

Higher orders in opacity in QGP tomography

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Motivation

- Hard probes are one of the main tools for understanding and characterizing the Quark-Gluon Plasma, created at RHIC and the LHC.
- Interactions of these probes with QGP constituents are dominantly described by **energy loss, where radiative is one of the most important mechanisms at high-pt.**
- pQCD approaches are typically used to analytically compute radiative energy loss.
- Typical assumptions: **optically thick or optically thin medium!**

Optically thick

Approximation of a jet experiencing an infinite number of scatterings with medium constituents.



While such an approximation would be adequate for QGP created in the early universe (Big Bang), Little Bangs are characterized by short, finite-size droplets of QCD matter..



Optically thin

Approximation of a jet experiencing one scattering with medium constituents.



The medium created in Little Bangs is typically several fms ($\lambda \approx 1\text{fm}$), so that considering several scattering centers in energy loss calculations is needed.



Two extreme limits to realistic situations considered in RHIC and LHC.



Relaxing these approximations to the case of a finite number of scattering centers is necessary.



Highly nontrivial problem, where previous attempts to address it have not yet provided conclusive or complete solutions.

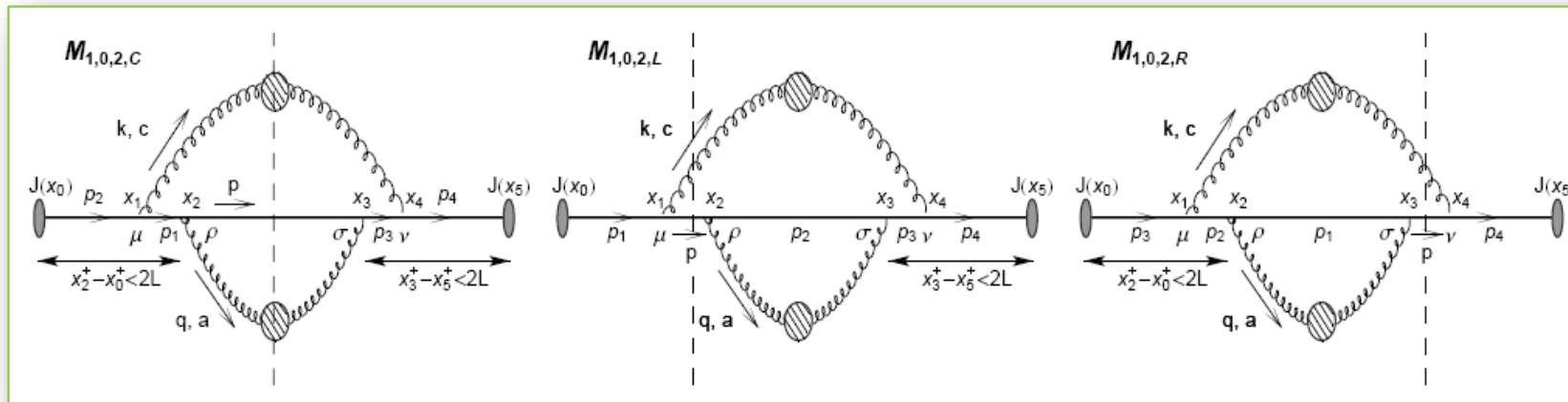


Have to implement this in **both analytical calculations and numerical framework.**

The dynamical radiative energy loss

Has the following unique features:

- *Finite size finite temperature QCD medium of dynamical (moving) partons.*
- Based on finite T field theory and generalized HTL approach.
- *Applicable to both light and heavy flavor.*
- Finite magnetic mass effects (M. D. and M. Djordjevic, PLB 709:229 (2012))
- Running coupling (M. D. and M. Djordjevic, PLB 734, 286 (2014)).



However, this radiative energy loss is developed up to the first order in opacity.

To improve its applicability for QGP tomography, it is necessary to relax this approximation!

Analytical calculations

We start from a closed-form expression (Wicks, 0804.4704)

$$x \frac{dN^{(n)}}{dx d^2\mathbf{k}} = \frac{C_R \alpha_s}{\pi^2} \int_0^L dz_1 \cdots \int_{z_{n-1}}^L dz_n \int \prod_{i=1}^n \left(d^2\mathbf{q}_i \frac{\bar{v}^2(\mathbf{q}_i) - \delta^2(\mathbf{q}_i)}{\lambda(z)} \right) \\ \times \left(-2 \mathbf{C}_{(1,\dots,n)} \cdot \mathbf{B}_n \left[\cos \sum_{k=2}^n \omega_{(k,\dots,n)} \Delta z_k - \cos \sum_{k=1}^n \omega_{(k,\dots,n)} \Delta z_k \right] \right)$$

derived for static QCD medium (i.e., (D)GLV case) but applicable for a generalized form of effective potential and mean free path λ .

