A microscopic model of charmonia production in heavy ion collisions

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Hard Probes

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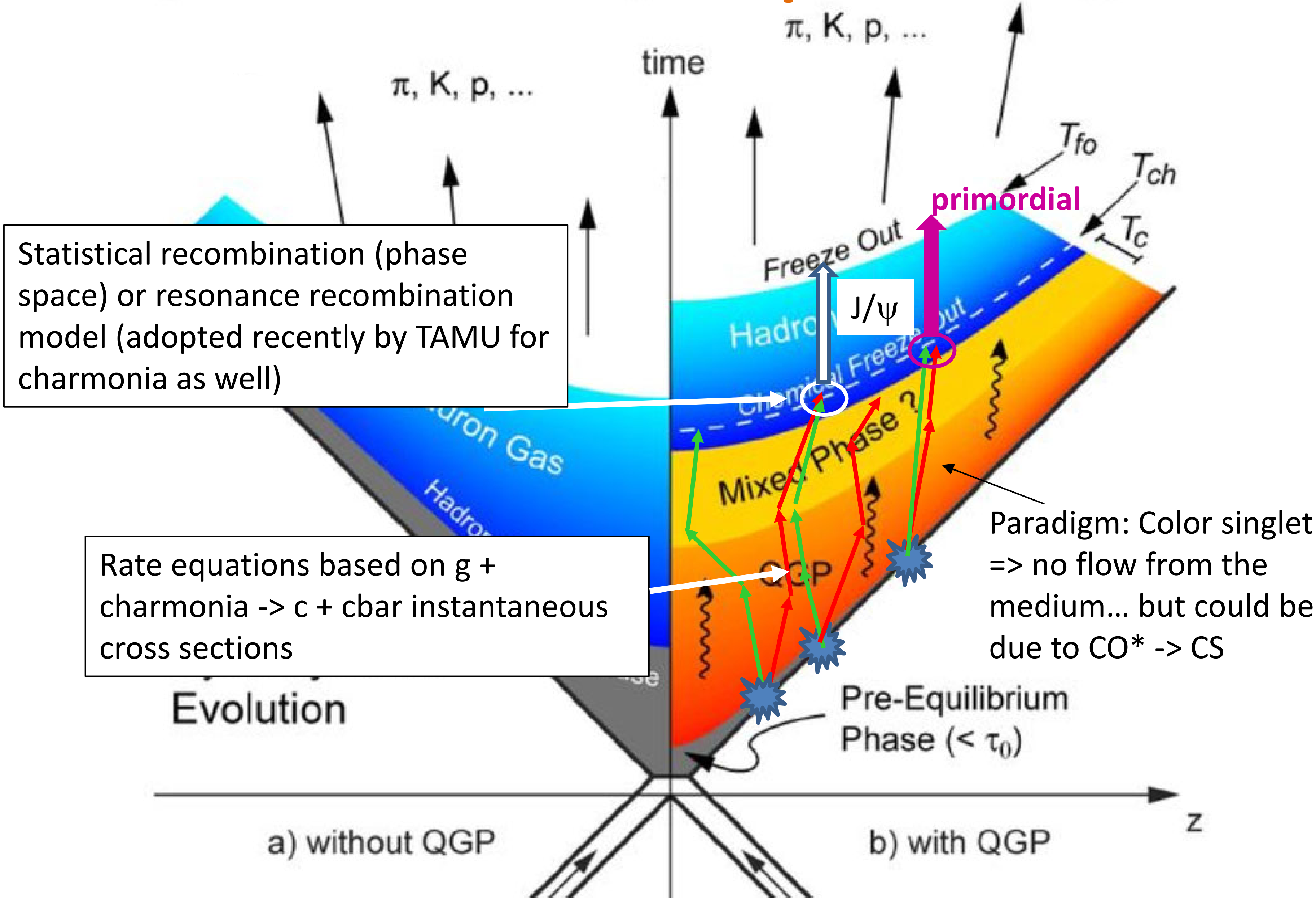
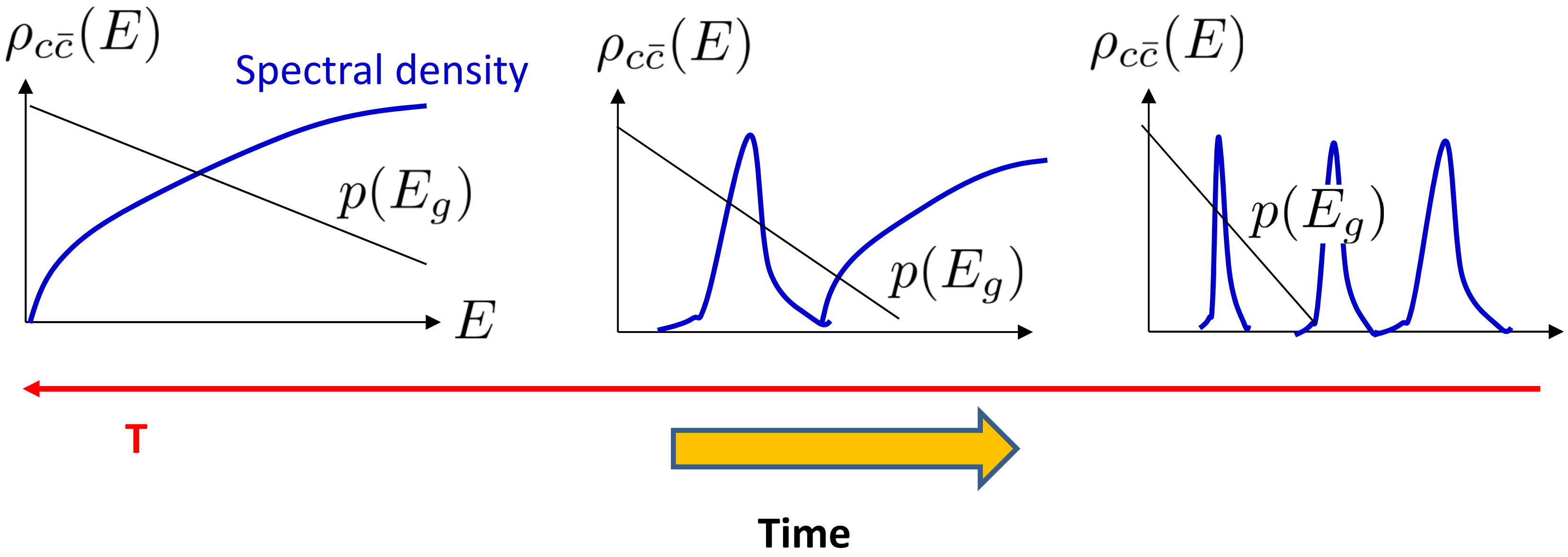
and Pays de la Loire

Charmonia in transport models



Charmonia in a microscopic theory
Several regimes / effects



Multiple scattering on quasi free states

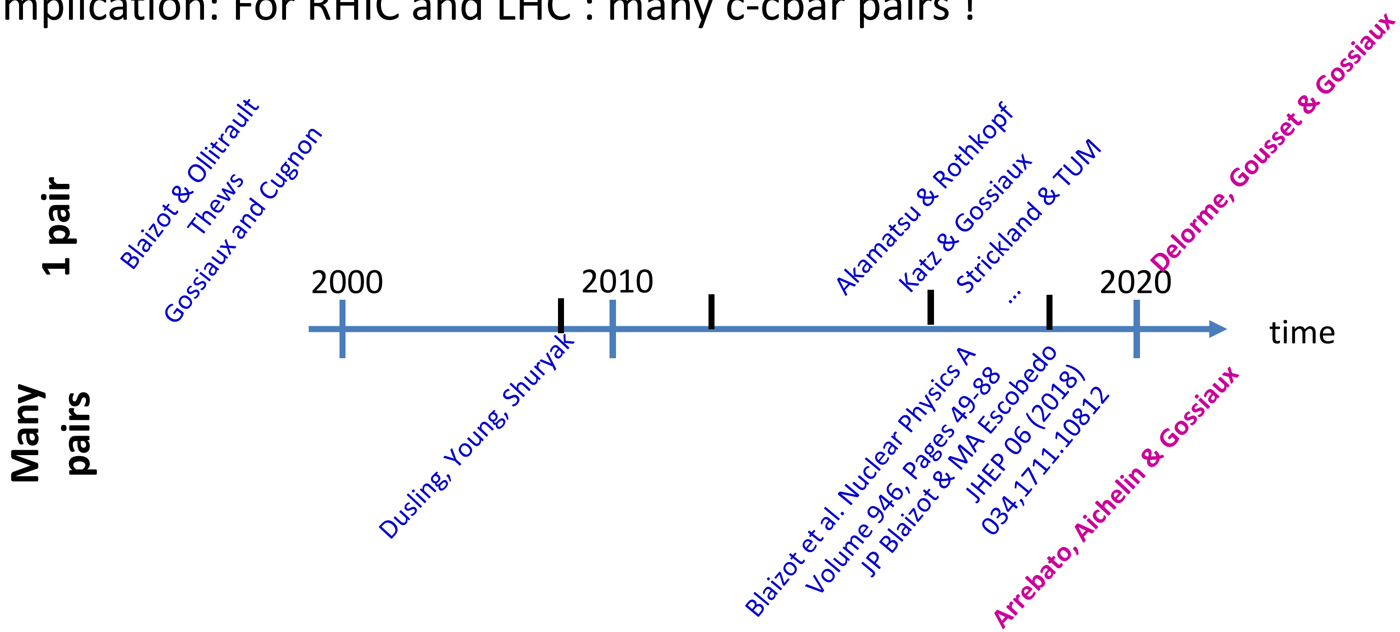
Gluo-dissociation of well identified levels by scarce "high-energy" gluons (dilute medium => cross section ok)

Well identified formalisms (Quantum Master Equation, Boltzmann transport, Stochastic equations,…) in well identified regimes, but continuous evolution and no unique framework continuously applicable (to my knowledge)

Yet, still a need to define the equivalent of a formation – dissociation rate

Several motivations to go microscopic & quantum

- The in-medium quarkonia are not born as such. One needs to develop an **initial compact state** to fully bloomed quarkonia
- The dissociation-recombination reactions affecting quarkonia are **not instantaneous**… In dense medium, the notion of cross section should be replaced by the more rigorous open-quantum system approach (**continuous transitions**)
- Better suited for **« from small to large »**
- Extra complication: For RHIC and LHC : many c-cbar pairs !



Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs (NRQCD) => mixed Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

The spirit of the method...

$$P^{\Psi}(t) = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\hat{\rho}_N(t))\right]$$

Simply taken at the end of the evolution (ideal world)

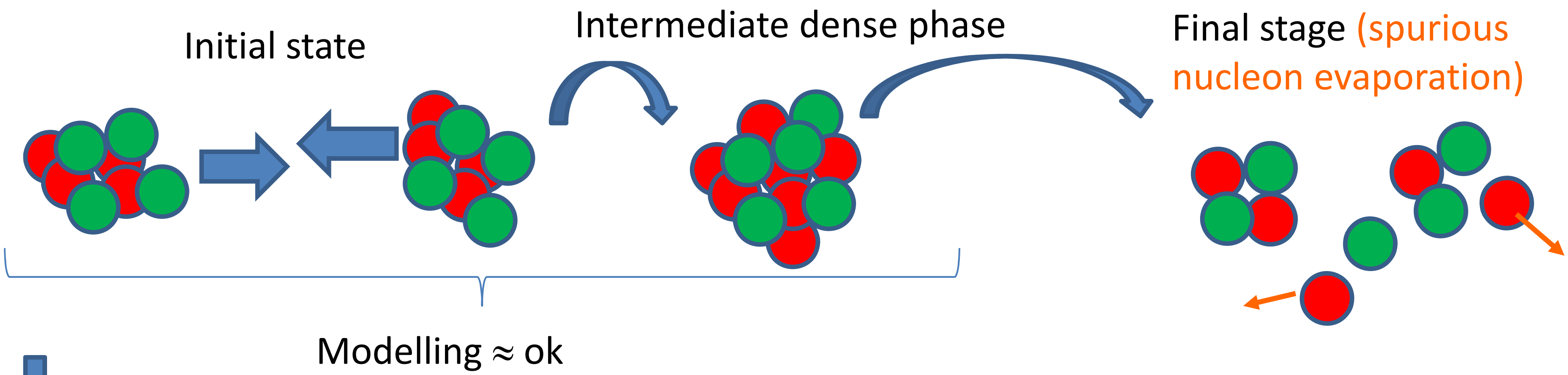
$$\hat{\rho}^{\Psi_i}_{Q\bar{Q}} = \sum_i |\Psi^i_{Q\bar{Q}}\rangle\langle\Psi^i_{Q\bar{Q}}|$$

$$\frac{d\hat{\rho}_N(t)}{dt} = -i\left[\hat{H}_N, \hat{\rho}_N(t)\right]$$

Various Quarkonia bound states (in vacuum)

Unfortunately... all N-body practionners know that **modelling the full system up to the last stage is quite challenging !** Issues of stability, energy conservation,...

