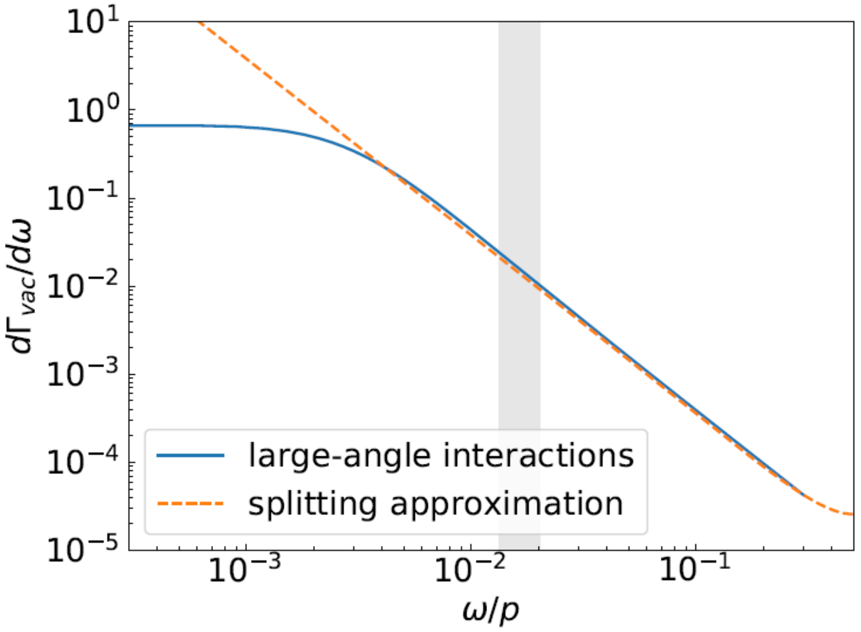


- $\omega > \mu_\omega$, $\mu_\omega \lesssim$ T
- Described with emission rates (obtained from AMY integral equations)
- Leading order calculation

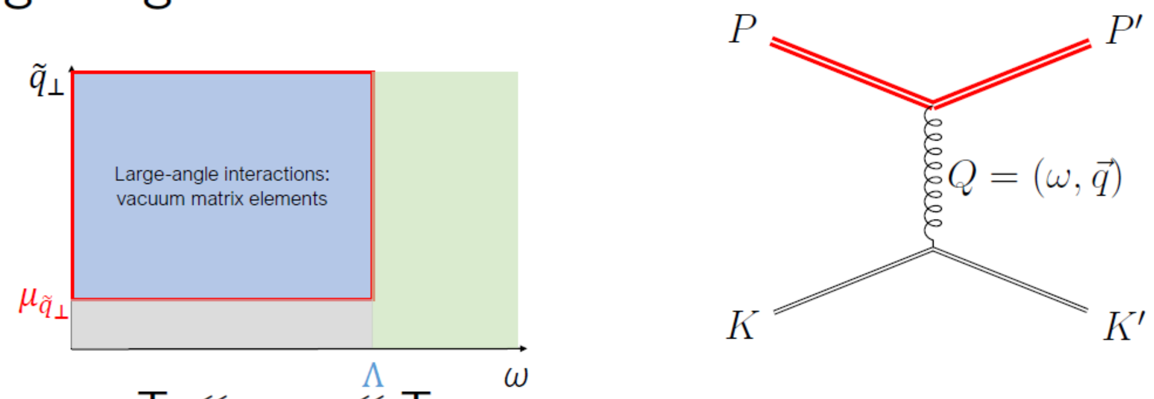


Hard 2 ↔ 2 Interactions

- Leading order vacuum pQCD matrix elements
- Keep to $\mathcal{O}(\frac{T}{p})$

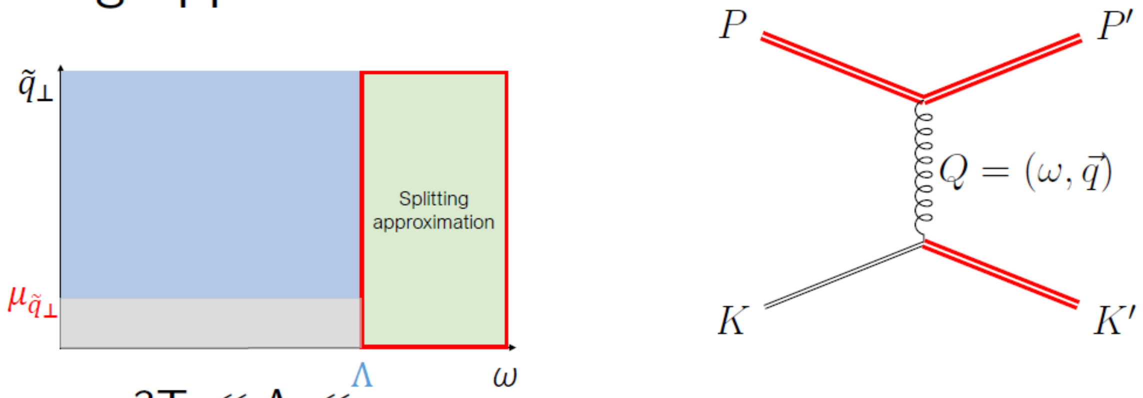


Large-angle interactions:



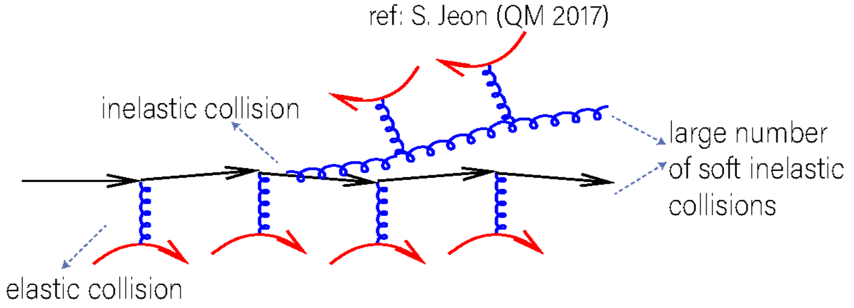
$gT \ll \mu_{\tilde{q}_\perp} \ll T$

Splitting approximation:



$3T \ll \Lambda \ll p$

When can we use diffusion?



Stochastic description of soft interactions

- Absorb non-perturbative effects
- Numerically more efficient
- Data-driven constraining

In pQCD, $\mu_\omega \lesssim T$, $gT < \mu_{\tilde{q}_\perp} < T$.
Beyond pQCD, how to choose the cutoffs?

Range of the Fokker-Planck equation applicability

Expand the Boltzmann equation:

$$\partial_t f(p,t) = \langle\omega\rangle f^{(1,0)}(p,t) + \frac{1}{2}\langle\omega^2\rangle f^{(2,0)}(p,t) + \frac{1}{6}\langle\omega^3\rangle f^{(3,0)}(p,t) + \ldots,$$

where $\langle\omega^k\rangle = \int d\omega \omega^k \frac{d\Gamma}{d\omega}$
For a valid Fokker-Planck description, we expect

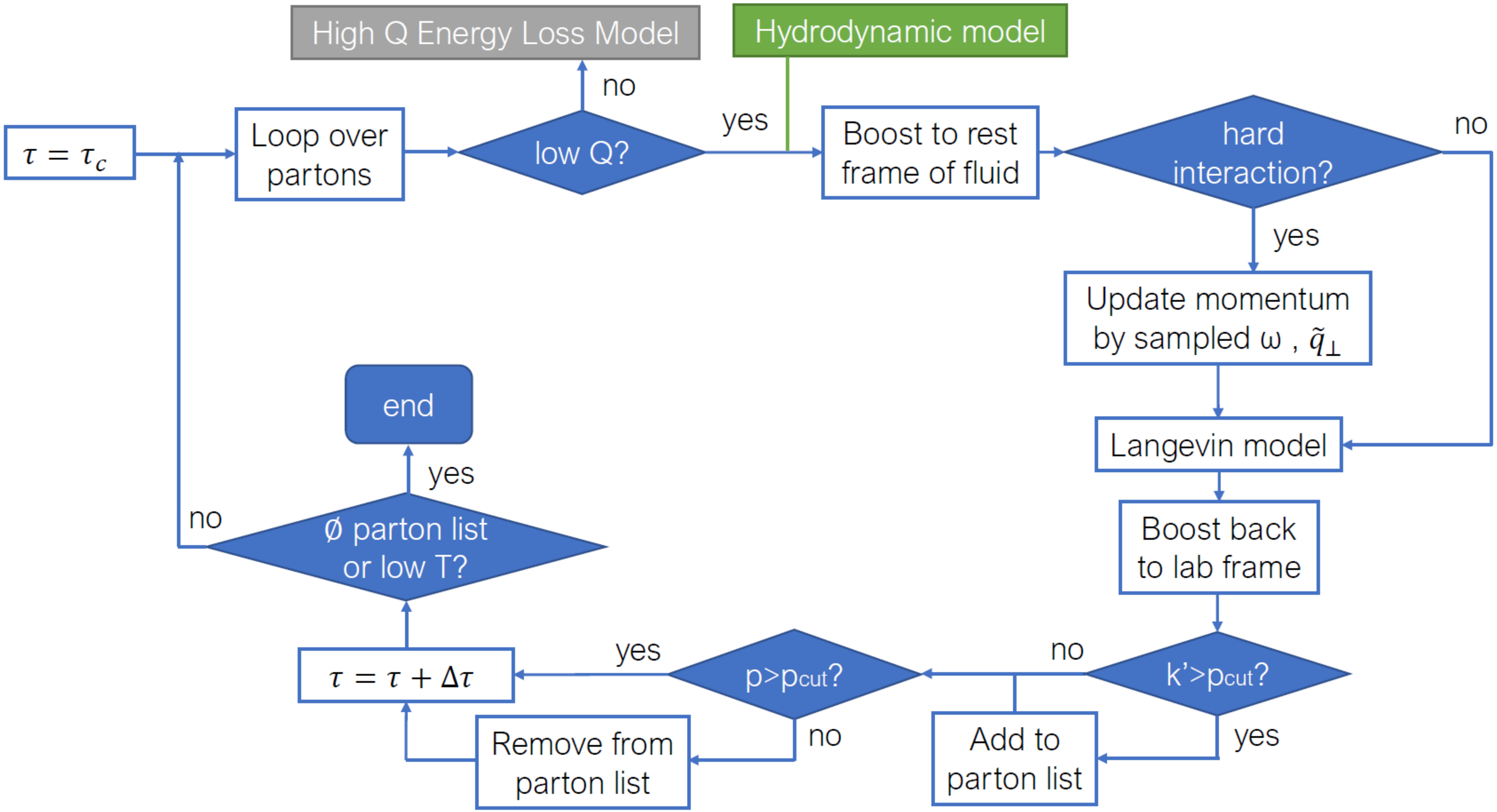
$$\mathcal{R} = \frac{\frac{1}{6}\langle\omega^3\rangle f_{FP}^{(3,0)}(p,t)}{\frac{1}{2}\langle\omega^2\rangle f_{FP}^{(2,0)}(p,t)} \ll 1$$

Define the scale (rough estimation):

$$\mathcal{S} = \frac{\langle\omega^3\rangle}{\langle\omega^2\rangle^{3/2}} \frac{1}{\sqrt{t}}$$

$S \ll 1$, valid stochastic description.