Despite much more involved analytical calculations, we showed that the radiative energy loss in a dynamical medium has the same form as in the static medium, except for two substitutions in mean free path and effective potential:

$$\frac{1}{\lambda_{\text{stat}}} = 6 \frac{1.202}{\pi^2} \frac{1+n_f/4}{1+n_f/6} \frac{1}{\lambda_{\text{dyn}}}$$

$$\left[\frac{\mu_E^2}{\pi(\mathbf{q}^2 + \mu_E^2)^2} \right]_{\text{stat}} \rightarrow \left[\frac{\mu_E^2 - \mu_M^2}{\pi(\mathbf{q}^2 + \mu_E^2)(\mathbf{q}^2 + \mu_M^2)} \right]_{\text{dyn}}$$



The above expression can also be used in our case, with these modifications of effective potential and mean free path.



Derived full analytical expressions up to 4th order in opacity.

$$\omega_{(m,\dots,n)} = \frac{\chi^2 + (\mathbf{k} - \mathbf{q}_m - \dots - \mathbf{q}_n)^2}{2xE}$$

$$\mathbf{C}_{(i_1 i_2 \dots i_m)} = \frac{(\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \dots - \mathbf{q}_{i_m})}{\chi^2 + (\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \dots - \mathbf{q}_{i_m})^2}$$

$$\mathbf{H} = \frac{\mathbf{k}}{\chi^2 + \mathbf{k}^2}, \text{ and } \mathbf{B}_i = \mathbf{H} - \mathbf{C}_i$$

$$\chi^2 \equiv M^2 x^2 + m_g^2$$

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - 2/3n_f) \ln(Q^2/\Lambda_{QCD})}$$

Numerical calculations

An example of contribution to gluon radiation spectrum:

$$\left(\frac{dN_g^{(3)}}{dx}\right)_1 = \frac{8C_R}{\pi x} \int_0^L \int_{z_1}^L \int_{z_2}^L dz_1 dz_2 dz_3 \int dk_{\perp} k_{\perp} \iint \frac{q_{1\perp} dq_{1\perp} d\varphi_1}{\pi} \iint \frac{q_{2\perp} dq_{2\perp} d\varphi_2}{\pi} \iint \frac{q_{3\perp} dq_{3\perp} d\varphi_3}{\pi}$$

$$\alpha_s(Q_k^2) \frac{1}{\lambda_{dyn}^3} \frac{\mu_E^2 - \mu_M^2}{(q_{1\perp}^2 + \mu_E^2)(q_{1\perp}^2 + \mu_M^2)} \frac{\mu_E^2 - \mu_M^2}{(q_{2\perp}^2 + \mu_E^2)(q_{2\perp}^2 + \mu_M^2)} \frac{\mu_E^2 - \mu_M^2}{(q_{3\perp}^2 + \mu_E^2)(q_{3\perp}^2 + \mu_M^2)}$$

$$\left(\frac{\chi^2 \mathbf{q}_{3\perp} (\mathbf{q}_{3\perp} - \mathbf{k}_{\perp} + \mathbf{q}_{1\perp} + \mathbf{q}_{2\perp}) + \mathbf{q}_{3\perp} \mathbf{k}_{\perp} (\mathbf{k}_{\perp} - \mathbf{q}_{3\perp})^2 + (\mathbf{q}_{1\perp} + \mathbf{q}_{2\perp}) (\mathbf{k}_{\perp} (\mathbf{q}_{3\perp}^2 - 2\mathbf{k}_{\perp} \mathbf{q}_{3\perp}) + \mathbf{k}_{\perp}^2 \mathbf{q}_{3\perp})}{(\chi^2 + \mathbf{k}_{\perp}^2)(\chi^2 + (\mathbf{k}_{\perp} - \mathbf{q}_{3\perp})^2)(\chi^2 + (\mathbf{k}_{\perp} - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp} - \mathbf{q}_{3\perp})^2)} \right)$$

$$\sin \left(\frac{\chi^2 + (\mathbf{k}_{\perp} - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp} - \mathbf{q}_{3\perp})^2}{4xE} z_1 \right)$$

$$\sin \left(\frac{\chi^2 + (\mathbf{k}_{\perp} - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp} - \mathbf{q}_{3\perp})^2}{4xE} z_1 + \frac{\chi^2 + (\mathbf{k}_{\perp} - \mathbf{q}_{2\perp} - \mathbf{q}_{3\perp})^2}{2xE} z_2 + \frac{\chi^2 + (\mathbf{k}_{\perp} - \mathbf{q}_{3\perp})^2}{2xE} z_3 \right),$$

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Implemented efficient numerical calculations of the contributions to the gluon radiation spectrum.