Clear lesson from the « old » cascade and QMD codes for fragment formation



Replace « final » => « initial » + Sum of time steps and chop off at the appropriate time scale

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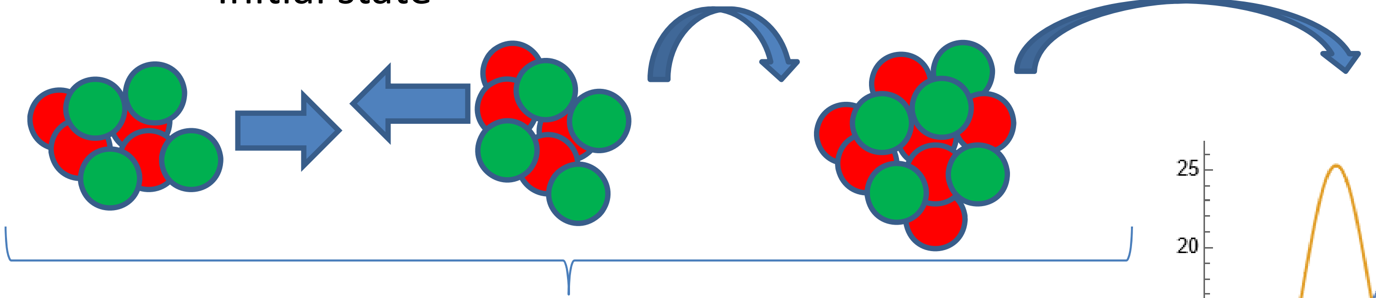
Initial state

Intermediate dense phase

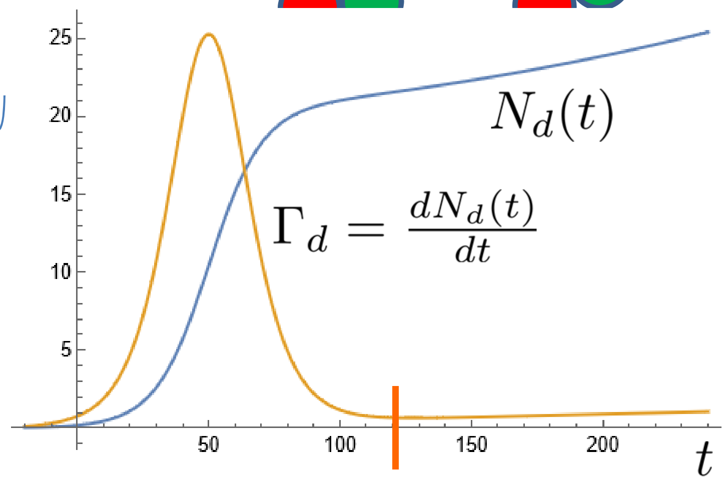
Final stage (spurious nucleon evaporation)



Modelling ≈ ok

Replace « final » => « initial » + Sum of time steps and chop off at the appropriate time scale

$$P^{\Psi}(t) = P^{\Psi}(t_0) + \int_{t_0}^{t} \Gamma(t')dt'$$

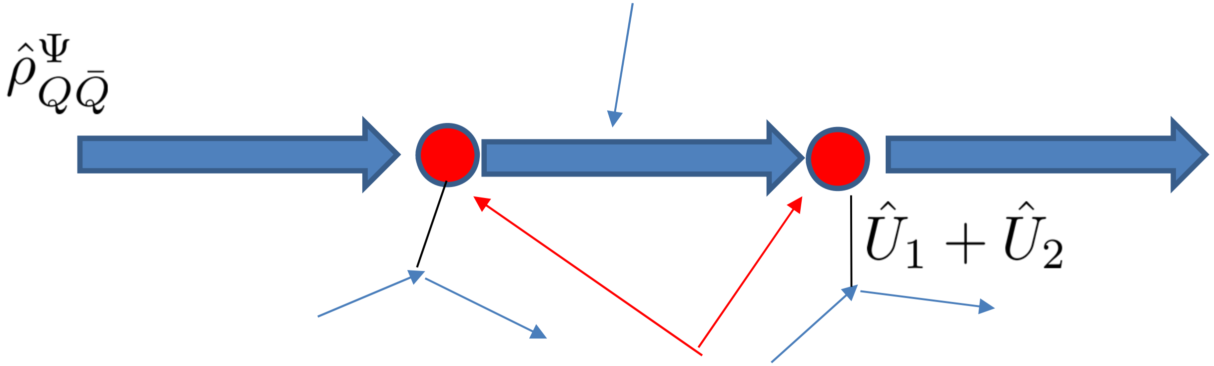


The spirit of the method…

Dealing with the dynamics ?

If eigenstates of the « internal » 2-body (QQbar) interaction

Free evolution => Γ=0



No « late time » evolution in this eigenbasis

Interaction with a 3rd body => modification of the $\hat{\rho}^{\Psi}_{Q\bar{Q}}$

… $\Gamma^{\Psi}(t) = -iTr[\hat{\rho}^{\Psi}[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$

Total interaction of Q and Qbar with all light partons

Source of « destruction » ⇔ imaginary potential

Remler Formalism at work

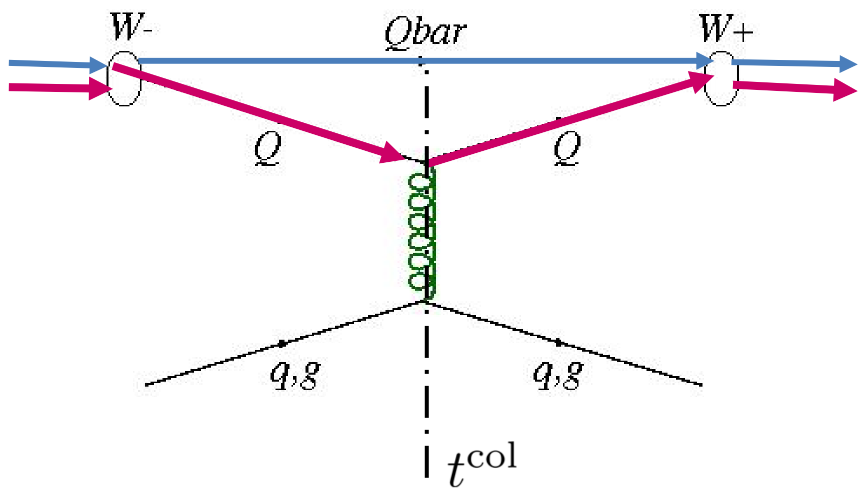
E.A. Remler, ANNALS OF PHYSICS 136, 293-316 (1981)

Level of the modelling : **semi classical** for the Q-Qbar evolution => Wigner distributions instead of density operators

Combining the expression of the Wigner's distribution and substituting in the **effective rate equation** :

$$\Gamma^{\Psi}(t) \approx \sum_{i=1,2} \sum_{j\geq 3} \delta(t-t_{ij}^{\mathrm{col}}) \int \frac{d^3p_i d^3x_i}{h^3} \left[W_{Q\bar{Q}}^{\Psi}(p_1,x_1;p_2,x_2)\Big|_{t+\epsilon} - W_{Q\bar{Q}}^{\Psi}(p_1,x_1;p_2,x_2)\Big|_{t-\epsilon} \right]$$

- The quarkonia production in this model is a three body process; the HQs interact only by collisions with the QGP !!!
- The "details" of H_{int} between HQ and bulk partons are incorporated into the evolution of W_N after each collision / time step (nice feature for the MC simulations)
- Dissociation and recombination treated in the same scheme



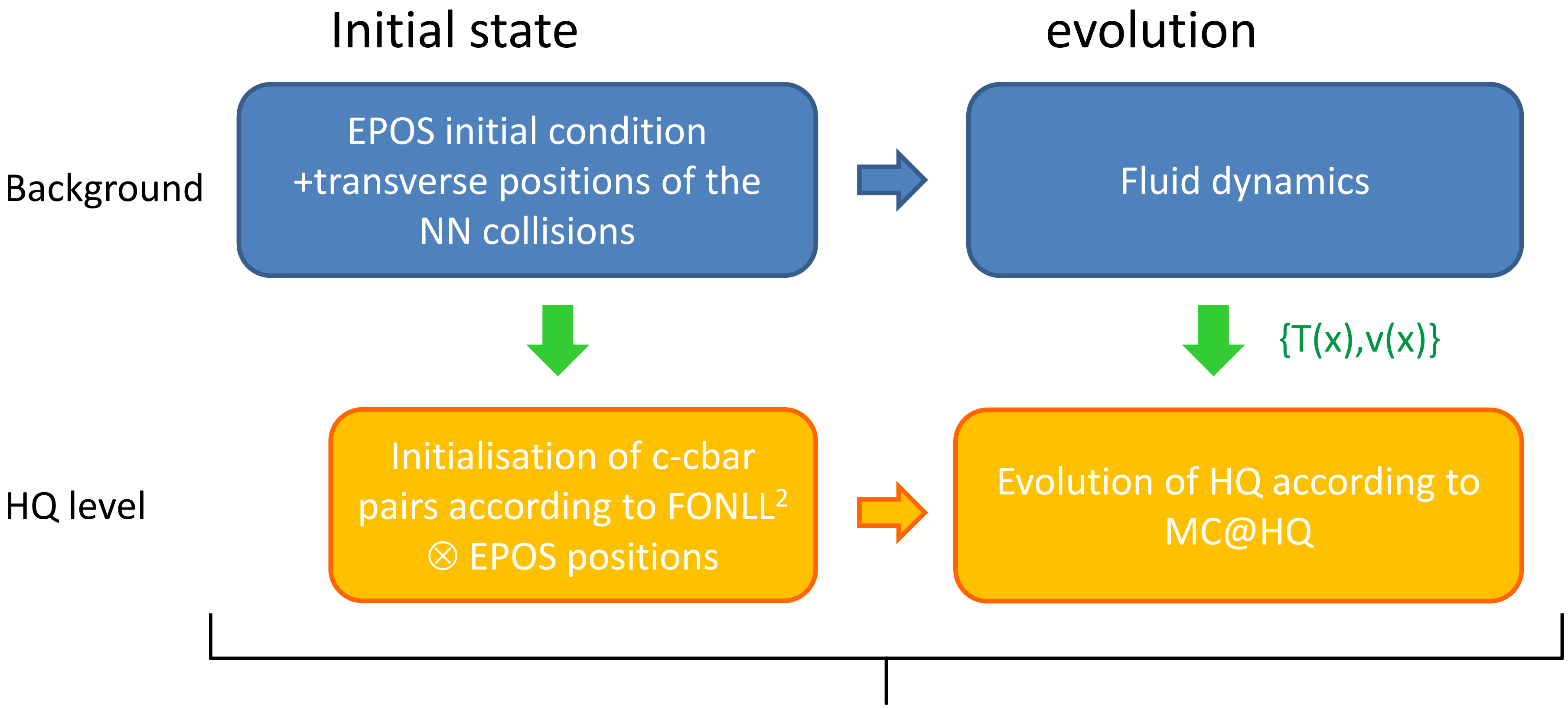
Then: $P^{\Psi}(t) = P^{\Psi}(t_0) + \int_{t_0}^{t} \Gamma(t')dt'$

NB: Also possible to generate similar relations for differential rates

Interaction of HQ with the QGP are carried out by EPOS2+MC@HQ (good results for D and B mesons production)