Numerical implementation in JETSCAPE



pcut: We only track hard partons with $p > p_{cut}$

Hard-soft factorized parton energy loss model

Transport of a high energy particle in QGP can be described using a Boltzmann function:

$$(\partial_t + \vec{v} \cdot \nabla_x) f^a(\vec{p}, \vec{x}, t) = -C[f]$$

$$C[\mathrm{f}] = \int d^3k [\omega(p+k, k) f(p+k) - \omega(p, k) f(p)]$$

Boltzmann function is linearized for high energy particles.

For small momentum transfer, Boltzmann equation reduce to Fokker-Planck equation:

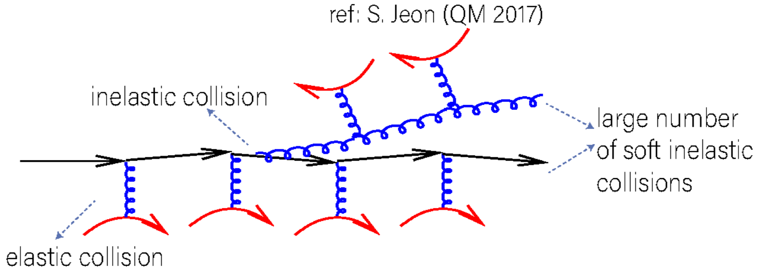
$$C^{\mathrm{diff}}[f] = -\frac{\partial}{\partial p^i}\left[\eta_D(p) p^i f(p)\right] - \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i \hat{p}^j \hat{q}_L(p) + \frac{1}{2}\left(\delta^{ij} - \hat{p}^i \hat{p}^j\right)\hat{q}(p)\right) f(p)\right]$$

Soft transport coefficients: η_D, $\hat{q}_L$, $\hat{q}$.

Fokker-Planck equation can be realized using Langevin model:

$$\frac{\Delta x}{\Delta t} = \frac{p}{E},$$

$$\frac{\Delta p}{\Delta t} = -\eta_D p + F^{\mathrm{thermal}}(t).$$



AMY resumed integral

$$\left.\frac{d\Gamma(p,\omega)}{d\omega}\right|^{1\leftrightarrow 2} = \frac{g^2}{16\pi p^3 \omega^2 (p-\omega)^2}\left[1 \pm n(\omega)\right]\left[1 \pm n(p-\omega)\right]$$

$$\times P^a_{bc}(z) \int \frac{d^2h}{(2\pi)^2} 2\mathbf{h} \cdot \mathrm{Re}\mathbf{F}(\mathbf{h},p,\omega)$$

$$P^a_{bc}(z) = \begin{cases} C_F \dfrac{1+(1-z)^2}{z}, & q \to gq \\ C_A \dfrac{1+z^4+(1-z)^4}{z(1-z)}, & g \to gg\ . \\ \dfrac{d_F C_F}{d_A}\left[z^2+(1-z)^2\right], & g \to q\bar{q} \end{cases}$$

$$2\mathbf{h} = i\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h}) + g_s^2 \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2}\mathcal{C}(\mathbf{q}_\perp)\left\{(C_s - C_A/2)\left[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-k\mathbf{q}_\perp)\right]\right.$$

$$\left. + (C_A/2)\left[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}+P\mathbf{q}_\perp)\right] + (C_s - C_A/2)\left[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-(p-k)\mathbf{q}_\perp)\right]\right\}.$$

Hard-soft factorization of parton energy loss

Weakly-coupled effective kinetic formalism

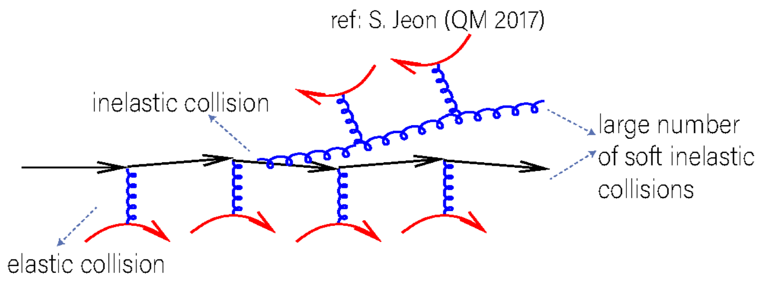
Leading-order realizations (e.g. MARTINI):

$$(\partial_t + \vec{v}\cdot\nabla_x) f^a(\vec{p},\vec{x},t) = -\mathcal{C}_a^{2\leftrightarrow 2}[f] - \mathcal{C}_a^{1\leftrightarrow 2}[f]$$

Hard-soft factorization:

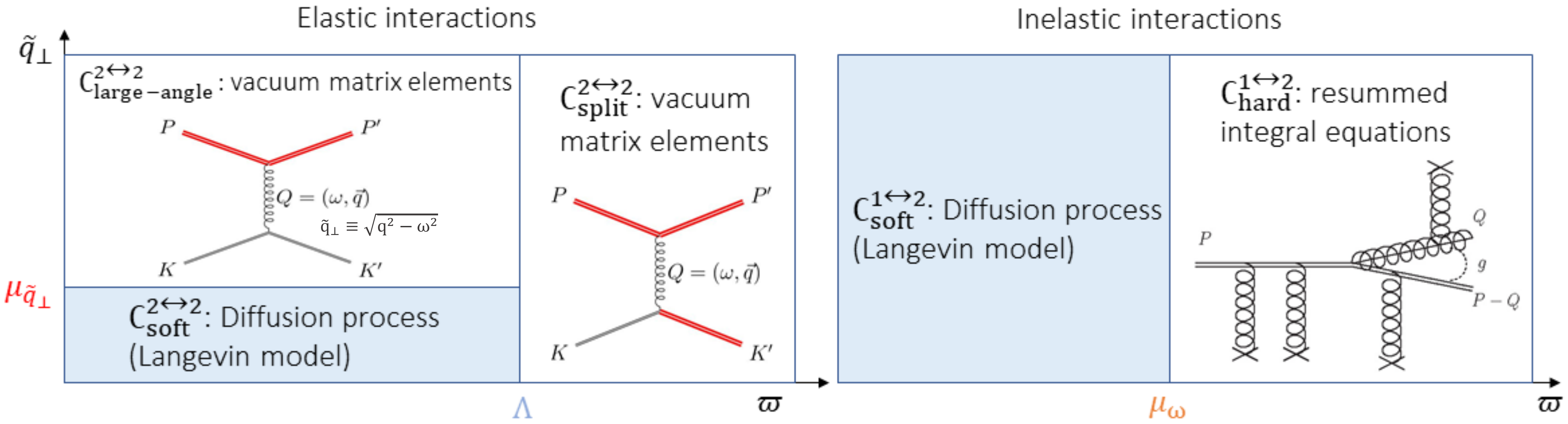
$$\mathcal{C}_a^{2\leftrightarrow 2} + \mathcal{C}_a^{1\leftrightarrow 2}$$

$$= \mathcal{C}_a^{\text{large-angle}}(\mu_{\tilde{q}_\perp}, \Lambda) + \mathcal{C}_a^{\text{split}}(\Lambda) + \mathcal{C}_a^{\text{large-}\varpi}(\mu_\varpi) + \mathcal{C}_a^{\text{diff}}(\mu_{\tilde{q}_\perp}, \mu_\varpi)$$



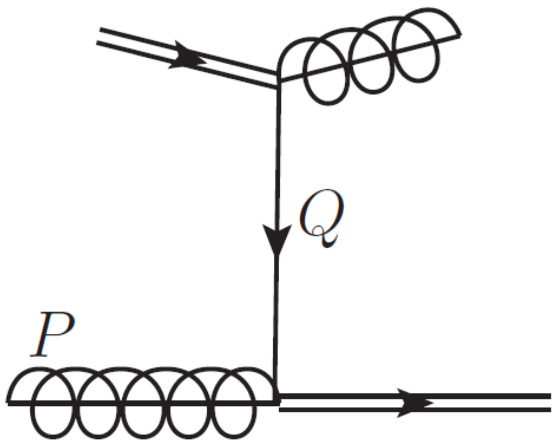
Interactions with the medium:

