Heavy quark diffusion coefficient during hydrodynamization - non-equilibrium vs. equilibrium

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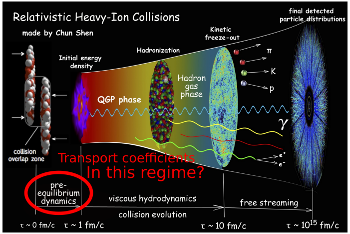
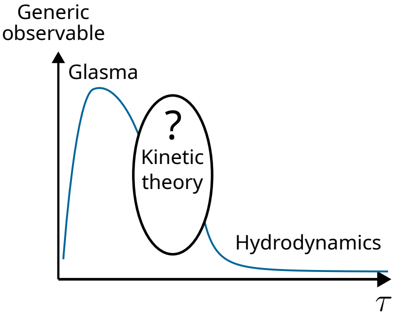
2303.12520 (κ)
2303.12595 ($\hat{q}$)

CoE QM

Heavy ions & transport coefficients in pre-equilibrium





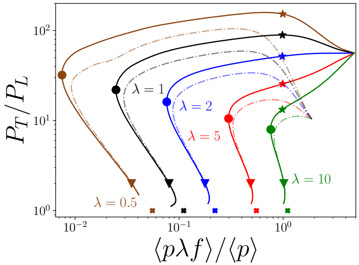
- Glasma stage can have a substantial impact on transport coefficients: Avramescu et. al: 2303.05599, Czajka et. al: PRC 105, 064910, PLB 834 (2022) 137464, NPA 1001 (2020) 121914, JP et. al: JHEP 2020, 77 (2020), D. Müller et. al: PLB 810 (2020) 135810, Ruggieri et. al: EPJP 137 (2022) 3, 307, PLB 798 (2019) 134933
- Evolution of coefficients during hydrodynamization poorly understood
- Aim of this work: close the gap, study heavy quark momentum diffusion coefficient κ during hydrodynamization using EKT.

Main physics questions of this talk

The relevant questions I'll aim to answer:

1. How large is κ during hydrodynamization?
2. How κ_T relates to κ_z during hydrodynamization?

Method: Effective kinetic theory & bottom-up thermalization PLB 502 (2001) 51-58 à la Kurkela & Zhu PRL 115 (2015) 18, 182301



- Star marker: maximum anisotropy / occupancy $\sim 1/\lambda$, $\lambda = 4\pi N_c \alpha_s$.
- Circle marker: minimum occupancy
- Triangle: Approximate isotropy $P_T/P_L = 2$.

Dof: gluon phase space density:

$$f(\boldsymbol{p}) = \frac{1}{\nu_g}\frac{\mathrm{d}N}{\mathrm{d}^3x\,\mathrm{d}^3\boldsymbol{p}}. \quad (1)$$

Dynamics: Boltzmann equation

$$-\frac{\partial f(\boldsymbol{p})}{\partial \tau} = \mathscr{C}_{1\leftrightarrow 2}[f(\boldsymbol{p})] + \mathscr{C}_{2\leftrightarrow 2}[f(\boldsymbol{p})] + \mathscr{C}_{\mathrm{exp}}[f(\boldsymbol{p})]. \quad (2)$$

Boost invariant expansion:

$$\mathscr{C}_{\mathrm{exp}}[f(\boldsymbol{p})] = -\frac{p_z}{\tau}\frac{\partial}{\partial p_z} f(\boldsymbol{p}). \quad (3)$$

Method: relevant observables

κ given by ($gQ \to gQ$, t-channel gluon exchange, PRC 71 (2005) 064904):

$$3\kappa = \frac{\langle \Delta k^2 \rangle}{\Delta t} = \frac{1}{2M} \int\limits_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{p}'} (2\pi)^3 \, \delta^3 \left(\boldsymbol{p} + \boldsymbol{k} - \boldsymbol{p}' - \boldsymbol{k}'\right) \times 2\pi\delta\left(k' - k\right) \boldsymbol{q}^2 \left[|\mathscr{M}_\kappa|^2 f(\boldsymbol{k})(1 + f(\boldsymbol{k}')) \right]. \tag{4}$$

k, k' gluon momenta, $q = k - k'$, p, p' heavy quark momenta.

$$|\mathscr{M}|^2_\kappa = \left[N_c C_H g^4\right] \frac{16 M^2 k^2 \left(1 + \cos^2 \theta_{\boldsymbol{k}\boldsymbol{k}'}\right)}{(q^2 + m_D^2)^2} \tag{5}$$