Subatech

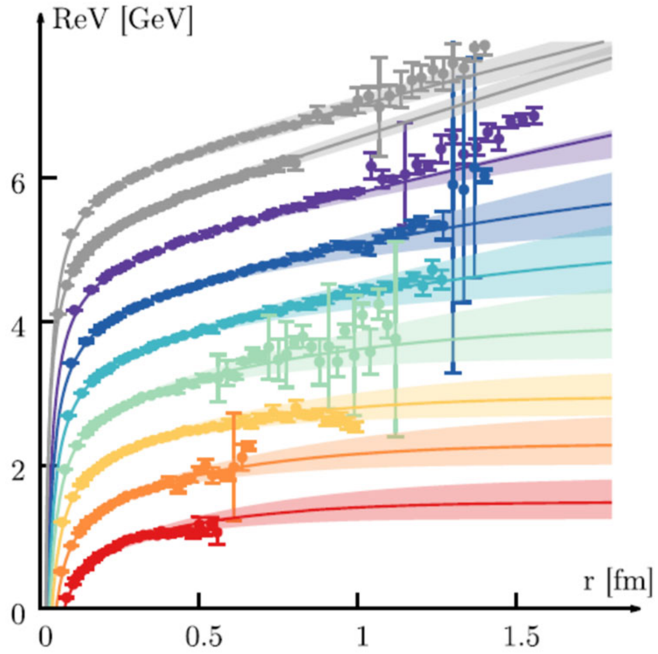
The 3 layers of the numerical modelling



Usual EPOS2 + MC@HQ used with some success to describe open heavy flavor production at LHC (see Eur. Phys. J. C (2016) 76:107)

Extension of the Remler formalism

- **Confining $Q\bar{Q}$ forces inside the MC evolution ; large impact on the # of close pairs… and correlated trajectories.**



(No internal potential in early applications dedicated to deuteron production in low energy AA collisions; advocated to be negligible… as only the « hot zone » was contributing

But for quarkonia, it turns out not to be the case => need for in-medium potential

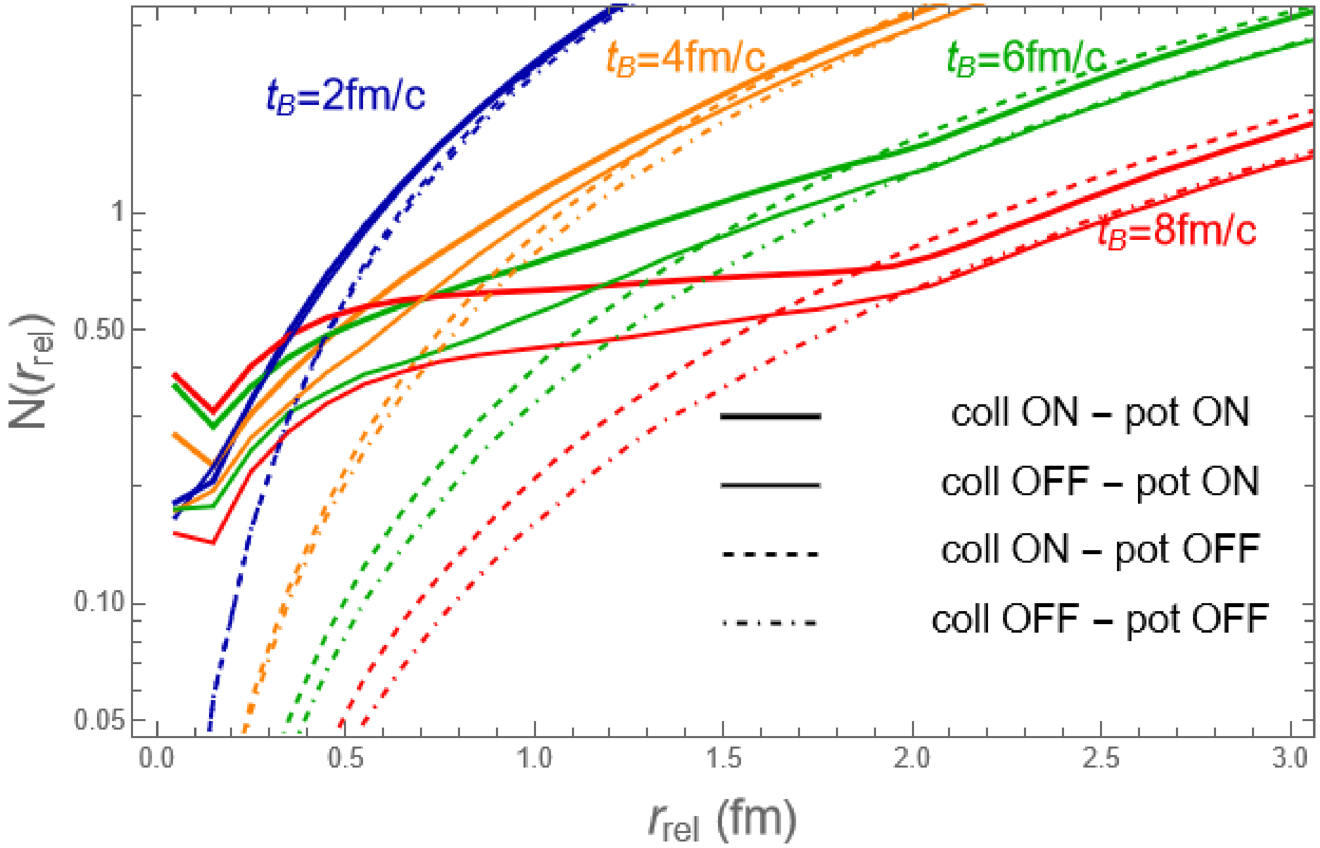
D. Lafferty and A. Rothkopf,
PHYS. REV. D 101, 056010 (2020)

The 3 layers of the numerical modelling

	Initial state		evolution
Background	EPOS initial condition +transverse positions of the NN collisions	→	Fluid dynamics
	↓		↓ {T(x),v(x)}
HQ level	Initialisation of c-cbar pairs according to FONLL[2] ⊗ EPOS positions	→	Evolution of HQ according to MC@HQ **+ c-cbar in-medium potential**
			↓
			Correlated SC trajectories

Complicated relativistic N-body problem… Only stable at « not too high » p_T

The dynamics of c-cbar correlation



- The c-cbar potential (« pot ON ») leads to a **huge increase of the c-cbar probability at close distance** at large times (not a random Poisson distribution !)…
- … Especially when the collisions with the QGP (« coll ») are switched ON as well

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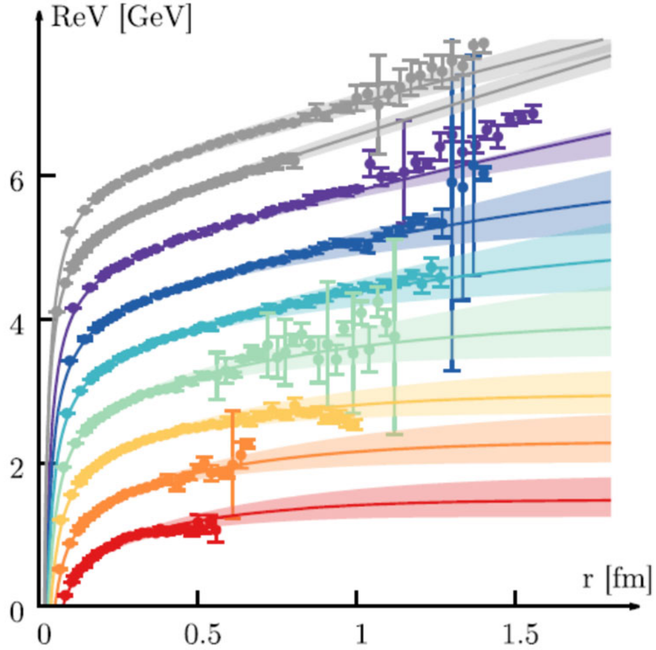
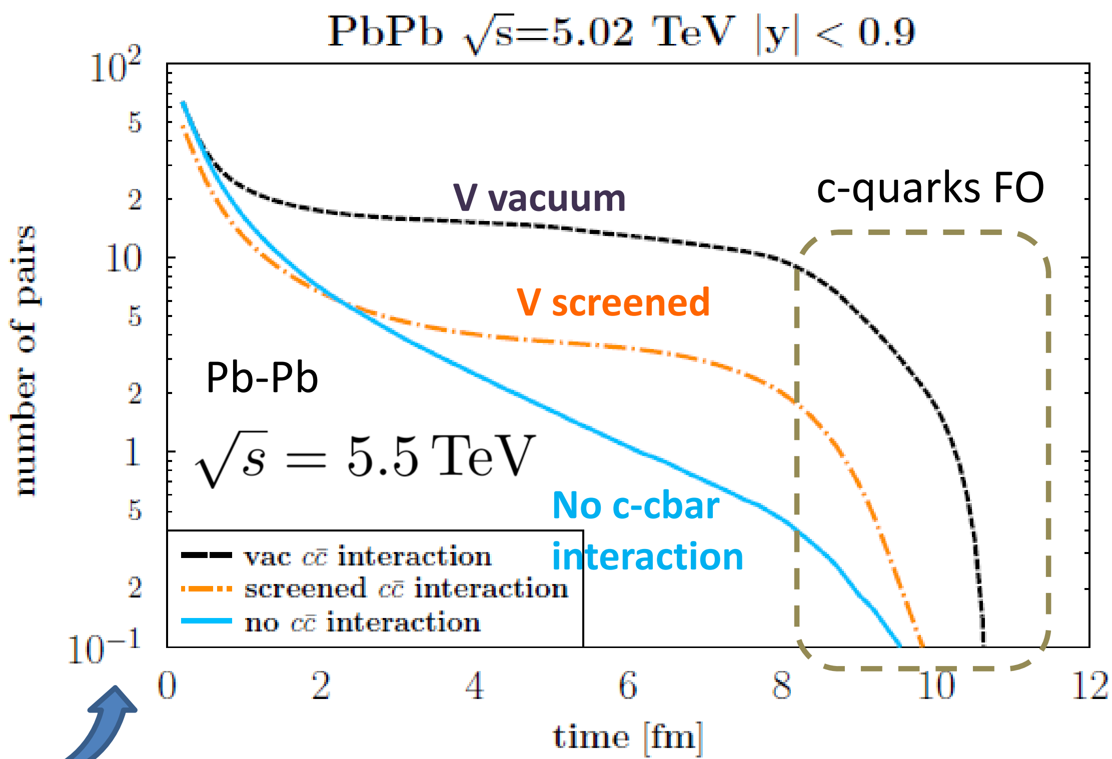


Instantaneous # of Q-Qbar at (invariant) distance < 1fm



- Extra source of Γ due to "local-T" basis evolution with time : Γ^{loc}
- Generalization to relativistic Wigner density (boosted quarkonium states)