- Large number of soft interactions
- Rare hard scatterings



Identity **Non-preserving** Soft Interactions - Soft Conversion

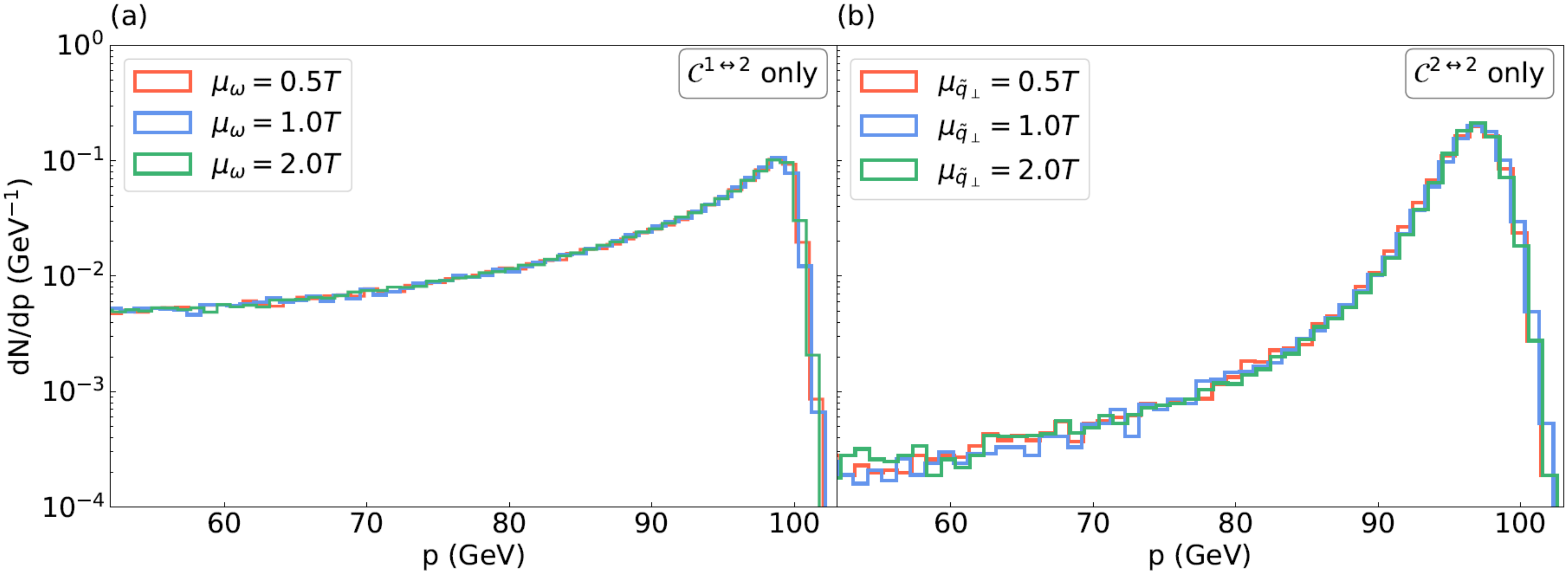
- Hard conversion is considered in rate-based approach
- Soft elastic conversion: soft fermion exchange
- Change parton identity by conversion rate
- Suppressed by T/p, include identity exchange but neglect energy loss



Hard-soft cutoff dependence of energy distribution

- $\mathcal{C}_{\rm soft} + \mathcal{C}_{\rm hard}$
- vacuum matrix elements for $\mathcal{C}_{\rm hard}{}^{2\leftrightarrow 2}$

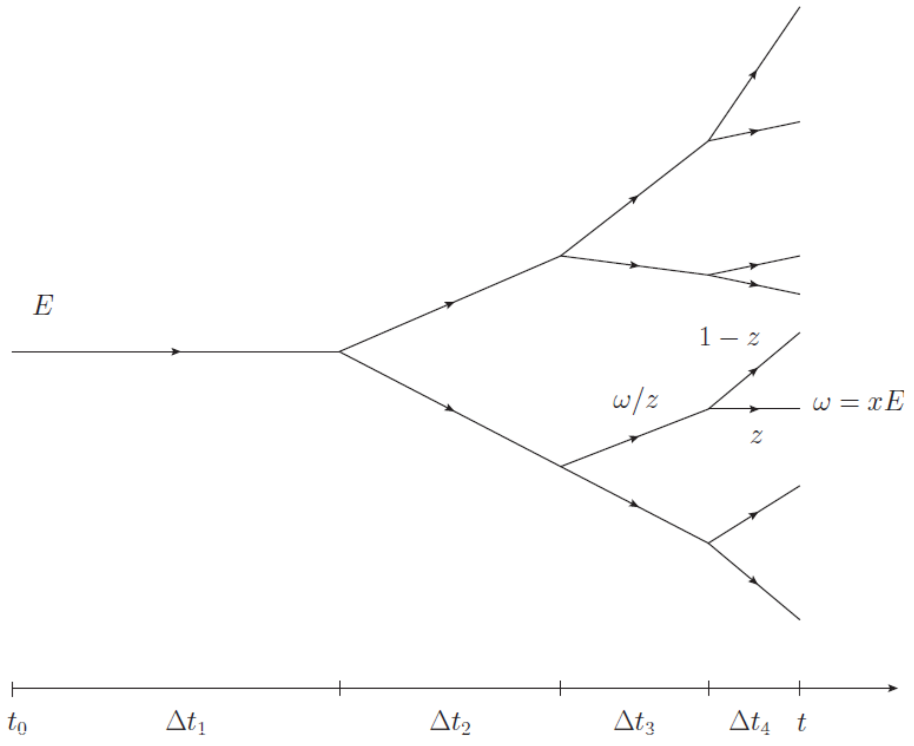
QGP medium, $\alpha_s = 0.005$, $T = 300$ MeV, $E_0 = 100$ GeV, time $t = (0.3/\alpha_s)^2$ fm/c



Parton energy distribution is independent on the hard-soft cutoffs.

In-medium gluon energy cascade

Successive medium induced quasi-democratic emissions
⇒ accumulation of gluons at thermal energy
⇒ a power-law scaling in the small energy region



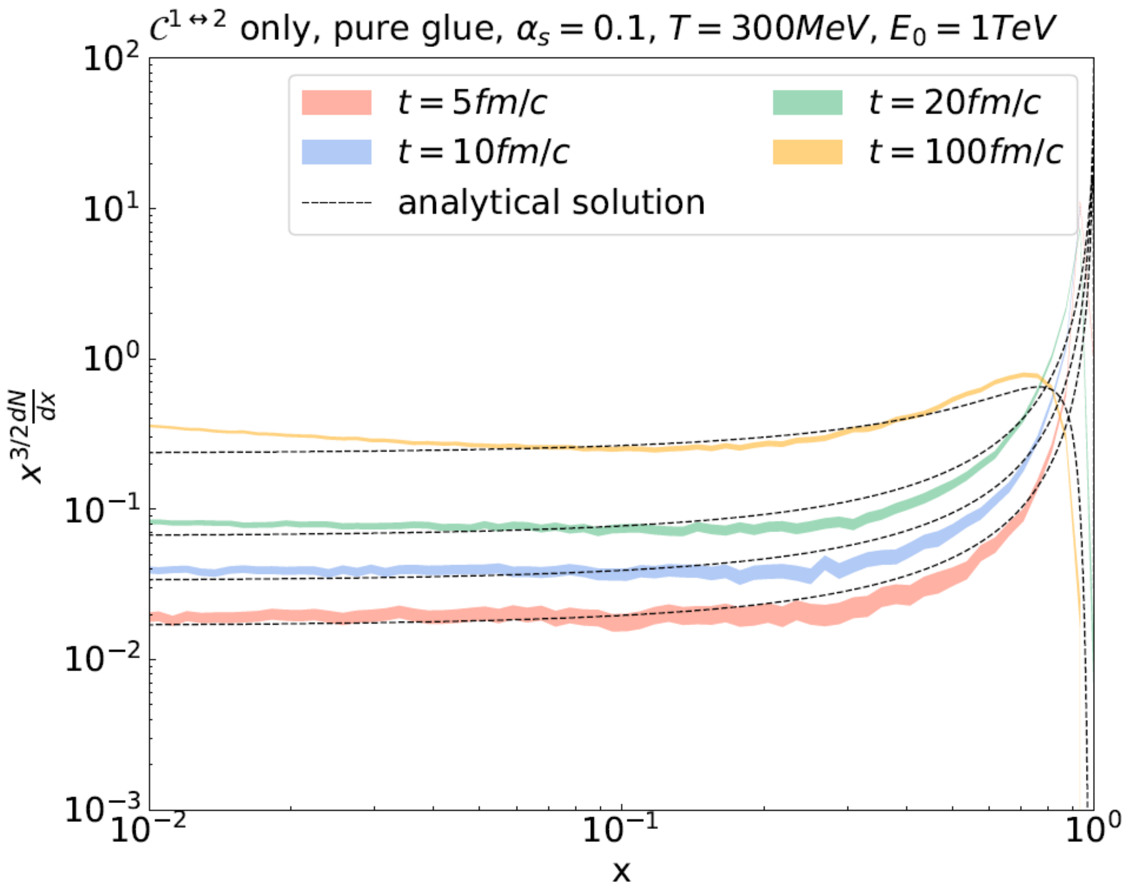
Blaizot, Iancu, Mehtar-Tani. 2013

Assumptions:

- independent successive branchings
- approximate inelastic differential rate valid in deep LPM region

$$\left.\frac{d\Gamma}{dz}\right|_{g \leftrightarrow gg} = \frac{\alpha_s N_c}{\pi} \frac{1}{[z(1-z)]^{3/2}} \sqrt{\frac{\hat{q}_{\text{eff}}}{p}}$$

In-medium gluon energy cascade - numerical comparison



Analytical: Blaizot, Iancu, Mehtar-Tani. 2013
Numerical: Dai, Paquet, Teaney, Bass. 2022

Analytical distribution of gluons:

$$x\frac{dN}{dx} = \frac{\tau}{\sqrt{x}(1-x)^{3/2}} e^{-\pi[\tau^2/(1-x)]}$$

where $x \equiv \omega/E_0$ and $\tau \equiv \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{E}}t$.