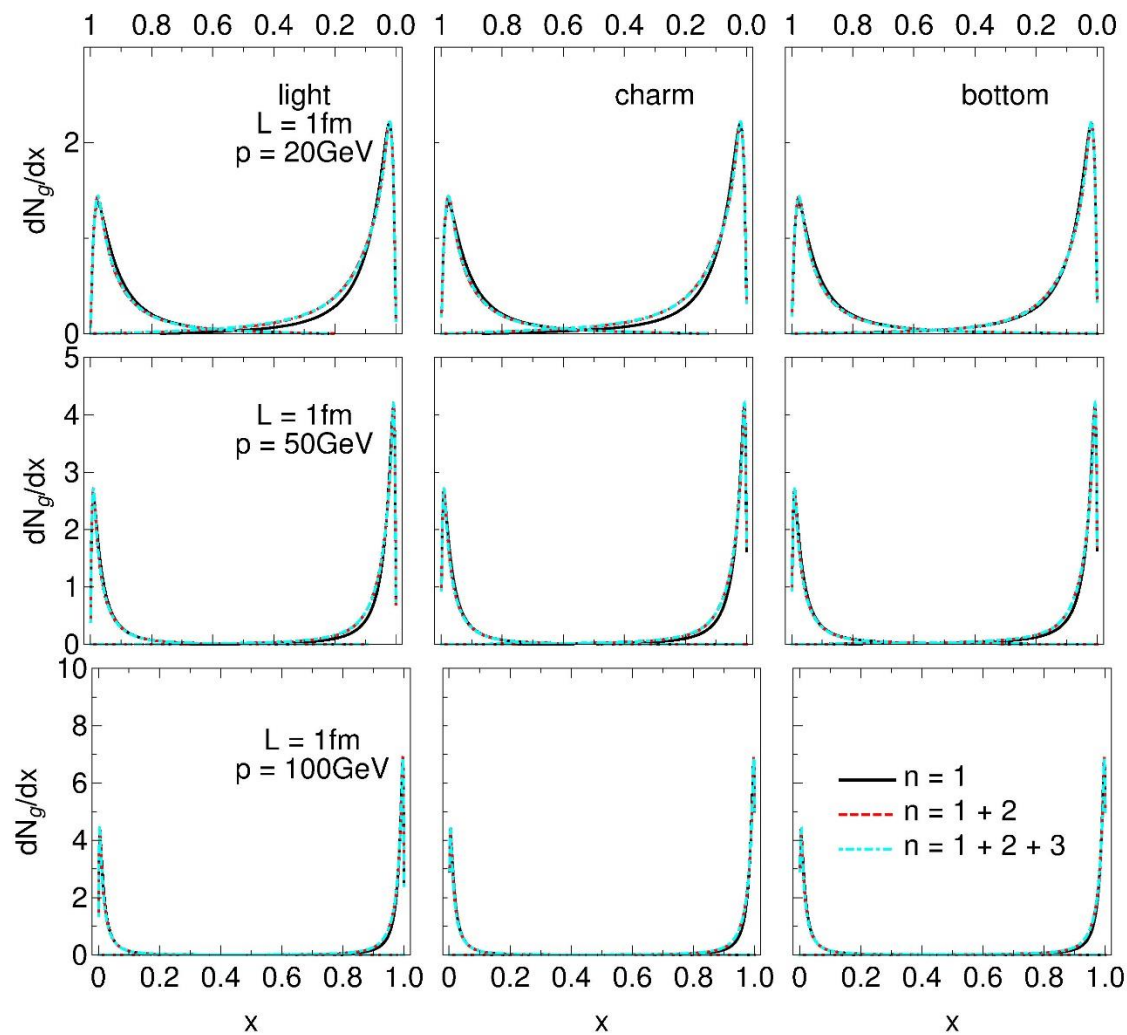
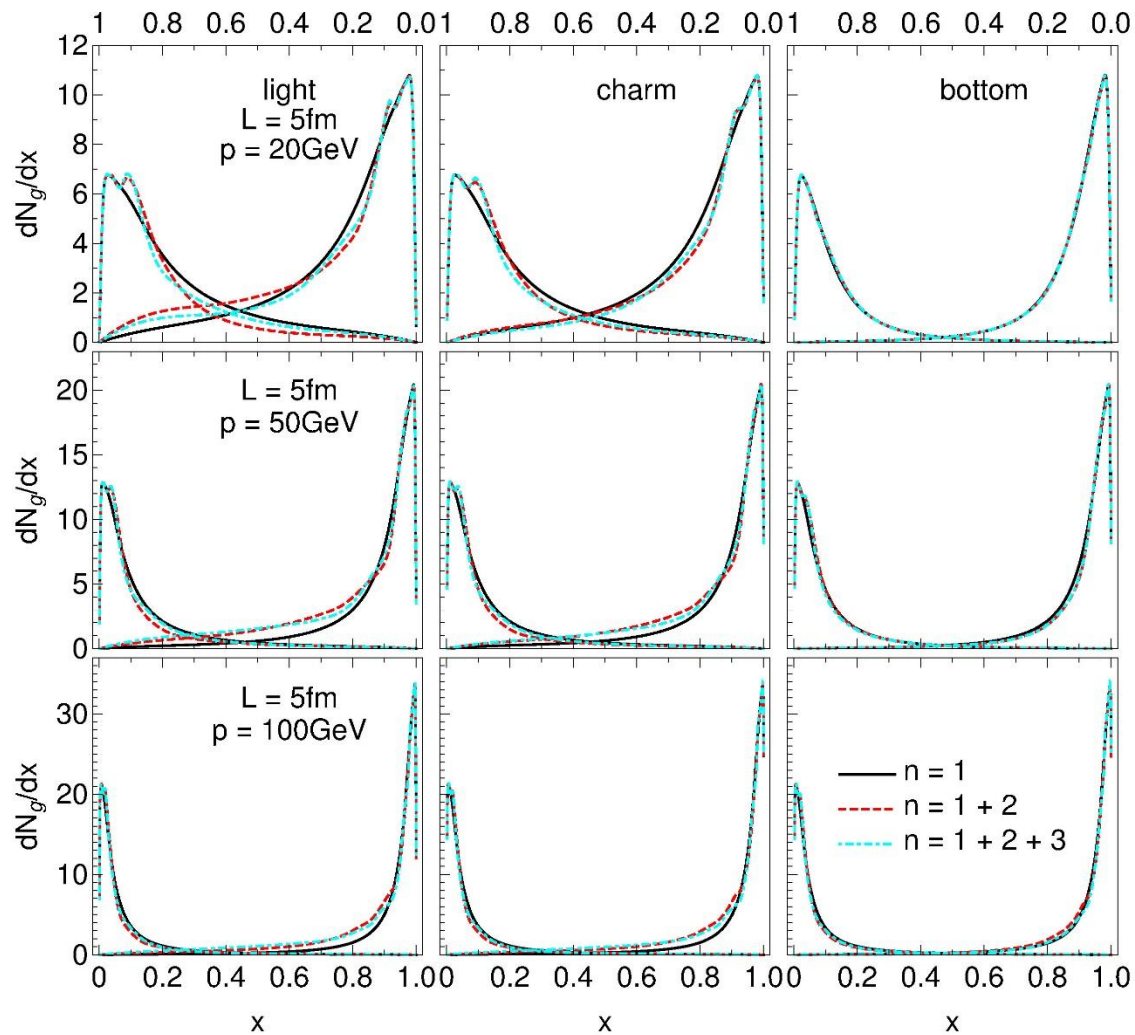
Highly oscillatory and hard to converge.