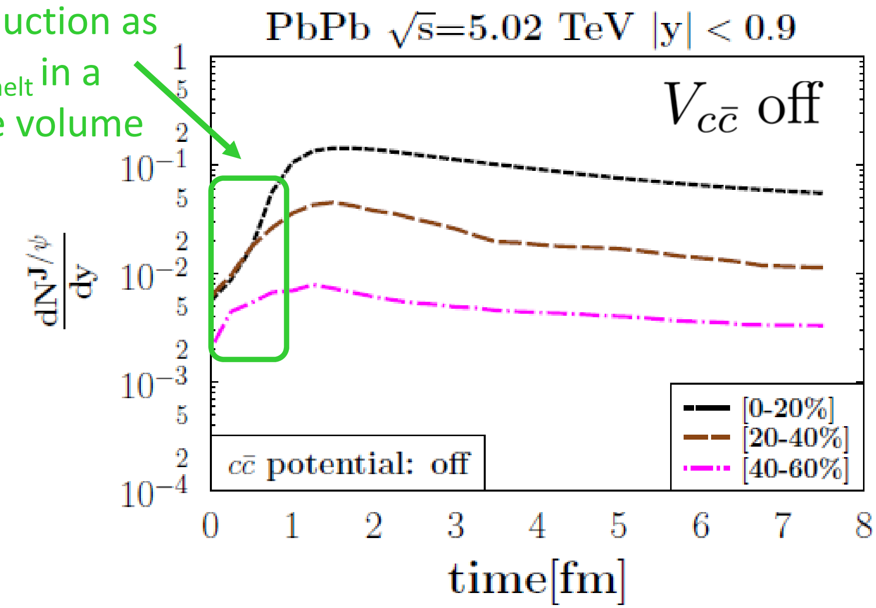
The 3 layers of the numerical modelling

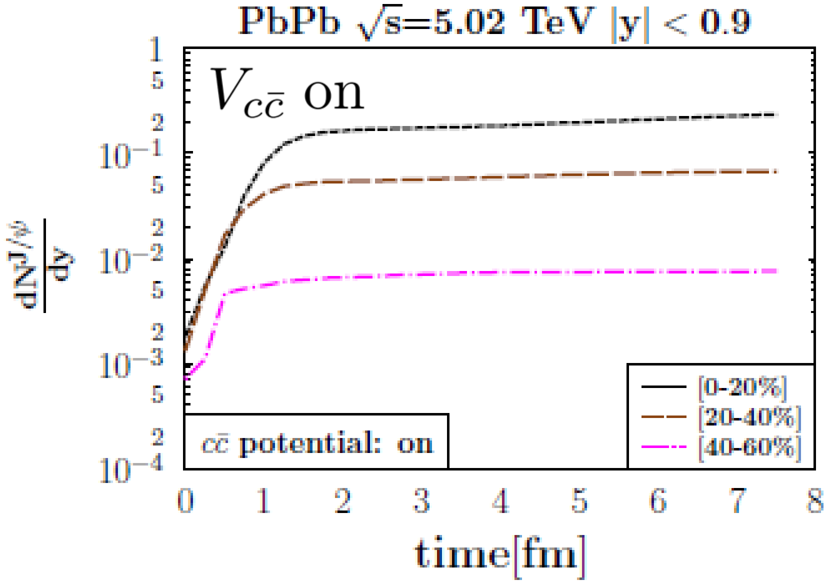
	Initial state		evolution	
Background	EPOS initial condition +transverse positions of the NN collisions	→	Fluid dynamics	
	↓		↓	$\{T(x), v(x)\}$
HQ level	Initialisation of c-cbar pairs according to FONLL[2] ⊗ EPOS positions	→	Evolution of HQ according to MC@HQ **+ c-cbar in-medium potential**	
	↓		↓	Correlated SC trajectories
Quarkonia level	Initial $J/\psi(T)$ $P(t_0)$ when $T(x) < T_{diss}$	→	Evaluation of Γ according to improved Remler + $\Gamma_{loc.}$	

We do not have J/ψ quasi particles in our approach, just correlated c-cbar trajectories

Results : J/ψ production vs time

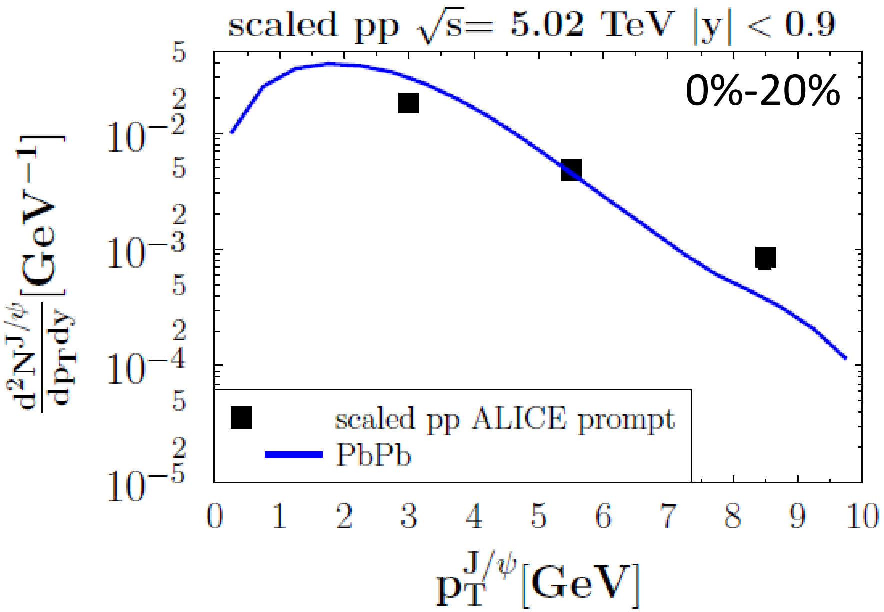
Delayed initial production as $T>T_{melt}$ in a large volume

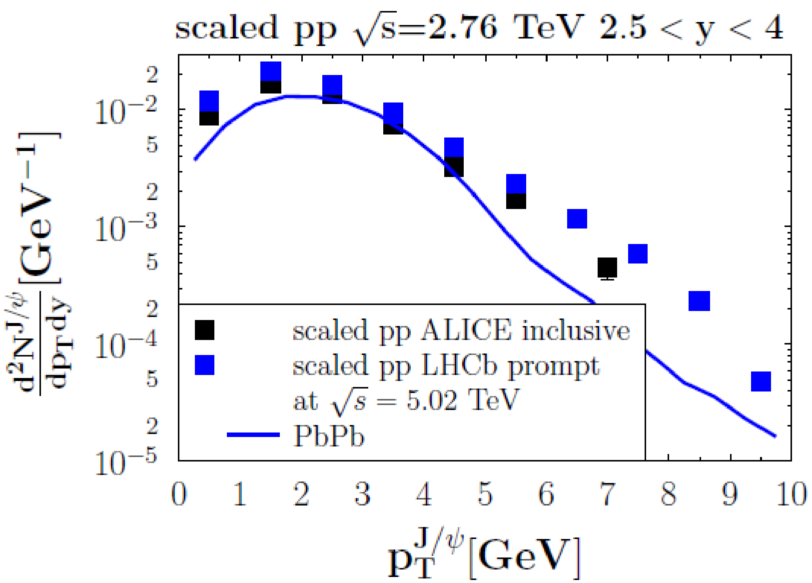




- Without interaction potential between c and cbar, the collisions with the medium manage to destroy the native J/ψ (left)
- With the interaction potential between c and cbar « on », one observes a steady rate of J/ψ creation (increase of Γ^{col}, increase of Γ^{local})… No adiabaticity, but **no instantaneous formation** either.

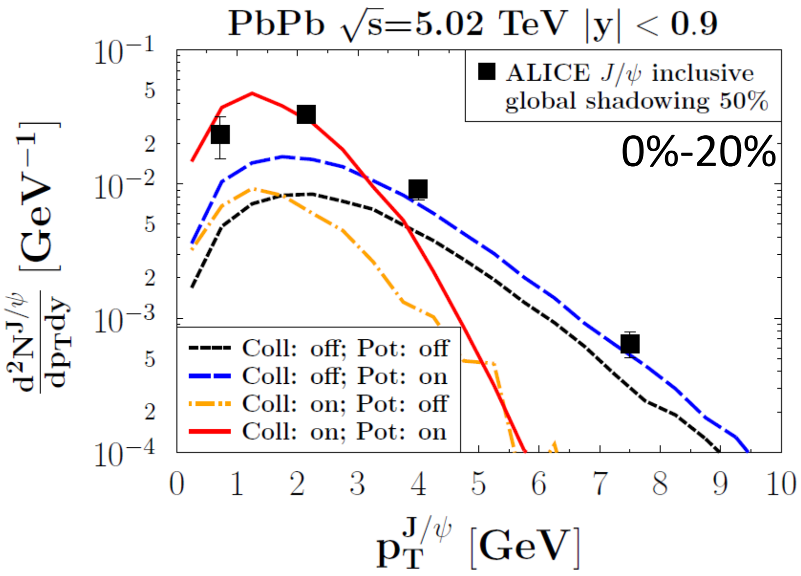
Results : J/ψ production vs p_T

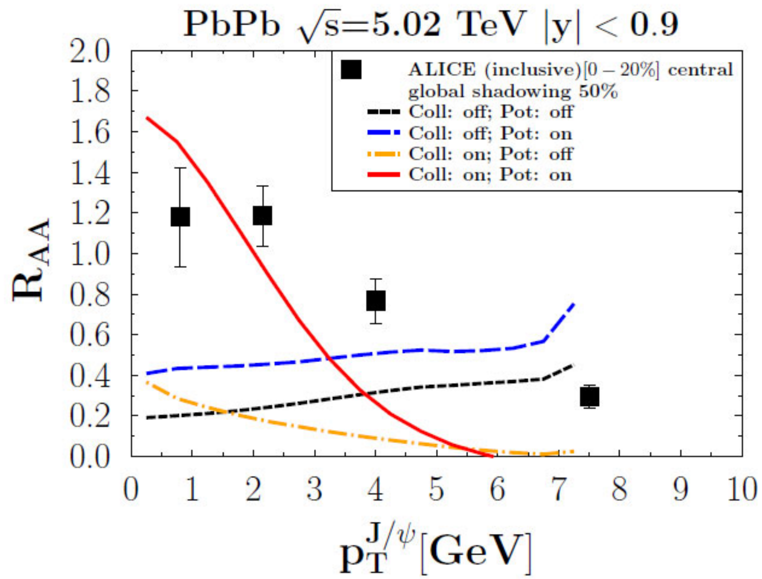


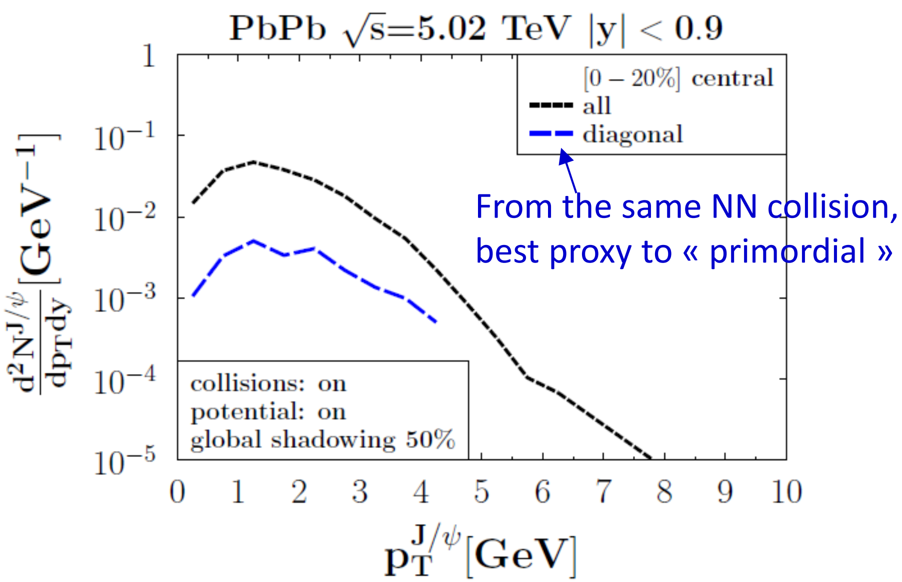


- **Equivalent pp production** (the denominator of the R_{AA}) : c-cbar according to FONLL[2] without any correlation, then coalescence with the Wigner distribution.
- No feed-down from higher states (to be implemented)
- Acceptable for p_T < 5 GeV/c, but deviations for higher p_T.
- To investigate : more appropriate scheme for c-cbar production, including c-cbar correlation : EPOS4