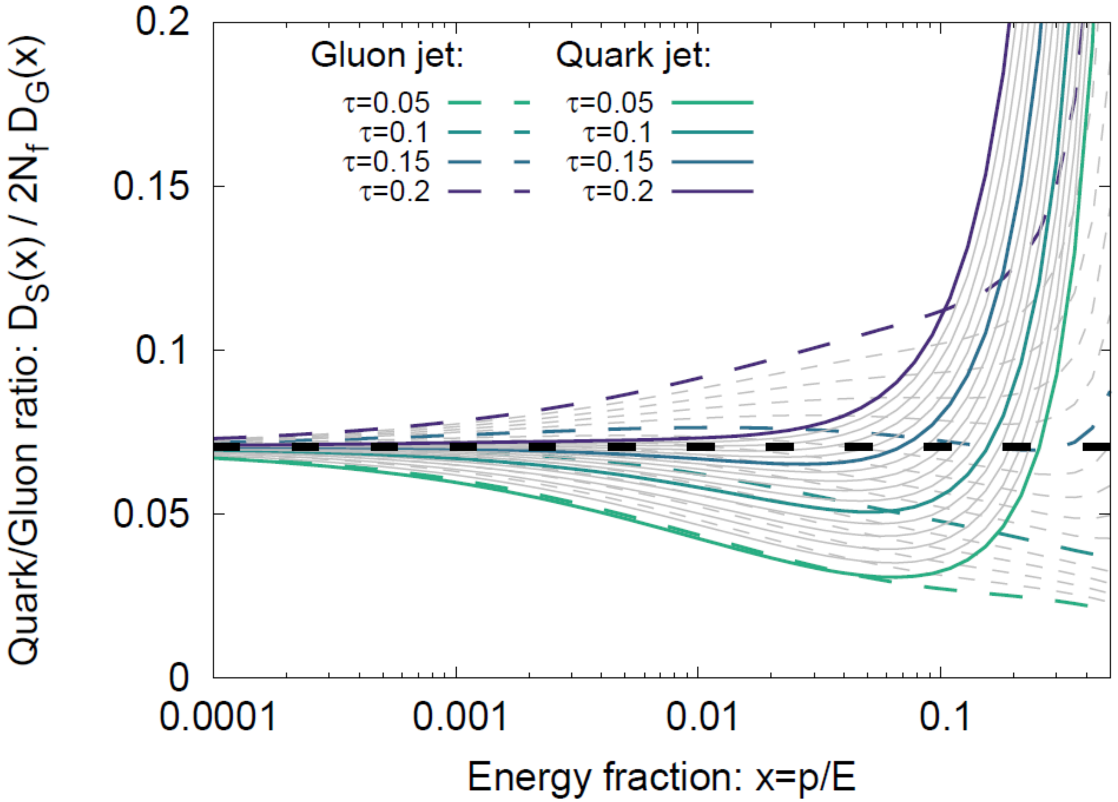
- In the small-x region, xdN/dx scales as $1/\sqrt{x}$.
- The analytical solution is well-reproduced by the full QCD numerical model.

In-medium fermion number cascade - analytical approximation

In the small-x and large-τ region, the quark to gluon ratio of the soft fragments:

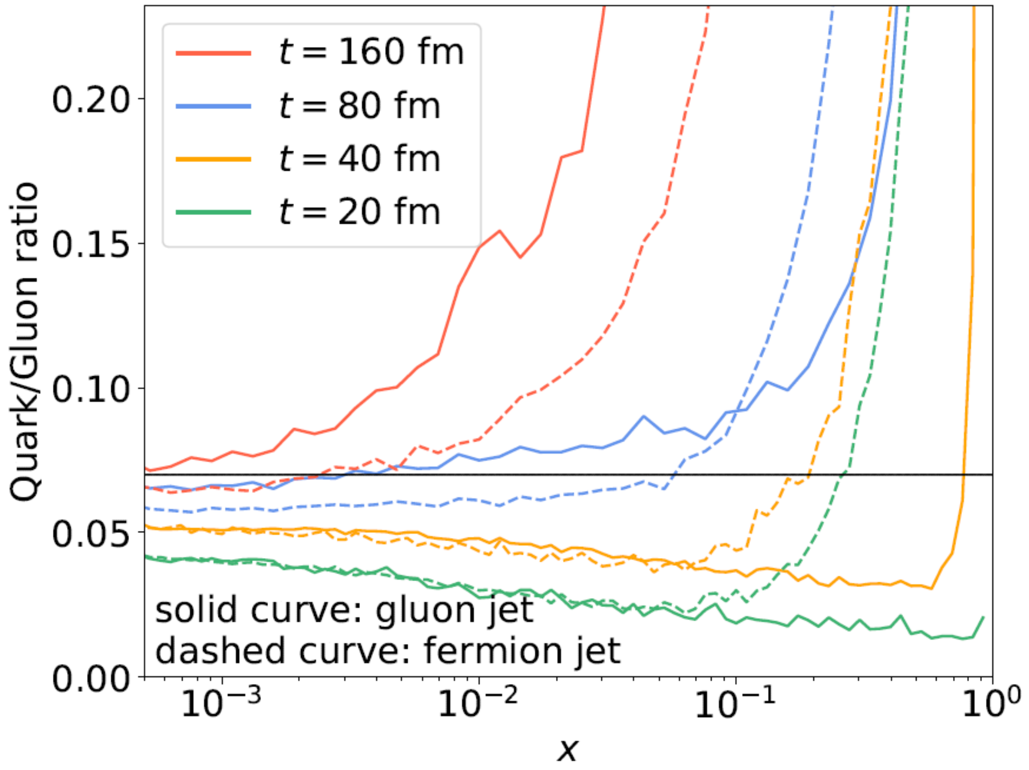
$$\frac{Q}{2N_f G} = \frac{1}{2N_f}\frac{\int_0^1 dz z\mathcal{K}_{qg}(z)}{\int_0^1 dz z\mathcal{K}_{gq}(z)} \approx 0.07$$

$G \equiv \sqrt{x} D_g(x) = x^{3/2} dN_g/dx$
$Q \equiv \sqrt{x} \sum_{i=1}^{N_F} \left(D_{q_i} + D_{\bar{q}_i}\right)$
$\mathcal{K}_{qg}$, $\mathcal{K}_{gq}$: the splitting function of $g \to q\bar{q}$ and $q \to gq$



Mehtar-Tani and Schlichting. 2018

In-medium fermion number cascade - numerical comparison



$\mathcal{C}^{1\leftrightarrow 2}$ only, $N_f = 3$, $\alpha_s = 0.3$
$T = 300$ MeV, $p_0 = 10$ TeV.

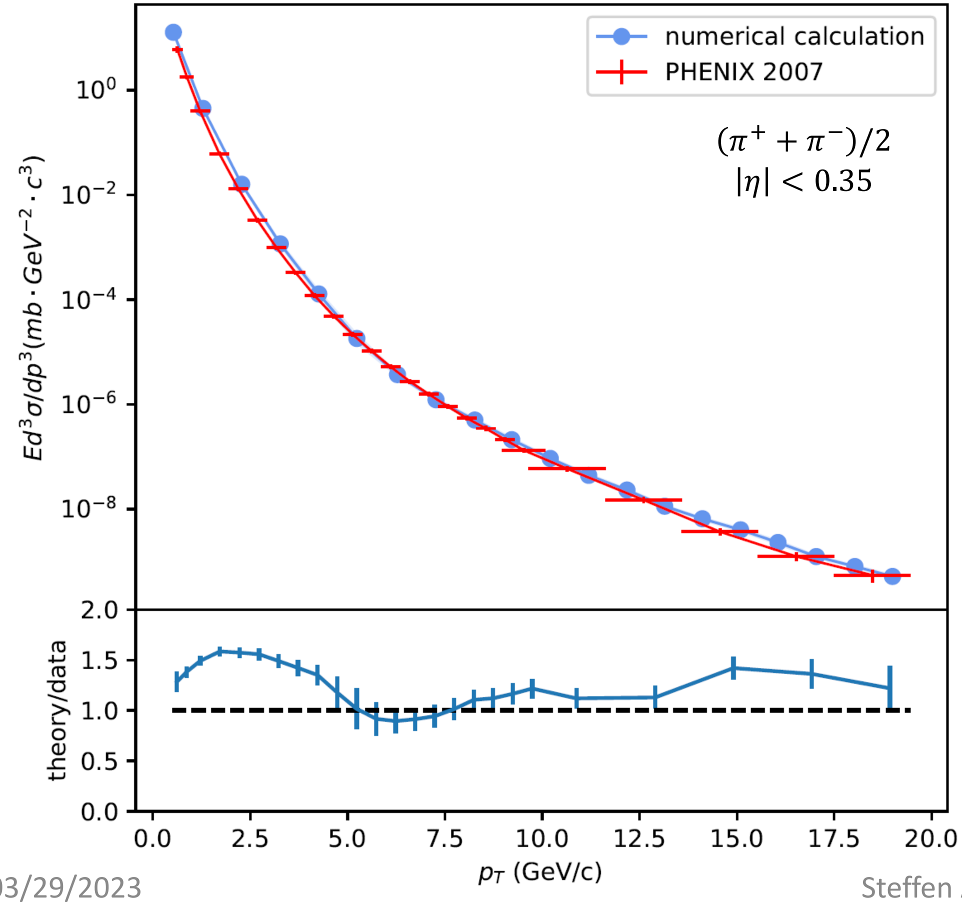
$$\frac{D_S}{2N_f D_g} = \frac{1}{2N_f} \frac{\int_0^1 dz z \mathcal{K}_{qg}(z)}{\int_0^1 dz z \mathcal{K}_{gq}(z)} \approx 0.07$$

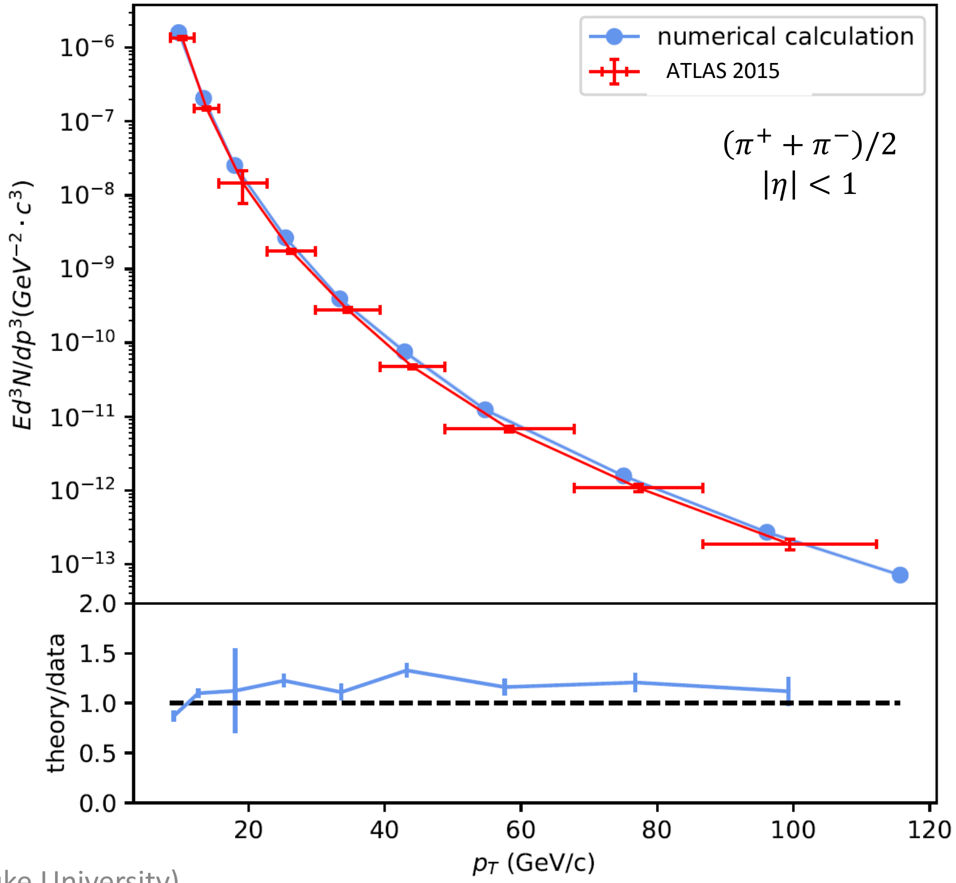
In the numerical calculation, the quark to gluon ratio also reaches to the same constant constraint, for either gluon jet or fermion jet.

