On the momentum broadening of in-medium jet evolution using a light-front Hamiltonian approach

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In collaboration with: Tuomas Lappi[2,3], Xingbo Zhao[4,5], and Carlos A. Salgado[1]

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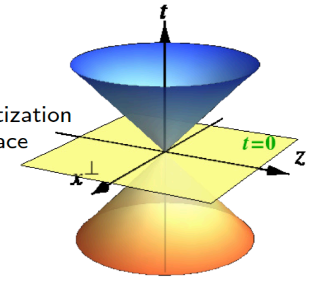
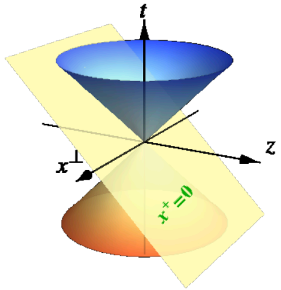
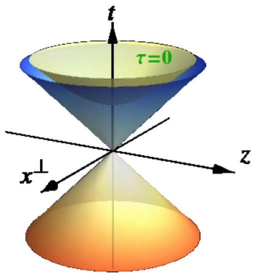
Methodology: Time-dependent Basis Light-Front Quantization (tBLFQ)[1]

- Light-front quantization
  - The quantum field is quantized on the equal light-front time surface $x^+(\equiv x^0 + x^3)$=0

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = mv^\mu \ (v^2 = 1)$

- Hamiltonian formalism
  - The state obeys the Schrödinger equation

$$\frac{1}{2}P^-(x^+)|\psi(x^+)\rangle = i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle$$

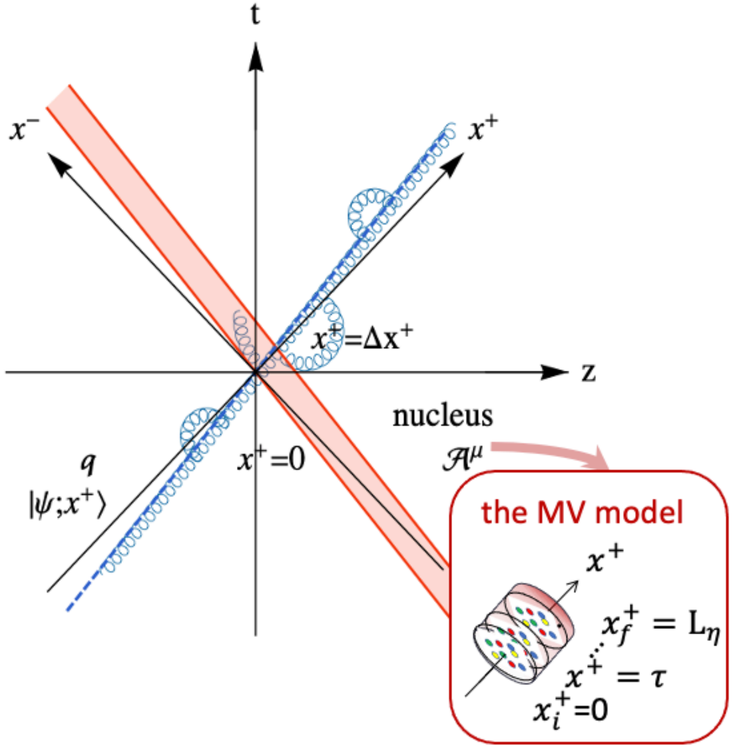
A nonperturbative treatment: numerically evaluating the evolution in sequential small timesteps

- Basis representation
  - Optimal basis has the same symmetries of the system, and it is the key to numerical efficiency

1. J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, C. Yang., Phys. Rev. C81, 035205 (2010); X. Zhao, A. Ilderton, P. Maris, and J. P. Vary, Phys. Rev. D88, 065014 (2013).

Methodology: A. The light-front Hamiltonian $P^-(x^+)$

We consider scattering of a high-energy quark moving in the positive z direction, on a high-energy nucleus moving in the negative z direction.



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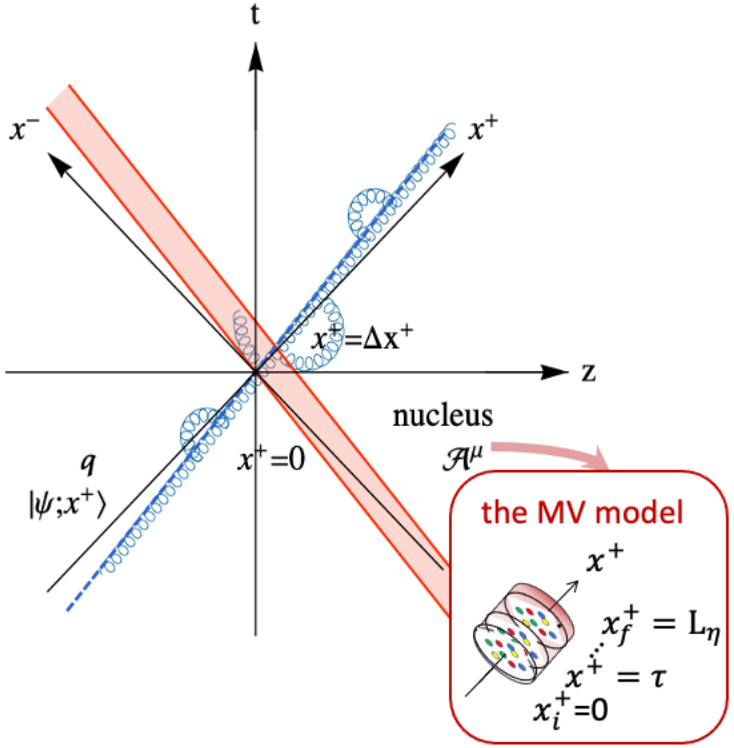


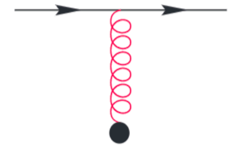
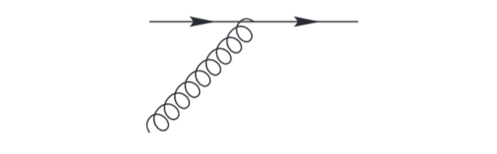
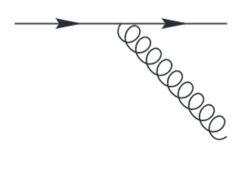
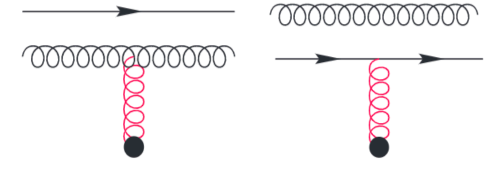
- **The light-front Hamiltonian** is derived from the QCD Lagrangian with a background color field $\mathcal{A}_\mu$,