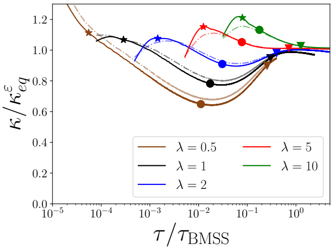
Other relevant observables:

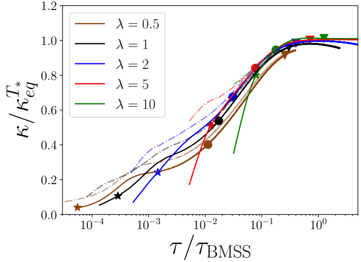
$$T_* = \frac{2\lambda}{m_D} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} f(p)(1 + f(p)), m_D^2 = 4 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{\lambda f(p)}{p}, T_\epsilon \sim \sqrt[4]{\epsilon}. \tag{6}$$

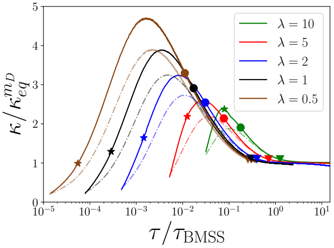
Equilibrium: use thermal distribution f_{BE}

Comparing equilibrium to non-equilibrium

- Try: match for the same m_D, T_* and ε:
- $\tau_{\mathrm{BMSS}} = \alpha_s^{13/5}/Q_s$ thermalization timescale (PLB 502 (2001) 51-58).
- Note: corresponding thermal system changes during t-evolution!



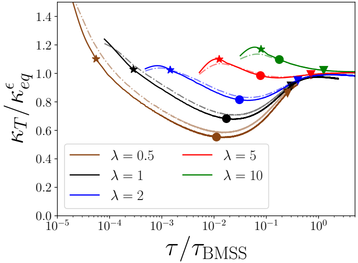


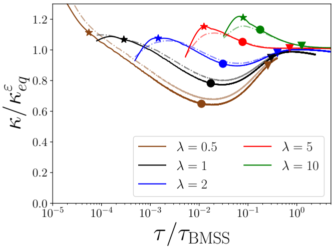


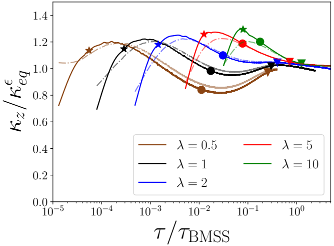
- Match for the same ε ($\sim$ Landau matching).

κ_{eq} vs κ during hydrodynamization (Question 1)

- $\kappa_{T,z}$ behave qualitatively similarly to κ (except κ_z at early times)
- Small λ → larger deviations (small λ → bottom-up reproduced better).

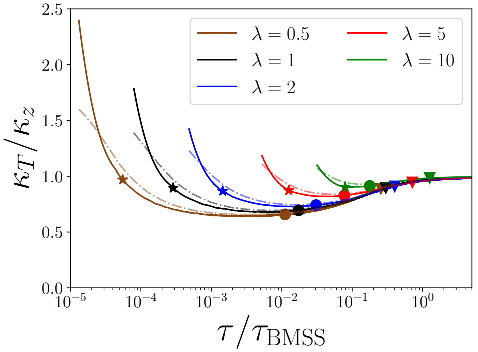






1 A: κ deviates from its equilibrium value by $\sim 30\%$ during hydrodynamization.

Transverse vs. longitudinal (κ_T vs. κ_z, Question 2)



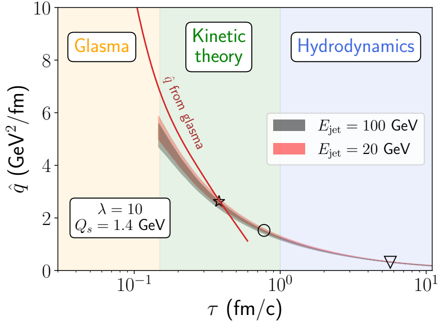
- Transverse diffusion coefficient enhanced initially.
- Longitudinal coefficients dominates during underoccupation.

2 A: At the maximum anisotropy transverse diffusion coefficient dominates. At the underoccupied phase longit. coefficient is larger.