Proton-proton baseline

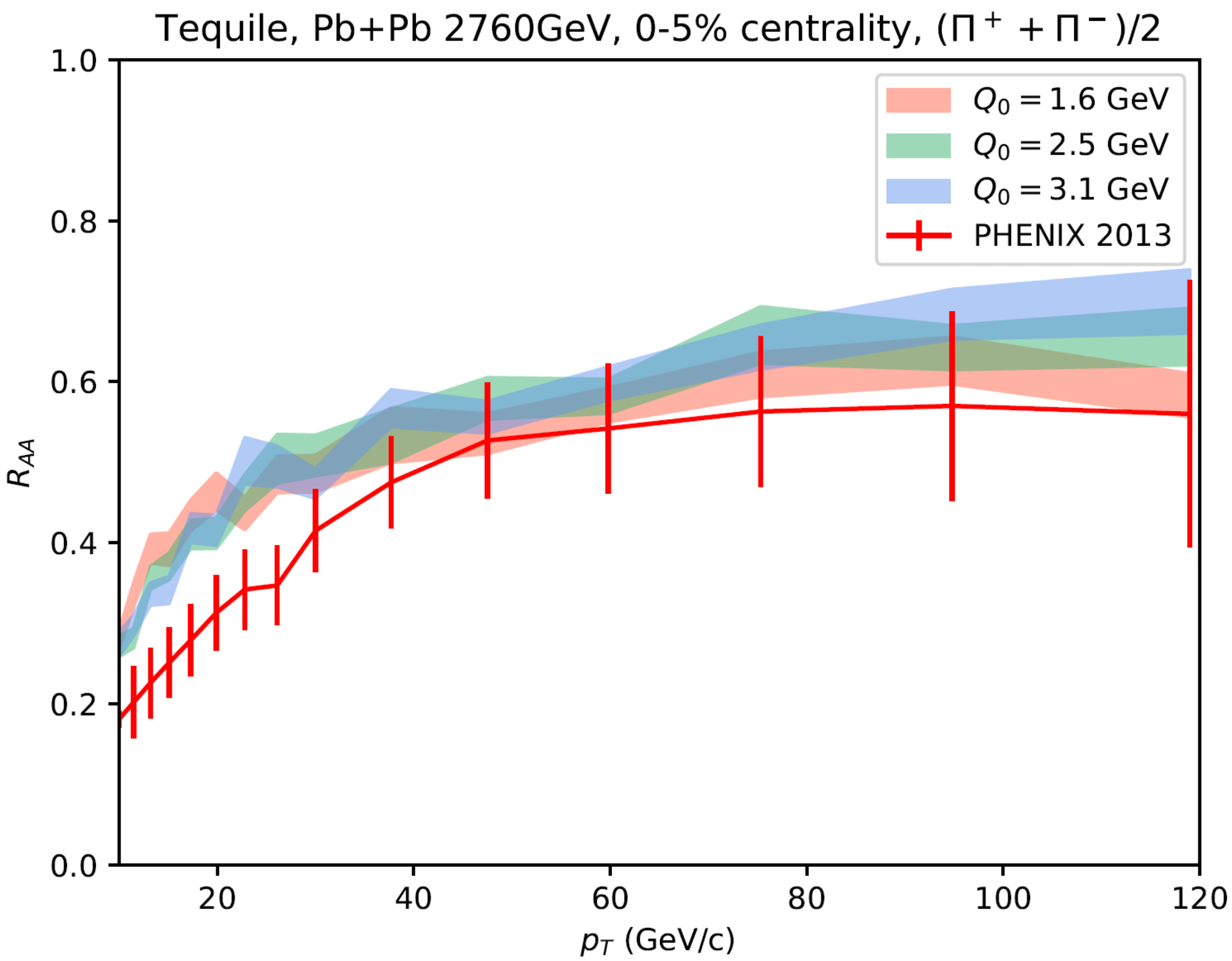
JETSCAPE PP19 tune

- Initial collisions: PYTHIA
- Parton evolution in vacuum: MATTER in vacuum





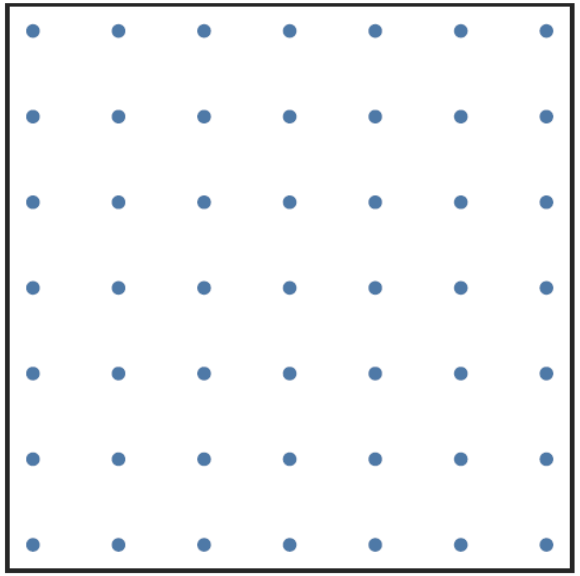
PbPb RAA with different Q0



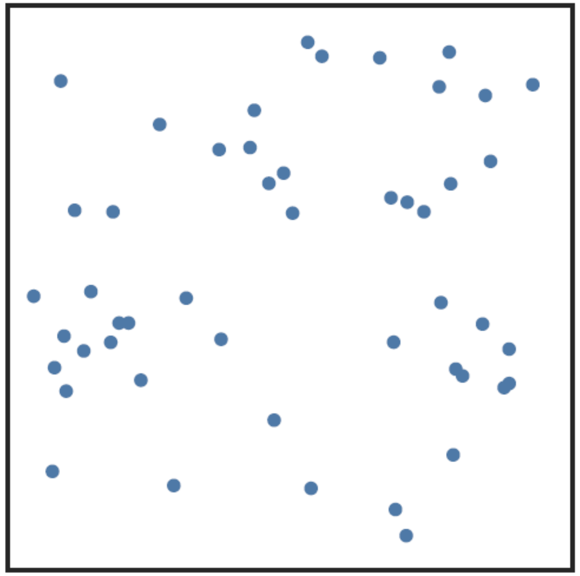
Sampling in parameter space

How to distribute points in parameter space for optimal emulator performance?

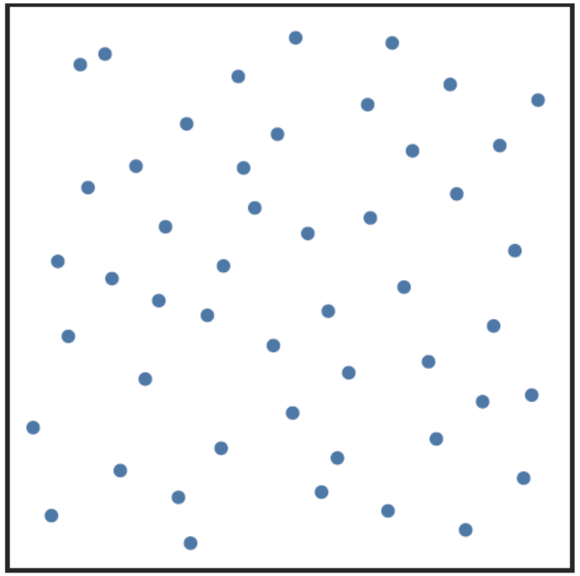
Factorial (uniform grid)



Random



Latin hypercube

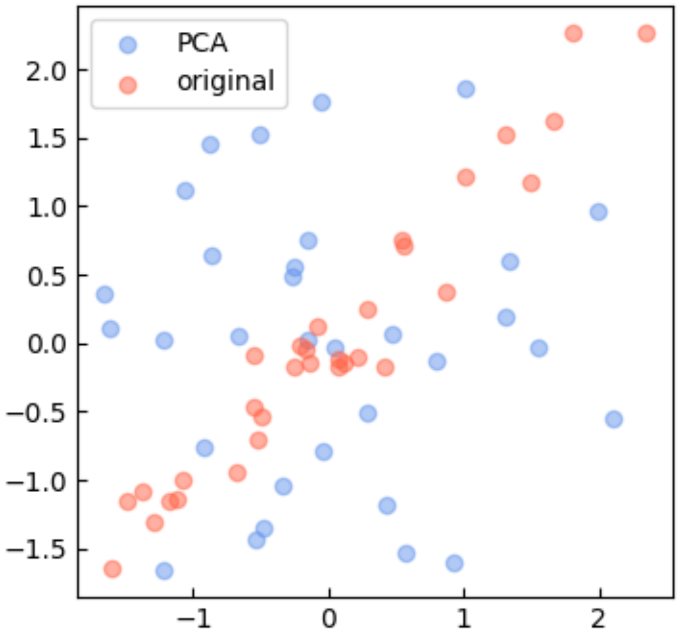


Required number of points grows *exponentially* — not viable in high dimensions

Tends to create large gaps and tight clusters

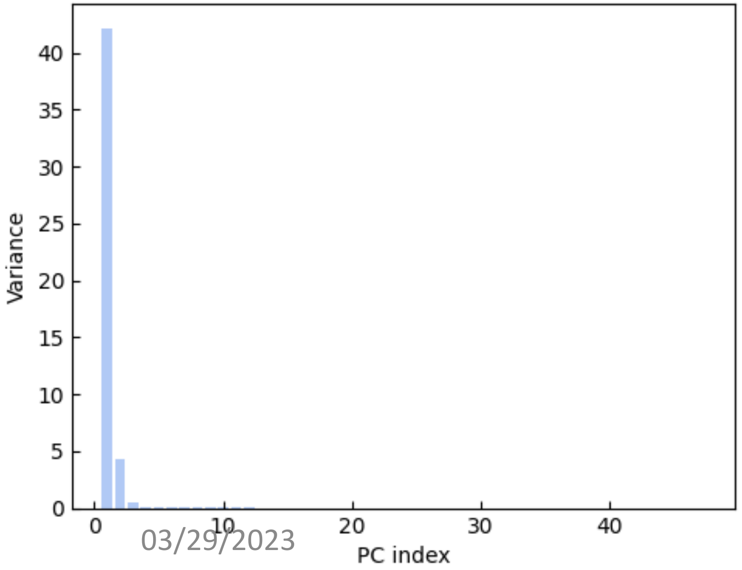
- Semi-random, space-filling
- Required number of points grows *linearly*
- “Efficient scaffolding” for emulator