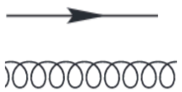
$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}{}_a F^a_{\mu\nu} + \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

where $D_\mu = \partial_\mu + ig(A_\mu + \mathcal{A}_\mu)$. In the $|q\rangle + |qg\rangle$ space, it includes the kinetic energy, the interaction with the background field, and gluon emission/absorption:

$$\mathrm{P}^-(x^+) = \mathrm{P}^-_{KE} + V(x^+)$$

The interaction matrix $V(x^+) = V_{qg} + V_{\mathcal{A}}(x^+)$

Fock sector	$\|q\rangle$	$\|qg\rangle$
$\langle q\|$		
$\langle qg\|$		



quark
gluon

gluon from the background field

Methodology: A. The light-front Hamiltonian $P^-(x^+)$

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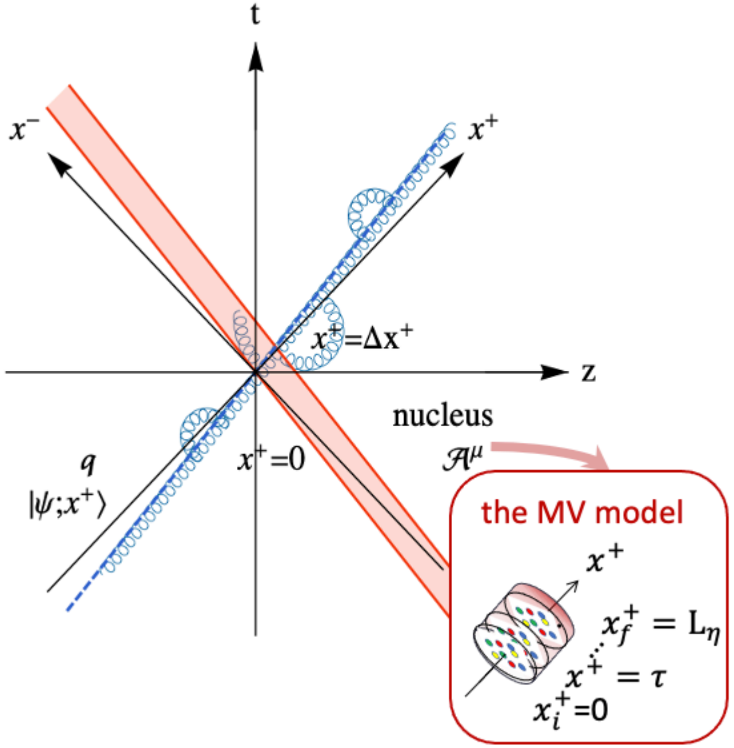

- **The background color field $\mathcal{A}_\mu$** is a classical gluon field described by the color glass condensate[1]
  - Color charges are stochastic variables with correlations

$$< \rho_a(x^+,\vec{x}_\perp)\rho_b(y^+,\vec{y}_\perp) > = g^2\tilde{\mu}^2\delta_{ab}\delta^2(\vec{x}_\perp - \vec{y}_\perp)\delta(x^+ - y^+)$$

  - The color field is solved from

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(x^+,\vec{x}_\perp) = \rho_a\,(x^+,\vec{x}_\perp)$$

    where m_g is a chosen infrared (IR) regulator.

  - The color sources of different layers are independent of each other, simulating the quarks from different nucleons of the heavy ion
    - The duration of the interaction: $x^+ = [0, L_\eta]$
    - Number of layers: N_η
    - The duration of each layer: $\tau = L_\eta/N_\eta$
  - Saturation scale:

$$Q_s^2 = C_F\,(g^2\tilde{\mu})^2 L_\eta/(2\pi)$$

Methodology: B. Basis representation

- The quantum state is expanded in **the basis states**:

$$|\psi; x^+\rangle = \sum_\beta c_\beta(x^+)|\beta\rangle, \qquad |q\rangle: |\beta_q\rangle; \quad |qg\rangle: |\beta_{qg}\rangle = |\beta_q\rangle \otimes |\beta_g\rangle$$

Each single particle state carries five quantum numbers: $\beta_l = \{k_l^x, k_l^y, k_l^+, \lambda_l, c_l\}, (l = q, g)$
the transverse momenta, the longitudinal momentum, helicity, and color

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$$|\psi; x^+\rangle = \sum_\beta c_\beta(x^+)|\beta\rangle,$$

$|q\rangle$: $|\beta_q\rangle$; $|qg\rangle$: $|\beta_{qg}\rangle = |\beta_q\rangle \otimes |\beta_g\rangle$

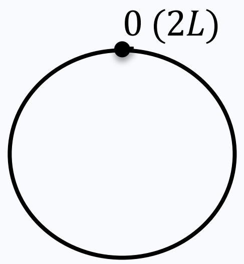
Each single particle state carries five quantum numbers: $\beta_l = \{k_l^x, k_l^y, k_l^+, \lambda_l, c_l\}, (l = q, g)$
the transverse momenta, the longitudinal momentum, helicity, and color

○ The longitudinal space

- $x^- = [0, 2L]$
- $p_l^+ = \frac{2\pi}{L} k_l^+$

  $k_q^+ = \frac{1}{2}, \frac{3}{2}, \ldots, K + \frac{1}{2}$

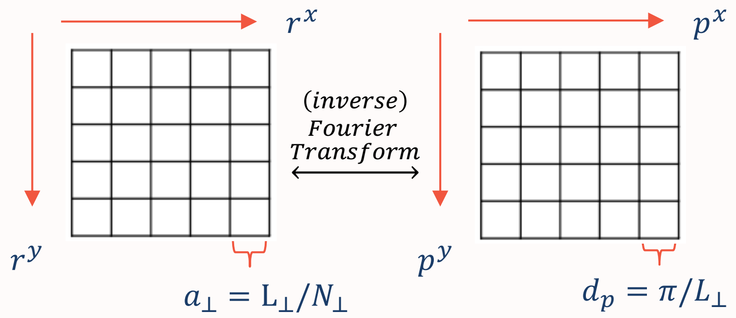
  $k_g^+ = 1, 2, \ldots, K$



Basis size: $N_{tot} = (2N_\perp)^2 \times 2 \times 3 + K \times (2N_\perp)^4 \times 4 \times 24$