Jet quenching factor $\hat{q}$ (F. Lindenbauer, 2303.12595)

Convention: $\hat{x}$ jet direction, $\hat{z}$ beam direction, $\hat{q}^{ij} = \frac{\mathrm{d}\langle q^i q^j \rangle}{\mathrm{d}L}$.

$$\hat{q}^{ij} = \frac{1}{4d_R} \lim_{|\mathbf{p}|\to\infty} \int_{\substack{\mathbf{k}\mathbf{k}'\mathbf{p}' \\ q_\perp < \Lambda_\perp}} q_\perp^i q_\perp^j (2\pi)^4 \delta^4(P + K - P' - K') \frac{\left|\mathcal{M}_{ag}^{ag}\right|^2}{|\mathbf{p}|} f_{\mathbf{k}} (1 + f_{\mathbf{k}'})$$



- Quark jet, (g differs by a Casimir factor), elastic scatterings off gluons.
- Match ε to glasma (D. Müller et. al: PLB 810, 135810 (2020)) at the IC (value of Q_s).
- Match $\hat{q}$ to JETSCAPE result (PRC 104 (2021) 2, 024905) at the triangle (choose $\Lambda_\perp$).
- Bands: different cutoff models and IC's.

Summary / answers & conclusions

The relevant answers:

1. How large is κ during hydrodynamization? **Within 30 % from κ^{eq} for the same ε!**
2. How κ_T relates to κ_z during hydrodynamization? **Initially κ_T dominates. At underoccupation κ_z is larger.**

These are potentially useful for:

- Phenomenological descriptions of heavy quark diffusion and quarkonium dynamics.

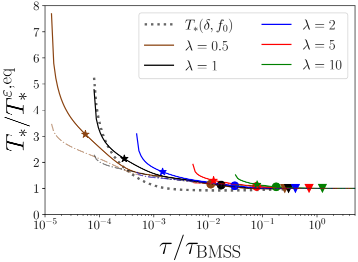
Future plans:

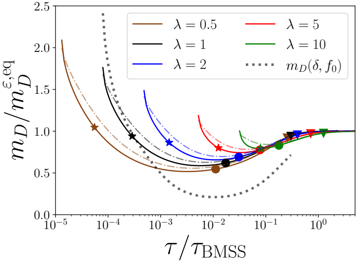
- More ongoing work on $\hat{q}$ (by F. Lindenbauer) & attractors.

Related talks at HP 2023:

- See talks by: D. Avramescu, K. Boguslavski & M. Martinez

Evolution of m_D and T_*

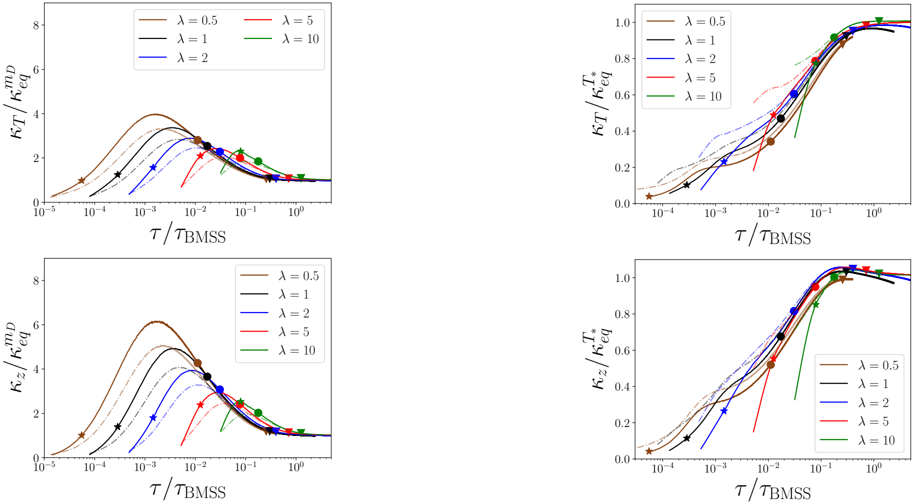




- Initially large $f_0 \to$ enhancement
- At underoccupation f dominates over $f^2 \to$ ratio becomes 1.