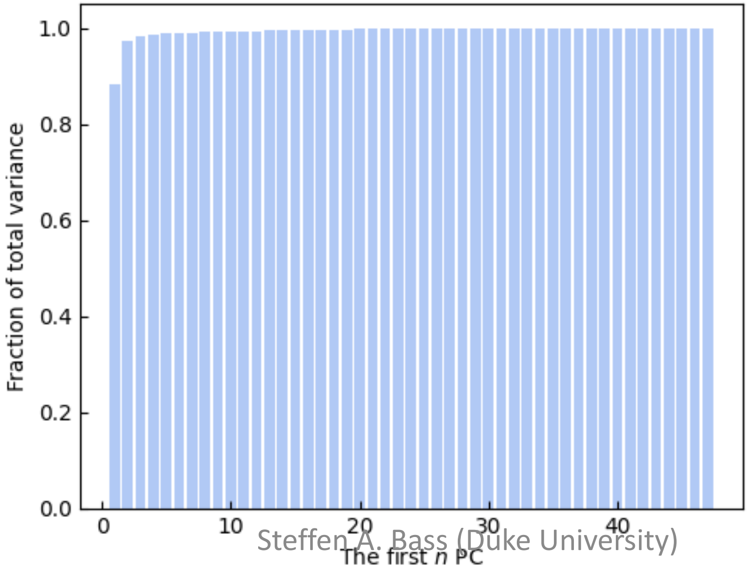
Principal component analysis of model outputs



PCA to reduce high dimensional output space:

- Multi-dimensional of the model outputs.
- Gaussian process generates 1-d output.
- Model outputs are highly-correlated.
- Model uncertainties is uncorrelated.





SVD decomposition of the model outputs.

$$\tilde{Y}_{n\times m} = U_{n\times n} S_{n\times m} V^T_{m\times m}$$

Diagonal of S encodes PC's varaince.

Cumulative variance to estimate how much information is preserved by the first few PCs.

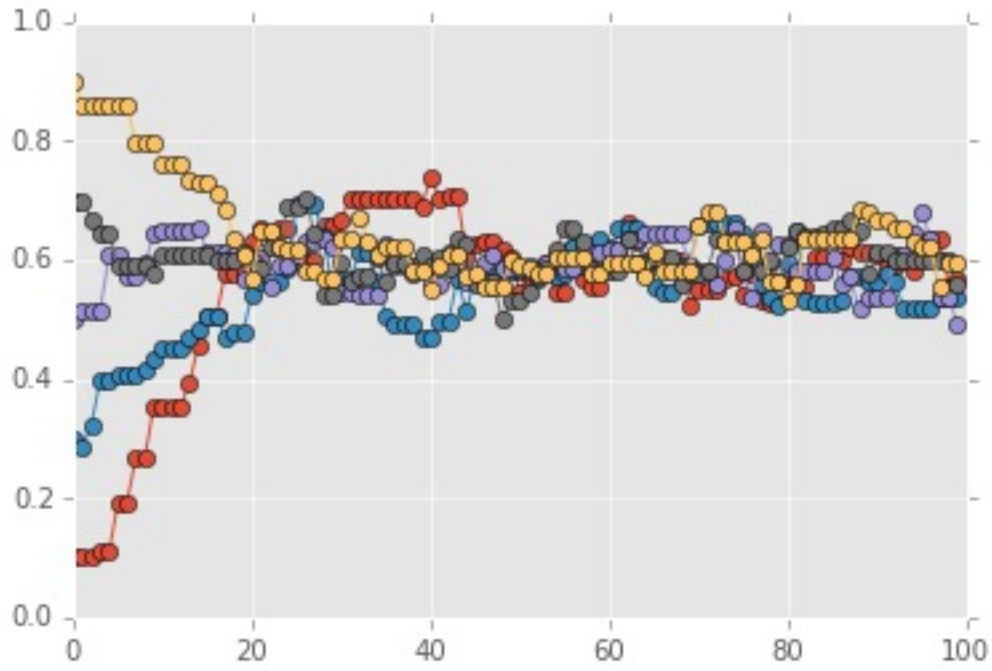
Posterior of model parameters

Markov chain Monte Carlo (MCMC) to sample the posterior of the parameter:

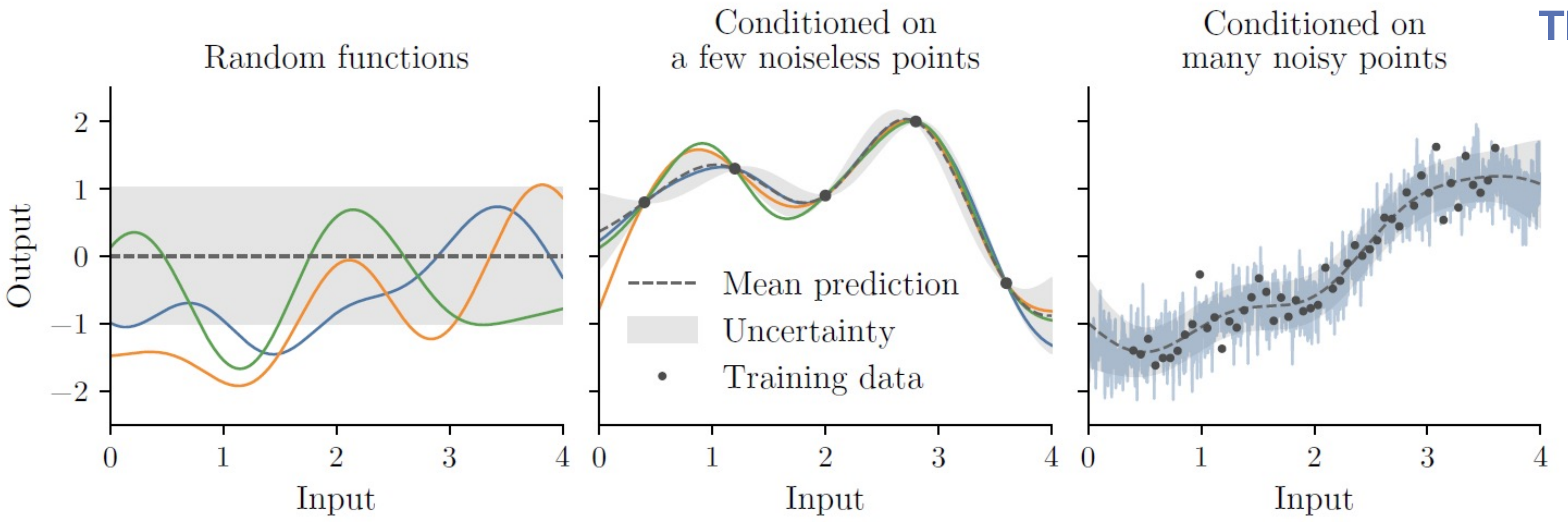
- The sampler performs weighted random walk in the parameter space.
- At each state x_t, a new position x_{new} is sampled.
- The acceptance rate of x_{new} is

$$r(x_t, x_{new}) = \min\left(1, \frac{\mathcal{L}(y|x_{new})P'(x_{new})}{\mathcal{L}(y|x_t)P'(x_t)}\right).$$

- After the "burn-in" steps, the distribution of the accepted samples is the posterior distribution.



Gaussian process emulator



A non-parametric regressor to fast-predict model outputs:

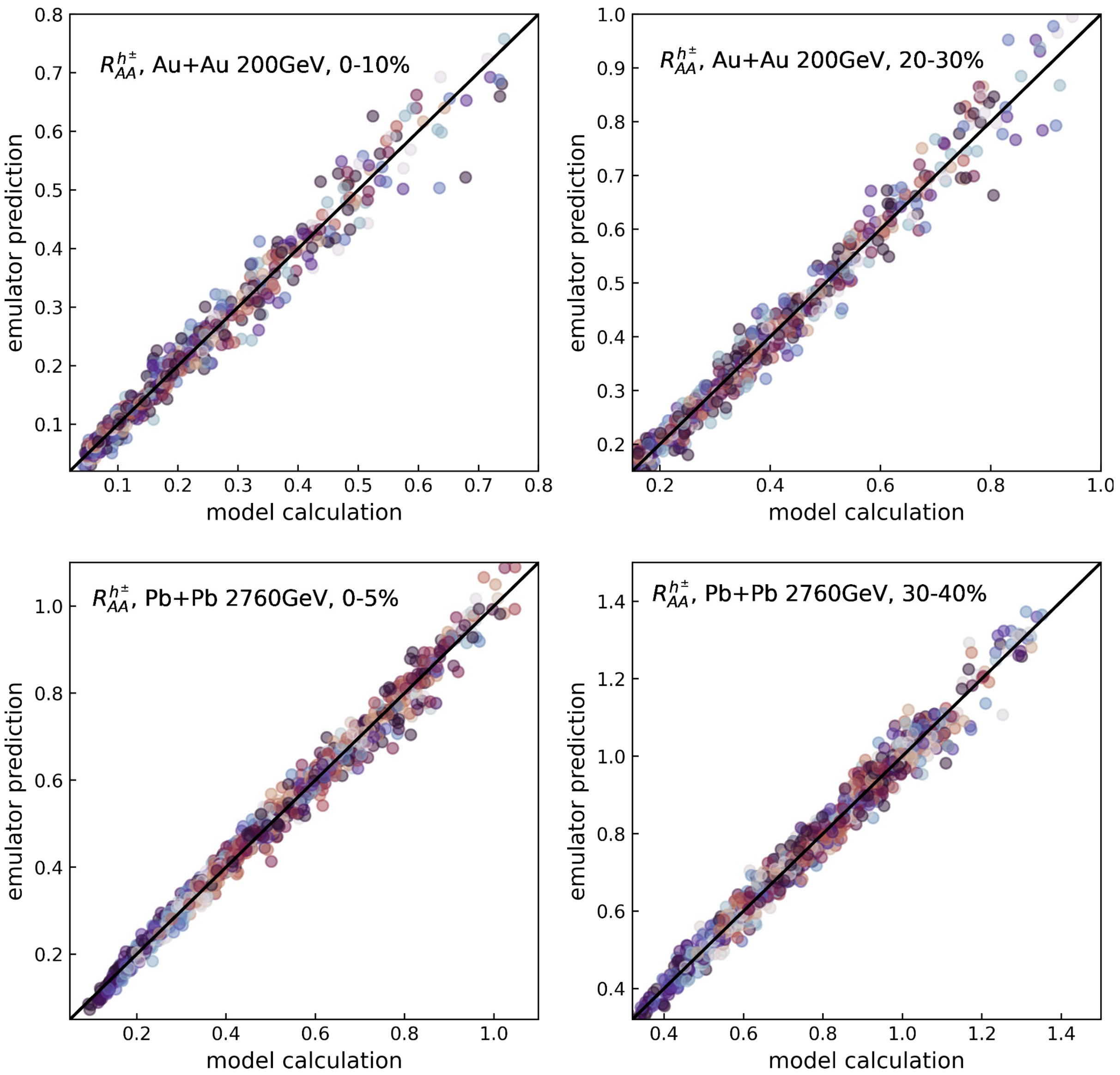
- A model of infinite-dimensional multivariate normal distributions.
- Require minimal assumption about the model.
- Emulate a distribution over functions.