○ The transverse space

- $r_l^\perp = [-N_\perp, \ldots, N_\perp - 1]\, L_\perp / N_\perp$
- $p_l^\perp = \frac{2\pi}{2L_\perp} k_l^\perp, k_l^\perp = -N_\perp, \ldots, N_\perp - 1$



Methodology: C. Time evolution

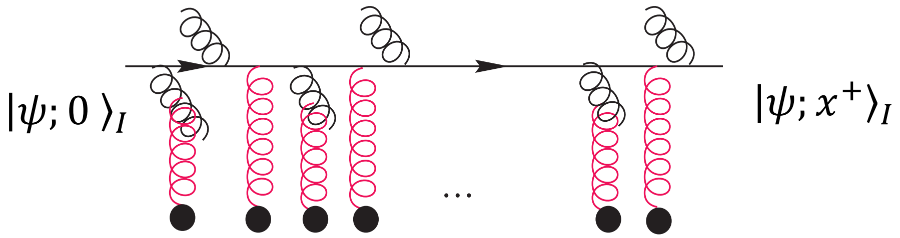
➢ **Solve the time-evolution equation** in the interaction picture

$$\frac{1}{2}V_I(x^+)|\psi;x^+\rangle_{\rm I} = i\frac{\partial}{\partial x^+}|\psi;x^+\rangle_{\rm I}$$

- P^-_{KE} as a phase factor: $|\psi;x^+\rangle_{\rm I} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}x^+}|\psi;x^+\rangle$, $\mathrm{V_I(x^+)} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}x^+}V(x^+)e^{-\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}x^+}$
- Time evolution as a product of many small timesteps

$$|\psi;x^+\rangle_{\rm I} = \mathcal{T}_+\exp\{-\frac{\mathrm{i}}{2}\int_0^{x^+}dz^+V_I(z^+)\}\,|\psi;0\,\rangle_{\rm I}$$

$$= \lim_{n\to\infty}\prod_{k=1}^n\mathcal{T}_+\exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k}dz^+V_I(z^+)\}\,|\psi;0\,\rangle_{\rm I}$$



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$$\frac{1}{2}V_I(x^+)|\psi; x^+\rangle_{\rm I} = i\frac{\partial}{\partial x^+}|\psi; x^+\rangle_{\rm I}$$

- P_{KE}^- as a phase factor: $|\psi; x^+\rangle_{\rm I} = e^{\frac{i}{2}\mathrm{P}_{KE}^- x^+}|\psi; x^+\rangle$, $\mathrm{V_I(x^+)} = e^{\frac{i}{2}\mathrm{P}_{KE}^- x^+}V(x^+)e^{-\frac{i}{2}\mathrm{P}_{KE}^- x^+}$
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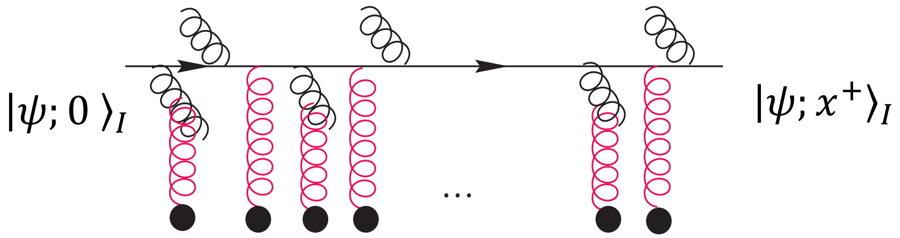

Each timestep contains two successive operations, and we choose suitable numerical method for the two different interactions:

$$\mathcal{T}_+ \exp\{-\frac{i}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ V_I(z^+)\} = \mathcal{T}_+ \exp\{-\frac{i}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ V_{\mathcal{A},I}(z^+)\}\times\,\mathcal{T}_+ \exp\{-\frac{i}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ V_{qg,I}(z^+)\}$$

matrix exponential in coordinate space + Fast Fourier Transform, $\sim O(N_{tot}\log N_{tot})$

4th-order Runge-Kutta method, $\sim O(N_{tot})$

The total computational complexity of each timestep is $O(N_{tot}\log N_{tot})$.

Results: Momentum broadening

- With the obtained light-front wavefunction, physical quantities can be evaluated directly with the corresponding operators

$$\langle p_\perp^2(x^+)\rangle = \langle \psi; x^+ | p_\perp^2 | \psi; x^+ \rangle$$

- The quark state transfers to different momentum modes through evolution. Such momentum broadening can be characterized by the quenching parameter:

$$\hat{q} \equiv \frac{\Delta \langle p_\perp^2(x^+)\rangle}{\Delta\, x^+}$$

Results: Momentum broadening

a) The single quark state $|q\rangle$

- Eikonal, $p^+ = \infty$

  The momentum broadening can be calculated analytically using Wilson lines:

$$\left\langle p_\perp^2(x^+)\right\rangle_{Eik} = \frac{Q_s^2}{2L_\eta}\left[\log\frac{\lambda_{UV}^2+m_g^2}{m_g^2} - \frac{\lambda_{UV}^2}{\lambda_{UV}^2+m_g^2}\right] x^+,$$

$$\hat{q}_{Eik} = \frac{Q_s^2}{2L_\eta}\left[\log\frac{\lambda_{UV}^2+m_g^2}{m_g^2} - \frac{\lambda_{UV}^2}{\lambda_{UV}^2+m_g^2}\right], \lambda_{UV} = \pi/a_\perp.$$