Results : J/ψ production vs p_T





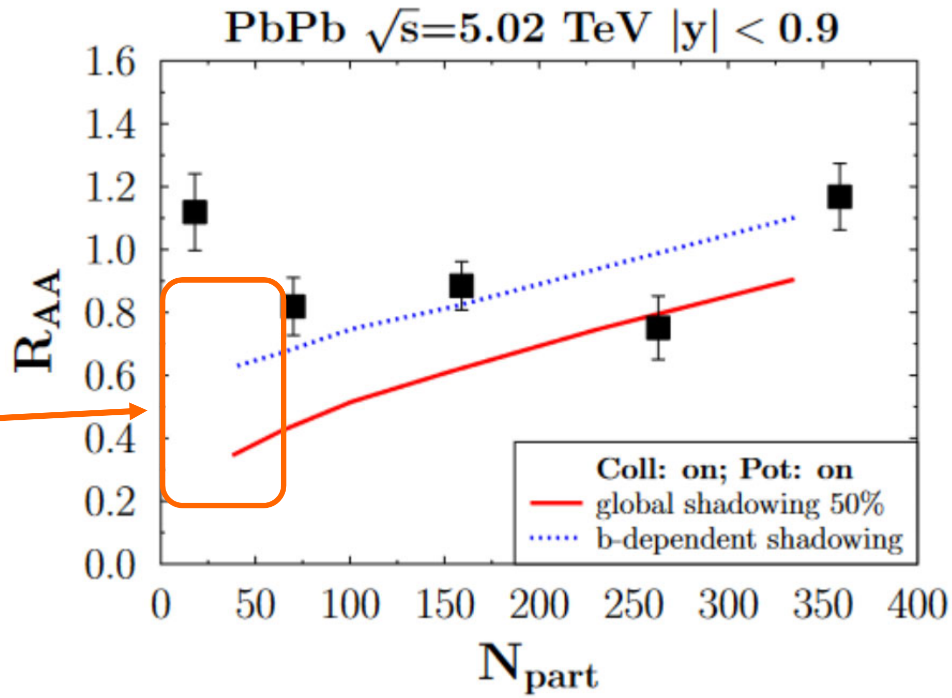


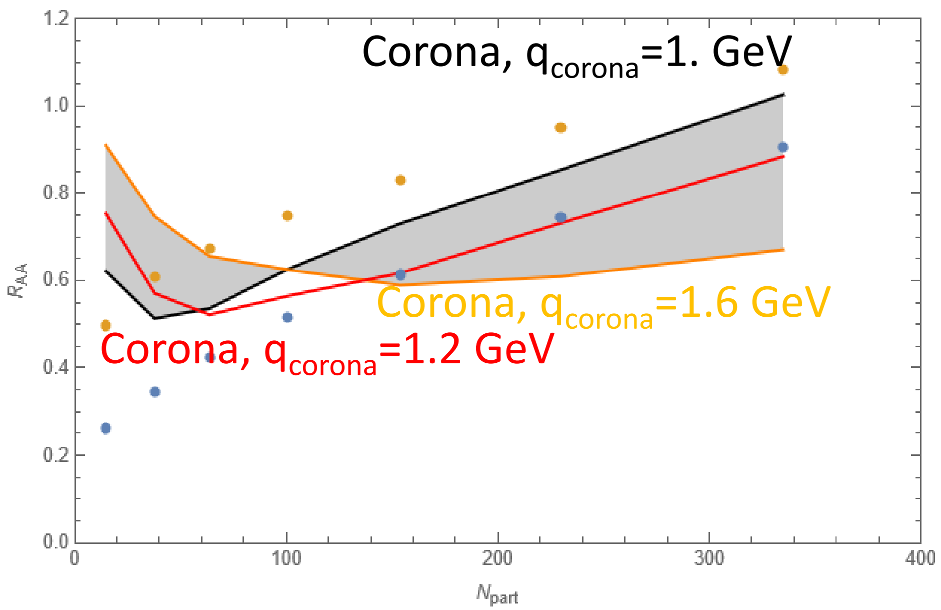
- Dynamical recombination is quite effective at low p_T
- At higher p_T, we are missing J/ψ as compared to the experimental value.
- Several possible reasons, under investigation:
  - in terms of transport model : « primordial too much suppressed »
  - lack of c-cbar correlation in the IS
  - …

R_{AA} vs N_{part}

<u>Caveat</u> : too crude modelling of the thermalization in the bulk… assumed to happen after 0.35 fm/c independent of the centrality

=> c and cbar created at t≈0 have the time to diffuse away => reduction of the production

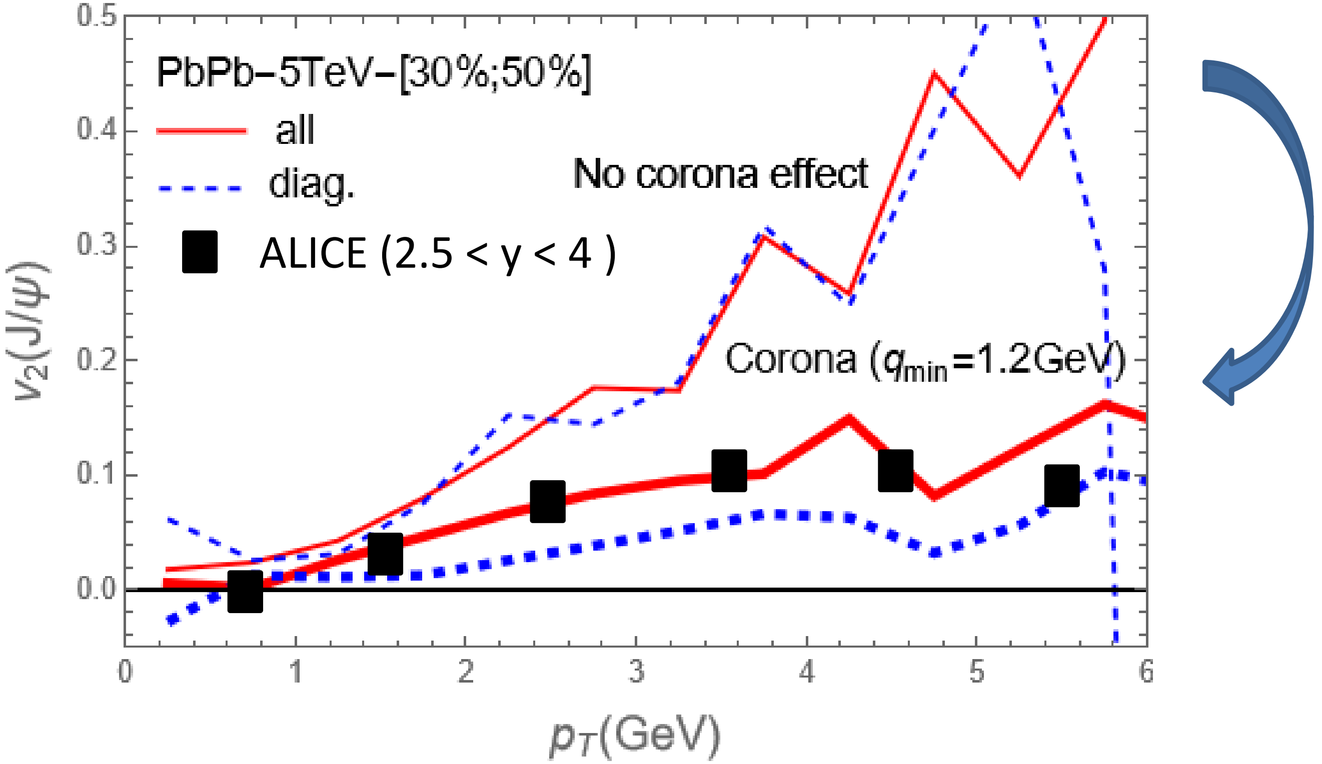




<u>One possible solution</u> : core – corona model for c-quarks : c-quarks with momentum transfer < q_{corona} are considered to combine -> quarkonia as in vacuum…

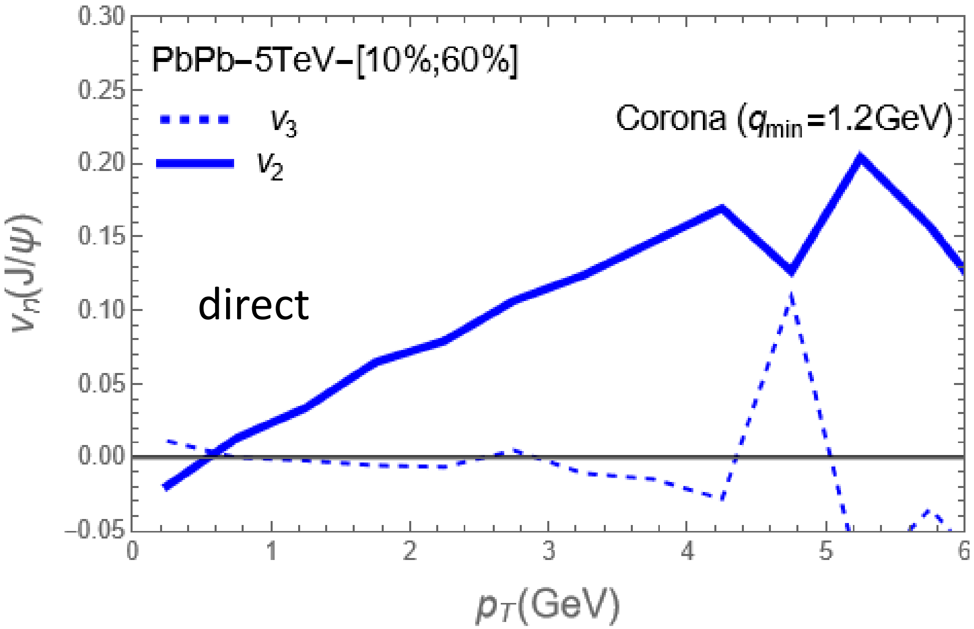
Optimal value : Corona, q_{corona}=1.2 GeV

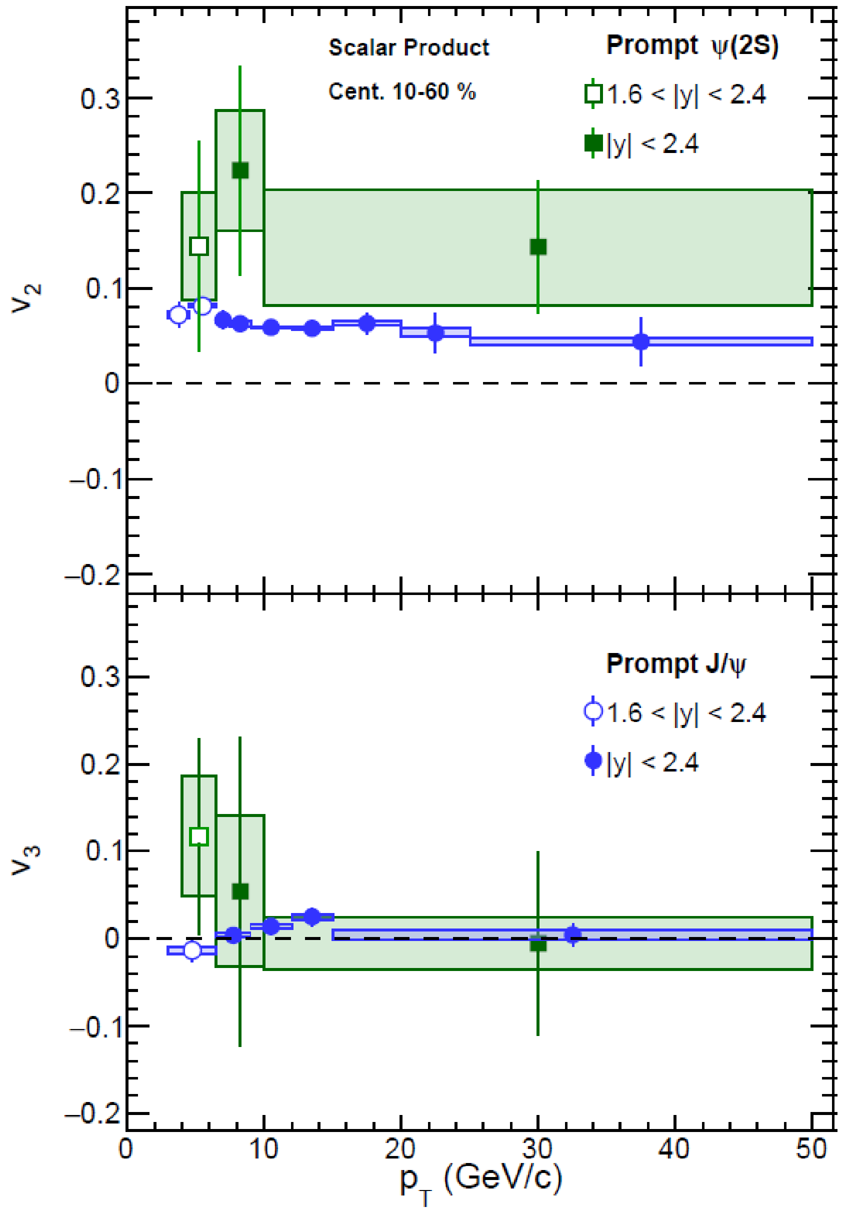
Results : J/ψ v_2



- v_2 excess as compared to experimental data (**late formation** of the J/ψ due to binding potential under restoration)
- Without corona, the « diagonal » contribution shows no difference wrt the full production, what is a bit conter-intuitive
- Corona has large effect on v2, even with moderate q_{thresh}. As the corona mostly affects the diagonal part, one recovers $v_2^{\rm diag} < v_2^{\rm all}$

Results : J/ψ v_n





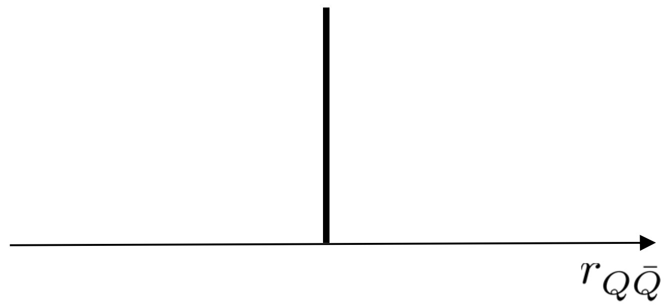
- Calculation for CMS centrality
- !!! Our model mostly applies at low and intermediate p_T.