Model output:

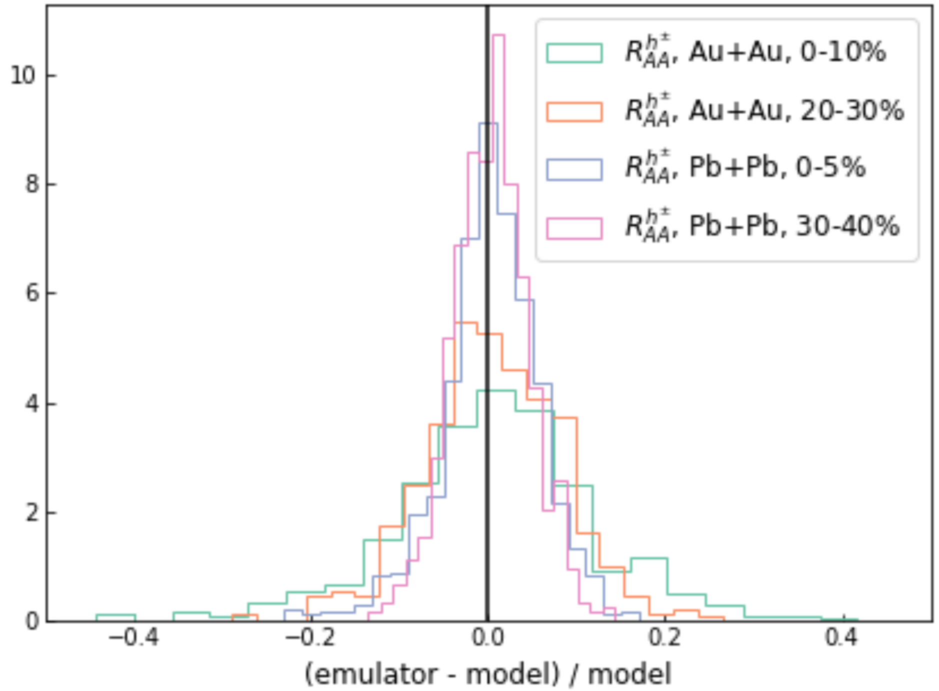
$$y \sim \mathcal{N}(\mu(x), \Sigma(x, x')),$$

$$\mathrm{cov}(y_i, y_j) = \exp\left(-\frac{|x_i - x_j|^2}{2l^2}\right) + \sigma_n^2 \delta_{ij}$$

Validation of Gaussian process emulator

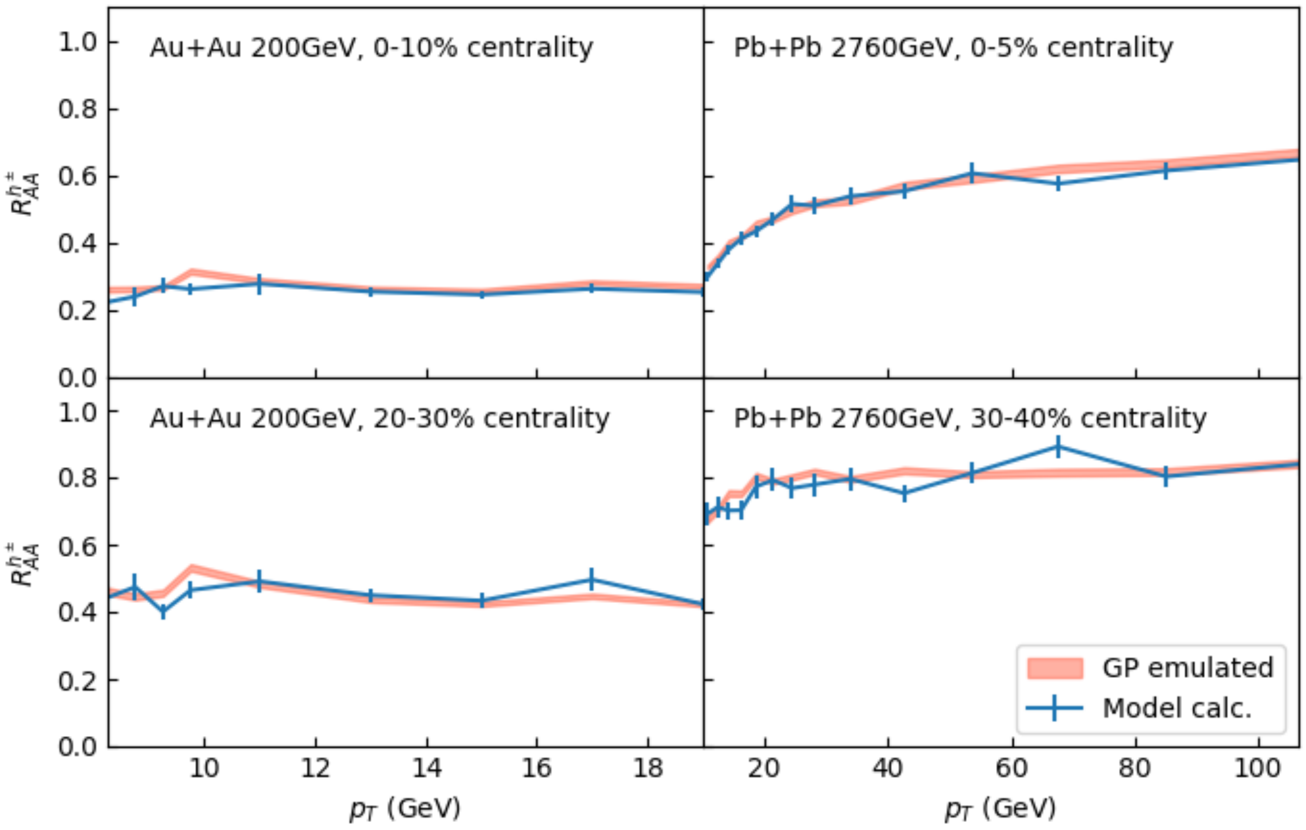


- Emulator works well.
- Relative difference between the model calculation and the emulator prediction is normally distributed.



Validation of Gaussian process emulator

- Choose a validation point.
- Calculate R_{AA} on the validation point in different collision systems.
- Train the Gaussian process emulator using the training data.
- Validate the trained emulator on the validation point.
- The emulator well predict the model outputs.



Bayesian inference

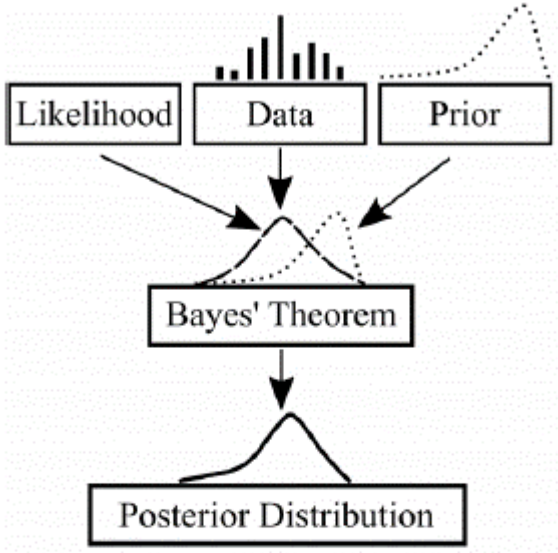
Estimate the probability distribution of the model parameters given the experimental observations based on Bayes' theorem:

$$P(x|\mathcal{D}) = \frac{\mathcal{L}(\mathcal{D}|x)p(x)}{\int \mathcal{L}(\mathcal{D}|x)dx} \propto \mathcal{L}(\mathcal{D}|x)P(x)$$

assuming a uniform distributed prior.