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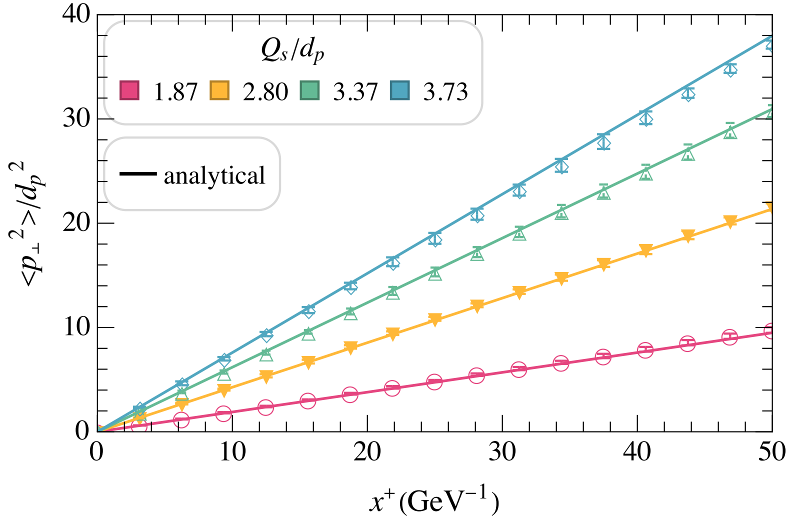
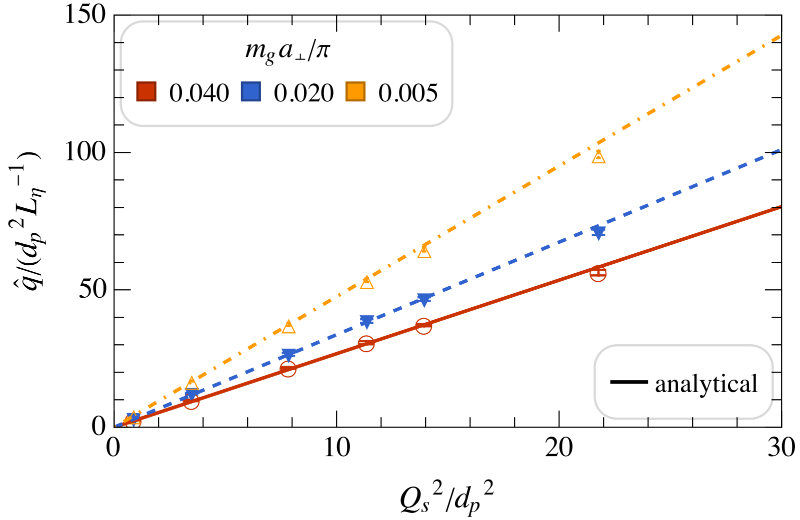
✓ Numerical results are verified





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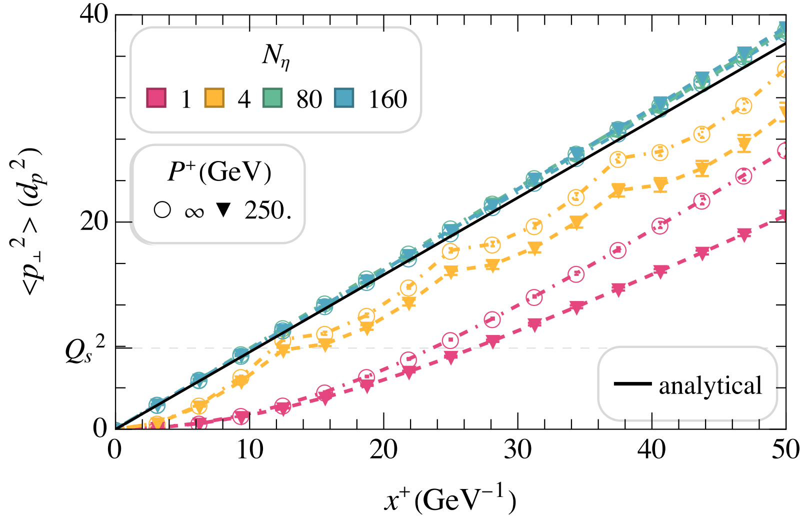
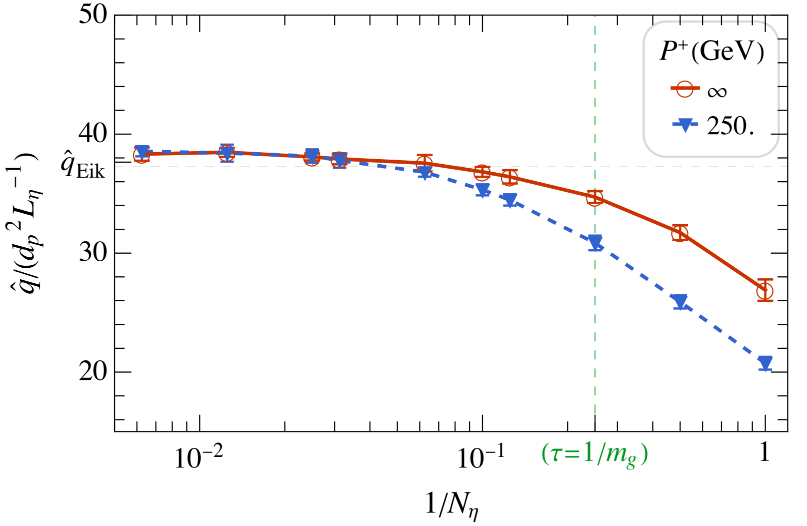
  ✓ Numerical results are verified

- Non-eikonal, $p^+ < \infty$
  1) ``layer'' effect, $\langle p_\perp^2(x^+)\rangle \propto (x^+)^2$ within layer, $\langle p_\perp^2(x^+)\rangle \propto x^+$ across layers
  2) $\hat{q}$ is smaller in the non-eikonal case with finite layers





Results: Momentum broadening

b) The quark-gluon state $|qg\rangle$

- Eikonal, $p^+ = \infty$

  The momentum square can be calculated analytically using Wilson lines:

$$\langle p_\perp^2(x^+)\rangle_{qg,c;Eik} = \langle \vec{p}_{q,\perp}^2(x^+)\rangle_{Eik} + \langle \vec{p}_{g,\perp}^2(x^+)\rangle_{Eik} + 2\,\langle \vec{p}_{q,\perp}(x^+)\cdot\vec{p}_{g,\perp}(x^+)\rangle_{c;Eik}$$

Same as that of a single quark (gluon)

$$\langle \vec{p}_{q,\perp}(x^+)\cdot\vec{p}_{g,\perp}(x^+)\rangle_c = -\int d^2v_\perp f_{Rel}(\vec{v}_\perp) s_{12}(v_\perp) \begin{cases} 0, & c = 3\otimes 8 \\ -\frac{3\sqrt{2}}{2}, & c = 3 \\ -\frac{\sqrt{2}}{2}, & c = \bar{6} \\ \frac{\sqrt{2}}{2}, & c = 15 \end{cases}$$