Integrated into modified DREENA-C (Dynamical Radiative and Elastic ENergy loss Approach, C-constant T medium) framework to provide predictions for high-pt observables.

Needed 70000 CPUh to generate predictions up to 3rd order.

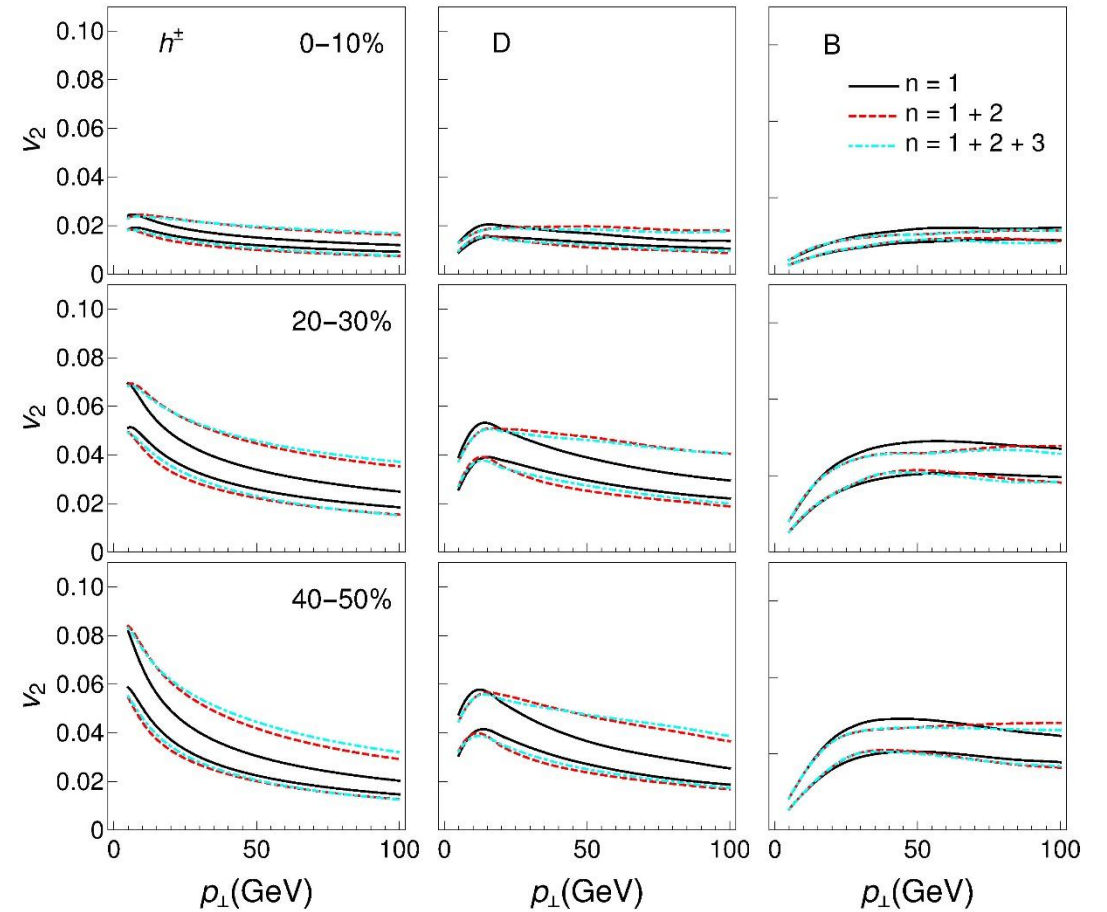
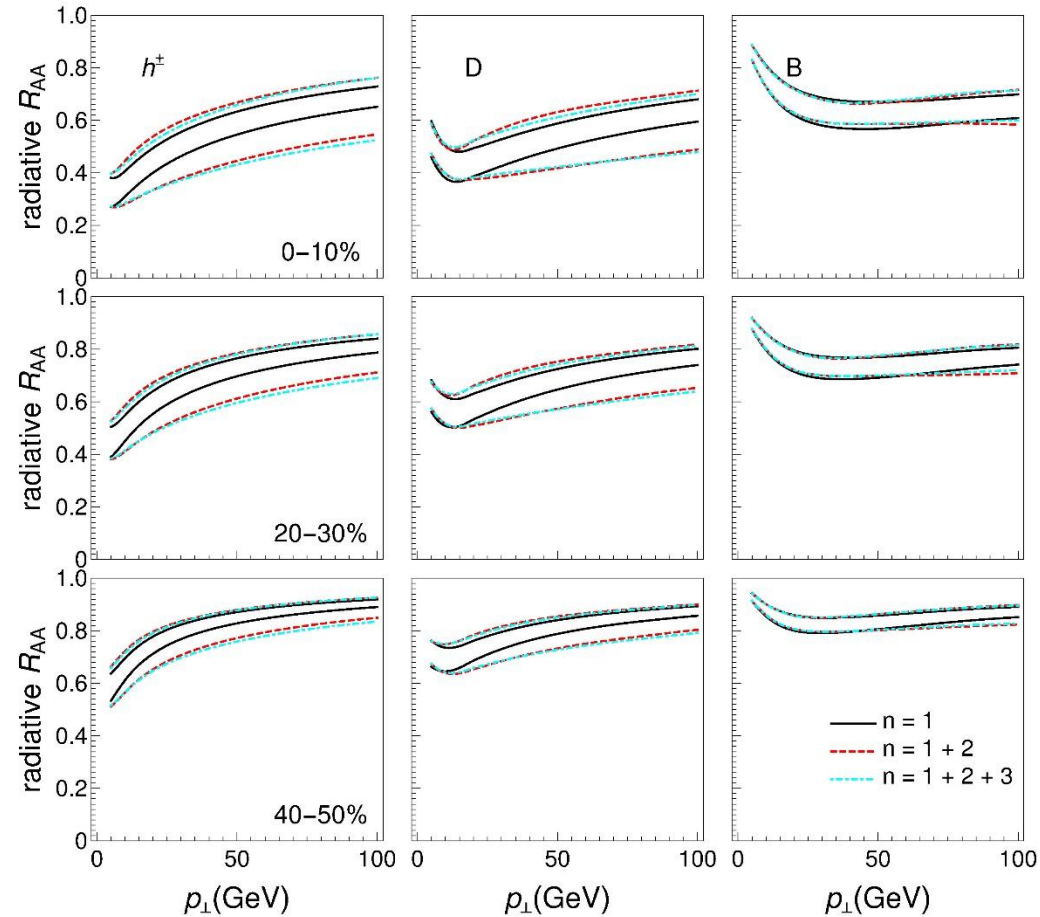


Gluon Radiation spectrum



The importance of higher orders of opacity decreases with the increase of jet energy and mass, and with decreasing the size of the medium.

Higher orders in opacity effects on radiative R_{AA} and v_2



Effects on RAA and v_2 are quantitatively and qualitatively similar.

Effects are smaller for more peripheral collisions. Negligible for B mesons, while increasing with decreasing mass.

Unexpectedly, for different magnetic mass limiting cases, these effects are opposite in sign:

- for $\mu_M/\mu_E=0.6$, the effects are small and reduce suppression.
- for $\mu_M/\mu_E=0.4$, opposite in sign and larger in magnitude.

What is the reason behind these unexpected results?

We examine the effective potential $v(\mathbf{q})$ in a dynamic QCD medium, expressed as the difference between electric ($v_L(\mathbf{q})$) and magnetic ($v_T(\mathbf{q})$) contributions, given by:

$$v(\mathbf{q}) = v_L(\mathbf{q}) - v_T(\mathbf{q})$$

where

$$v_L(\mathbf{q}) = \frac{1}{\pi} \left(\frac{1}{(\mathbf{q}^2 + \mu_{pl}^2)} - \frac{1}{(\mathbf{q}^2 + \mu_E^2)} \right), \quad v_T(\mathbf{q}) = \frac{1}{\pi} \left(\frac{1}{(\mathbf{q}^2 + \mu_{pl}^2)} - \frac{1}{(\mathbf{q}^2 + \mu_M^2)} \right)$$

μ_E , μ_M , and $\mu_{pl} = \mu_E/\sqrt{3}$ denote the electric, magnetic, and plasmon masses, respectively.