Conclusions and Perspectives

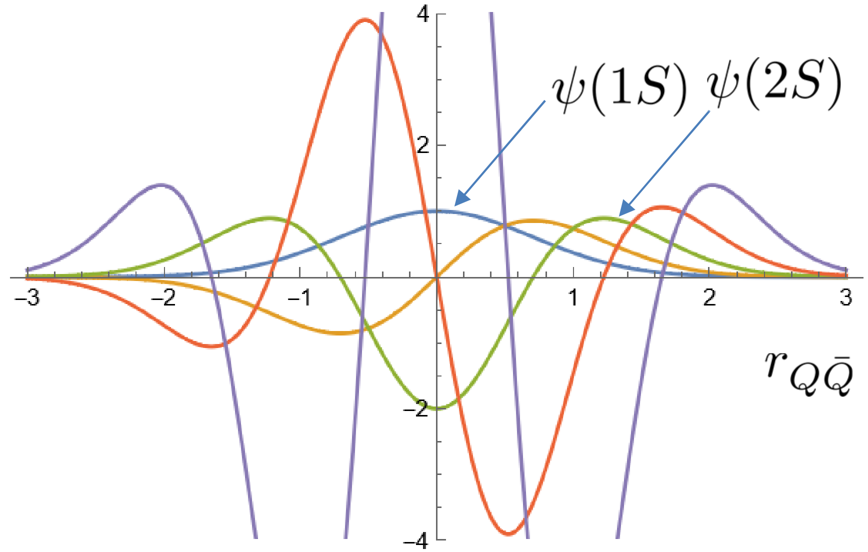
- First move towards a microscopic approach based on individual c and cbar dynamics implementing some of the open quantum systems features for charmonia production in realistic HI conditions : **dynamical coalescence**
- Encouraging results, but still many features to be refined
- Rooms for improvement :
  - Include excited states decay
  - Including color transparency and more generally color dynamics
  - More reliable treatment of the fully relativistic evolution of a N-body coupled system (under construction)
  - Upgrading to EPOS4 => More realistic « initial state » for the c-cbar pairs, including correlations at intermediate p_T.
  - Late interactions with hadronic phase
  - … (suggestions welcome)

Back up

Quantum coherence at early time



$= \sum$



Initial compact state : δ

Dissociation rate: $\Gamma(r_{Q\bar{Q}}) \propto \alpha_S T \times \Phi(m_D r_{Q\bar{Q}}) \sim \alpha_S^2 T^3 \times r_{Q\bar{Q}}^2$

Coherence

Neglect of coherence

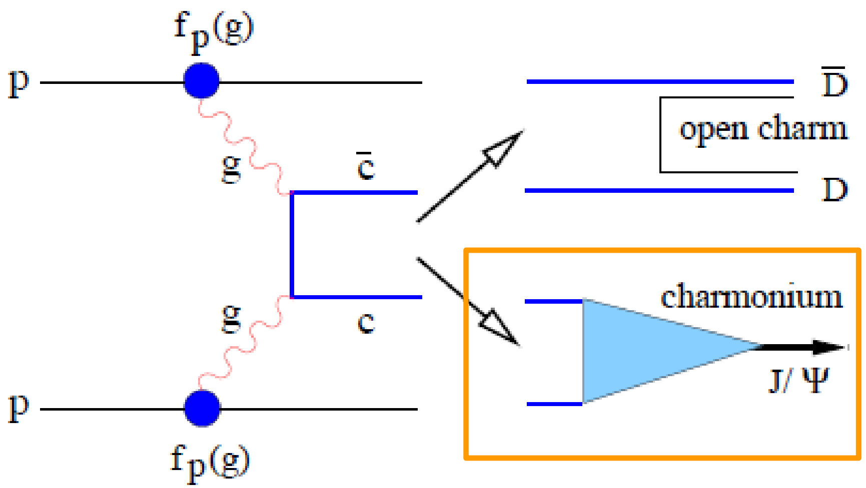
$$\Gamma(r_{Q\bar{Q}}) \approx 0 \propto \sum c_j^\star c_i \langle\psi_j|r^2|\psi_i\rangle \longrightarrow \Gamma \propto \sum_i |c_i|^2 \langle\psi_i|r^2|\psi_i\rangle \approx \sum_i |c_i|^2 \Gamma_i \neq 0$$

Crucial to include coherence !

N.B. : one can model this effect by phenomenological formation time, but lack of control

Quantum coherence

Picture behind transport theory :



Open heavy flavor and quarkonia assumed to be uncorrelated

Formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

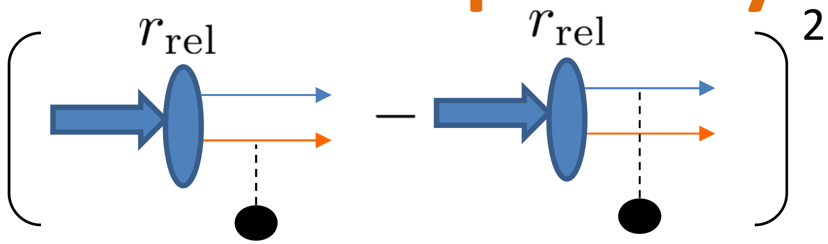
Common belief in the transport community:

Quarkonia initially « formed » in QGP and survive with an *individual* survival probability

$$S(t) = e^{-\int_{\tau_f}^{t} \Gamma(T(t'))dt'}$$

The two sides of color transparency

Q-Qbar propagation in QGP. $\sigma \propto$



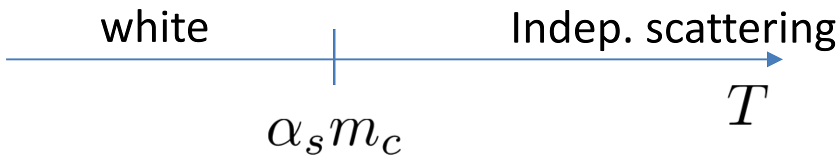
If $r_{rel} << l_{correl}$: **white object** => no Energy loss

If $r_{rel} >= l_{correl}$: 2 HQ interact individually with QGP.

$$l_{\rm correl} \sim \frac{1}{m_D}$$ (soft modes)

Small T : $r_{\rm rel} \approx \frac{1}{\alpha_s m_c}$

Large T : $r_{\rm rel} \gtrsim \frac{1}{m_D} \approx \frac{1}{gT}$



- Most of the transport models have considered up to now that primordial charmonia can just be destroyed (with a small probability), but not deflected.
- In our approach, we have investigated the consequences of considering the opposite limit… with too large v_2 resulting from this prescription…

Remler's formalism for dynamical coalescence

Generic idea : describe charmonia (Ψ) production using density matrix

$$P^{\Psi}(t) = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\hat{\rho}_N(t))\right]$$

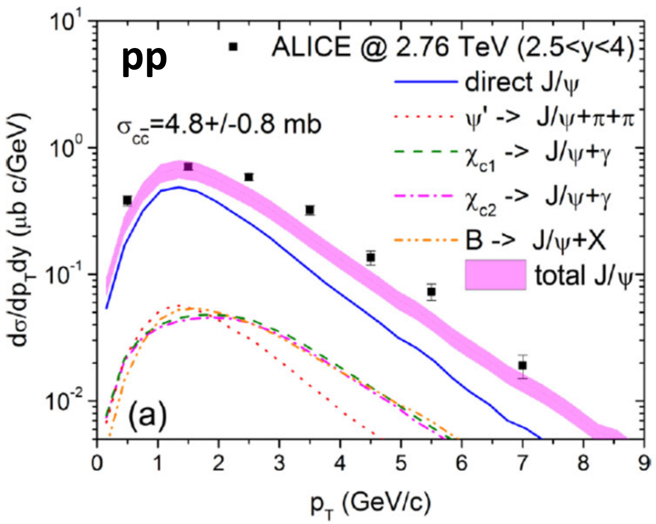
$$\hat{\rho}^{\Psi_i}_{Q\bar{Q}} = \sum_i |\Psi^i_{Q\bar{Q}}\rangle\langle\Psi^i_{Q\bar{Q}}|$$

Single quarkonia density operator

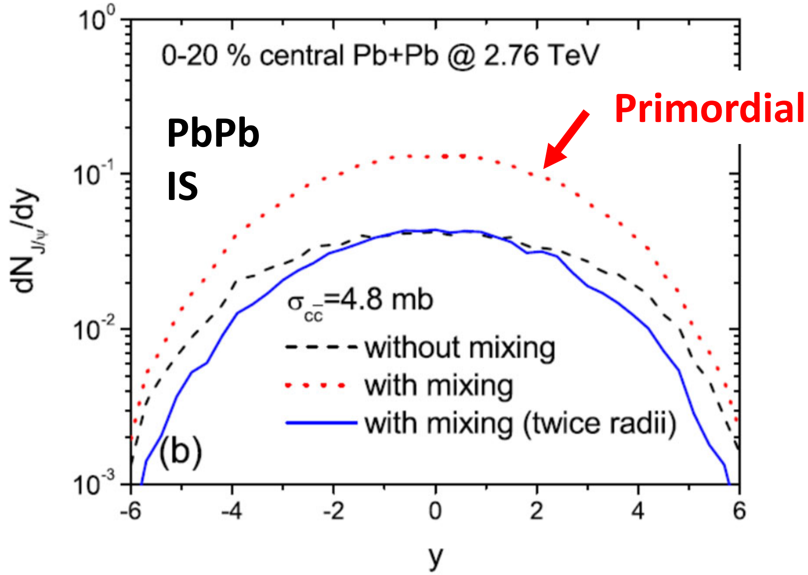
N-body density matrix (bulk partons + many c and many cbar)

"Just" looking at the initial stage brings interesting features:

T. Song, J.Aichelin and E.Bratkovskaya, PRC 96. 014907 (2017)



Good reproduction of pp -> J/ψ + x !!!

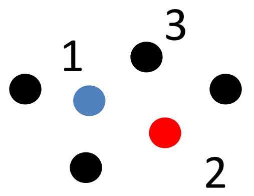


considerable enhancement of primordial J/ψ (in the **initial** state): **large off-diagonal contributions**

Remler formalism at work

Combining the rate definition + VN equation : $\Gamma^{\Psi}(t) = -iTr[\rho^{\Psi}[H_N, \rho_N(t)]]$

Generic case where $H_N = \Sigma_i K_i + \Sigma_{i>j} V_{ij}$



1 & 2 : $c \,\&\, \bar{c}$

3, 4, …… : light quarks

Strictly speaking, not QCD. Important process partly missing : gluo-dissociation

$$H_N = H_{1,2} + H_{N-2} + U_1 + U_2$$

$c\bar{c}$ Internal Hamiltonian

Light quarks

$\Sigma_{i>2} V_{i1}$ $\Sigma_{i>2} V_{i2}$

Heavy – light interaction

$$\Gamma^{\Psi}(t) = -iTr[\rho^{\Psi}[H_N, \rho_N(t)]] = -iTr[\rho_N(t)[\rho^{\Psi}, H_N]]$$

Only U_1 and U_2 => ≠0 (as $[\rho^{\Psi}, H_{1,2}] = 0$)

$$\Gamma^{\Psi}(t) = -iTr[\hat{\rho}^{\Psi}[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

Sub-part of the VN equation, still impossible to deal with exactly at the quantum n-body level

Remler formalism at work

Passing to the Wigner representation:

$$W_N(\{r\},\{p\}) = \int \Pi d^3y e^{ipy} \langle r - \frac{y}{2} | \hat{\rho}_N | r + \frac{y}{2} \rangle$$

Direct space

$$\partial \rho_N(t)/\partial t = -i\Sigma_j [K_j, \rho_N(t)] -i\Sigma_{j>k}[V_{jk}, \rho_N(t)]$$

Wigner space….