qg color configuration

qg relative transverse coordinate $\vec{v}_\perp$

Results: Momentum broadening

b) The quark-gluon state $|qg\rangle$

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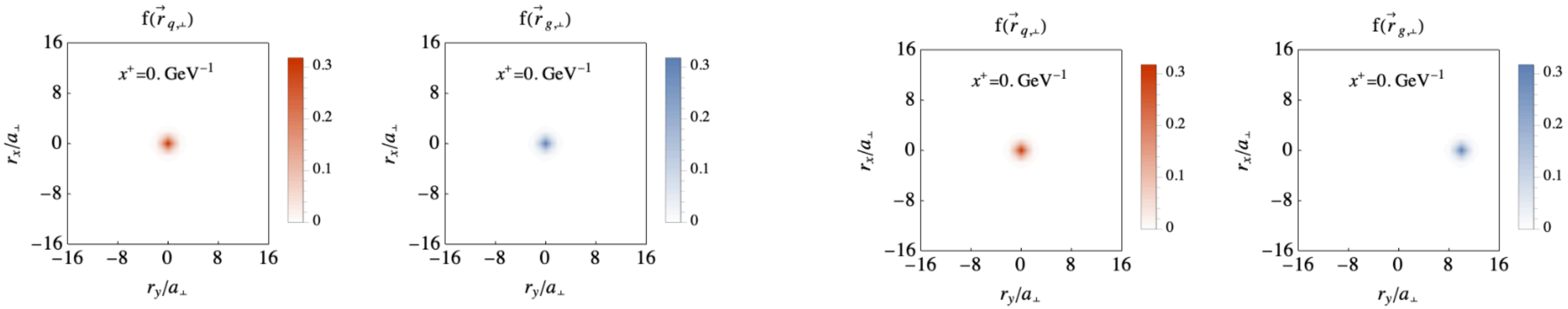
➨ Consider a quark-gluon initial state with small and large separation



small qg-separation

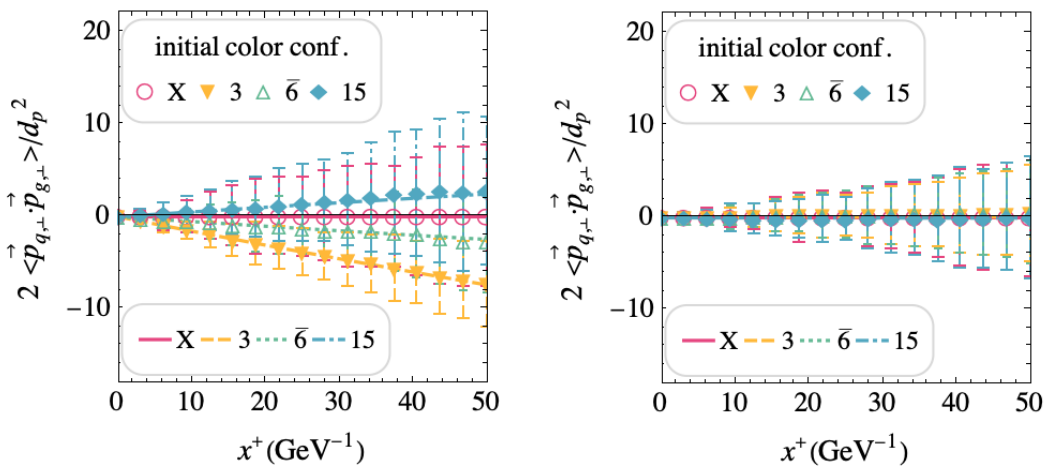
large qg-separation

Results: Momentum broadening

b) The quark-gluon state $|qg\rangle$

- Eikonal, $p^+ = \infty$
  - ✓ Numerical results agree with analytical

Larger qg-separation



Results: Momentum broadening

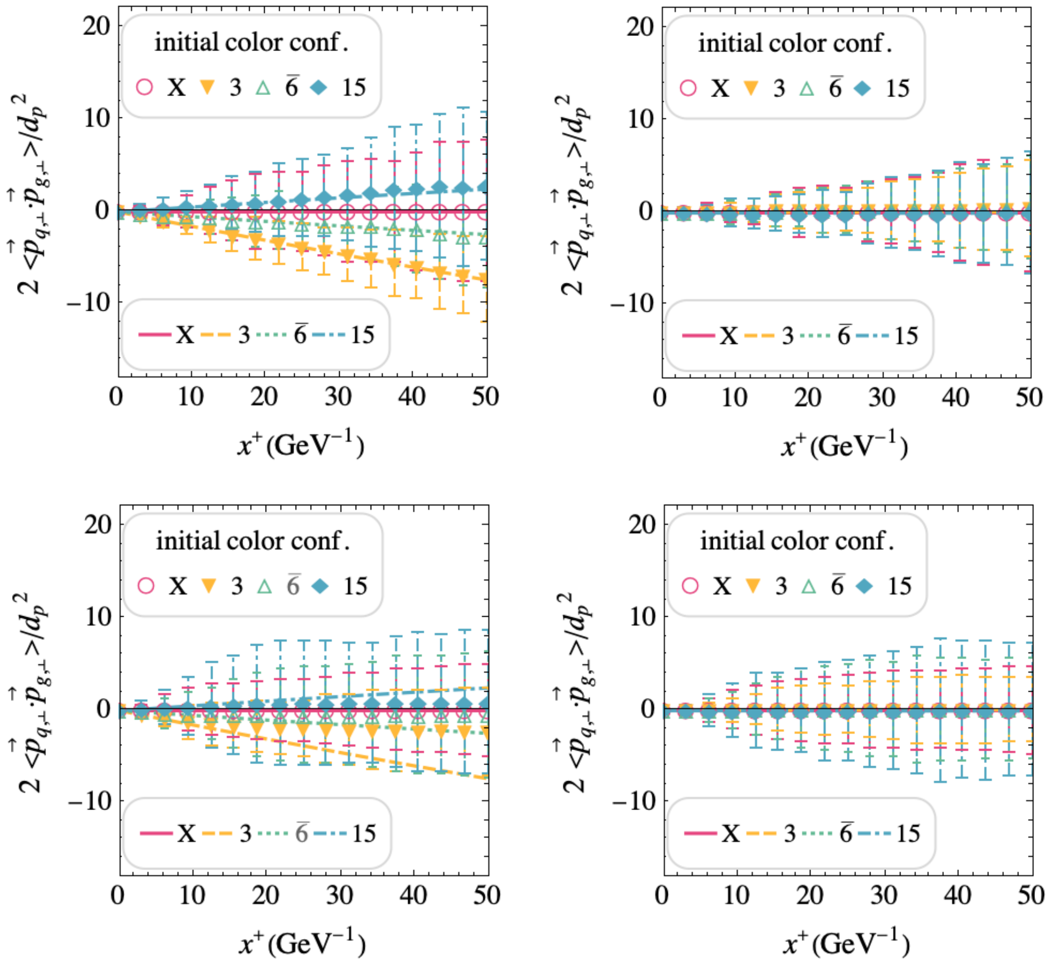
b) The quark-gluon state $|qg\rangle$

Larger qg-separation

- Eikonal, $p^+ = \infty$
  - ✓ Numerical results agree with analytical

- Non-eikonal, $p^+ = 1.5$ GeV
  correlation is suppressed due to space diffusion