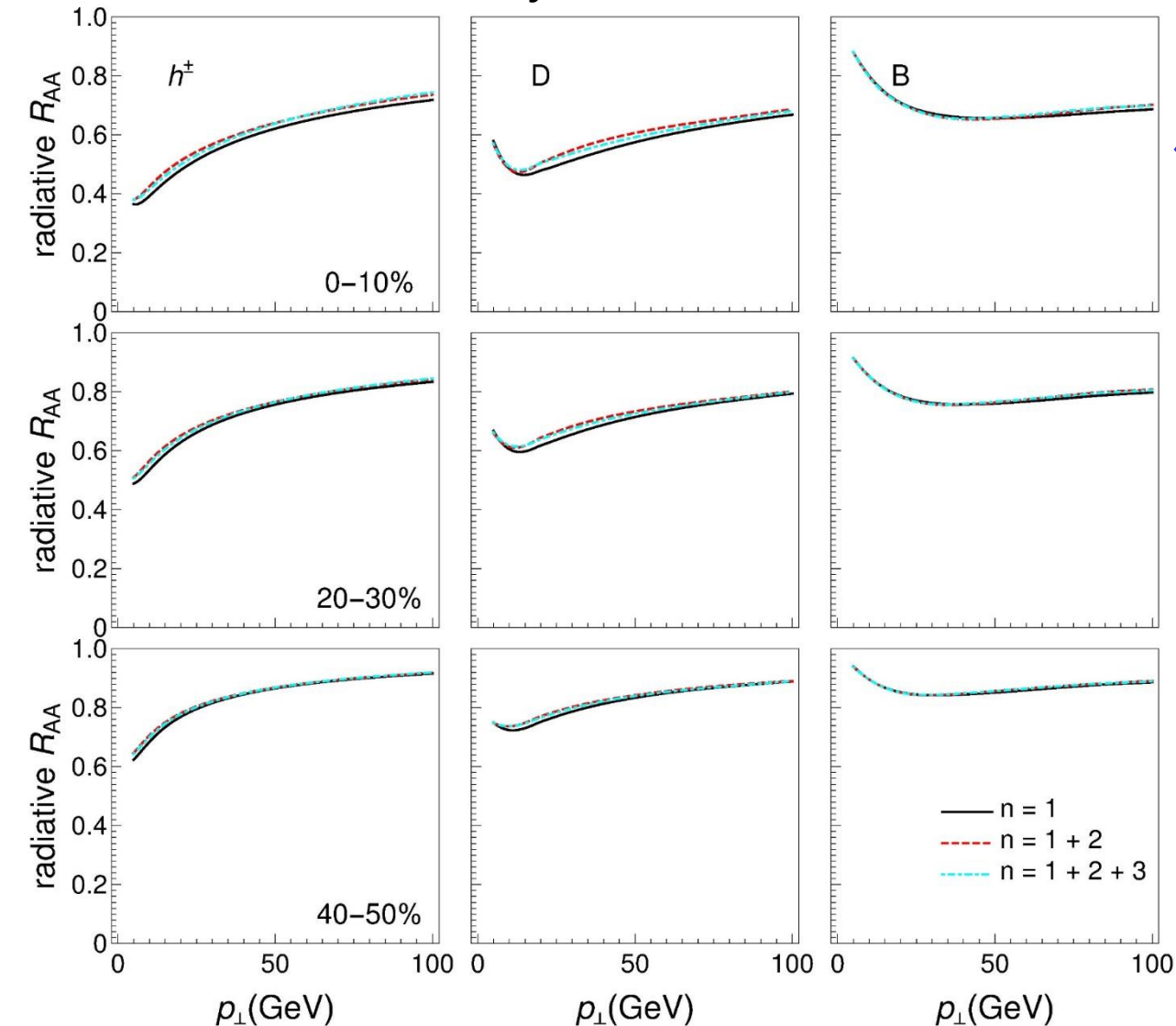
The electric contribution is always positive due to $\mu_{pl} < \mu_E$.

The magnetic contribution has a non-trivial dependence on the magnetic mass:

- for $\mu_M > \mu_{pl}$, the magnetic contribution decreases the energy loss.
- for $\mu_M < \mu_{pl}$, it increases the energy loss and, consequently, suppression.

This observation may intuitively explain the results observed in the previous slide.

Electric dynamic contribution



μ_E is well-defined from lattice QCD, with consistent perturbative calculations. So, the electric potential is well-defined in dynamical energy loss, enabling separate testing of higher opacity orders.

Surprisingly, higher orders in opacity have a negligible impact on the electric contribution to energy loss!

Therefore, the higher orders mainly affect the magnetic contribution to energy loss, with the sign of the effect being dependent on the value of the magnetic mass.