$$\partial W_N(t)/\partial t = \langle \Sigma_i v_i \cdot \partial_r W_N(\mathbf{r},\mathbf{p},t)\rangle + \langle \Sigma_{i\geq j} \Sigma_n \delta(t - t_{ij}(n)) \times (W_N(\mathbf{r},\mathbf{p},t+\epsilon) - W_N(\mathbf{r},\mathbf{p},t-\epsilon))\rangle$$

One to one correspondance

… treated at the semi-classical level :

Wigner distribution ⬄ {trajectories in phase space}

$[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]$ can be modelized from the trajectories evolution in Wigner space

Remler formalism at work

The effective rate for quarkonia state creation (dissociation) in the medium is

$$\Gamma^{\Psi}(t) = -iTr[\hat{\rho}^{\Psi}[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

Working in the phase space through Wigner distribution

$$W^{\Psi_i}_{Q\bar{Q}} = \int d^3y e^{ipy} \langle r - \frac{y}{2} | \Psi^i \rangle \langle \Psi^i | r + \frac{y}{2} \rangle$$

Quarkonia: Double Gaussian approximation

$$W^{\Psi}_{Q\bar{Q}}(r_{\rm rel}, p_{\rm rel}) = C e^{r_{\rm rel}^2 \sigma^2} \times e^{\frac{p_{\rm rel}^2}{\sigma^2}}$$

Parameter: The Gaussian width $\sigma \approx 0.35$ fm

$$[\frac{\hbar^2}{2\mu}\nabla^2 + V(r)]\Psi_{QQ}(r) = E_{QQ}\Psi_{QQ} \longrightarrow \langle r^2 \rangle \longrightarrow W^{\Psi}$$

W_N : Semi-classical approach

$$W_N = \Pi_i \hbar^3 \delta(x_i - x_{i0}(t))\delta(p_i - p_{i0}(t))$$

… but no explicit description of W_N required (as it appears in the trace)

and (less trivial) : generalisation at finite 4-velocity u; fully relativistic… to warrant orthogonality of states

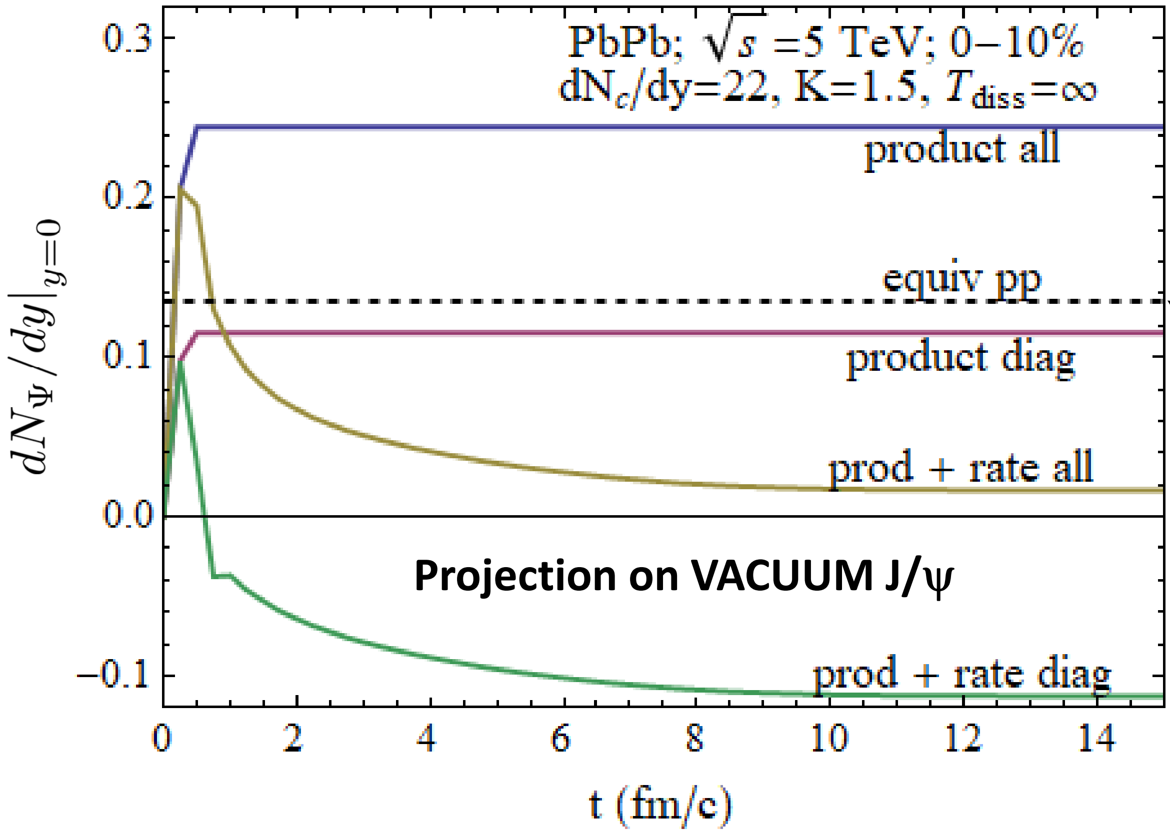
$$\mathrm{Tr}[W_u^{J/\psi} W_u^{\psi'}] = 0$$

Subatech

Preliminary results for J/ψ production in Pb-Pb

Word of caution: Exploratory phase => not meant to have an exact comparison with exp. data

$$P^{\Psi}(t) = P^{\Psi}(t_0) + \int_{t_0}^{t} \Gamma(t')dt'$$



Cumulated « production » (if no rate equation), indeed oveshoots pp due to off-diagonal contributions

The denominator in the R_{AA}

The full production (i.e. the numenator in the R_{AA})

First answer to puzzle found in Song et al: the primordial production is killed rather fast by the « loss » rate.

Remler formalism for the QGP : last ingredient

Combining the rate definition + VN equation : $\Gamma^{\Psi}(t) = -iTr[\rho^{\Psi}[H_N, \rho_N(t)]]$

$$H_N = H_{1,2} + H_{N-2} + U_1 + U_2$$

$c\bar{c}$ Internal Hamiltonian … … …

In QGP, 2 body T-dependent effective potential => ….

$$\Gamma^{\Psi}(t) = -iTr[\rho^{\Psi}[H_N, \rho_N(t)]] = -iTr[\rho_N(t)[\rho^{\Psi}, H_N]]$$

$$[\rho^{\Psi}, H_{1,2}(T)] = 0$$

$$= -iTr[\hat{\rho}^{\Psi}(T)[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

One only preserves the structure of the Remler « collisional rate » if one works in the « local » basis ρ^{Ψ} (T) !!!

Accessible for T > T_{dissoc}^{Ψ} (=0.4 GeV for J/ψ)

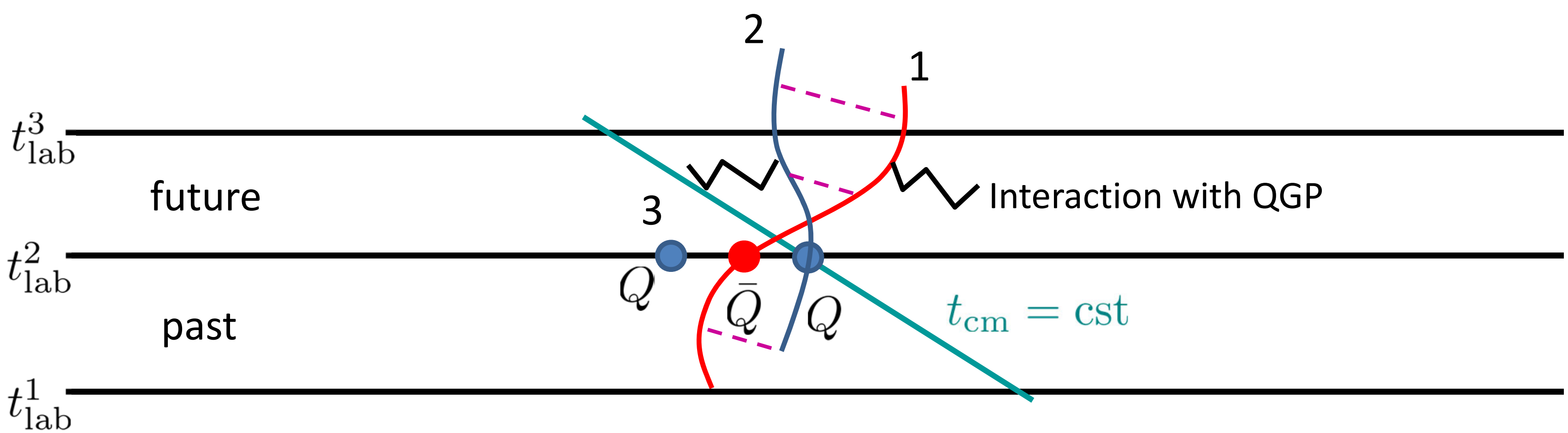
Back to the rate : $\Gamma^{\Psi}(t) = \frac{dP^{\Psi}(t)}{dt} = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}} \frac{d\hat{\rho}_N(t)}{dt}\right]$

$$\Gamma^{\Psi}(t) = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}(T(t)) \frac{d\hat{\rho}_N(t)}{dt}\right] + \frac{dT}{dt}\mathrm{Tr}\left[\frac{\partial \hat{\rho}^{\Psi}_{Q\bar{Q}}(T)}{\partial T} \hat{\rho}_N(t)\right]$$

New contribution to the rate (so-called « local rate »)

The Q-Qbar dynamics… the CM strategy

- Main objective : evaluate the propagation towards future of N Q-Qbar interacting pairs
- Strategy (adopted presently) : For each time step in the laboratory frame, pass to the cm frame and perform the evolution in the cm frame (where the potential is well defined)



- "Issue" : slicing the global time evolution (usual strategy in MC) is not 100% compatible with passing to c.m. frame as 2 particles are usually not at the same relative time in both frames (residual glitches ⇔ numerical noise)

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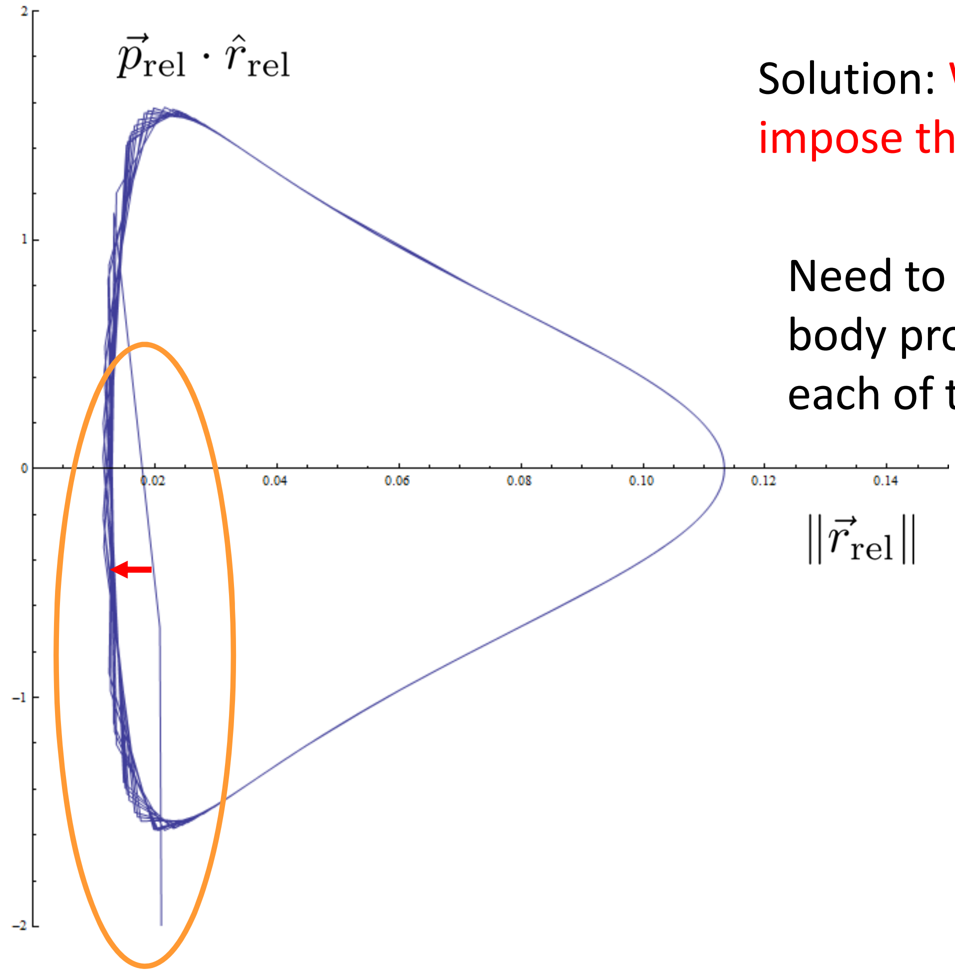
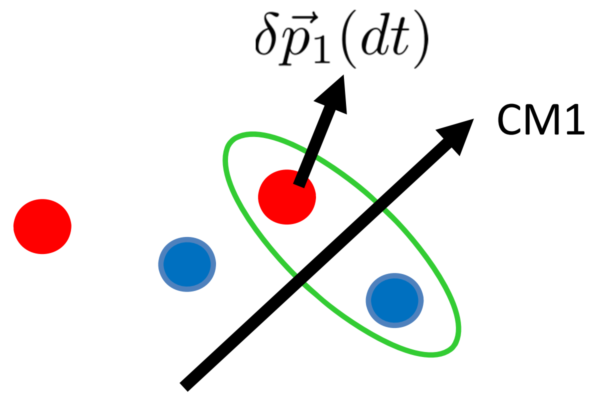
The Q-Qbar dynamics… the CM strategy