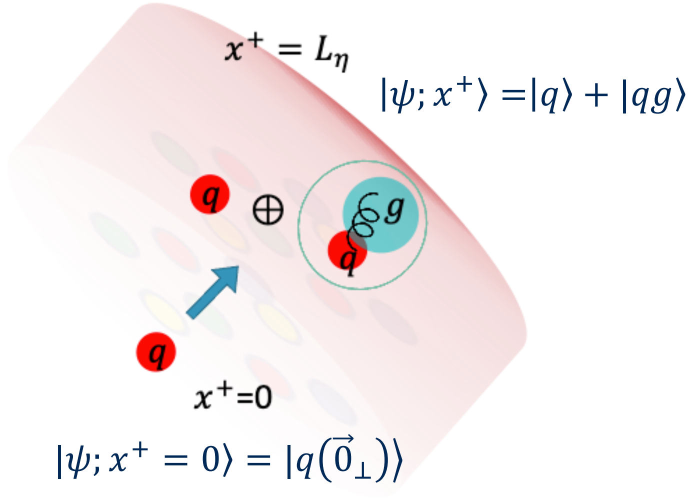
Smaller p^+



Results: Momentum broadening

c) The dressed quark state $|q\rangle+|qg\rangle$

- In the $|q\rangle + |qg\rangle$ space, consider the initial state as a single quark state,



$$\langle p_\perp^2(x^+)\rangle_{total} = \langle q|p_\perp^2(x^+)|q\rangle + \langle qg|p_\perp^2(x^+)|qg\rangle$$

Results: Momentum broadening

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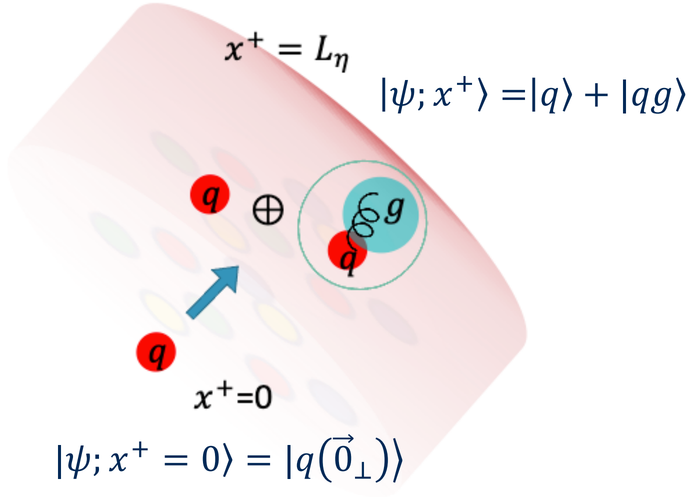

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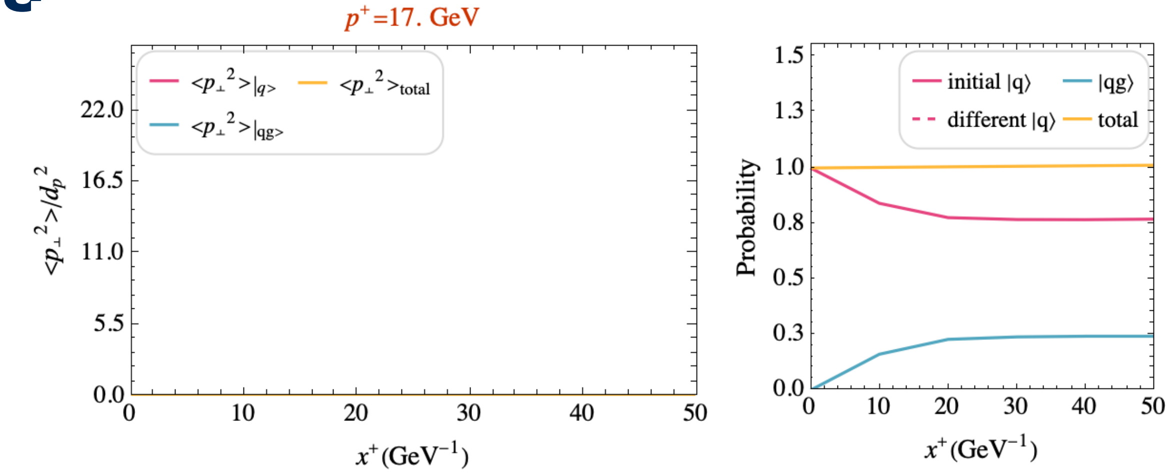
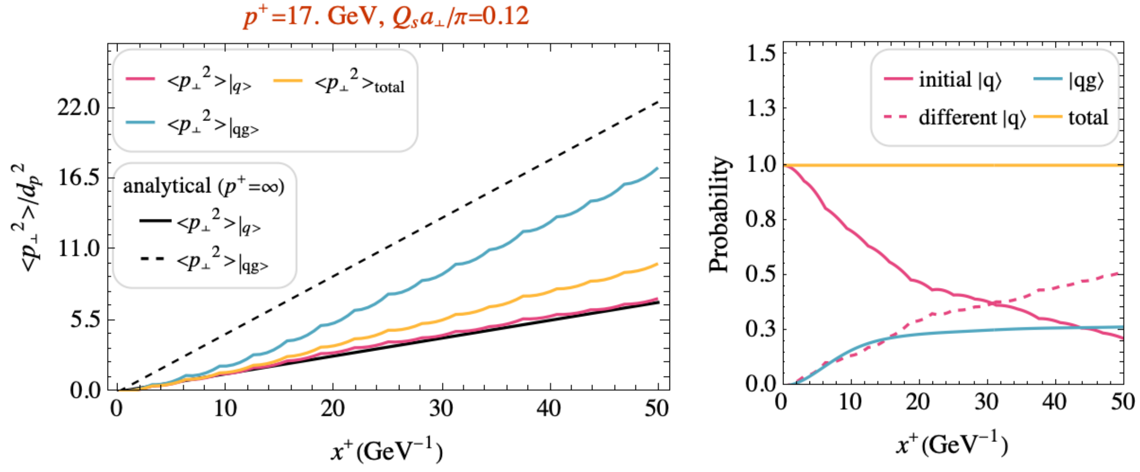