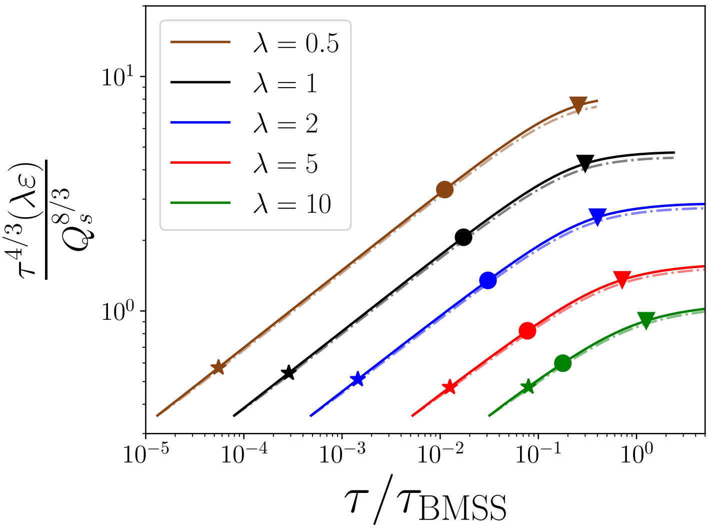
- Initial enhancement due to large occupation number.
- Suppression due to underoccupation.

Transverse and longitudinal κ matched for other quantities



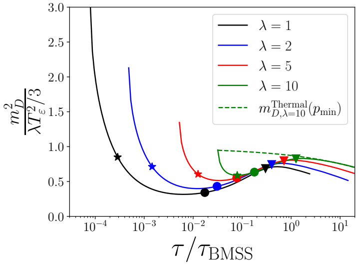
- Similar to the results for the full coefficient

Time-evolution of energy density



Discretization effects

- By far the most important parameter: UV cutoff p_{min}.



$$m_D^2(p_{\mathrm{min}}) = \frac{8\lambda}{(2\pi)^2}\int_{p_{\mathrm{min}}}^{\infty} \mathrm{d}p\, p f(p)$$

$$= \frac{2\lambda T}{\pi^2}\left(T\,\mathrm{Li}_2\left(e^{-\frac{p_{\mathrm{min}}}{T}}\right) - p_{\mathrm{min}}\log\left(1-e^{-\frac{p_{\mathrm{min}}}{T}}\right)\right)$$

- Compare non-equilibrium quantities to thermal result, with the same UV cutoff p_{min}.
- Left: Effect illustrated for m_D
- Redefine thermal T_* and κ similarly.
- Residual discretization effects cause ratios to deviate from equilibrium values at late times.

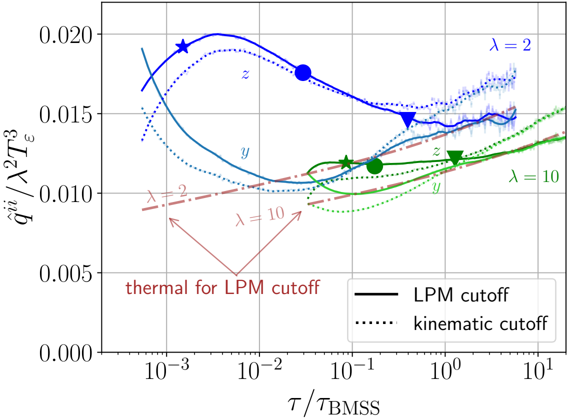
Details of $\hat{q}$ extraction 2303.12595

Dynamical cutoffs

$$\Lambda_{\perp}^{\mathrm{LPM}}(E_{\mathrm{jet}}, T) = \zeta^{\mathrm{LPM}} g \times (E_{\mathrm{jet}} T^3)^{1/4}$$
$$\Lambda_{\perp}^{\mathrm{kin}}(E_{\mathrm{jet}}, T) = \zeta^{\mathrm{kin}} g \times (E_{\mathrm{jet}} T)^{1/2}.$$

- E_{jet} fixed, T decreases during evolution.
- Match energy density of the glasma at $Q_s\tau = 1$, yields $Q_s = 1.4\mathrm{GeV}$
- Match parameters ζ at triangle marker, where $T_\varepsilon = 0.21Q = 295\mathrm{MeV}$ and $\lambda = 10$ to the median value for $\hat{q}_{\mathrm{therm}}$ in the LBT parametrization of the JETSCAPE collaboration PRC 104 (2021) 2, 024905.

$\hat{q}$ in different directions 2303.12595



- Similarly to κ: direction along beam axis is enhanced.
- Dashed vs. solid: different cutoff models.
- Data smoothedned with filter (unfiltered in the background)