- "Minor problem" #1: Classical equations of motion are **unstable** (in the CM):



Solution: Work in Hamilton – Jacobi coordinates or impose the conserved quantities (L and Etot)

Need to factorize the N-body problem as an {} of 2-body problems for some evolution over time step dt, each of them to be solved in the CM



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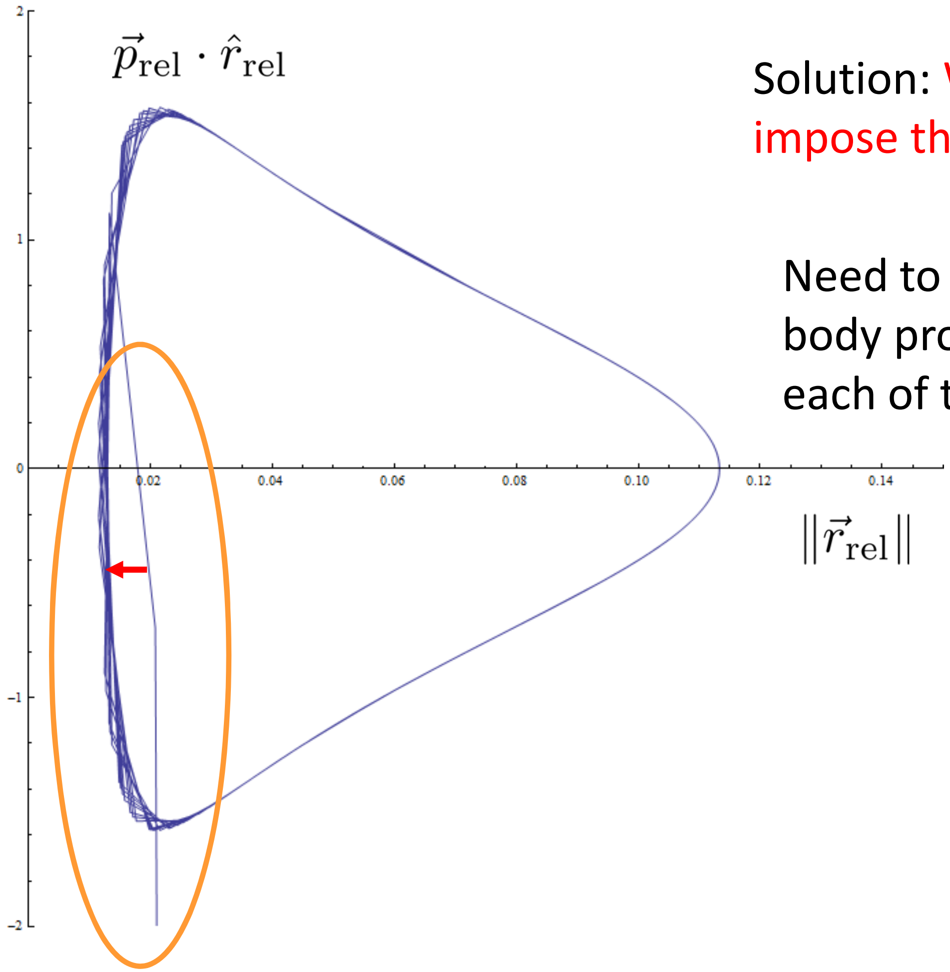

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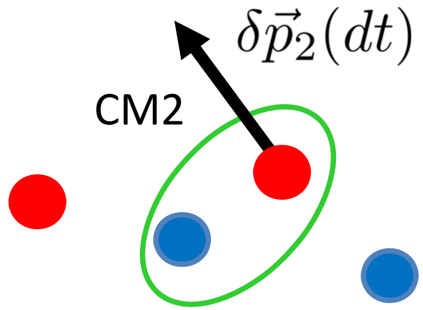

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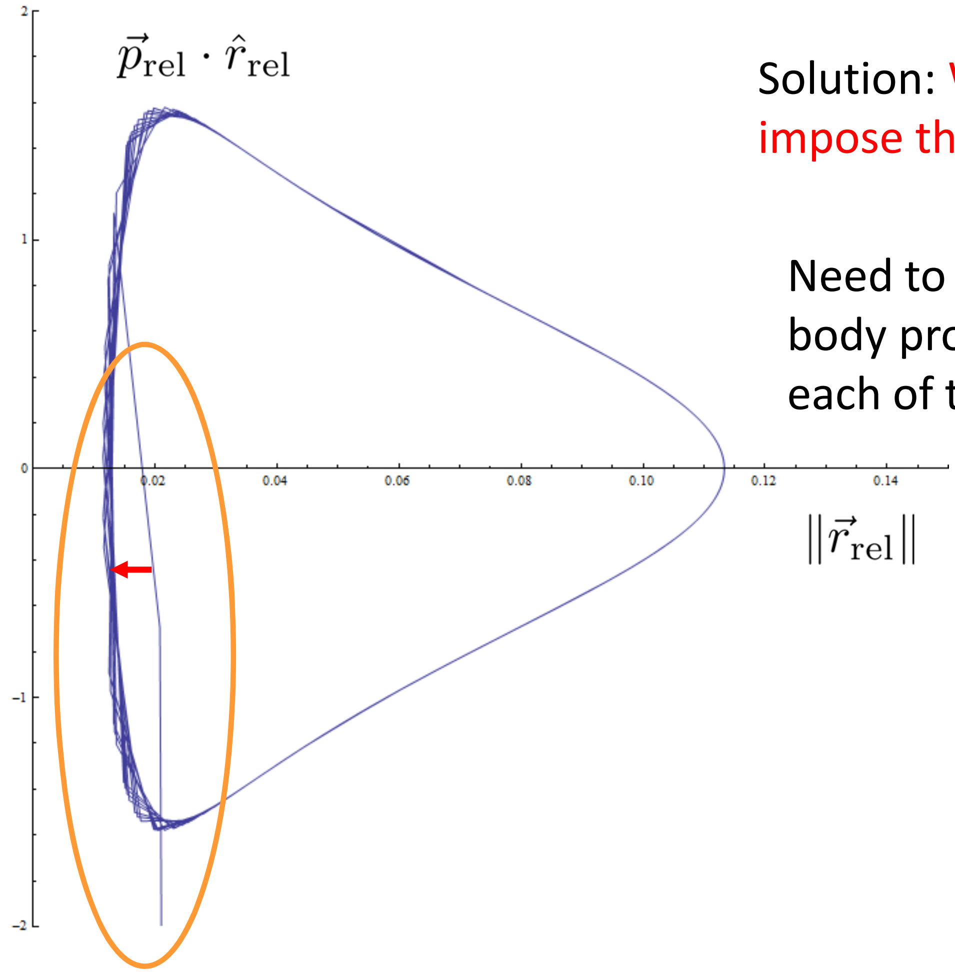

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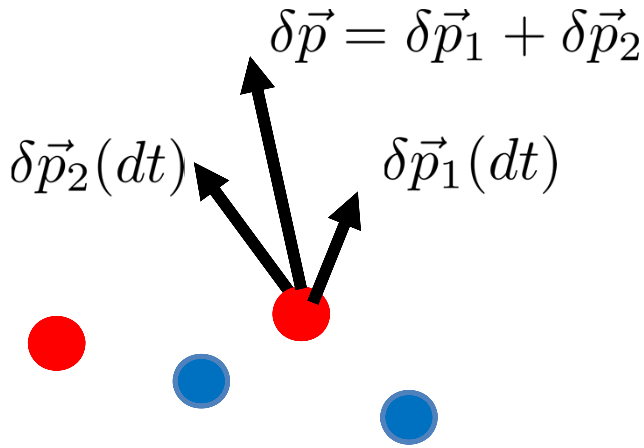

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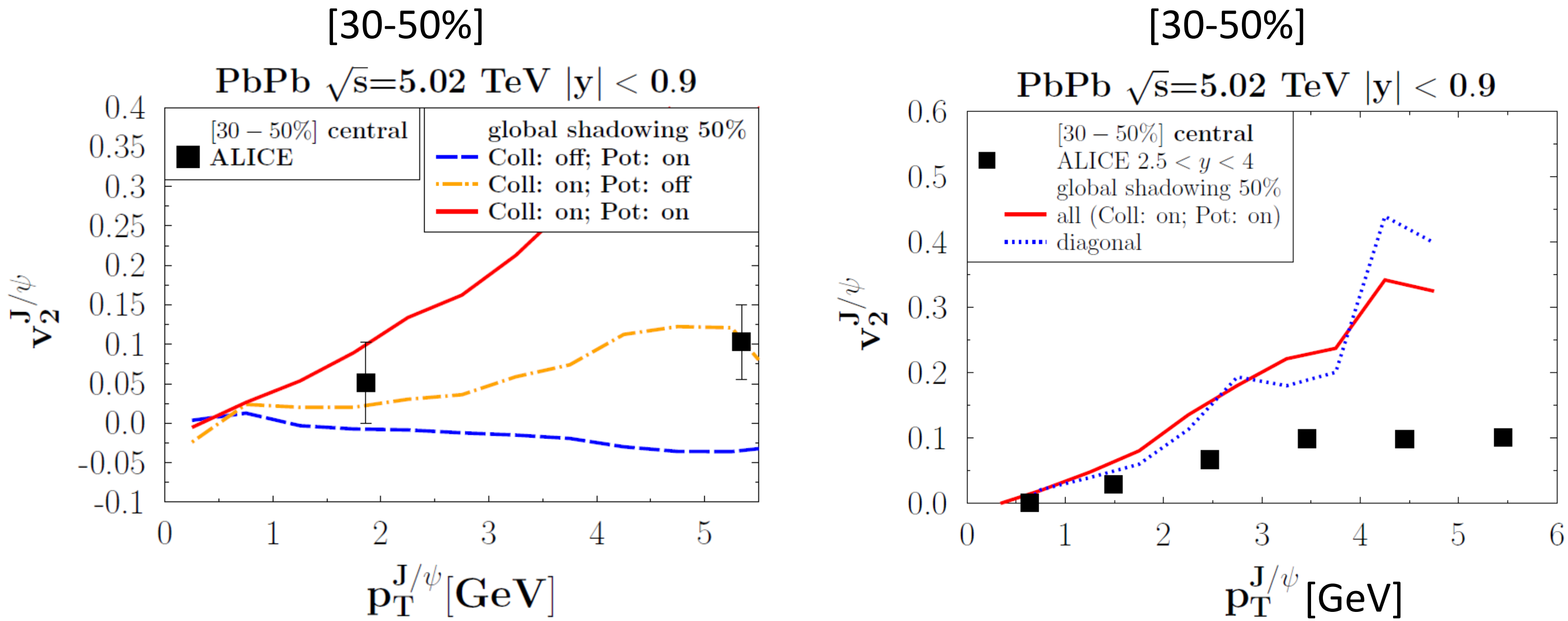
Results : J/ψ v_2



- v_2 excess as compared to experimental data (**late formation** of the J/ψ due to binding potential under restoration)
- The « diagonal » contribution shows no difference wrt the full production, what is a bit conter-intuitive