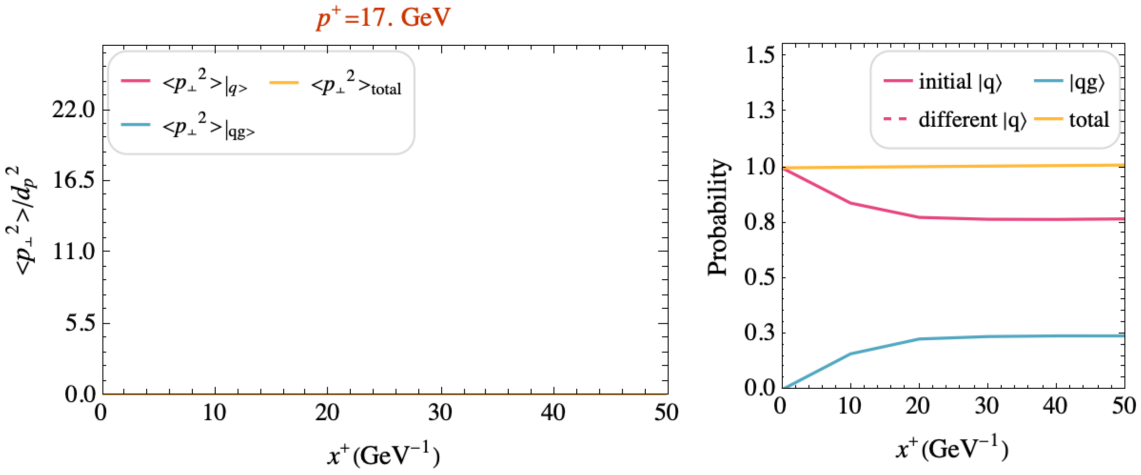
vacuum



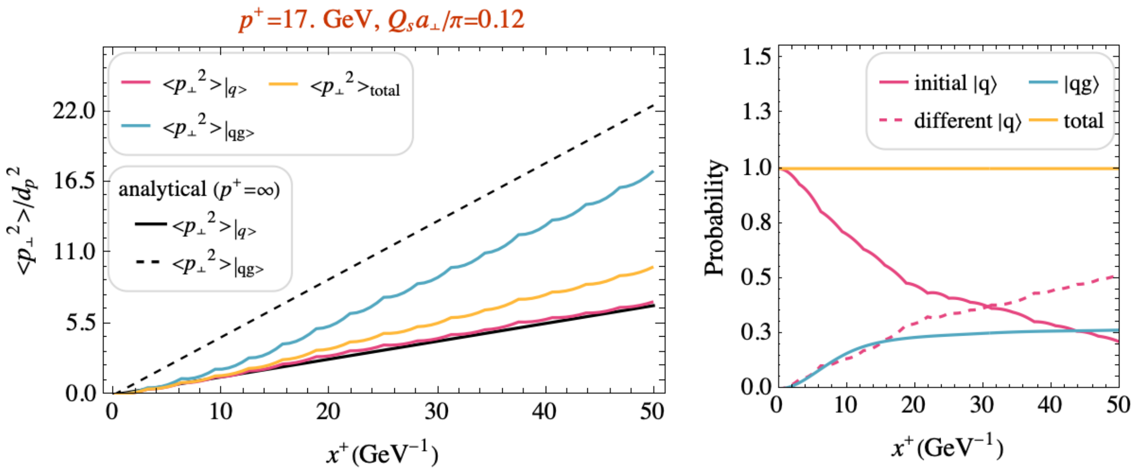
medium

Results: Momentum broadening

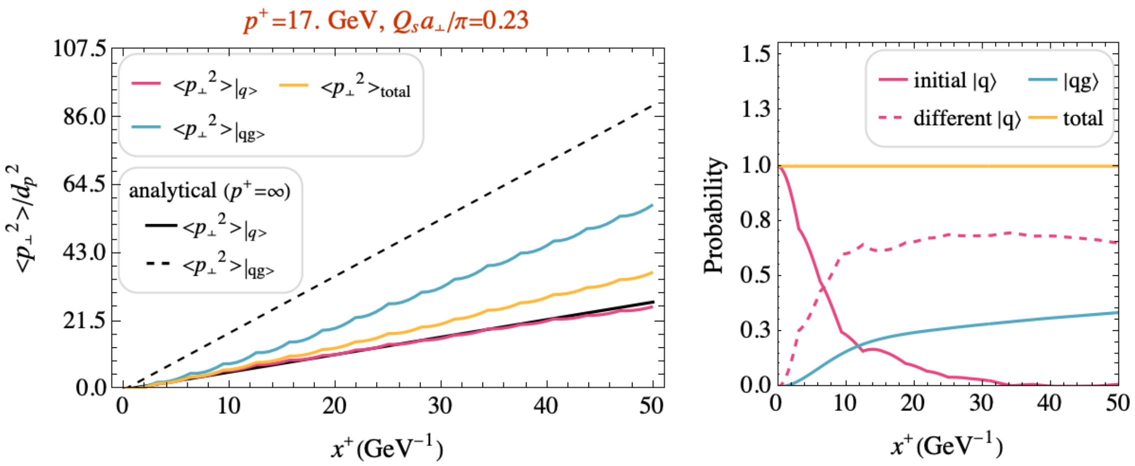
c) The dressed quark state $|q\rangle + |qg\rangle$



vacuum



medium



denser medium

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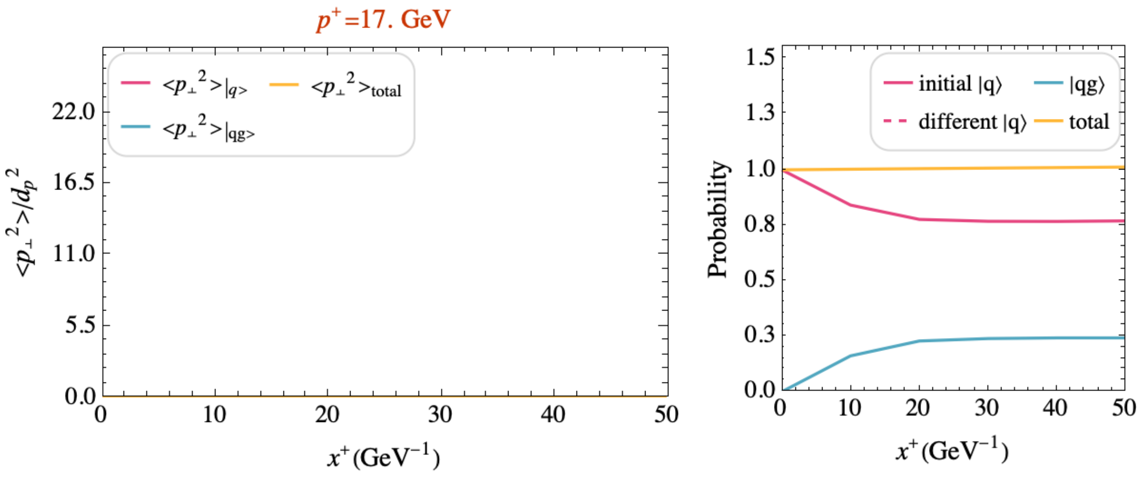
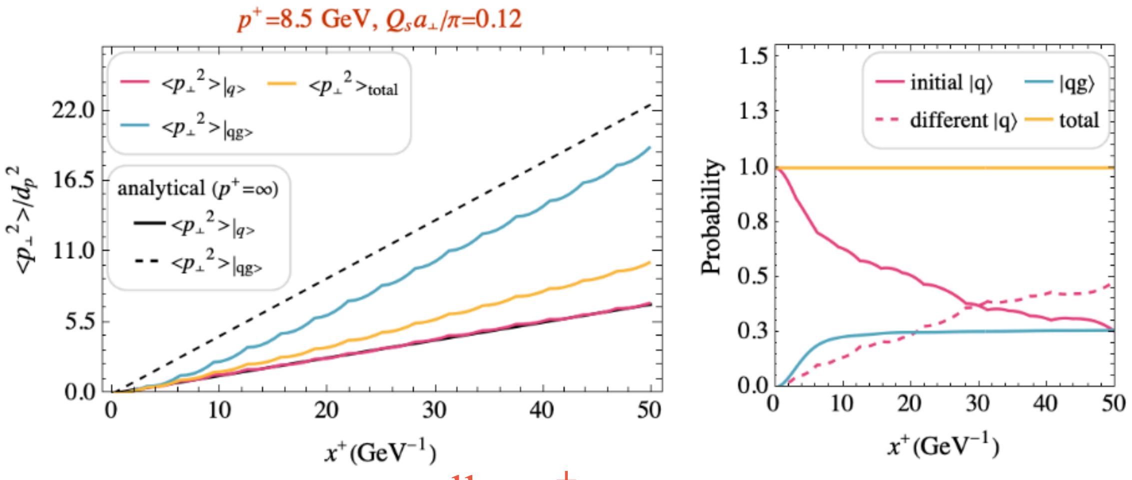
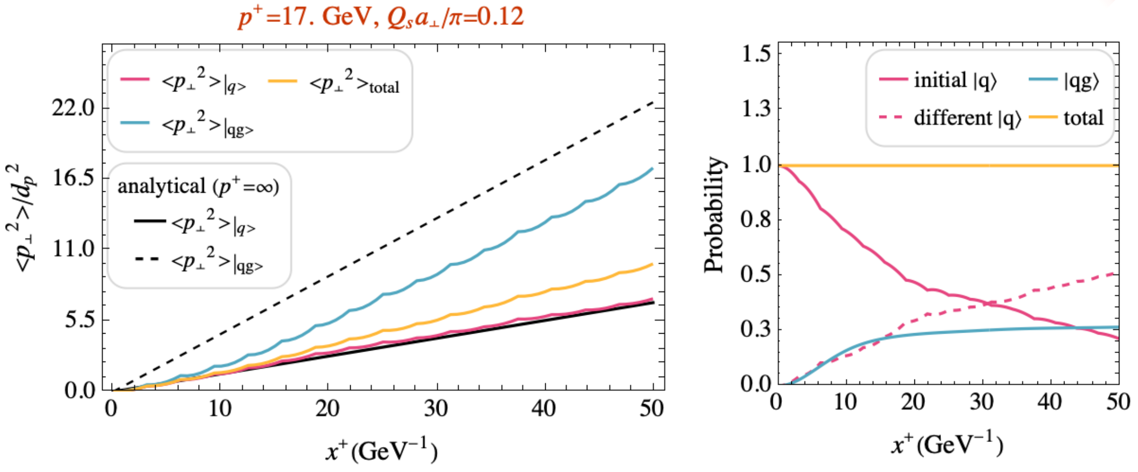


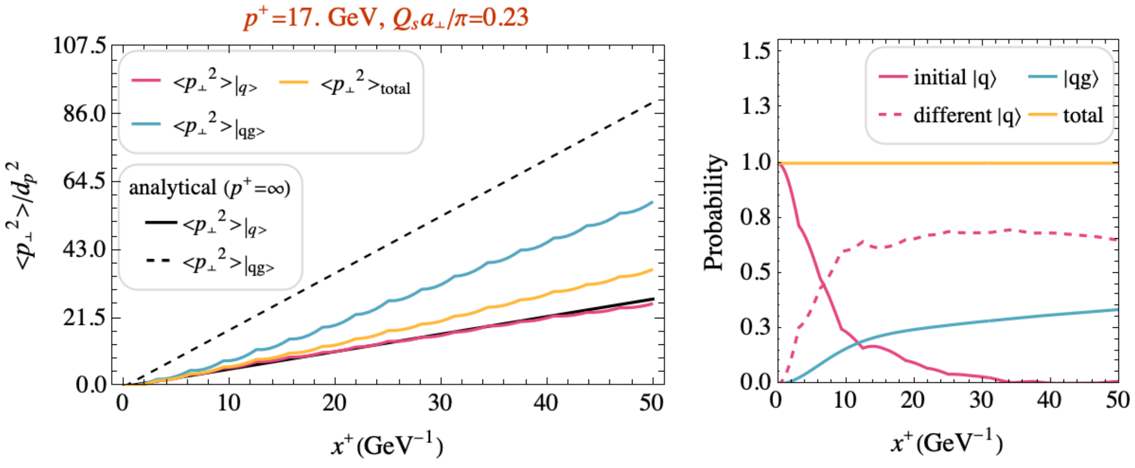
vacuum



smaller p^+



medium



denser medium

Summary and outlooks

- Following the non-perturbative light-front Hamiltonian formalism developed in our preceding work [1],
  1. We investigated the momentum broadening of an in-medium quark jet in both the eikonal and **non-eikonal** regimes [2].
  2. A main advantage of this method: one can smoothly vary the separate magnitudes of kinetic energy, gluon emission, and medium effects.
- Ongoing study:
  - A different scenario: The quark coming from outside the medium, described by the fully developed wave function that contains a gluon cloud.
- Parallel study: **Quantum Simulation of jet evolution in a medium [3]** **[Wednesday, 10:50, Wenyang Qian]**

[1] M. Li, T. Lappi, and X. Zhao, Phys. Rev. D 104, 056014 (2021)
[2] M. Li, T. Lappi, X. Zhao, and Carlos A. Salgado, *to appear*
[3] J. Barata, X. Du, M. Li, W. Qian, and C. A. Salgado, Phys. Rev. D 106 (2022) 7, 074013; and ongoing works

Thank you!