- For $\mu_M/\mu_E=0.4$, μ_M is smaller than μ_{pl} , resulting in significant effects that increase the suppression.
- For $\mu_M/\mu_E=0.6$, μ_M is slightly larger than μ_{pl} , leading to small effects that reduce suppression.

S. Stojku, B. Ilic, I. Salom, MD, arXiv:2303.14527

The latest 2+1 flavor lattice QCD results with physical quark masses limit magnetic screening to $0.58 < \mu_M/\mu_E < 0.64$ (Borsanyi, *et al.*, 2015). **Thus, for this magnetic screening range, higher-order opacity effects are small ($\sim 5\%$) in a dynamical QCD medium and can be safely neglected!**

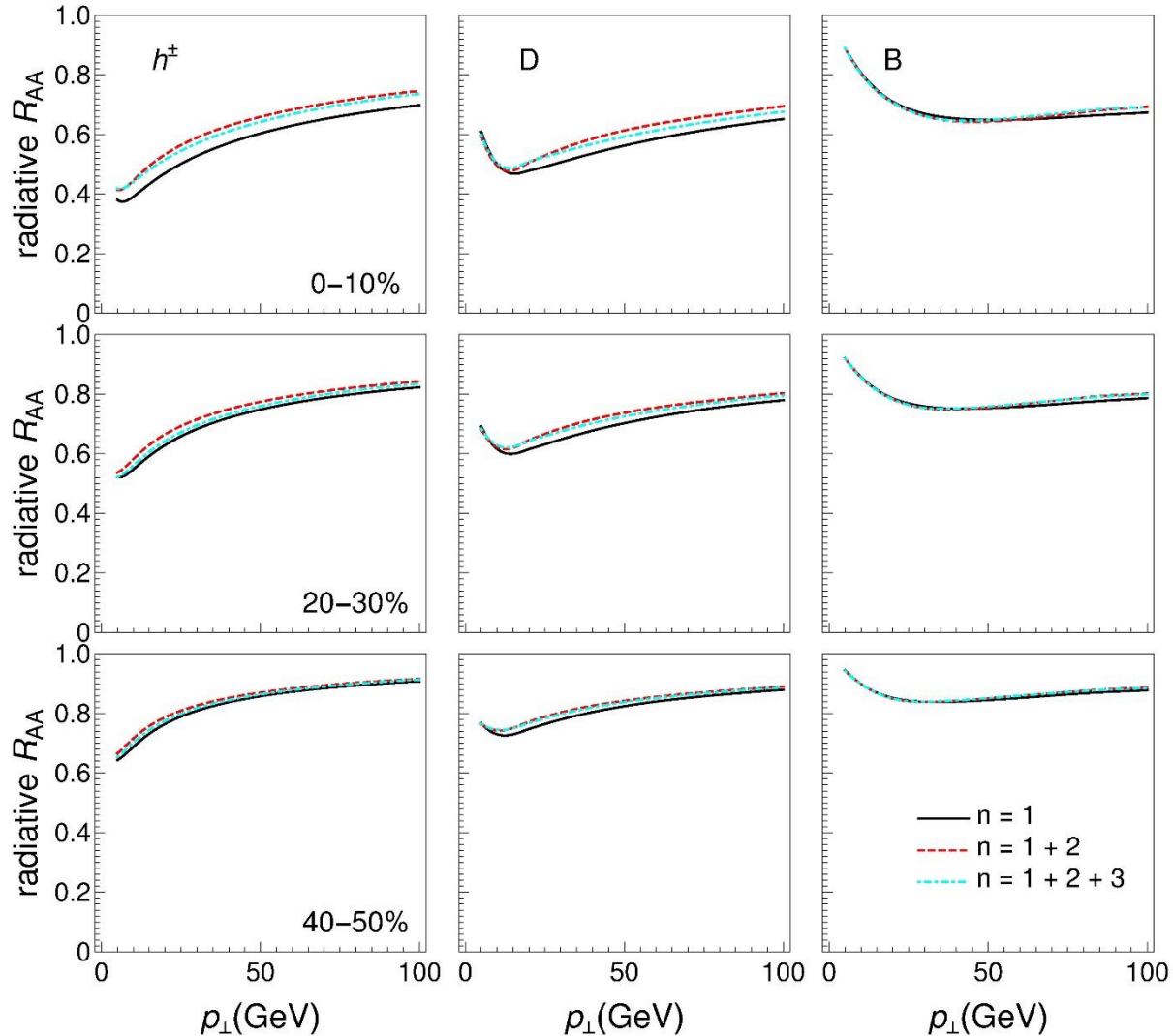
Can higher orders in opacity effects be also neglected under the static QCD medium approximation, where only the electric contribution exists?

(does not have to be the case, as electric potential differs significantly in a static versus dynamic QCD medium)

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(Electric) static contribution



Higher order in opacity effects in static QCD medium.

Larger than effects for electric contribution in dynamical QCD medium, but still small (less than 6%).

For optically thin medium models with static approximation, including multiple scattering centers has a small effect on the numerical results, i.e., these effects can also be neglected.