Given the normally distributed model uncertainty, the likelihood function is written as:

$$\mathcal{L}(y = y_{exp}|x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\frac{(f(x) - y_{exp})^2}{\sigma_M^2 + \sigma_{exp}^2}\right]$$



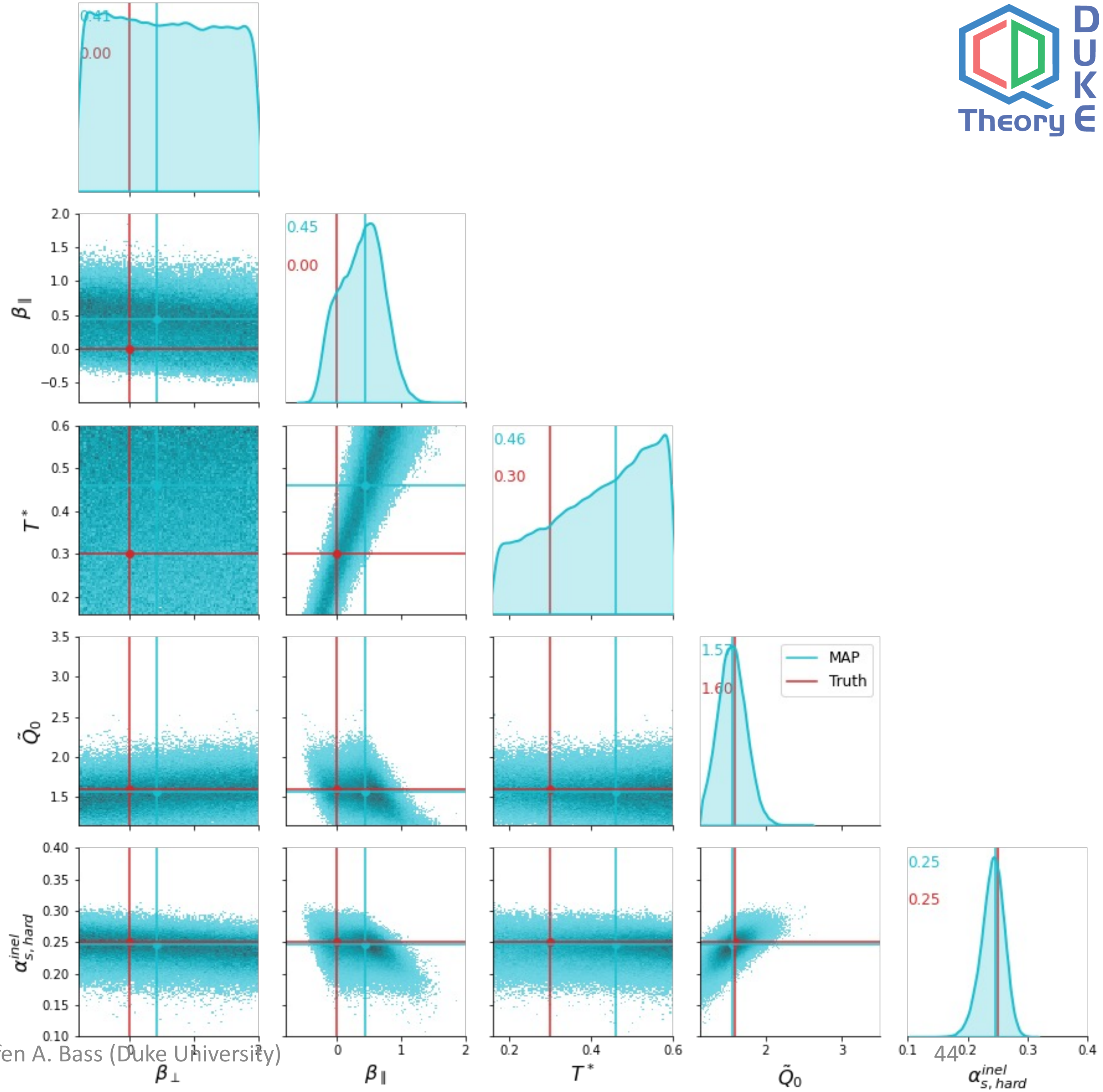
Estimate the true parameter value:
- Maximum likelihood estimation (MLE)
- Maximum A Poseriori (MAP)

Closure test - posterior

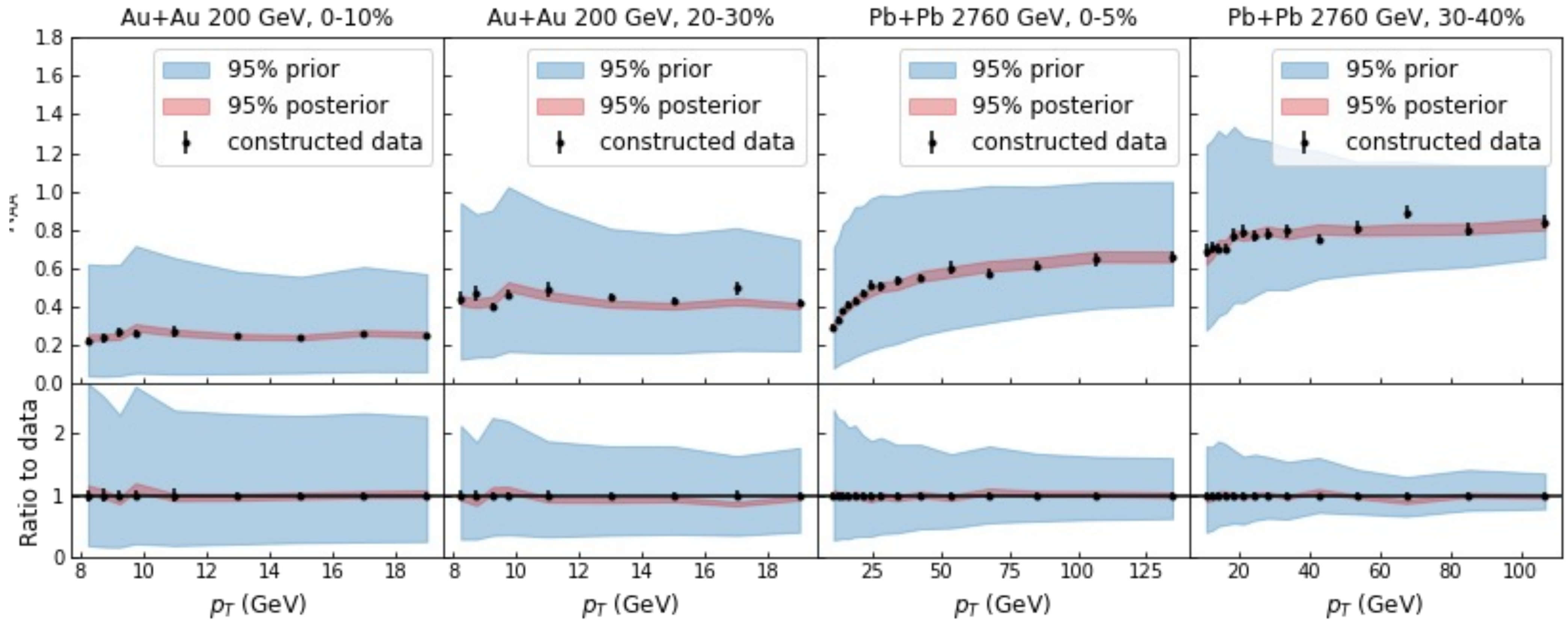
Closure test:
- Assume a set of parameters as “true values”.
- Calculate “constructed data” using “true values”.
- Perform Bayesian analysis on “constructed data” to evaluate the performance of the Bayesian analysis.

The posterior distribution constrained using Bayesian model-to-data comparison:

- R_{AA} is not sensitive to $\beta_\perp$: no constrain on $\beta_\perp$.
- $\beta_\parallel$ correlated to T^* : weaker constrain, need more comparison to evaluate.
- $\tilde{Q}_0$ and $\alpha^{inel}_{s,hard}$ are well-constrained around “true values”.



Closure test – observables emulation

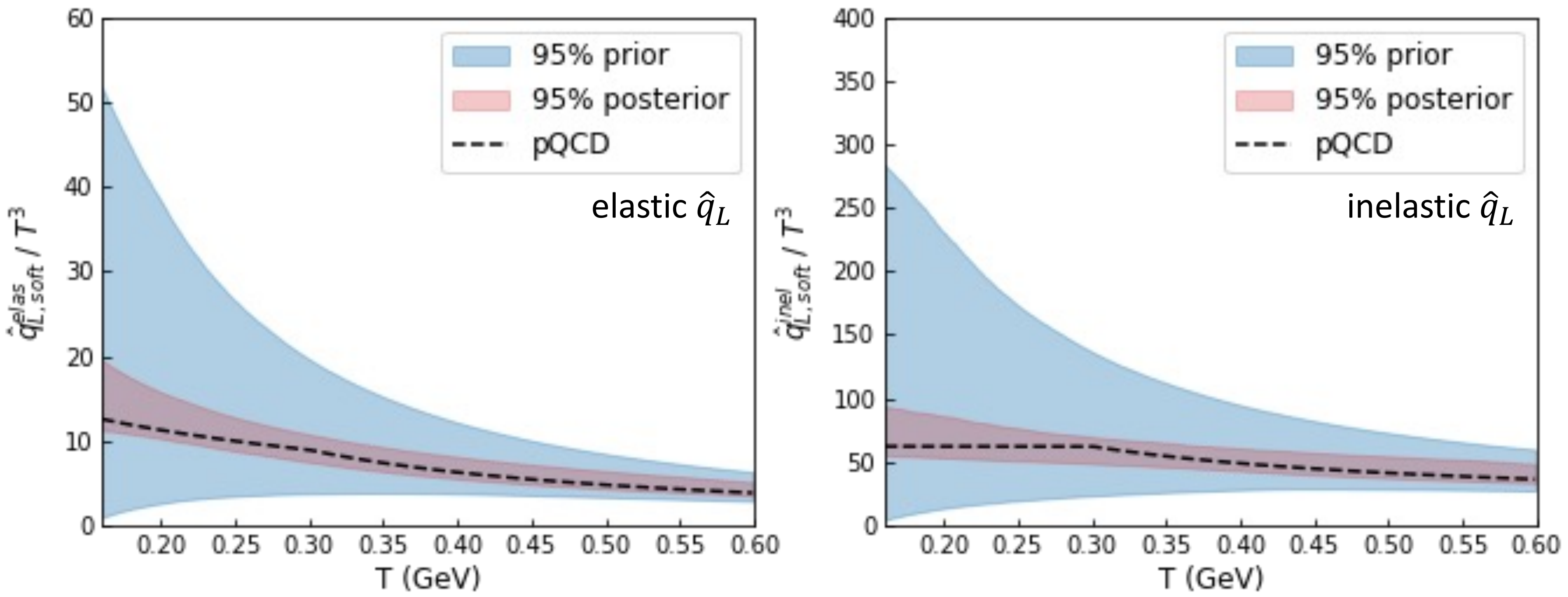


Observables emulated using posterior parameters are close to “constructed data”.

Closure test - posterior

Apply posterior of $\beta_{\parallel}$ and T^* to calculate $\hat{q}_L$: well-constrained around pQCD values.

$$\hat{q}_L\left(\beta_{\parallel}, T^*\right) = \hat{q}_{L,\text{ pQCD}}\left(1 + \beta_{\parallel}\frac{\Lambda_{QCD}}{T}\right)$$



Model-to-data comparison performs well.