S. Stojku, B. Ilic, I. Salom, MD, arXiv:2303.14527

Higher orders in opacity in evolving medium?

Including higher-order effects in evolving medium is very difficult task.



Can be partially assessed by studying how higher-order effects depend on the temperature, which changes in the evolving medium.



Study the effects of higher orders in opacity for three different temperature values $T=200, 400, 600\text{MeV}$.

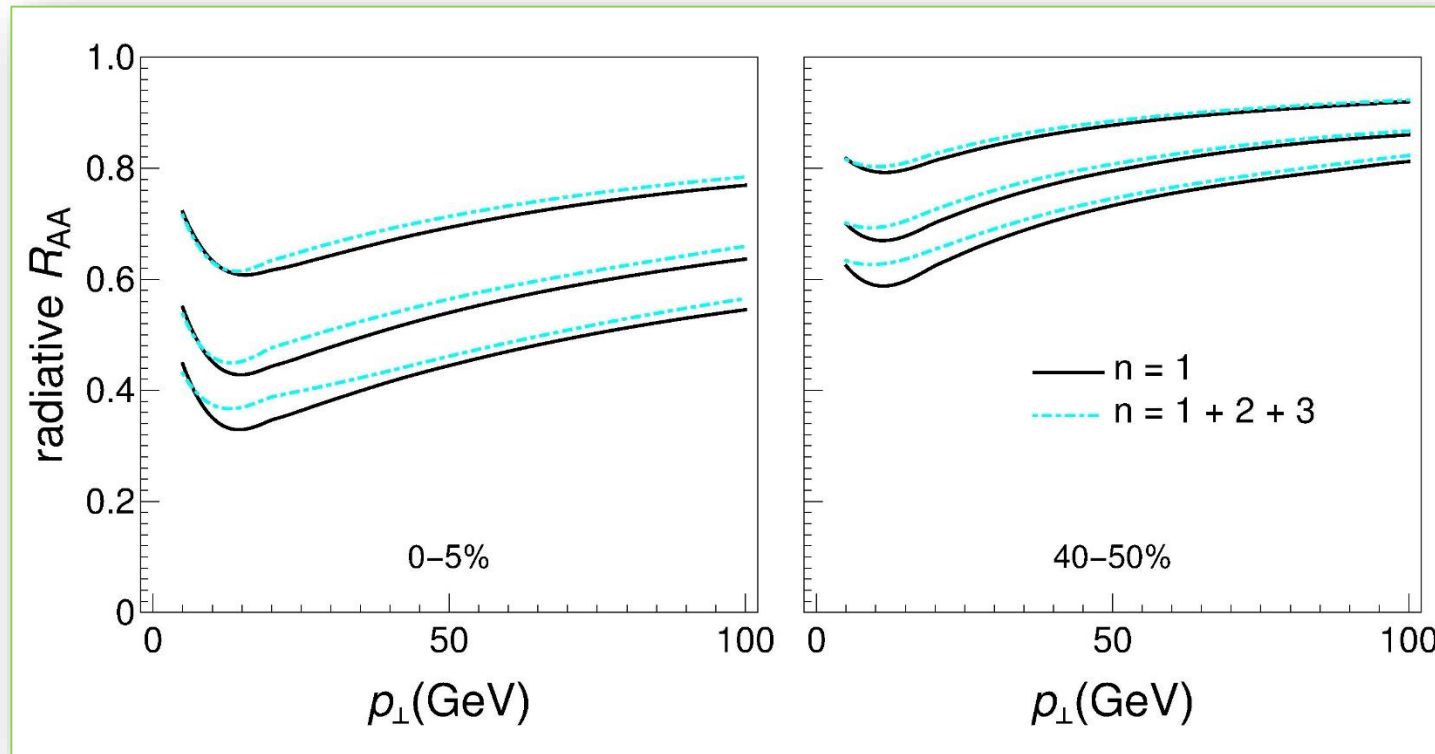
Higher-order effects are largely independent of these T values.



Thus, unlikely that obtained results will be significantly affected by the inclusion of medium evolution.



We expect the impact of multiple scattering centers will remain small.



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Summary

- We generalized dynamical energy loss and (D)GLV towards finite orders in opacity.
- Negligible impact on bottom quarks due to short gluon formation time, i.e., incoherent limit.
- Including the 2nd order in opacity is sufficient for charm and light quarks.
- Higher opacity orders have minimal effects on high-pt observables in the static QCD medium approximation.
- In a dynamical medium, electric contribution remains insensitive to opacity increase.
- The effects of magnetic screening on R_{AA} are opposite in sign in two limits ($\mu_M/\mu_E=0.4$ and $\mu_M/\mu_E=0.6$).
- For most current estimates of magnetic screening, these effects are less than 5% and can be neglected.
- The first order in opacity is a suitable approximation for finite-size QCD medium created in the RHIC and the LHC.



QGP tomography

Thank you for your attention!

Canyon of river DREENA in Serbia



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МИНИСТАРСТВО ПРОСВЕТЕ,
НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА